12-05-2023 Θεωρία Ελέγχου

B1: $A=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-2&1&0&0\\1&-2&0&0\end{bmatrix}$, $B=\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$, $C=[1\ \ -1\ \ 0\ \ 0]$

$$\Gamma_c=[B\ \ AB\ \ A^2B\ \ A^3B]$$

$$=\begin{bmatrix}0&1&0&-1\\0&1&0&-1\\1&0&-1&0\\1&0&-1&0\end{bmatrix},\ \text{Rank}(\Gamma_c)=2$$

$$\mathcal{X}_c=\left\langle\begin{pmatrix}0\\0\\1\\1\end{pmatrix},\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right\rangle$$

Ο πίνακας παρατηρησιμότητας:

$$\Gamma_o=\begin{bmatrix}C\\CA\\CA^2\\CA^3\end{bmatrix}=\begin{bmatrix}1&-1&0&0\\0&0&1&-1\\-3&3&0&0\\0&0&-3&+3\end{bmatrix}\quad \text{Rank}(\Gamma_o)=2<4$$

Ο μη παρατηρήσιμος υπόχωρος:

$$\mathcal{X}_{\bar{o}}=\ker(\Gamma_o)=\{\underline{x}\in\mathbb{R}^4:\ \Gamma_o\cdot\underline{x}=\underline{0}\}$$

$$=\{\underline{x}\in\mathbb{R}^4:\ x_1=x_2,\ x_3=x_4\}=$$

(Μία βάση του είναι) $\left\langle\begin{pmatrix}1\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\right\rangle$ Οι δύο υπόχωροι ταυτίζονται.

$$\Sigma(A,B,C)\overset{Q}{\sim}\Sigma(\underbrace{Q_o^{-1}AQ_o}_{\hat{A}},\underbrace{Q_o^{-1}B}_{\hat{B}},\underbrace{CQ_o}_{\hat{C}})$$

$$\hat{A} = \begin{bmatrix} \hat{A}_{11} & 0 \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} \hat{C}_1 & 0 \end{bmatrix}$$

$$Q_0 = [\langle x_0 \rangle \; \langle \bar{x}_0 \rangle]$$

$$Q_0 = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

↳ διαλέγω τις δύο πρώτες στήλες ώστε οι 4 να είναι γραμμικά ανεξάρτητες.

$$\det(Q_0) = \det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 1 \times 0 + 1 \times 1 = 1 \neq 0$$

Μη Ιδιάζων.

Εάν έχω →
$$\left[\begin{array}{c|c} A_1 & A_2 \\ \hline 0 & A_3 \end{array}\right]^{-1} = \begin{bmatrix} A_1^{-1} & X \\ 0 & A_3^{-1} \end{bmatrix}$$

$$A_1 X + A_2 A_3^{-1} = 0 \Rightarrow$$
$$X = -A_1^{-1} A_2 A_3^{-1}$$

$$Q_0^{-1} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

0 $\hat{A} = Q_0^{-1} A Q_0$, $\hat{B} = Q_0^{-1} B$, $\hat{C} = C Q_0$

$$\hat{A} = \left[\begin{array}{cc:cc} 3 & -4 & 0 & 0 \\ 3 & -3 & 0 & 0 \\ \hdashline -3 & 4 & 0 & 1 \\ -2 & 1 & -1 & 0 \end{array}\right] \qquad \hat{B} = \left[\begin{array}{c} 0 \\ 0 \\ \hdashline 0 \\ 1 \end{array}\right] \Big\} \hat{B_1}$$

$\hat{A_1}$, $\hat{A_2}$, $\hat{A_3}$, $\hat{A_4}$

$$\hat{C} = [\underbrace{1 \quad -1}_{\hat{C_1}} \;\vdots\; \underbrace{0 \quad 0}_{0}]$$

Κάνω αύσκη έναν μετασχηματισμό:

$$\Sigma(\hat{A_{11}}, \hat{B_1}, \hat{C_1}) \overset{Q_1}{\sim} \Sigma(\underbrace{Q_1^{-1}\hat{A_{11}}Q_1}_{\tilde{A_{11}}}, \underbrace{Q_1^{-1}\hat{B_1}}_{0}, \underbrace{\hat{C_1}\cdot Q_1}_{\tilde{C_1}})$$

$$\tilde{A_{11}} = \begin{bmatrix} 0 & 1 \\ * & * \end{bmatrix}, \quad \tilde{C_1} = [1 \quad 0]$$

$$\left.\begin{aligned} Q_1^{-1}\hat{A_{11}}Q_1 = \tilde{A_{11}} \iff \hat{A_{11}}Q_1 = Q_1\tilde{A_{11}} \\ \hat{C_1}\cdot Q_1 = \tilde{C_1} = [1, 0] \end{aligned}\right\}$$

$$Q_1 = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

$$[1 \quad -1]\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = [1, 0] \left\} \begin{aligned} q_{11} - q_{21} &= 1 \\ q_{12} - q_{22} &= 0 \\ q_{12} &= q_{22}. \end{aligned}\right.$$

$$\begin{bmatrix} 3 & -4 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ * & * \end{bmatrix}$$

$$\hat{A_{11}} = \begin{bmatrix} 3 & -4 \\ 3 & -3 \end{bmatrix}$$

έχουν το ίδιο χαρ. πολ.

Βρίσκω το χαρακτηριστικό πολυώνυμο:

$$\varphi_{\hat{A_{11}}}(\lambda) = \det(\lambda I_2 - \hat{A_{11}})$$

$$= \det \begin{bmatrix} \lambda-3 & 4 \\ -3 & \lambda+3 \end{bmatrix} = (\lambda-3)(\lambda+3) + 12$$

$$= \lambda^2 + \cancel{3\lambda} - \cancel{3\lambda} - 9 + 12$$

$$= \lambda^2 + 3$$

$$\tilde{A_{11}} = \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -4 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} q_{11} & q_{22} \\ q_{11}-1 & q_{22} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{22} \\ q_{11}-1 & q_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}$$

$(1,1)$: $3q_{11} - 4(q_{11}-1) = -3q_{22}$

$3q_{11} - 4q_{11} + 4 = -3q_{22}$

$-q_{11} + 4 = -3q_{22}$

$\Rightarrow 3q_{22} - q_{11} = -4$

$(1,2)$: $3q_{22} - 4q_{22} = q_{11} \Rightarrow q_{11} + q_{22} = 0$

$(2,1)$: $3q_{11} - 3(q_{11}-1) = -3q_{22}$

$\cancel{3q_{11}} - \cancel{3q_{11}} + 3 = -3q_{22} \Rightarrow q_{22} = -1$

και $q_{11} - 1 = 0$

$q_{11} = 1$

$$Q_1 = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$$ και ο $$Q^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \hat{A}_{11} & 0 \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} , \quad \hat{B} = \begin{bmatrix} \overset{\nearrow 0}{\hat{B}_1} \\ \hat{B}_2 \end{bmatrix}$$

$$\hat{C} = [\, \hat{C}_1 \quad 0 \,]$$

$$\tilde{A} = \begin{bmatrix} Q_1^{-1} \hat{A}_{11} Q_1 & 0 \\ \hat{A}_{21} Q_1 & \hat{A}_{22} \end{bmatrix} , \quad \tilde{B} = \begin{bmatrix} \overset{\nearrow Q_1^{-1} \cdot 0}{0} \\ \hat{B}_2 \end{bmatrix}$$

$$\tilde{C} = [\, \underset{[1,0]}{\hat{C}_1 Q_1} \quad 0 \,]$$

$n = 5$, $\Gamma_0 \in \mathbb{R}^{10 \times 5}$

B17. $\Sigma(A,B,C)$

$$\hat{\Gamma}_0 = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & -1 & 1 \end{bmatrix}$$

$$\mathcal{X}_{\bar{o}} = \ker(\Gamma_0) = \ker(\hat{\Gamma}_0) = \{ x \in \mathbb{R}^5 : \hat{\Gamma}_0 \cdot \underline{x} = \underline{0} \}$$

$$\hat{\Gamma}_0 \sim \begin{bmatrix} -1 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 0 & 1 & 1 \\ 0 & +1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \tilde{\Gamma}_0$$

$x : \tilde{\Gamma}_0 \underline{x} = 0$

$$-x_1 + x_2 + 0 \cdot x_3 + x_4 + \overset{\nearrow 0}{\cancel{x_5}} = 0$$
$$+x_2 - x_3 + x_4 = 0$$
$$x_5 = 0$$