## **Applied Survival Analysis Lab 11: Analysis of competing risks**

We analyse data involving treatment interruptions (TI) among HIV-infected patients in the presence of the competing risk of death or loss from follow-up. The data are included in file TI-data\_final.dta.

The data are as follows:

fail3	Freq.	Percent	Cum.
Alive   TI or new regimen   Death or lost to fup	10,738 1,376 2,048	75.82 9.72 14.46	75.82 85.54 100.00
	14,162	100.00	

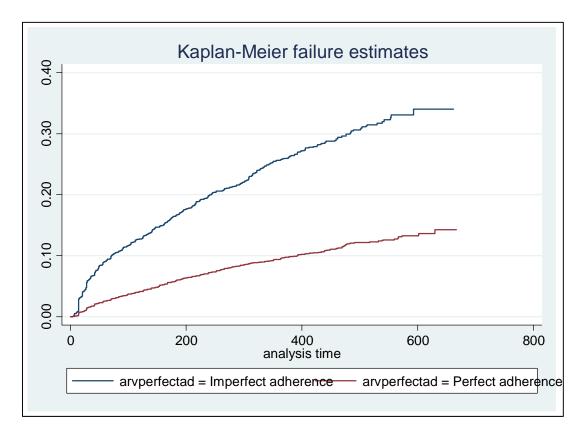
In other words, there are 1,376 individuals with the event of interest and 2,048 with the competing event. The question is: "What is the cumulative incidence of treatment interruption over time?"

We will perform an analysis of these data in a number of ways. First, we will perform the naïve analysis by considering, as the estimate of the cumulative incidence function, one minus the Kaplan Meier estimate of survival  $\hat{S}(t_i)$  at some time point  $t_i$ , i.e.,

$$\hat{F}_i(t_i) = 1 - \hat{S}(t_i)$$

We will also assess the effect of perfect ARV adherence on the hazard of TI. This is done as follows:

The result is as follows:



Now, we are told that this estimate probably overestimates the true cumulative incidence function. So let's see what we can do in order to perform a better estimation.

First of all, we will try performing the following steps:

- 1. Analyze the data considering any failure other than TI as censored data. This will produce the cause-specific hazard  $\hat{h}_1(t)$
- 2. Analyze the data considering any failure other than death or loss to follow-up as censored data. This will produce the cause-specific hazard  $\hat{h}_2(t)$
- 3. Estimate the discrete survival function  $\hat{S}(t) = \prod_{i:t_i < t} \{1 \hat{h}_1(t) \hat{h}_2(t)\}$
- 4. Estimate the cumulative incidence function as  $\hat{F}_{TI}(t) = \sum_{i:t_i < t} \hat{h}_1(t_i) \hat{S}(t_{i-1})$

Let's follow this approach.

STEP 1. Analyze the data considering any failure other than TI as censored data. This will produce the cause-specific hazard  $\hat{h}_1(t)^1$ 

. quietly stcox arvperfectad

<sup>&</sup>lt;sup>1</sup> Note that we have already set up the data to consider all failures other than TI as censored data (from the Kaplan-Meier analysis previously.

Predict the baseline hazard of TI (this is the same as the hazard for TI among those with imperfect ARV adherence:

```
. predict h_TI_0, basehc (12790 missing values generated)
```

Now we sort the data in ascending order by time and descending order by event. This will ensure that the censored times tied with event times will be placed after the event times:

```
. gsort _t - _d
. by _t: replace h_TI_0 = . if _n > 1
(994 real changes made, 994 to missing)
```

Note that, because we had 994 ties in the data with death or loss to follow-up as the event, we needed to remove any duplicate contributions of the hazards during all but the first of these tied event times. Now predict the hazard for TI among those with perfect adherence:

```
. gen h_TI_1 = h_TI_0*exp(_b[arvperfectad ])
(13784 missing values generated)
```

STEP 2. Analyze the data considering any failure other than death or loss to follow-up as censored data. This will produce the cause-specific hazard  $\hat{h}_2(t)$ 

```
. quietly stcox arvperfectad
```

Predict the baseline hazard of death or loss to follow-up (DL) (this is the same as the hazard for TI among those with imperfect ARV adherence):

```
. predict h_DL_0, basehc (12266 missing values generated)
```

Now we sort the data in ascending order by time and descending order by event. This will ensure that the censored times tied with event times will be placed after the event times:

```
. gsort _t - _d
. by _t: replace h_DL_0 = . if _n > 1
(1537 real changes made, 1537 to missing)
```

Note that, because we had 1,537 ties in the data with death or loss to follow-up as the event, we needed to remove any duplicate contributions of the hazards during all but the first of these tied event times. Now predict the hazard for death or loss to follow-up among those with perfect adherence:

```
. gen h_DL_1 = h_DL_0*exp(_b[arvperfectad ])
(13803 missing values generated)
```

Before going further, note that there are 513 observations with missing adherence measures:

STEP 2a. Generate an indicator to exclude observation without failure from any kind and replace missing hazards with zeros in cases where one or the other failure was observed.

```
. gen exclude=(missing(h_TI_0) & missing(h_DL_0))
```

Note that the following observations will be excluded:

. tab exclu	de fail3			
exclude	   Alive	fail3 TI or new	Death or	Total
0	0   10,738	378 998	359   1,689	737 13,425
Total	10,738	1,376	2,048	14,162

In addition to the 10,738 observations which are censored (since they have not experienced either type of failure) there are 994 ties and 4 observations with missing adherence measures among subjects with a TI and 1537 ties and 152 observations with missing adherence measures among patients who died or were lost to follow-up. These will also be excluded.

Now replace the remaining observations with missing only one of the two types of failure with zero hazard.

```
. replace h_TI_0 =0 if h_TI_0 ==. & h_DL_0 ~=.
(359 real changes made)

. replace h_TI_1 =0 if h_TI_1 ==. & h_DL_1 ~=.
(359 real changes made)

. replace h_DL_0 =0 if h_DL_0 ==. & h_TI_0 ~=.
(378 real changes made)
```

```
. replace h_DL_1 =0 if h_DL_1 ==. & h_TI_1 \sim=. (378 real changes made)
```

Note that there were 359 changes made among those experiencing a TI and 378 changes among those experiencing death or loss to follow-up.

STEP 3. Estimate the discrete survival function  $\hat{S}(t) = \prod_{i:t_i < t} \{1 - \hat{h}_1(t) - \hat{h}_2(t)\}^2$ 

```
. gsort -exclude _t
```

Note that we sorted the excluded variables first and the non-excluded last, so we can produce a running sum over them<sup>3</sup>:

```
. gen S_0 = exp(sum(log(1- h_DL_0 - h_TI_0))) if exclude==0
(13425 missing values generated)
. gen S_1 = exp(sum(log(1- h_DL_1 - h_TI_1))) if exclude ==0
(13425 missing values generated)
```

Note the trick where, instead of performing the product we sum the log survival and exponentiate, i.e., we use the property that,

$$\hat{S}(t) = \prod_{i:t_i < t} \{1 - \hat{h}_1(t) - \hat{h}_2(t)\} = \exp\left\{ \sum_{i:t_i < t} \log\left(1 - \hat{h}_1(t) - \hat{h}_2(t)\right) \right\}$$

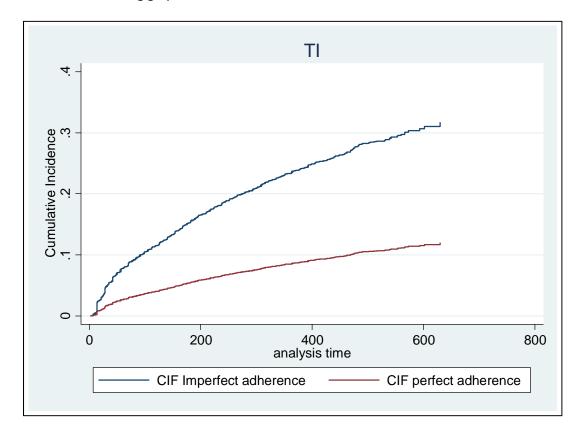
Finally, we calculate the estimated CIFS and graph:

```
. gen cif_TI_0 = sum(S_0[_n-1]*h_TI_0) if exclude==0
(13425 missing values generated)
. label var cif_TI_0 "CIF Imperfect adherence"
. gen cif_TI_1 = sum(S_1[_n-1]*h_TI_1) if exclude==0
(13425 missing values generated)
. label var cif_TI_1 "CIF perfect adherence"
. twoway line cif_TI* _t , connect(J J) sort ylab(0 0.1 0.2 0.3 0.4) title(TI)
> ytitle(Cumulative Incidence) xtitle(analysis time)
```

<sup>&</sup>lt;sup>2</sup> Note that 1,753 missing values will be generated for the observations with neither of the two failures (which will be excluded from the analysis as censored observations and thus observations providing no information).

<sup>&</sup>lt;sup>3</sup> If we did not do this, only values with successively available observations would be summed resulting in a virtually zero result!

This results in the following graph:



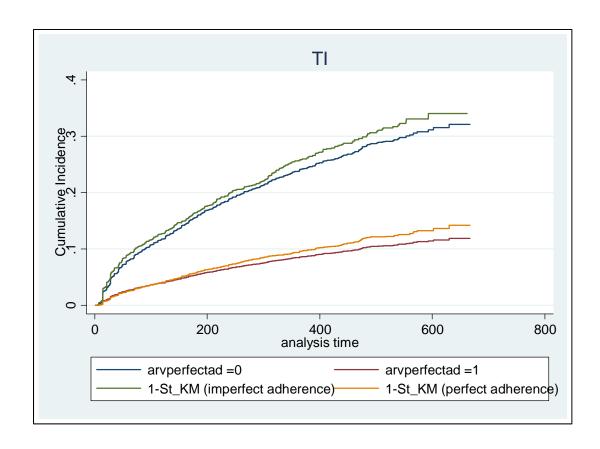
Now we will generate the Aalen-Johansen estimator of the cumulative incidence function. First we will perform the analysis of the effect of ARV adherence on TI in the presence of death or loss to follow-up.

```
. stset time2int , fail(fail3==1)
    failure event: fail3 == 1
obs. time interval:
                    (0, time2int]
 exit on or before:
                    failure
    14162 total obs.
       0 exclusions
   14162 obs. remaining, representing
    1376 failures in single record/single failure data
 3548482 total analysis time at risk, at risk from t =
                                                                 0
                            earliest observed entry t =
                                                                 0
                                  last observed exit t =
                                                               667
```

```
. stcrreg arvperfectad , compete(fail3 == 2)
       failure _d: fail3 == 1
  analysis time _t: time2int
Iteration 0: log pseudolikelihood = -12326.422
Iteration 1: log pseudolikelihood = -12326.263
Iteration 2: log pseudolikelihood = -12326.263
                                         No. of obs = 13649
No. of subjects = 13649
Competing-risks regression
                                                            1372
                                         No. failed =
Failure event : fail3 == 1
Competing event: fail3 == 2
                                         No. competing =
                                                             1896
                                                           10381
                                         No. censored
                                         Wald chi2(1)
                                                      =
                                                           418.43
                                                           0.0000
Log pseudolikelihood = -12326.263
                                         Prob > chi2
                       Robust
                SHR Std. Err. z  P>|z|  [95% Conf. Interval]
        _t |
arvperfectad .3258656 .0178622 -20.46 0.000
                                                .2926713
______
```

We note that perfect adherence is associated with an almost 2/3 reduction of the sub-hazard for TI adjusted for the effect of death and loss to follow-up. The CIF estimate based on the Aalen-Johansen estimator is produced as follows:

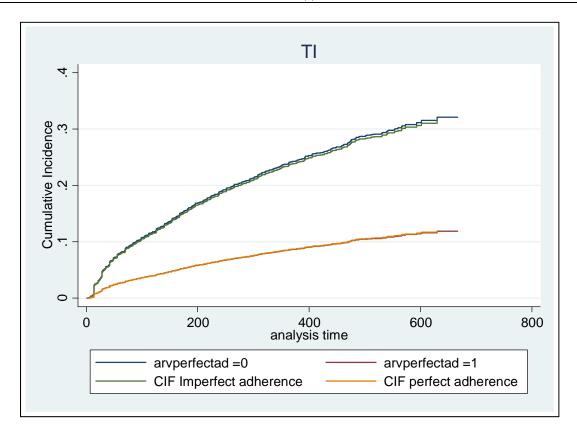
```
. sts gen St_KM=s, by(arvperfectad )
gen CIF_KM_0=1-St_KM if arvperfectad ==0
(11692 missing values generated)
. gen CIF_KM_1=1-St_KM if arvperfectad ==1
(2983 missing values generated)
. label var CIF_KM_1 "1-St_KM (perfect adherence)"
. label var CIF_KM_0 "1-St_KM (imperfect adherence)"
. stcurve, cif at1(arvperfectad =0) at2(arvperfectad =1) title(TI) ylab(0 0.1 0.2 0.3 > 0.4) addplot( line CIF_KM_0 _t , connect(J)||line CIF_KM_1 _t, connect(J))
```



We see how the Kaplan Meier estimate of the CIF for TI overestimates the true CIF (adjusted for death or loss to follow-up).

On the other hand, plotting the Aalen-Johansen estimator and the one manually produced by the Cox model produces the following graph:

```
. stcurve, cif at1(arvperfectad =0) at2(arvperfectad =1) title(TI) ylab(0 0.1 0.2 0.3
> 0.4) addplot( line cif_TI_0 _t , connect(J) | line cif_TI_1 _t, connect(J))
```



The agreement is remarkable!