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Compact operators and the geometric structure of C^*-algebras. (English summary)

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Let $\mathfrak{A}$ be a C^*-algebra. In this note the authors present geometric conditions that characterize those elements $a \in \mathfrak{A}$ which turn into compact operators when $\mathfrak{A}$ acts on a Hilbert space. To state one of them, we fix a subset M of the unit ball of $\mathfrak{A}$, put $\mathrm{cp}(M) = \{a \in \mathfrak{A}\colon\ \|a \pm m\| \leq 1$ for all $m \in M\}$, and call a norm-one element $a \in \mathfrak{A}$ geometrically compact if $\mathrm{cp}(\mathrm{cp}(\{x\}))$ is (norm-) compact. It turns out that a has this property if and only if there is a faithful representation π such that $\pi(a)$ is a compact operator. Moreover, a faithful representation π can be found such that $\pi(a)$ is finite-dimensional, if and only if the geometric rank $r_{\mathrm{g}}(x)$ of a, i.e. the dimension of the linear span of $\mathrm{cp}(\mathrm{cp}(\{x\}))$, is finite. The authors furthermore investigate the behaviour of the geometric rank, and point out that no obvious connection exists between the numbers $r_{\mathrm{g}}(a)$ and $\mathrm{rk}\,\pi(a)$.

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