

9-3-2023

Πολυμηχανική γραμμικότητα :

Εύσεξισης
μεταγνήσης ανεξάρτητων
μεταβλητών

dataset mr3

Model1 $y = b_0 + b_1 \underline{x_1} \quad (+\varepsilon)$

$$R^2 = 0.3751$$

$$\begin{aligned} H_0: b_1 &= 0, \quad H_1: b_1 \neq 0 \\ \hat{b}_1 &= 1.96 \end{aligned} \quad \boxed{P \approx 0}$$

x_1 : σταύρωση
μηχανικής
λαρισαϊστικής

Model3 : $y = b_0 + b_1 x_1$

$$R^2 = 0.2838,$$

$$H_0: b_3 \approx 0, \quad b_3 \neq 0$$

$$\boxed{P = 10^{-8} \approx 0}$$

Model 13 : $y = b_0 + b_1 x_1 + b_2 x_3$

$$R^2 = 0.377$$

$$H_0: b_1 = 0 \quad H_1: b_1 \neq 0$$

$$\boxed{P = 2 \cdot 10^{-4}}$$

$$H_0: b_3 = 0$$

$$H_1: b_3 \neq 0$$

$$\boxed{P = 0.843}$$

Model 1

$$SSR = SS(x_1) = 3142.6$$

Model 13

$$SS(x_1) = 3142.6$$

$$SS(x_3|x_1) = 15.9$$

$$SSR(x_1, x_3) = 3158.5$$

exce opacta
n orca
elocigis
(R)

Model 31

$$SS(x_3) = 2377.6$$

$$SS(x_1|x_3) = 780.9$$

$$SSR(x_3, x_1) = 3158.5$$

Model 3

$$SS(x_3) = 2377.6$$

Εργασία Φ για εμπειρογνωμόνα την
μοντέλο με πολλούς

$$Y = b_0 + b_1 X_1 + \cdots + b_k X_k$$

$$\#\beta = k+1$$

① F-test για ανάλογη πολλούς

$$H_0: b_1 = b_2 = \cdots = b_k = 0$$

$$H_1: \text{τουλάχιστον ένα } b_j \neq 0$$

F-test

$$\text{F-statistic} = \frac{\text{MSR}}{\text{MSE}}$$

$$\left(\text{MSR} = \frac{\text{SSR}}{\text{df}_{\text{reg}}} = \frac{\text{SSR}}{\#\beta - 1} = \frac{\text{SSR}}{k} \right)$$

$$\text{MSE} = \frac{\text{SSE}}{\text{df}_{\text{er}}} = \frac{\text{SSE}}{n - \#\beta} = \frac{\text{SSE}}{n - k - 1}$$

Εκπίδια: Αν ισχύει η $H_0 \Rightarrow$

$$F \sim F_{\text{df}_{\text{reg}}, \text{df}_{\text{er}}}$$

Accept H_0 : an $F \leq F_{\alpha, \text{dfreq}, \text{dfer}}$
 reject an $F > F_{\alpha, \text{dfreq}, \text{dfer}}$

(100% reject an $p < \alpha$)

A.X. Model B : $Y = b_0 + b_1 x_1 + b_2 x_3$
 $\underbrace{$ # d.f. = 3

$F = 29.35$ $n = 100$

(2, 97 d.f.)

$H_0: b_1 = b_2 = 0$

$P = 1 \cdot 10^{-10}$
 $H_1: b_1 \neq 0 \text{ or } b_2 \neq 0$
 in case 2a 2

A.X. Model 1

$Y = b_0 + b_1 x_1$

F test

$H_0: b_1 = 0$

$H_1: b_1 \neq 0$

$\hat{\wedge}$

t test (b_1)

$H_0: b_1 = 0$

$H_1: b_1 \neq 0$

$$F_{1,n} \sim t^2$$

Fia feorolægspunktis

$$\text{one. } F = \frac{MSR}{MSE} = \left(\frac{t_{\alpha/2}}{b_1} \right)^2$$

$$F_{\alpha/2, df_{\text{err}}} = t_{\alpha/2, df_{\text{err}}}^2$$

Nærðsíða

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 \leftarrow \text{Model 1}$$

b_1, b_2 age, bago
 b_3, b_4 með fæðvun
 ævifor

$$H_0: b_3 = b_4 = 0$$

$$H_1: \text{cov. eva} \neq 0$$

F test for part of model

Model 2 :
$$Y = b_0 + b_1 X_1 + b_2 X_2$$
 (part)

Model 1 :
$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4$$
 full model

Model 2 : nested in Model 1
(with coefficient)

F-test : $H_0: b_3 = b_4 = 0$ vs $H_1: \text{non-zero } f_0$
or model 1

$$F = \frac{\frac{SSR(\text{full}) - SSR(\text{part})}{df_r(\text{full}) - df_r(\text{part})}}{MSE(\text{full})}$$

$$\hat{=} \frac{\frac{SSR(x_3, x_4 | X_1, X_2)}{df_r(\text{full}) - df_r(\text{part})}}{MSE(x_1, x_2, x_3, x_4)}$$

R : `anova(model 1, model 2)`

↑
provides 2 nested models

Параллельная 8

MR3

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 \quad (\text{model 123})$$

$$H_0: b_2 = b_3 = 0 \quad H_1: b_2 \neq 0 \text{ or } b_3 \neq 0$$

full : model 123

part : model 1

$$F = 62,294$$

$$P = 2 \cdot 10^{-16}$$

$\Rightarrow \text{accept } H_0.$

Par. 3

$$\text{Model 123 : } Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3$$

$$H_0: b_3 = 0 \quad H_1: b_3 \neq 0$$

- a) t-test toward b_3 over Model 123
- b) F-test regarding Model 123
vs Model 12

- a) Aus summary (model 123) : t-test b_3

$t = 0.955, p = 0.342$
 ⑥ anova(model12, model123)
 $F = 0.913, P = 0.342$
 $\Rightarrow (0.955)^2$

F test type I, III

① Shows probabilities $x_1, x_2, x_3, \dots, x_k$.
Diagonally nested models

$Y \sim X_1$ F test (X_1) sequential
 F-tests

$Y \sim X_1 + X_2$ Ftest ($X_2 | X_1$) (Ftests)

$Y \sim X_1 + X_2 + X_3$ Ftest ($X_3 | X_1, X_2$) type I

⋮

$Y \sim X_1 + \dots + X_k$ Ftest ($X_k | X_1, \dots, X_{k-1}$)

②

to full model

$$Y = b_0 + b_1 X_1 + \dots + b_k X_k$$

$$F\text{-test } H_0: b_1 = 0 \quad H_1: b_1 \neq 0 \quad (\sim t\text{-test})$$

$$F\text{-test } X_1 \mid X_2, \dots, X_k \quad (\text{uses } s_1 \text{ unifong})$$

$$F\text{-test } H_0: b_2 = 0 \quad H_1: b_2 \neq 0$$

$$F\text{-test } X_2 \mid X_1, X_3, \dots, X_k$$

→ partial F-tests in F-tests type III

(divide a few t-tests
in full model)

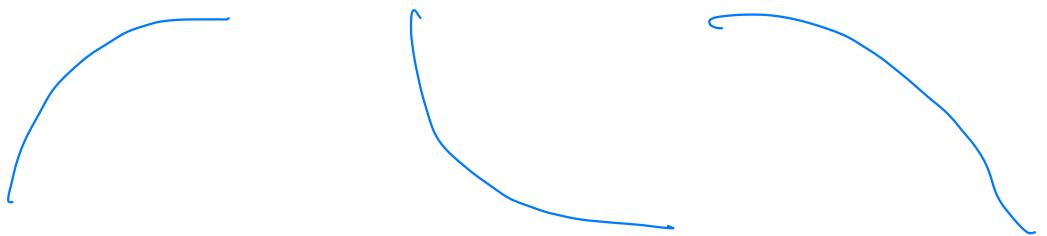
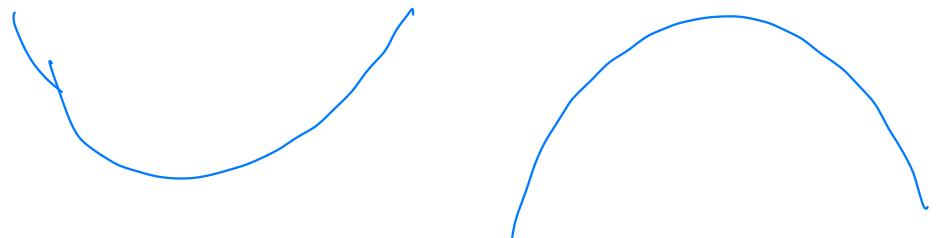
Παραγωγή Παραδόσεις

y, X

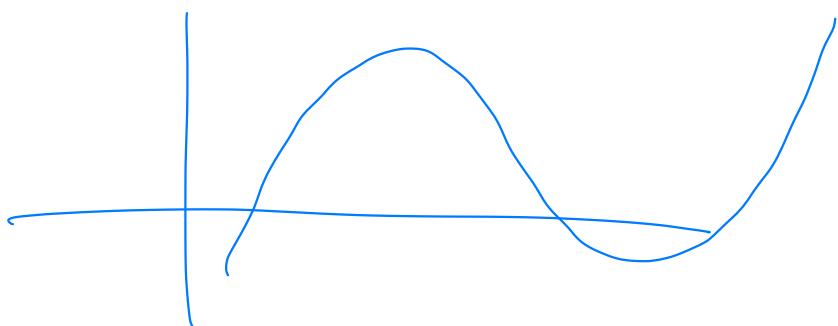
Παραγωγή Παραδόσεις

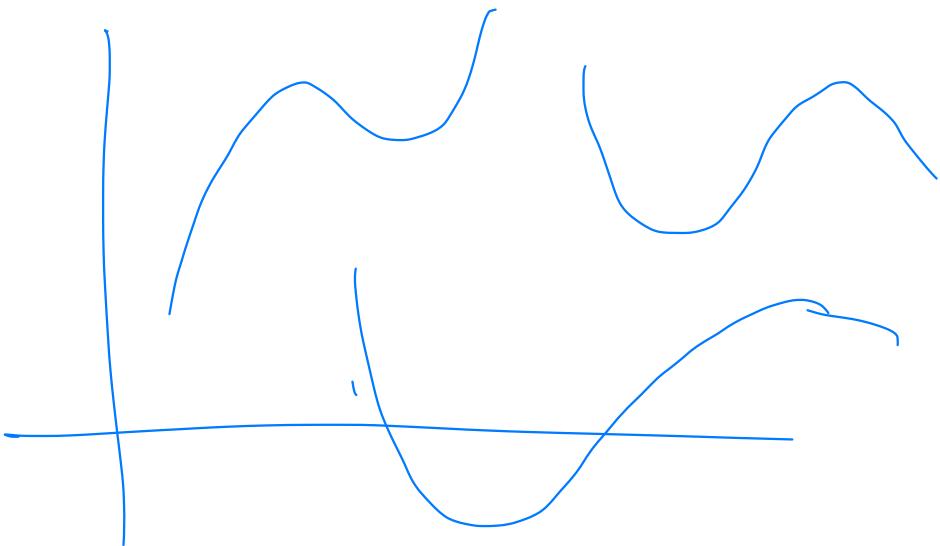
$$Y = b_0 + b_1 X + b_2 X^2 + b_3 X^3 + \dots + b_k X^k + \varepsilon$$

$$f(x) = b_0 + b_1 x + b_2 x^2$$

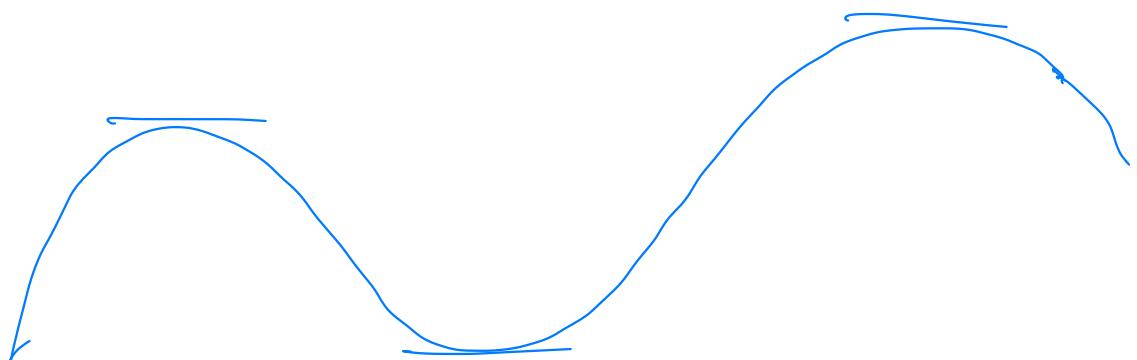


$$f(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$





$$f(x) = \dots + b_4 x^4$$



Параметры

dataset = ozone

$$Y = \text{ozone}$$

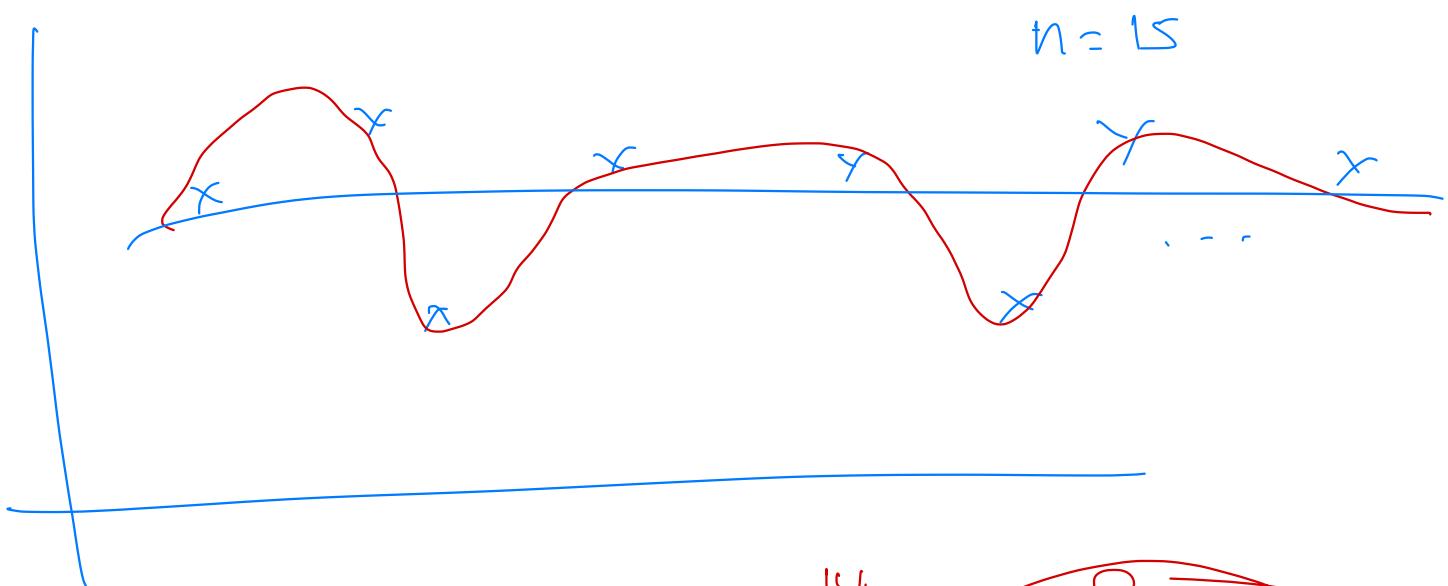
$$X = \text{wind.}$$

$$\textcircled{1} \quad Y = b_0 + b_1 X$$

$$\textcircled{2} \quad Y = b_0 + b_1 X + b_2 X^2$$

$$③ Y = b_0 + b_1 x + b_2 x^2 + \dots + b_7 x^7$$

$$Y \sim \text{poly}(x, 7)$$



$$Y = b_0 + b_1 x + \dots + b_{14} x^{14}$$

$$df_{\text{fer}} = 0$$

$$R^2 = 1$$

$$SSE = 0$$

$$MSE = \frac{0}{0} = \frac{SSE}{df_{\text{fer}}} = ?$$

overfitting !!

