605: On the material to be examined, 2020

The material to be examined is described in the slides which can be found in the eclass. It goes without saying that complete proofs may be required. The following items *will be considered known and will be used*, but proofs will not be required:

1. Remark on Fejér' sTheorem: More generally, if the one-sided limits $f(t_+)$ and $f(t_-)$ exist, then $\sigma_n(f,t) \to \frac{f(t_+)+f(t_-)}{2}$.
2. For each $a \in (0,1)$, one can construct a "Cantor-like set" C^a (i.e. a compact set, with empty interior and no isolated points) having measure a.
3. Luzin's Theorem: If $f : X \to \mathbb{R}$ is measurable, then for every $\epsilon > 0$ there exists a closed set $F_\epsilon \subseteq X$ with $\lambda(X \backslash F_\epsilon) < \epsilon$ so that the function $f|_{F_\epsilon}$ is continuous.
4. A bounded function $f : [a,b] \to \mathbb{R}$ is Riemann integrable if and only if it is almost everywhere continuous, that is, if its set of discontinuities has measure zero. Then f is Lebesgue integrable and the two integrals coincide.
5. The Riesz-Fischer theorem for L^p: It will be considered known and will be used, but the proof may only be required for $p = 1$.
6. A trigonometric series which is not the Fourier series of an $L^1(\mathbb{T})$ function: Only the statements and use of the Propositions and Lemmas in this subsection will be required, but not their proofs.