Microeconomic Theory I

Cost Curves

Stella Tsani

stsani@econ.uoa.gr

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Types of Cost Curves

- **A total cost curve is the graph of a firm's total cost function.**
- **A variable cost curve is the graph of a firm's variable cost function.**
- **An average total cost curve is the graph of a firm's average total cost function.**

Types of Cost Curves

- **An average variable cost curve is the graph of a firm's average variable cost function.**
- **An average fixed cost curve is the graph of a firm's average fixed cost function.**
- **A marginal cost curve is the graph of a firm's marginal cost function.**

Types of Cost Curves

- **How are these cost curves related to each other?**
- **How are a firm's long-run and short-run cost curves related?**

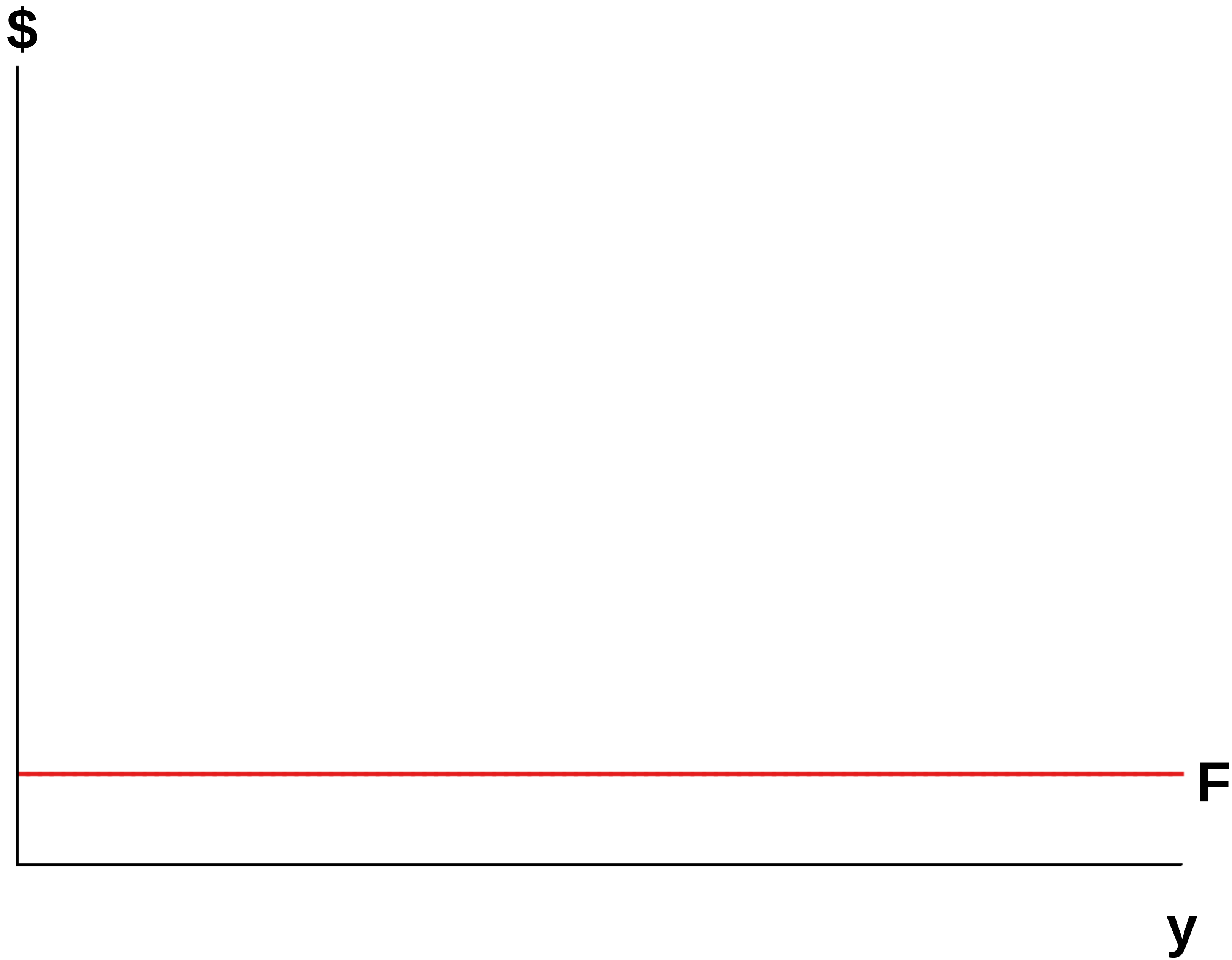
Fixed, Variable & Total Cost Functions

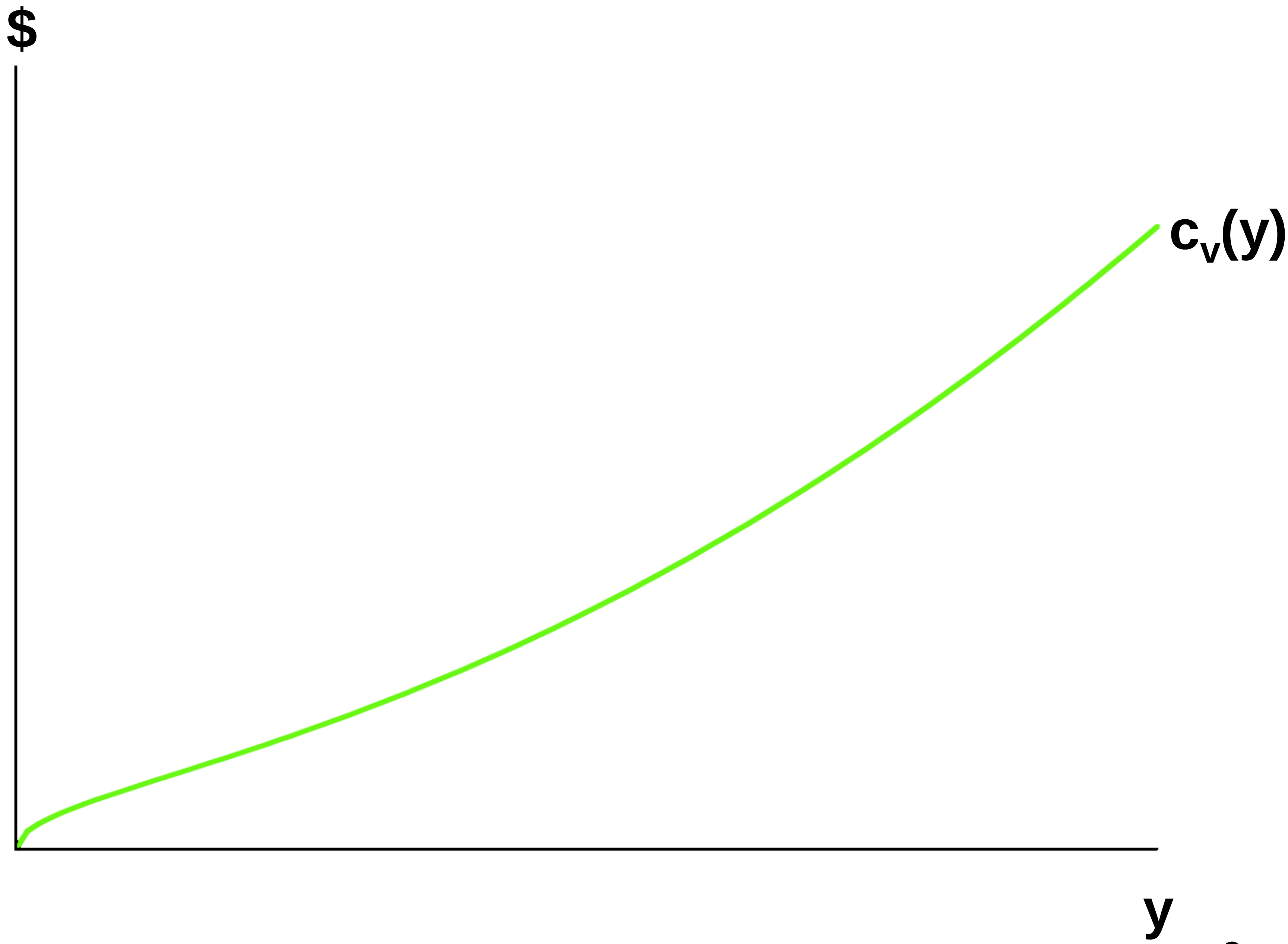
- **F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.**
- **$c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units. $c_v(y)$ is the firm's variable cost function.**
- **$c_v(y)$ depends upon the levels of the fixed inputs.**

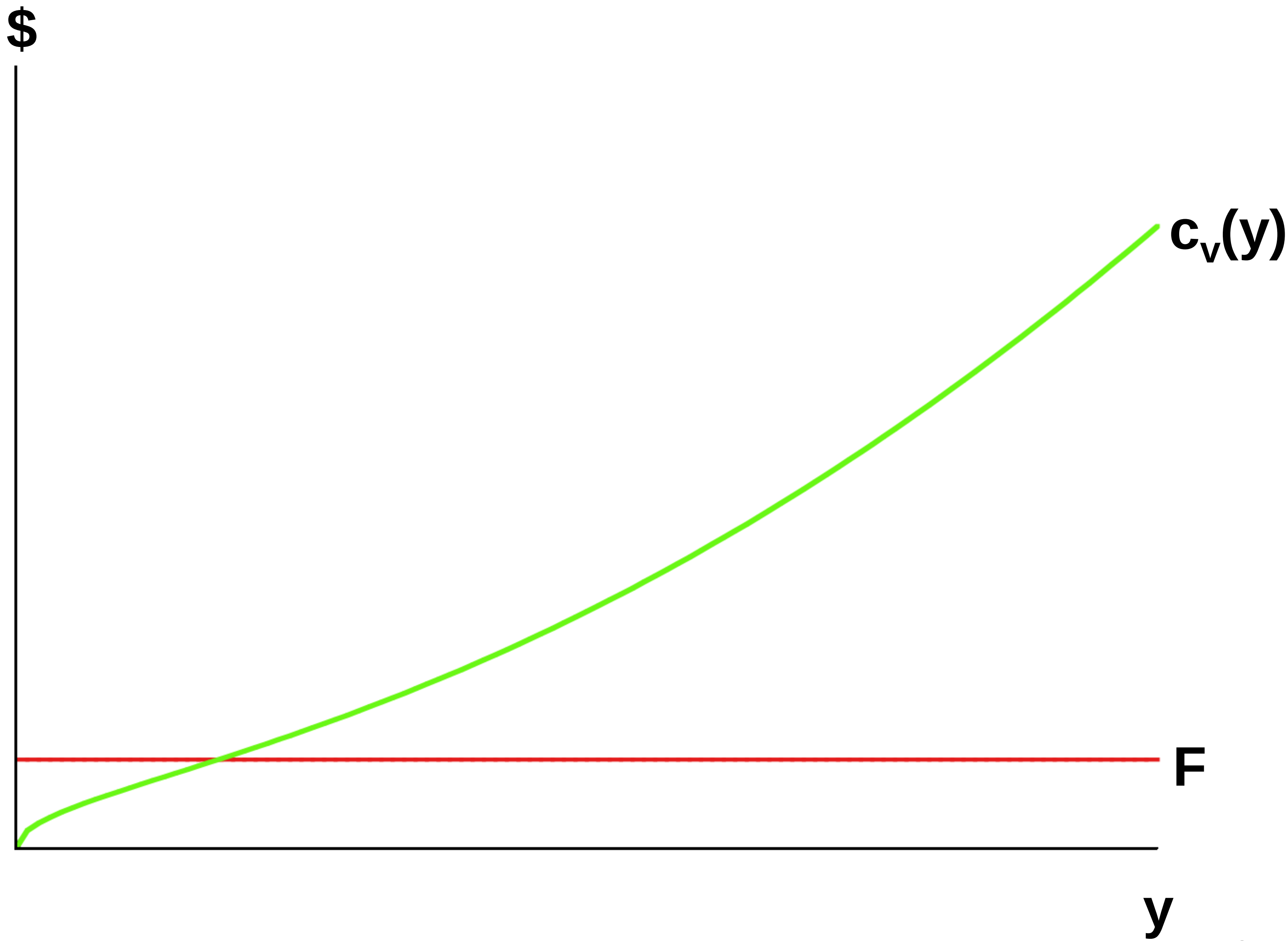
Fixed, Variable & Total Cost Functions

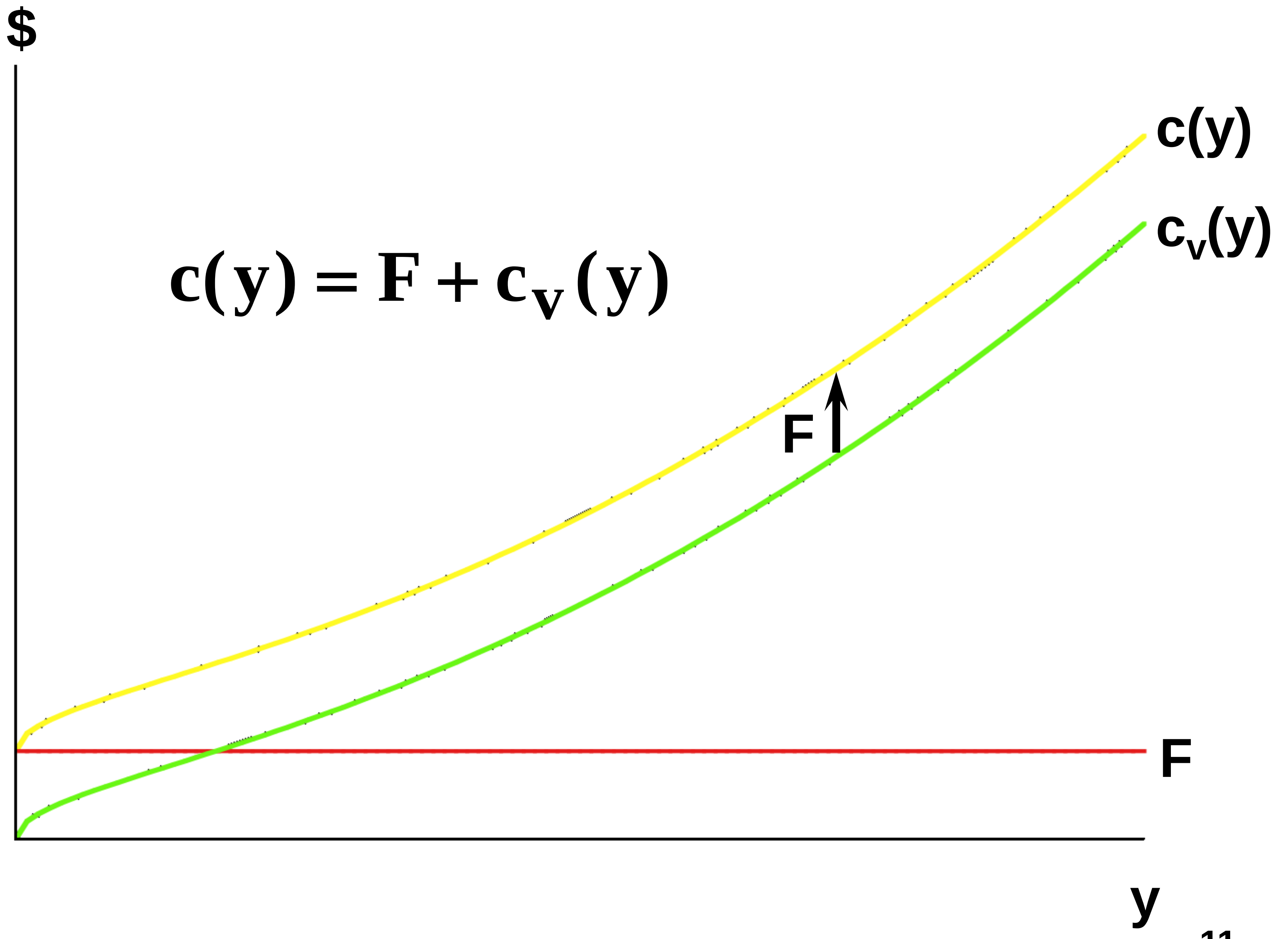
- c(y) is the total cost of all inputs, fixed and variable, when producing y output units. c(y) is the firm's total cost function;

$$c(y) = F + c_v(y).$$


$
F
y


$
$c_v(y)$
y


$
$c_v(y)$
F
y



$$c(y) = F + c_v(y)$$

Av. Fixed, Av. Variable & Av. Total Cost Curves

- **The firm's total cost function is**

$$c(y) = F + c_v(y).$$

For y > 0, the firm's average total cost function is

$$AC(y) = \frac{F}{y} + \frac{c_v(y)}{y}$$

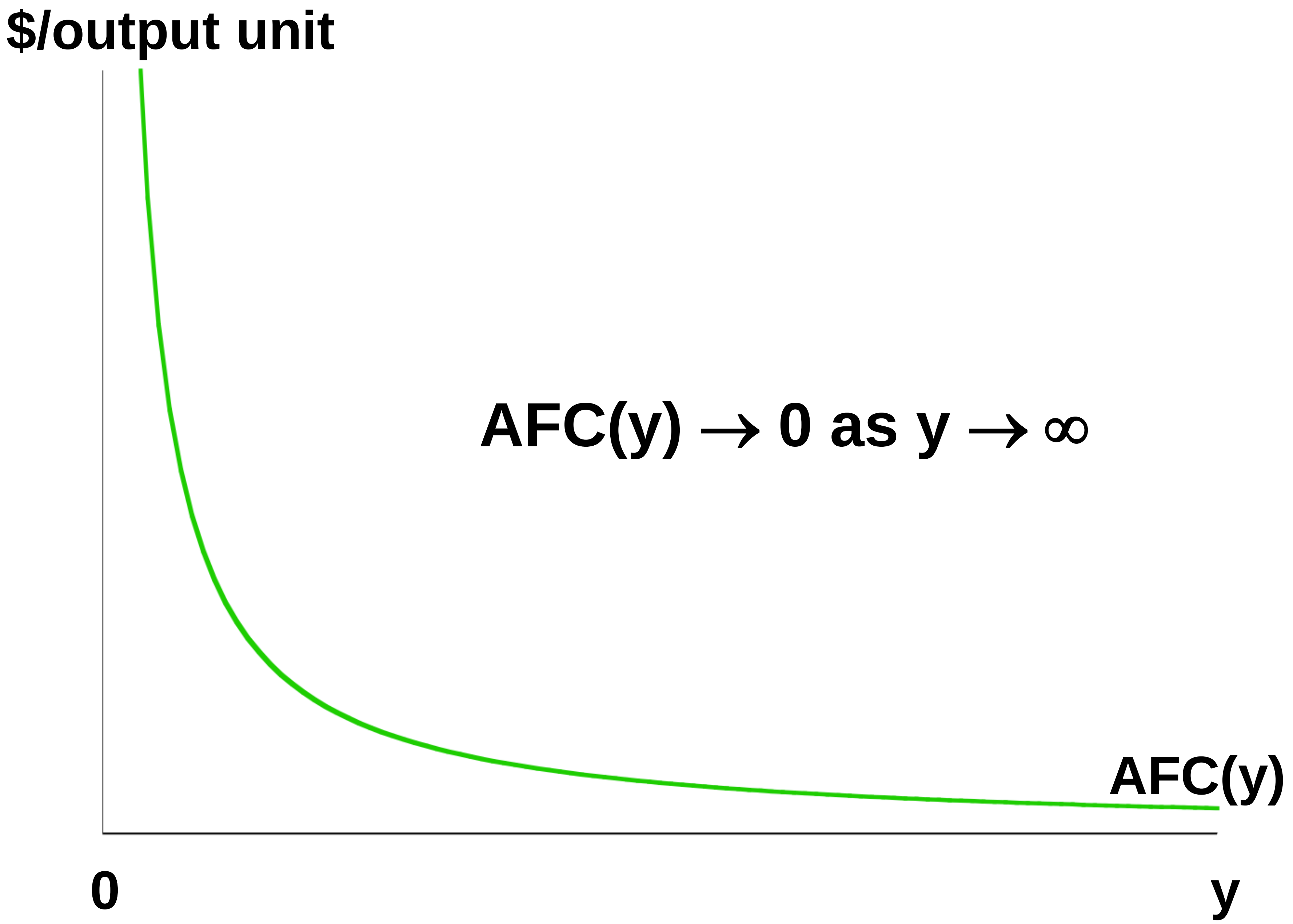
$$= AFC(y) + AVC(y).$$

Av. Fixed, Av. Variable & Av. Total Cost Curves

- **What does an average fixed cost curve look like?**

$$\mathbf{AFC(y) = \frac{F}{y}}$$

- **AFC(y) is a rectangular hyperbola so its graph looks like ...**


$/output unit
AFC(y) → 0 as y → ∞
AFC(y)
0
y

Marginal Cost Function

- **Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,**

$$MC(y) = \frac{\partial c_v(y)}{\partial y}.$$

Marginal Cost Function

- **The firm's total cost function is**

$$c(y) = F + c_v(y)$$

and the fixed cost F does not change with the output level y, so

$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$

- **MC is the slope of both the variable cost and the total cost functions.**

Marginal and Variable Cost Functions

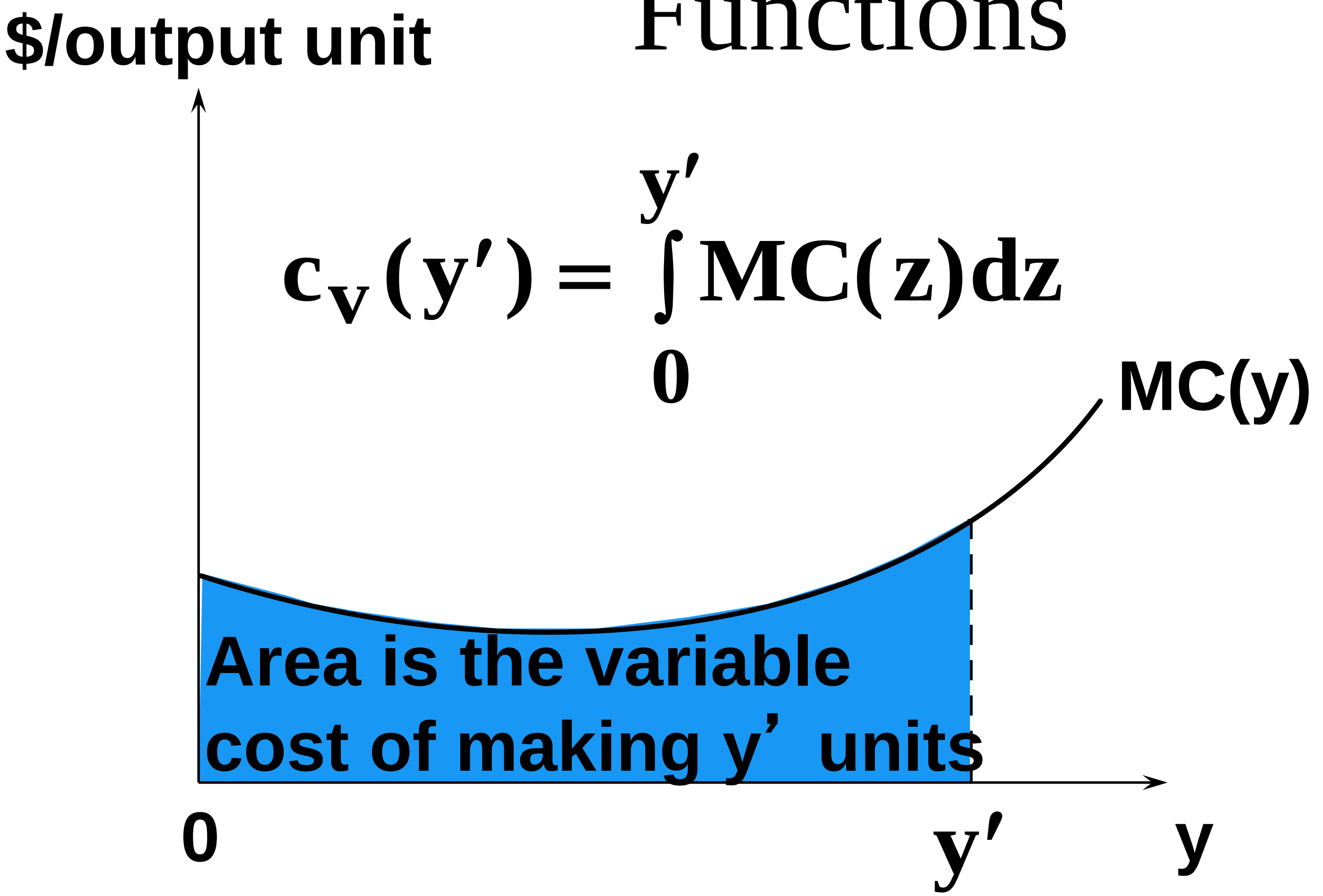
- **Since MC(y) is the derivative of $c_v(y)$, $c_v(y)$ must be the integral of MC(y). That is,**

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

$$\Rightarrow \quad c_v(y) = \int_0^y MC(z)dz.$$

Marginal and Variable Cost Functions

$$c_v(y') = \int_0^{y'} MC(z)\,dz$$



Marginal & Average Cost Functions

- **How is marginal cost related to average variable cost?**

Marginal & Average Cost Functions

Since $$\mathbf{AVC(y) = \frac{c_v(y)}{y}},$$

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c_v(y)}}{\mathbf{y^2}}.$$

Marginal & Average Cost Functions

Since $$\mathbf{AVC(y) = \frac{c_v(y)}{y}},$$

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c_v(y)}}{\mathbf{y^2}}.$$

Therefore,

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{0} \quad \textbf{as} \quad \mathbf{y \times MC(y)} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{c_v(y)}.$$

Marginal & Average Cost Functions

Since $$\mathbf{AVC(y) = \frac{c_v(y)}{y},}$$

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c_v(y)}}{\mathbf{y^2}}.$$

Therefore,

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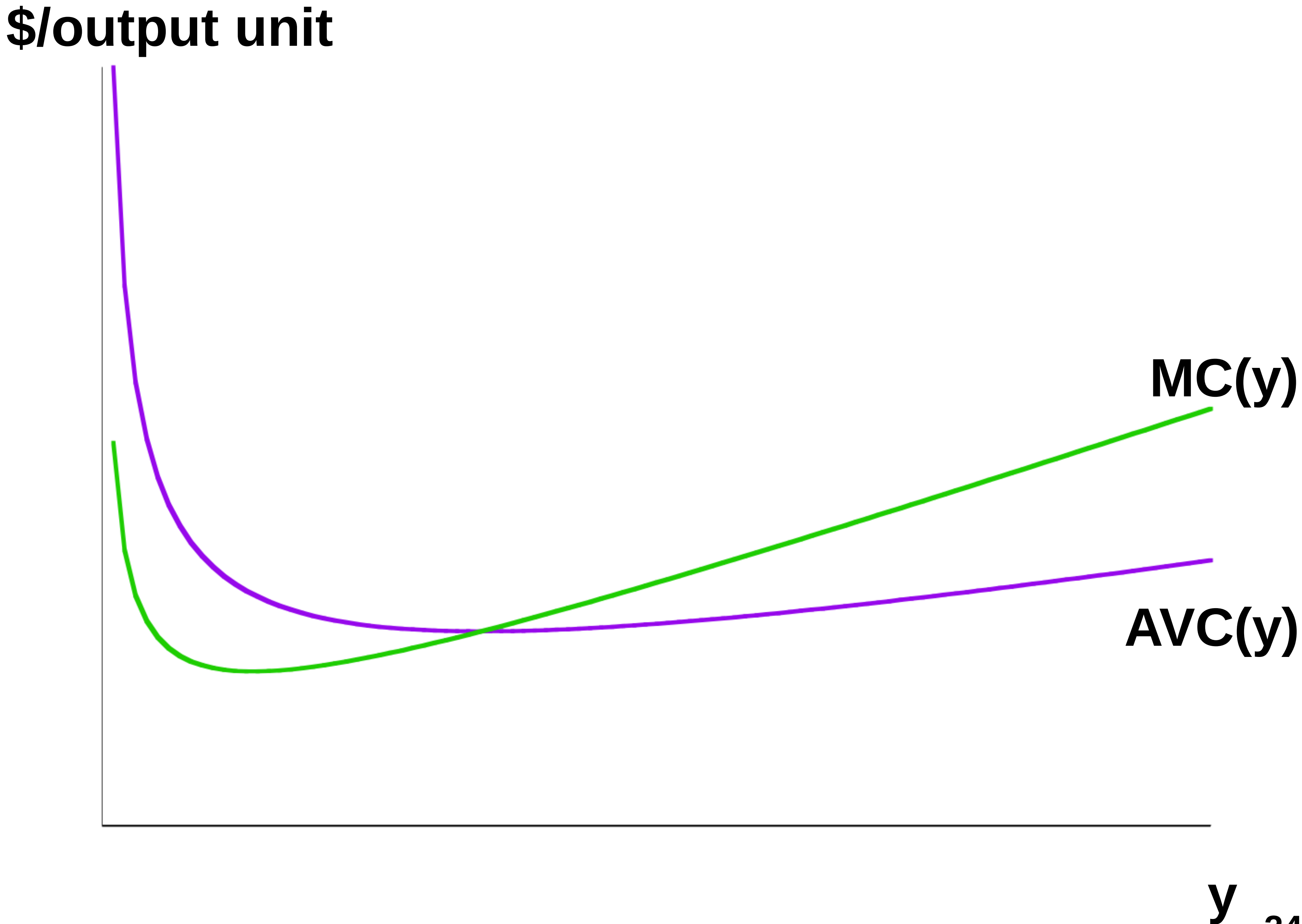
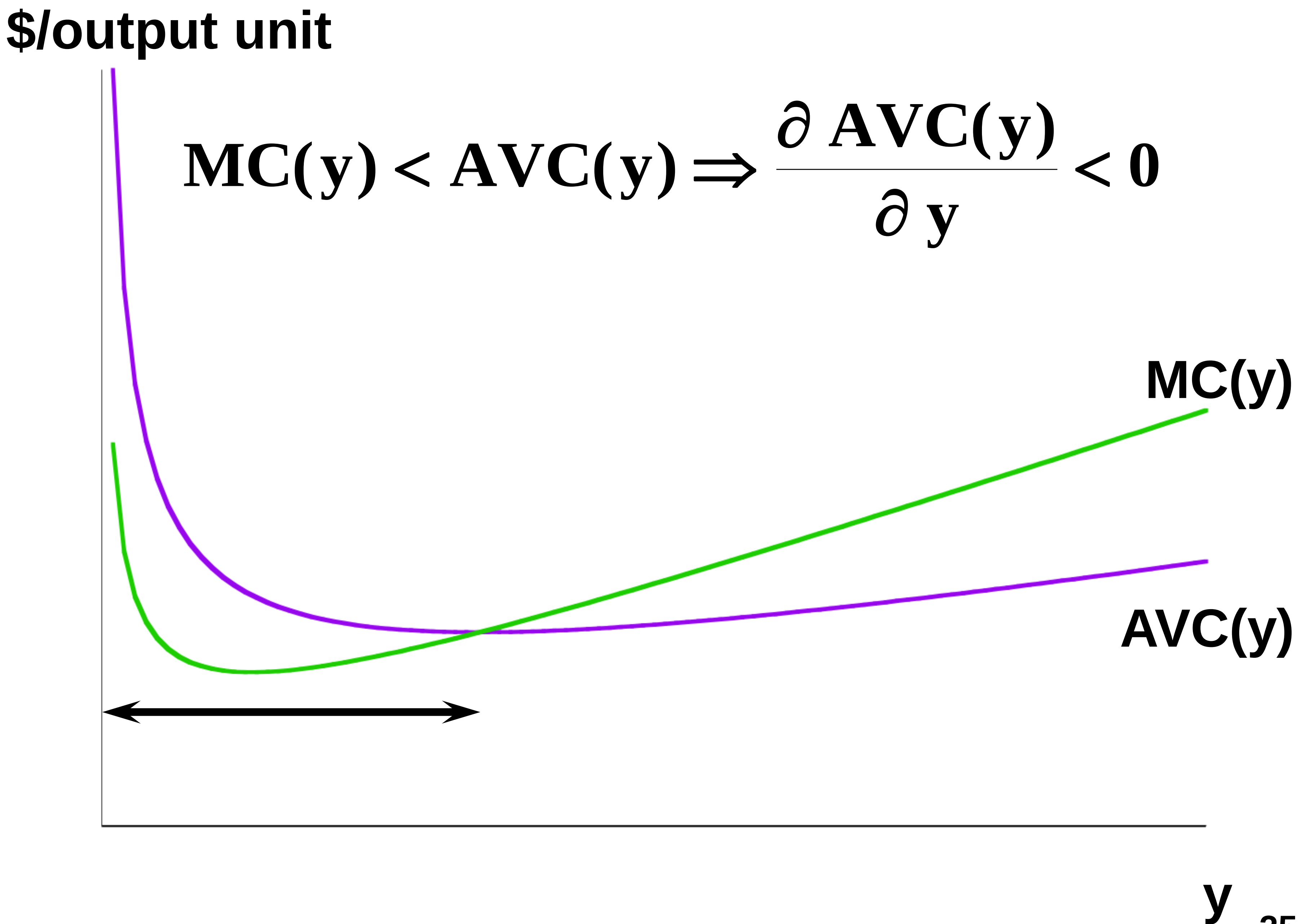
$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad \mathbf{MC(y)} \begin{matrix} > \\ = \\ < \end{matrix} \frac{\mathbf{c_v(y)}}{\mathbf{y}} = \mathbf{AVC(y)}.$$

$$\frac{\partial \, \mathbf{AVC(y)}}{\partial \, \mathbf{y}} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{0} \;\; \textbf{as} \;\; \mathbf{MC(y)} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{AVC(y)}.$$

Marginal & Average Cost Functions

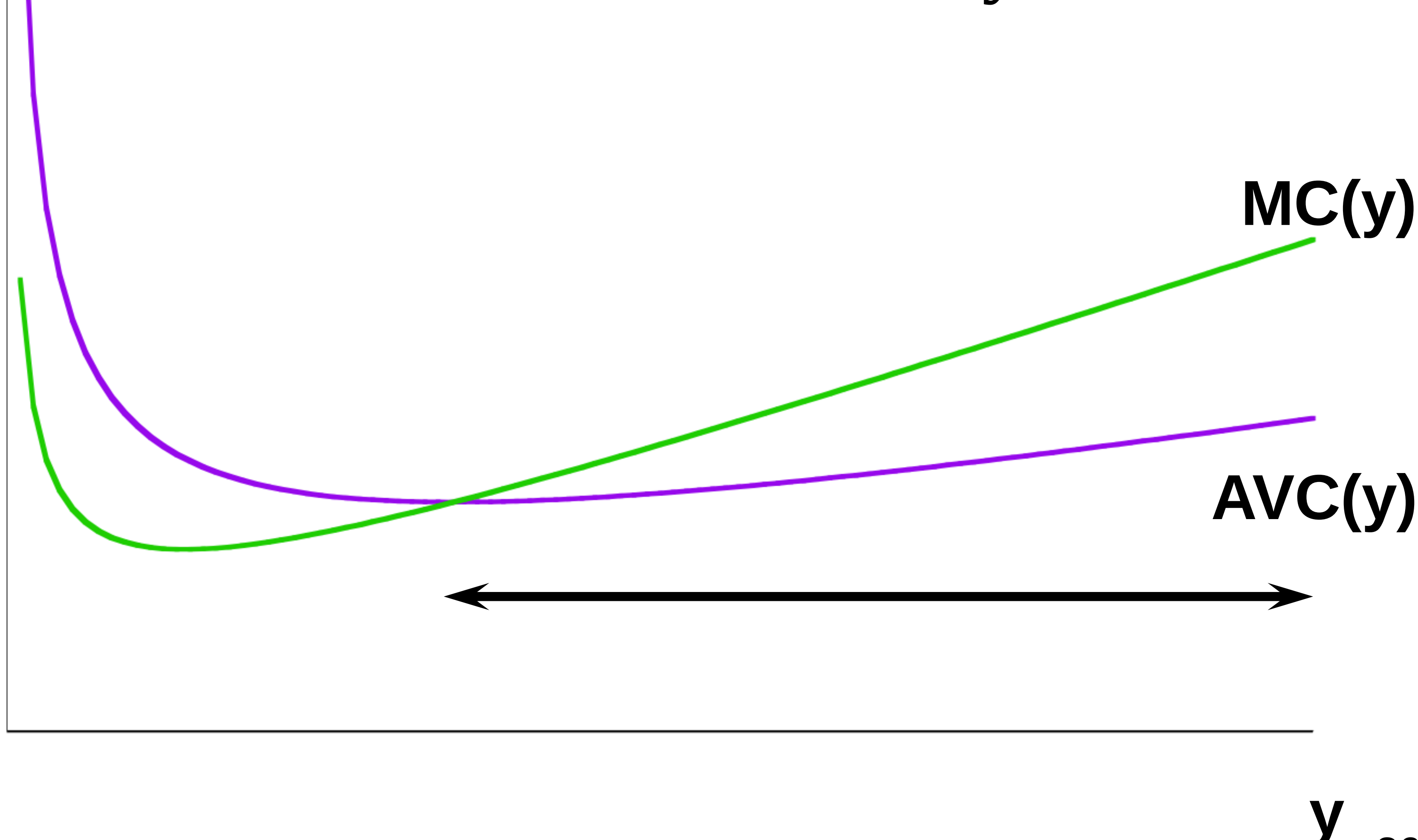

$/output unit
MC(y)
AVC(y)
y

$$\mathbf{MC(y) < AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} < 0}$$

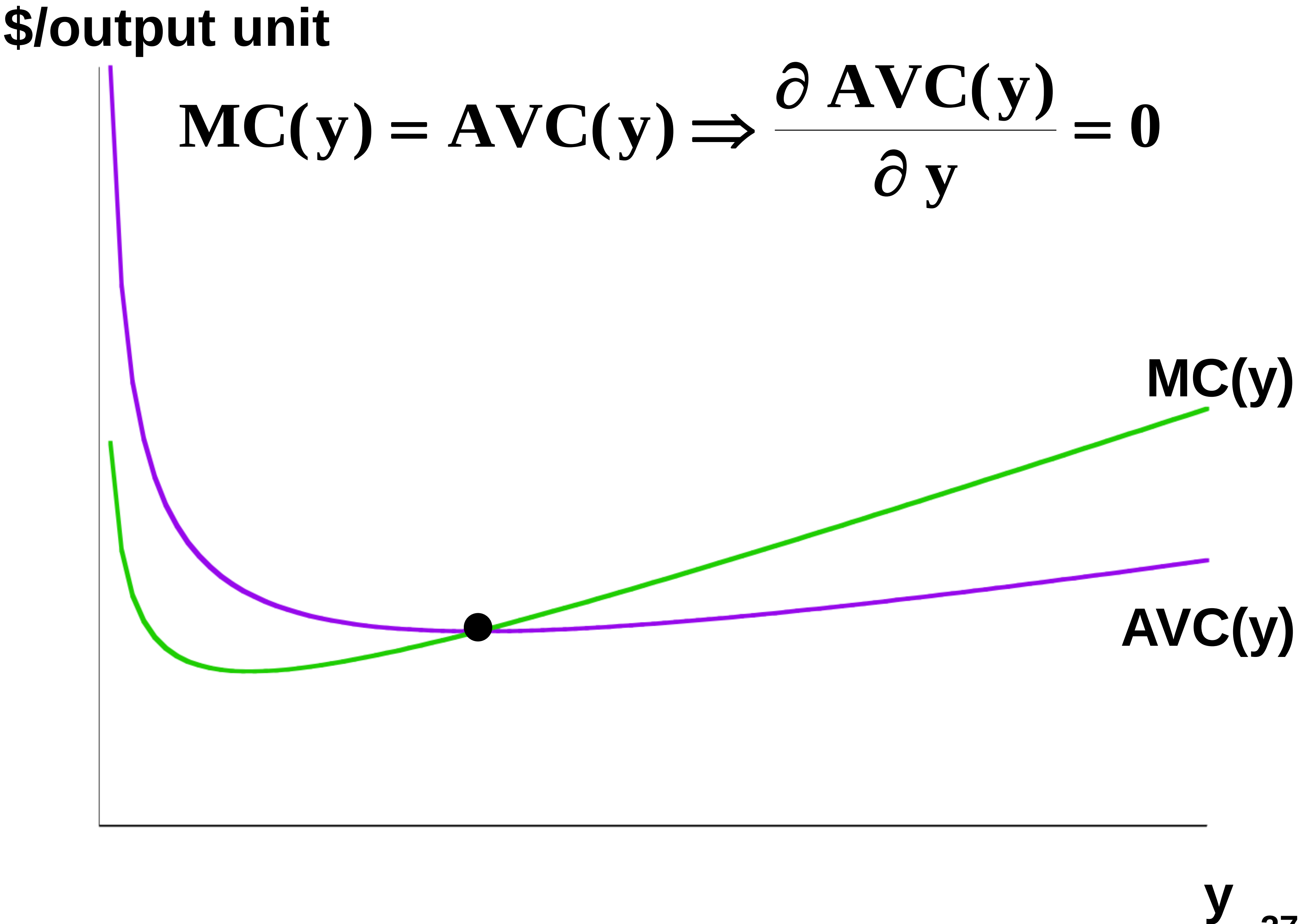


$/output unit

$$MC(y) > AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} > 0$$



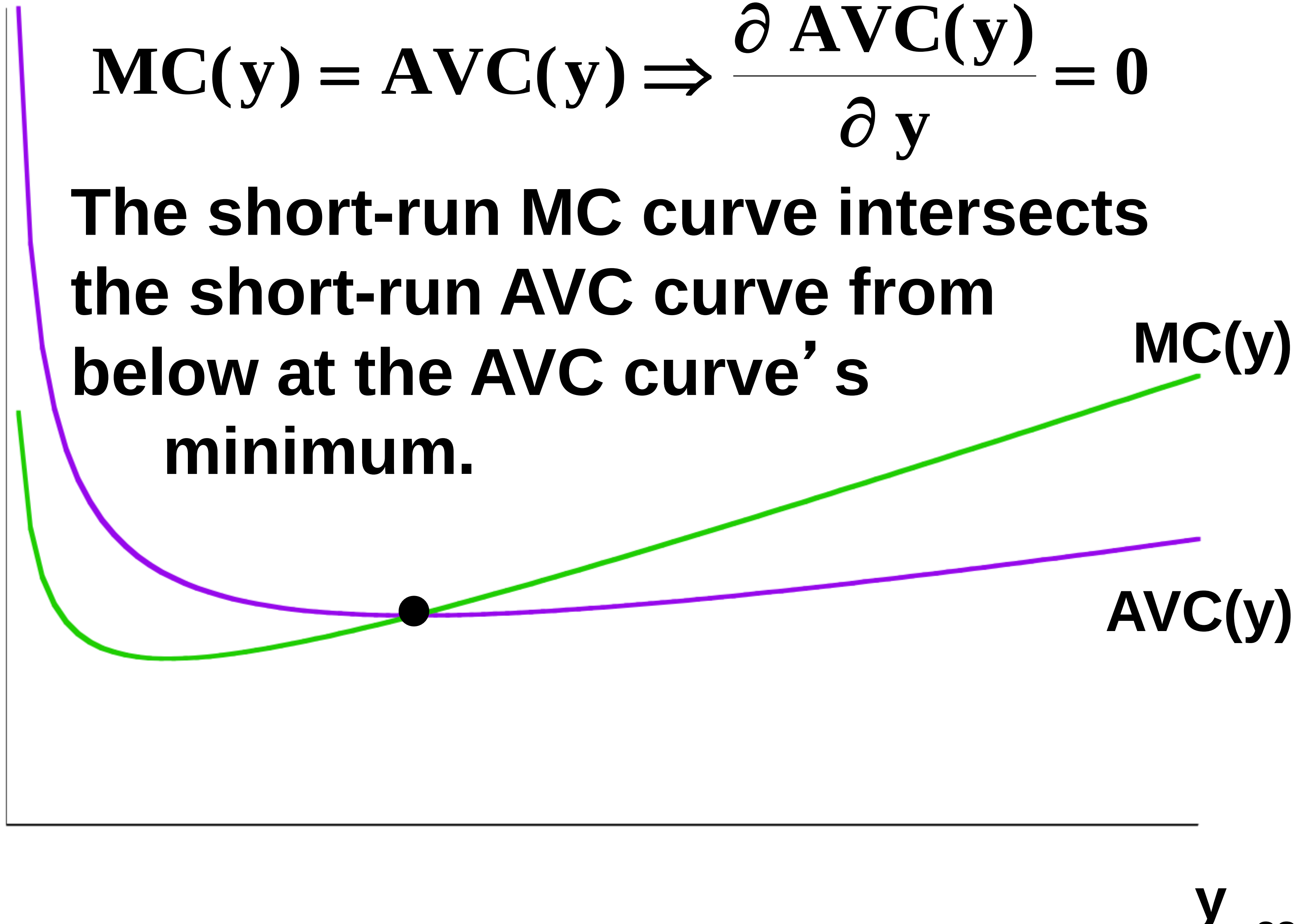
$$MC(y) = AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} = 0$$



$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial\, AVC(y)}{\partial\, y} = 0$$

The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.



Marginal & Average Cost Functions

Similarly, since $\mathbf{ATC(y) = \dfrac{c(y)}{y}}$,

$$\frac{\partial\, \mathbf{ATC(y)}}{\partial\, \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c(y)}}{\mathbf{y^2}}.$$

Marginal & Average Cost Functions

Similarly, since $$\mathrm{ATC}(y) = \frac{c(y)}{y},$$

$$\frac{\partial\, \mathrm{ATC}(y)}{\partial\, y} = \frac{y \times \mathrm{MC}(y) - 1 \times c(y)}{y^2}.$$

Therefore,

$$\frac{\partial\, \mathrm{ATC}(y)}{\partial\, y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textbf{as} \quad y \times \mathrm{MC}(y) \begin{matrix} > \\ = \\ < \end{matrix} c(y).$$

Marginal & Average Cost Functions

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Therefore,

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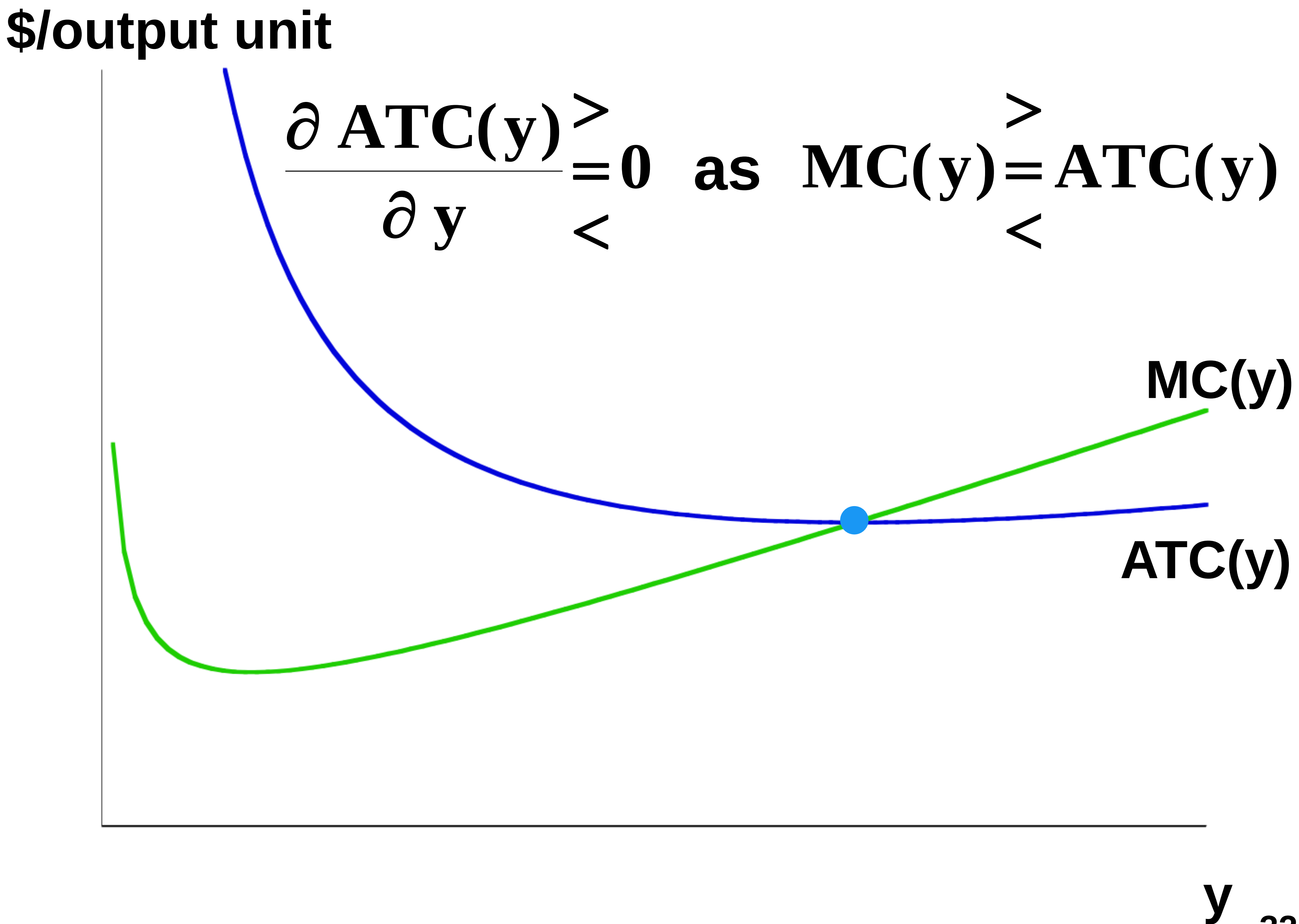
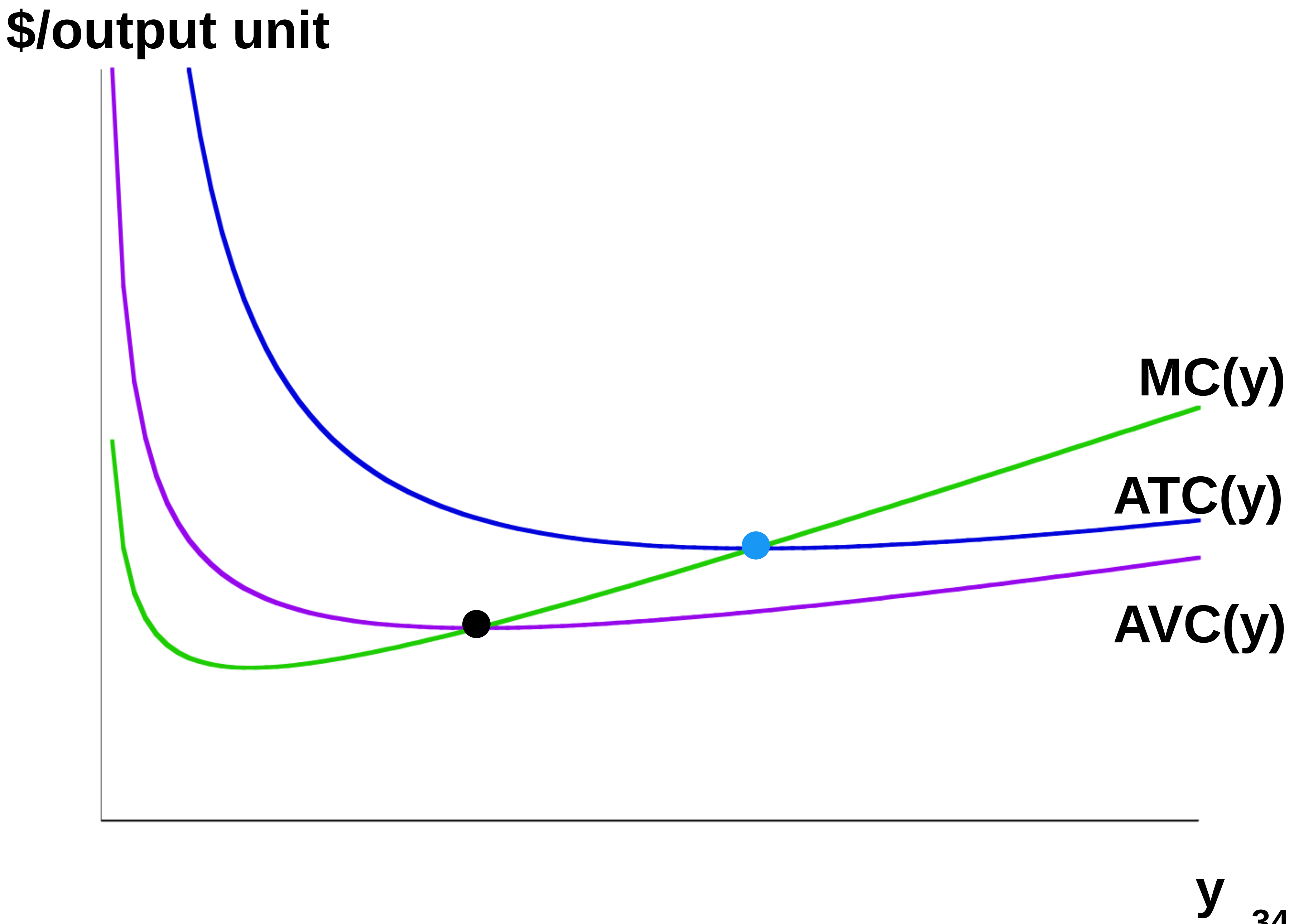
$$\frac{\partial\, ATC(y)}{\partial\, y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c(y)}{y} = ATC(y).$$

$$\frac{\partial\, ATC(y)}{\partial\, y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ as } MC(y) \begin{matrix} > \\ = \\ < \end{matrix} ATC(y)$$



Marginal & Average Cost Functions

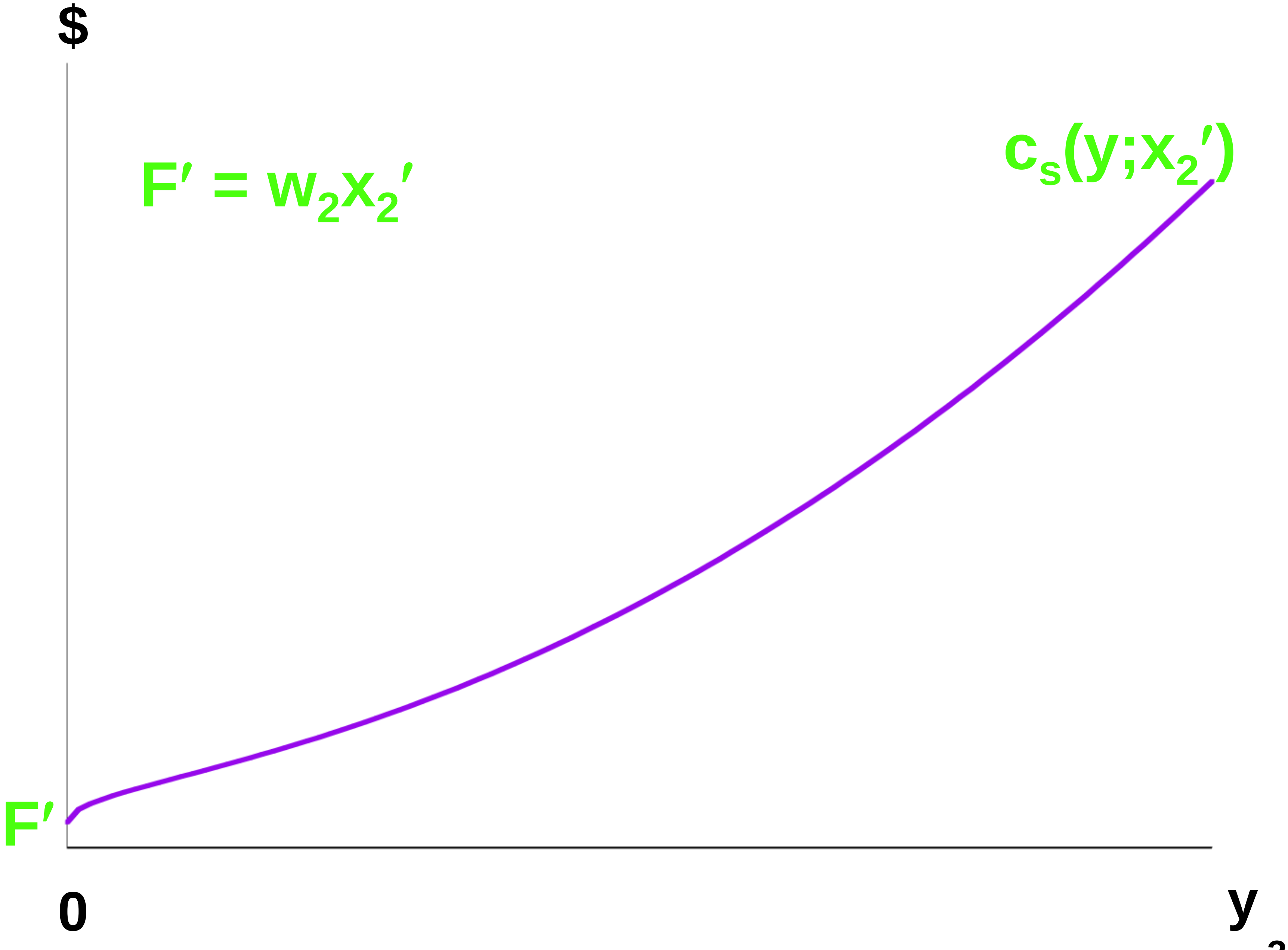
- **The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.**
- **And, similarly, the short-run MC curve intersects the short-run ATC curve from below at the ATC curve's minimum.**

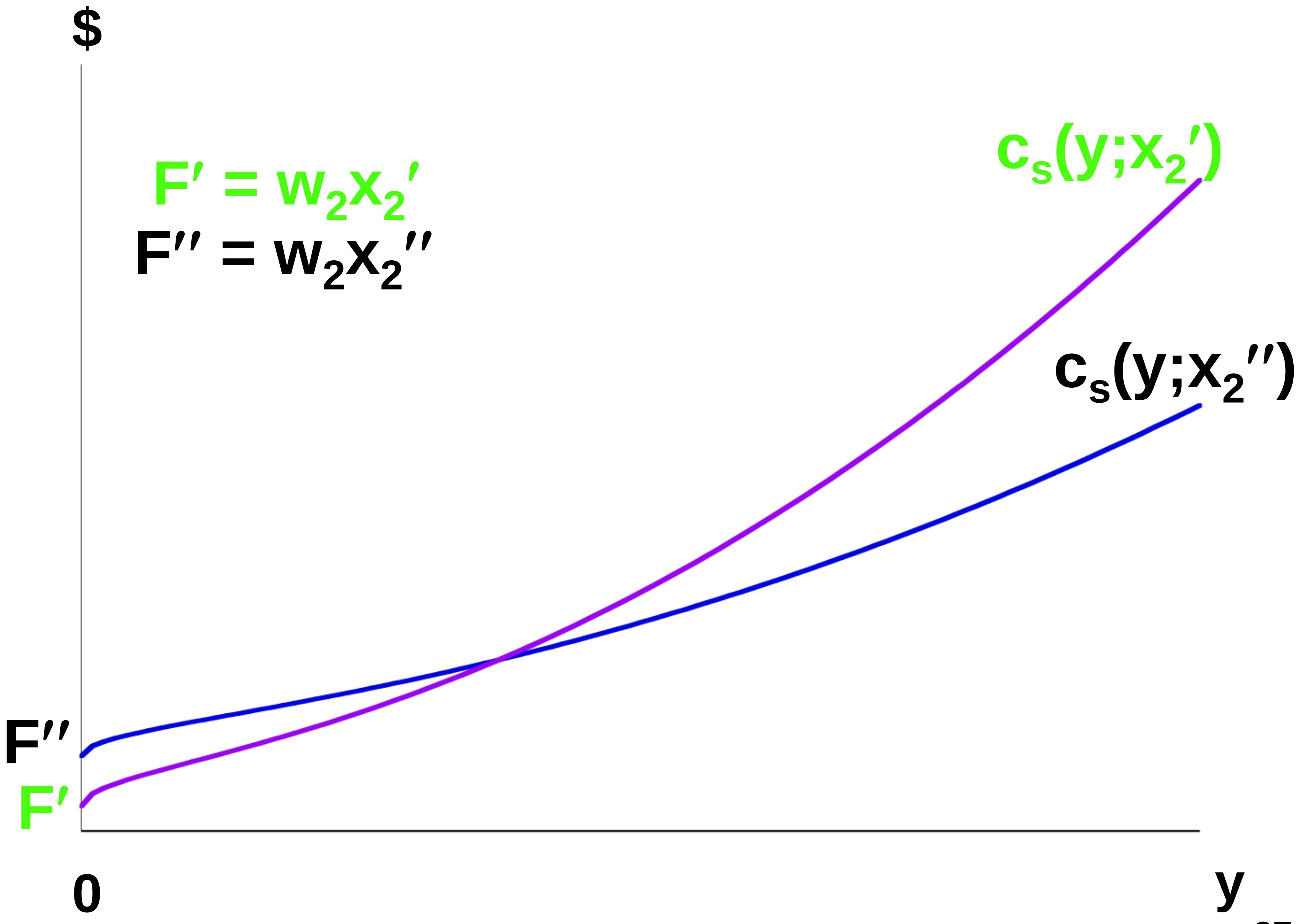

$/output unit
MC(y)
ATC(y)
AVC(y)
y

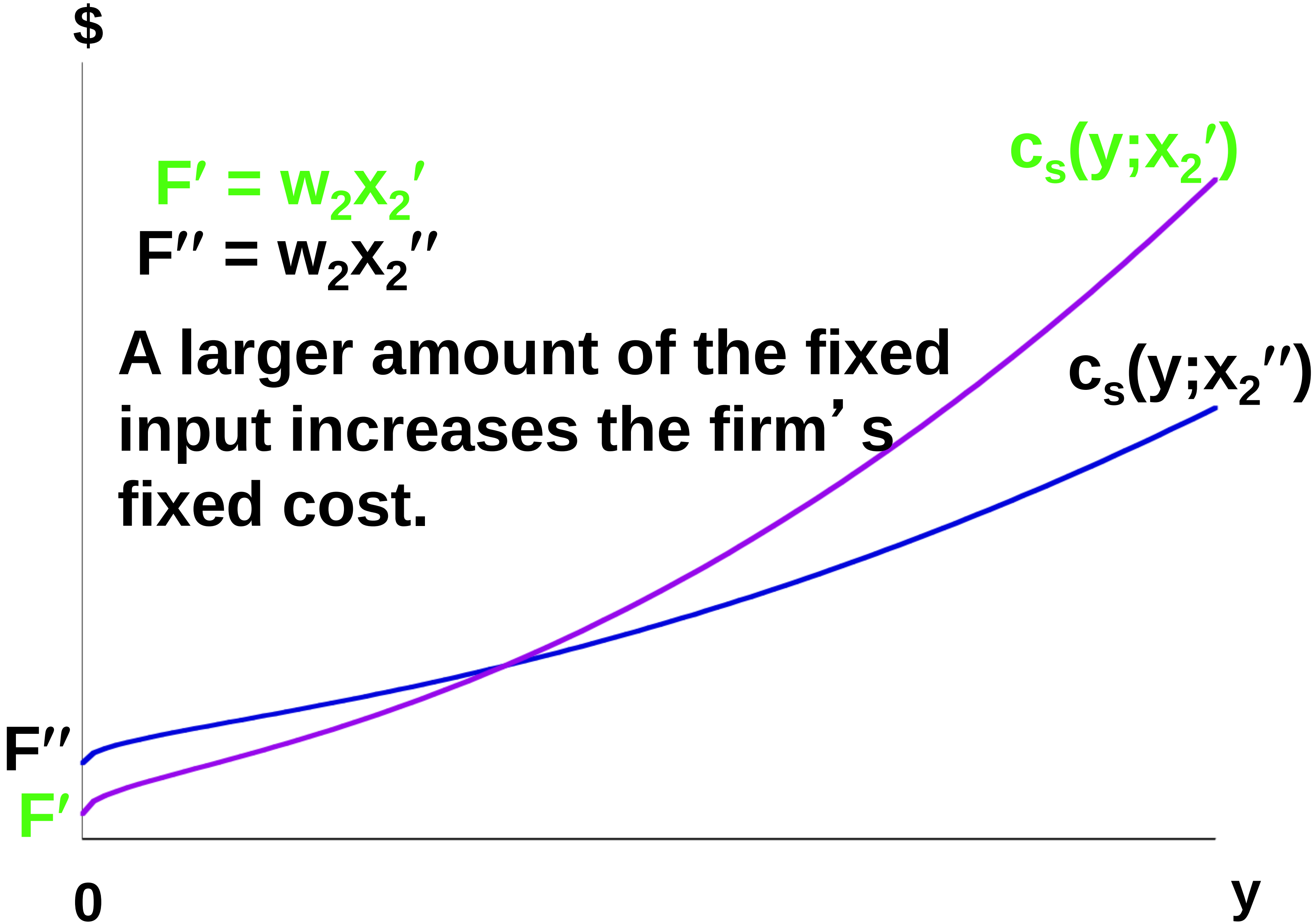
Short-Run & Long-Run Total Cost Curves

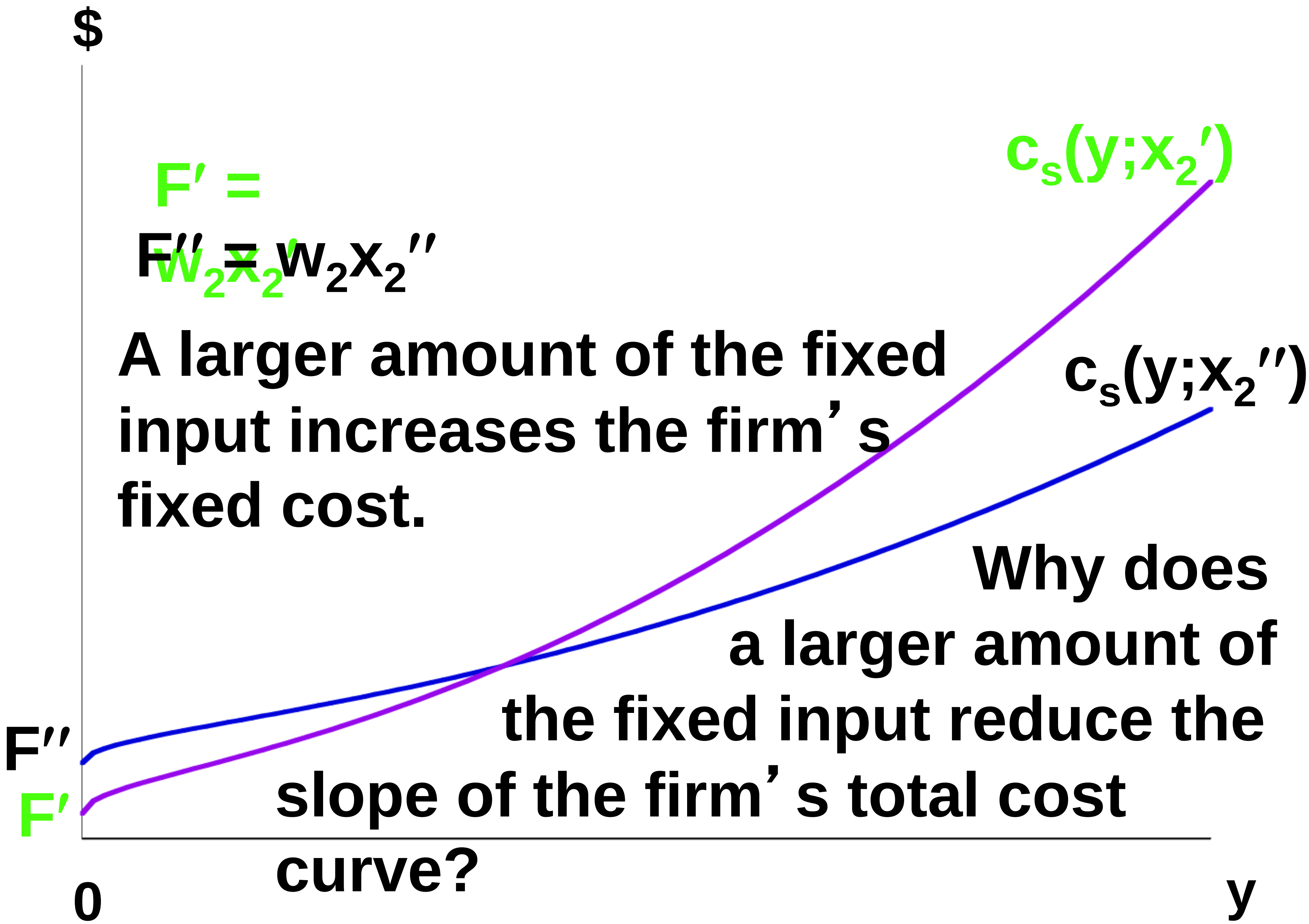
- **A firm has a different short-run total cost curve for each possible short-run circumstance.**
- **Suppose the firm can be in one of just three short-runs;**

$x_2 = x_2'$
or $x_2 = x_2''$ $x_2' < x_2'' < x_2'''$.
or $x_2 = x_2'''$.


$
F′ = w2x2′
cs(y;x2′)
F′
0
y


$
F′ = w₂x₂′
F″ = w₂x₂″
cs(y;x₂′)
cs(y;x₂″)
F″
F′
0
y


$
F′ = w2x2′
F″ = w2x2″
A larger amount of the fixed input increases the firm’s fixed cost.
cs(y;x2′)
cs(y;x2″)
F″
F′
0
y



A larger amount of the fixed input increases the firm's fixed cost.

Why does a larger amount of the fixed input reduce the slope of the firm's total cost curve?

Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is

Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is

$1/MP_1$ units of input 1.

Short-Run & Long-Run Total Cost Curves

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Therefore, the extra amount of input 1 needed for 1 extra output unit is
$1/MP_1$ units of input 1.
Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is

Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is

$1/MP_1$ **units of input 1.**

Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is $$MC = \frac{w_1}{MP_1}.$$

Short-Run & Long-Run Total Cost Curves

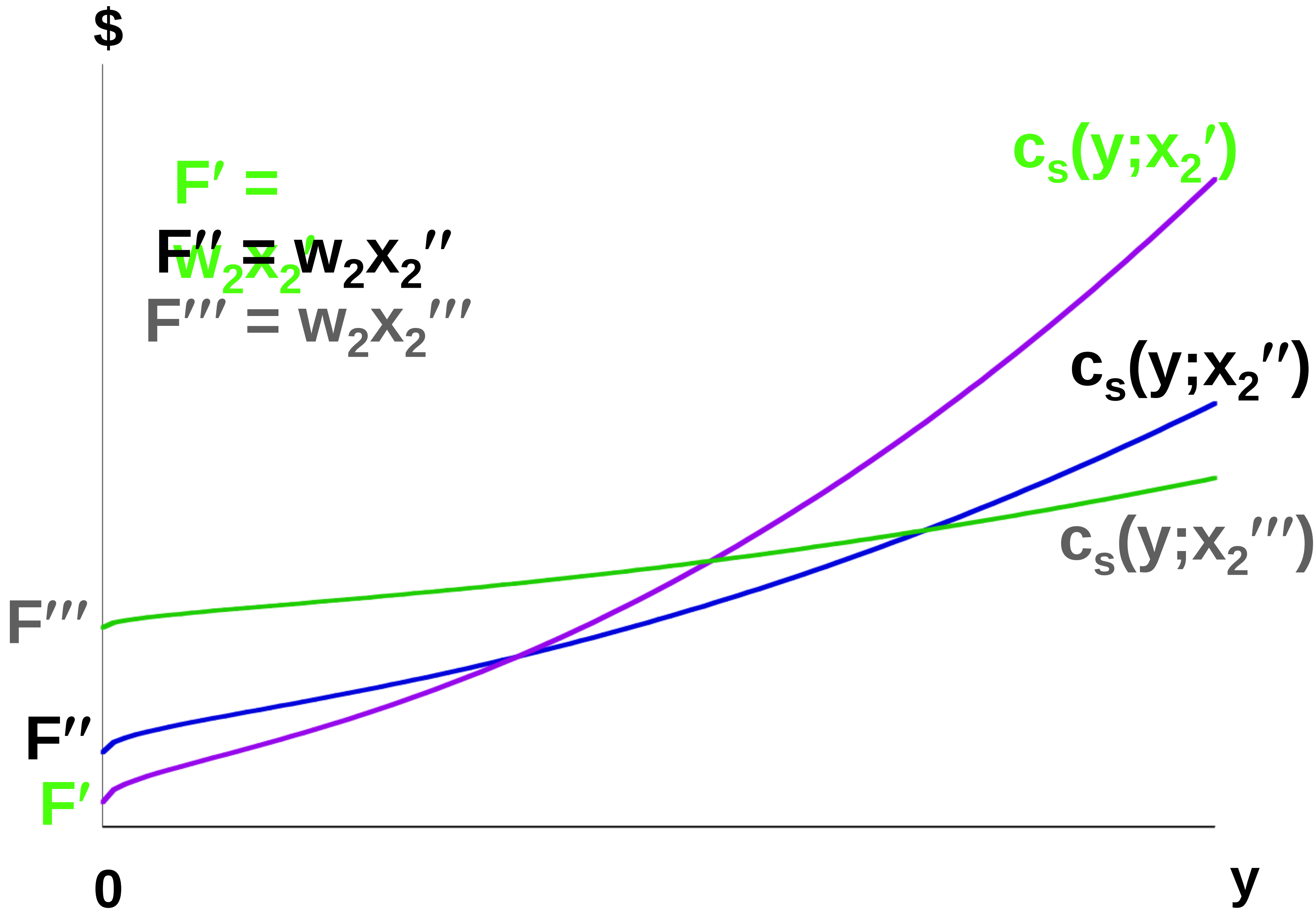
$$MC = \frac{w_1}{MP_1}$$ **is the slope of the firm's total cost curve.**

Short-Run & Long-Run Total Cost Curves

$$MC = \frac{w_1}{MP_1}$$ **is the slope of the firm's total cost curve.**

If input 2 is a complement to input 1 then MP_1 is higher for higher x_2.
Hence, MC is lower for higher x_2.

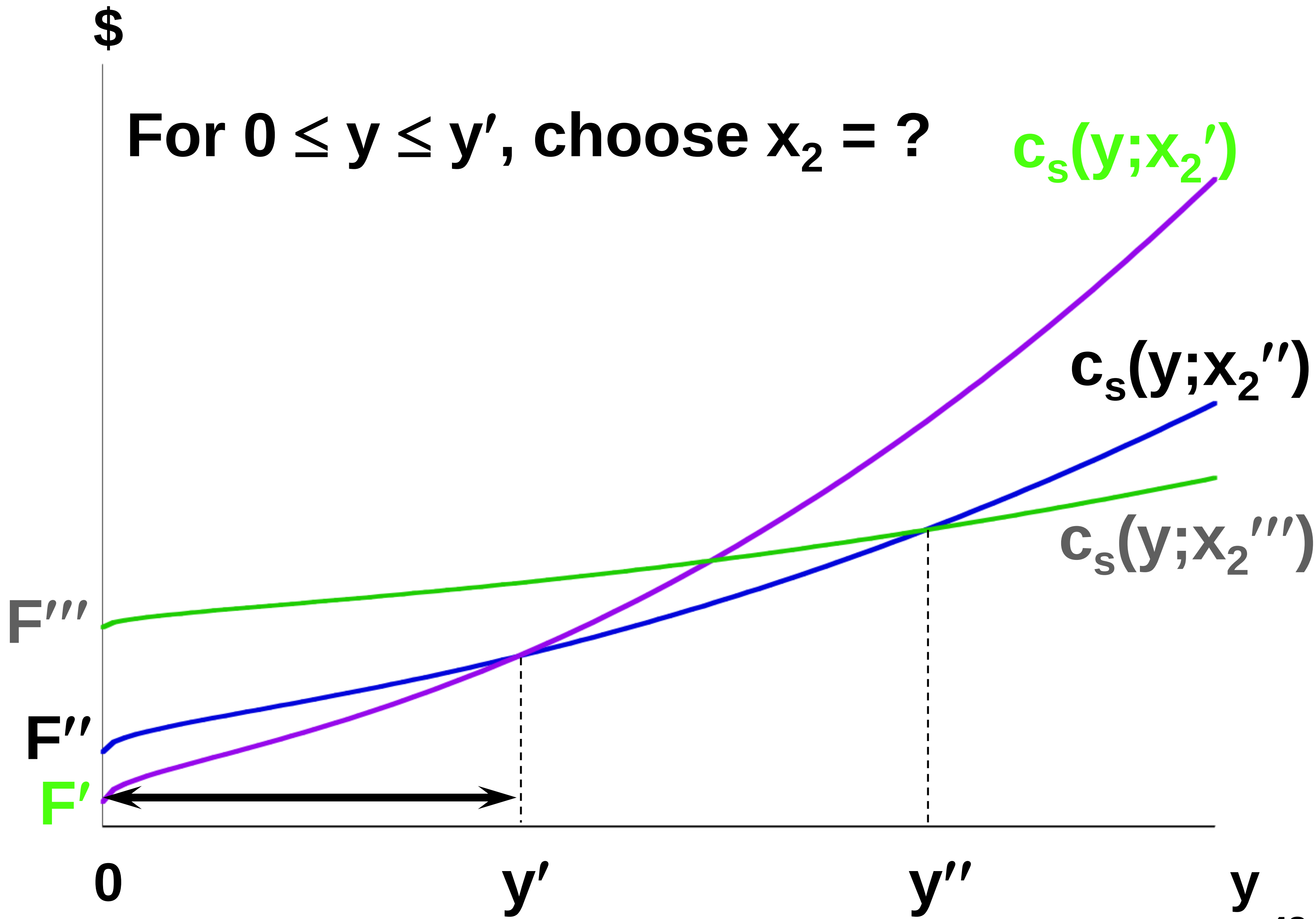
That is, a short-run total cost curve starts higher and has a lower slope if x_2 is larger.

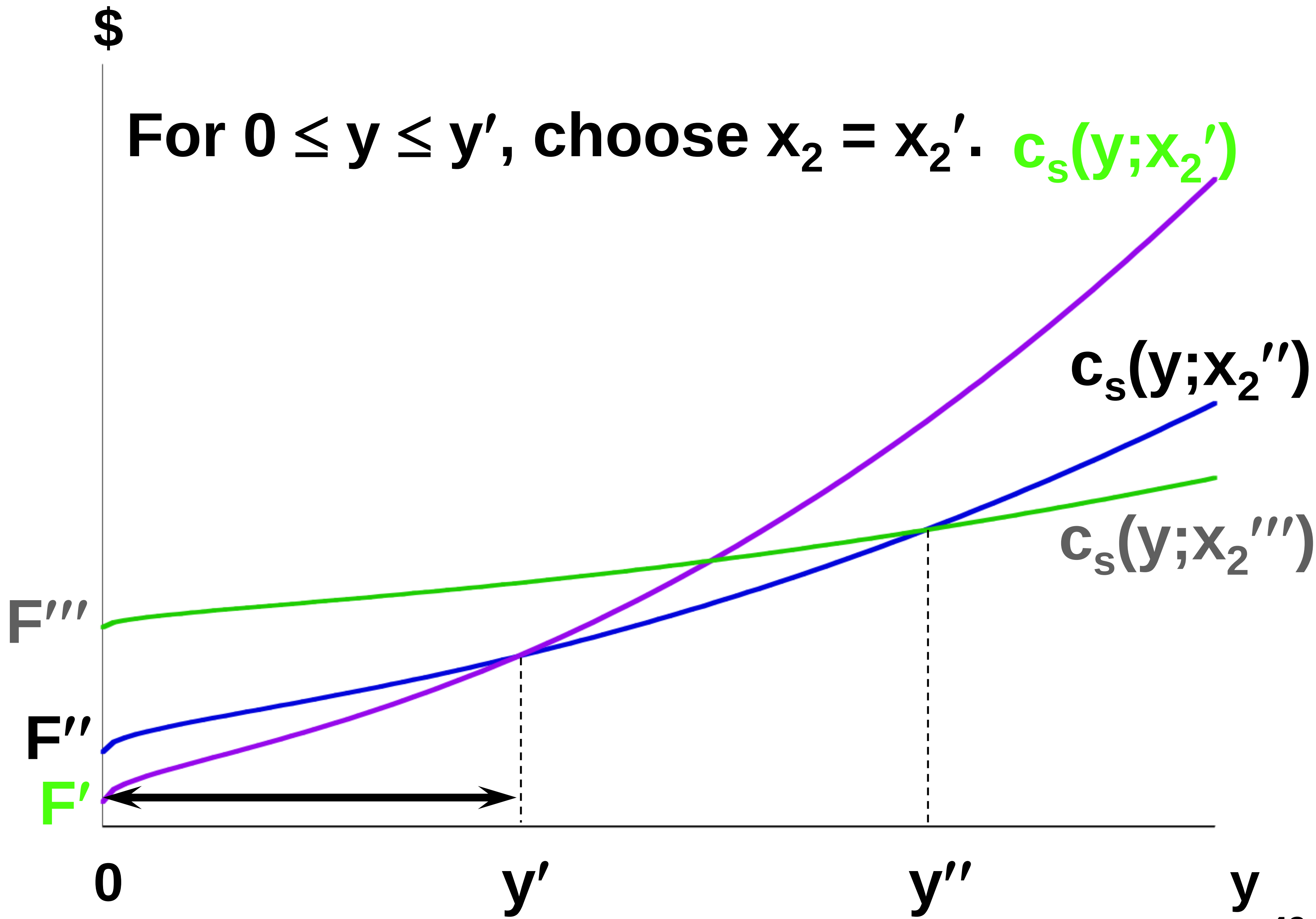

$
F′ = $w_2x_2′$
F″ = $w_2x_2″$
F‴ = $w_2x_2‴$
$c_s(y;x_2′)$
$c_s(y;x_2″)$
$c_s(y;x_2‴)$
F‴
F″
F′
0
y

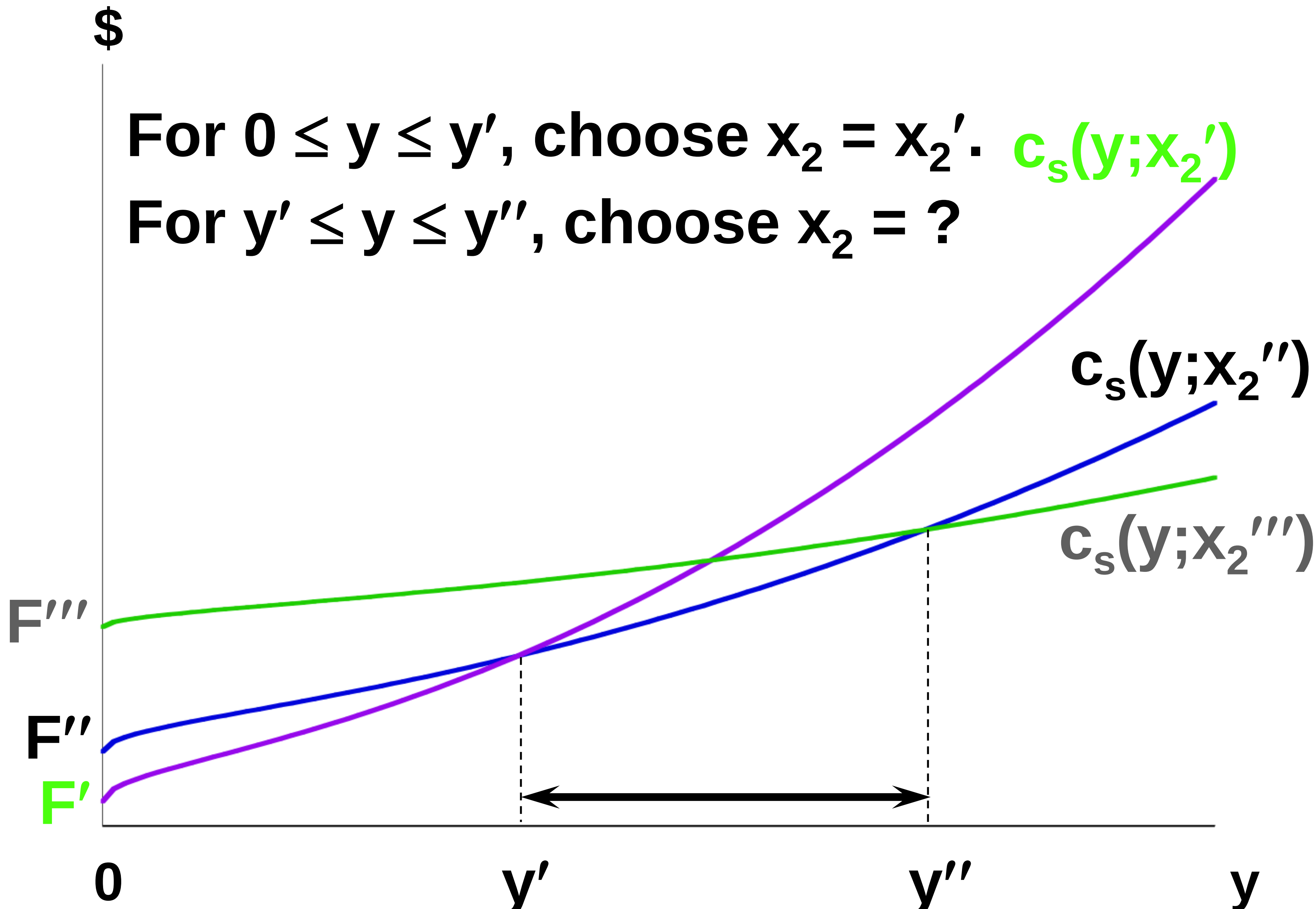
Short-Run & Long-Run Total Cost Curves

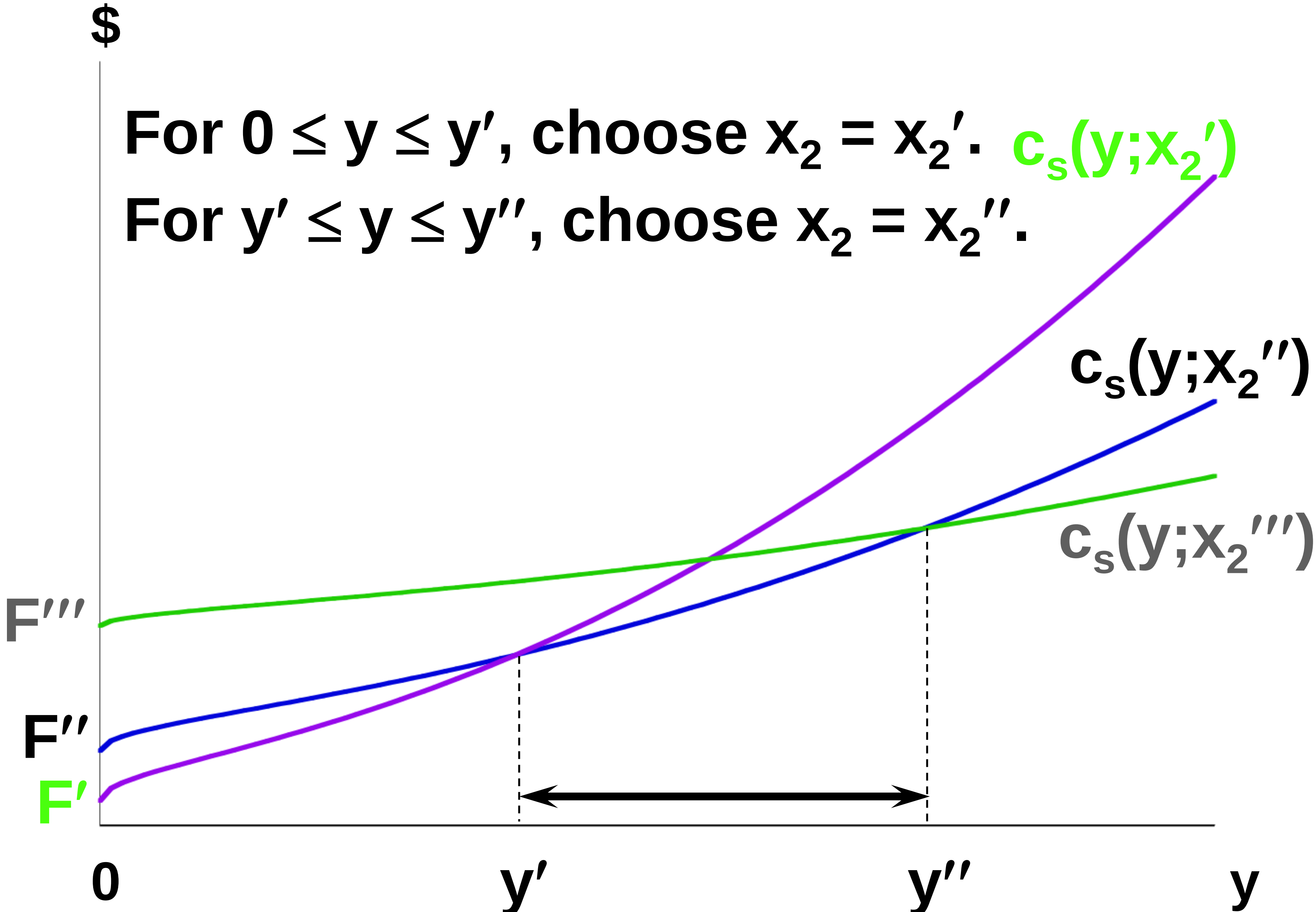
- **The firm has three short-run total cost curves.**
- **In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2', x_2'', or x_2'''.**
- **How does the firm make this choice?**

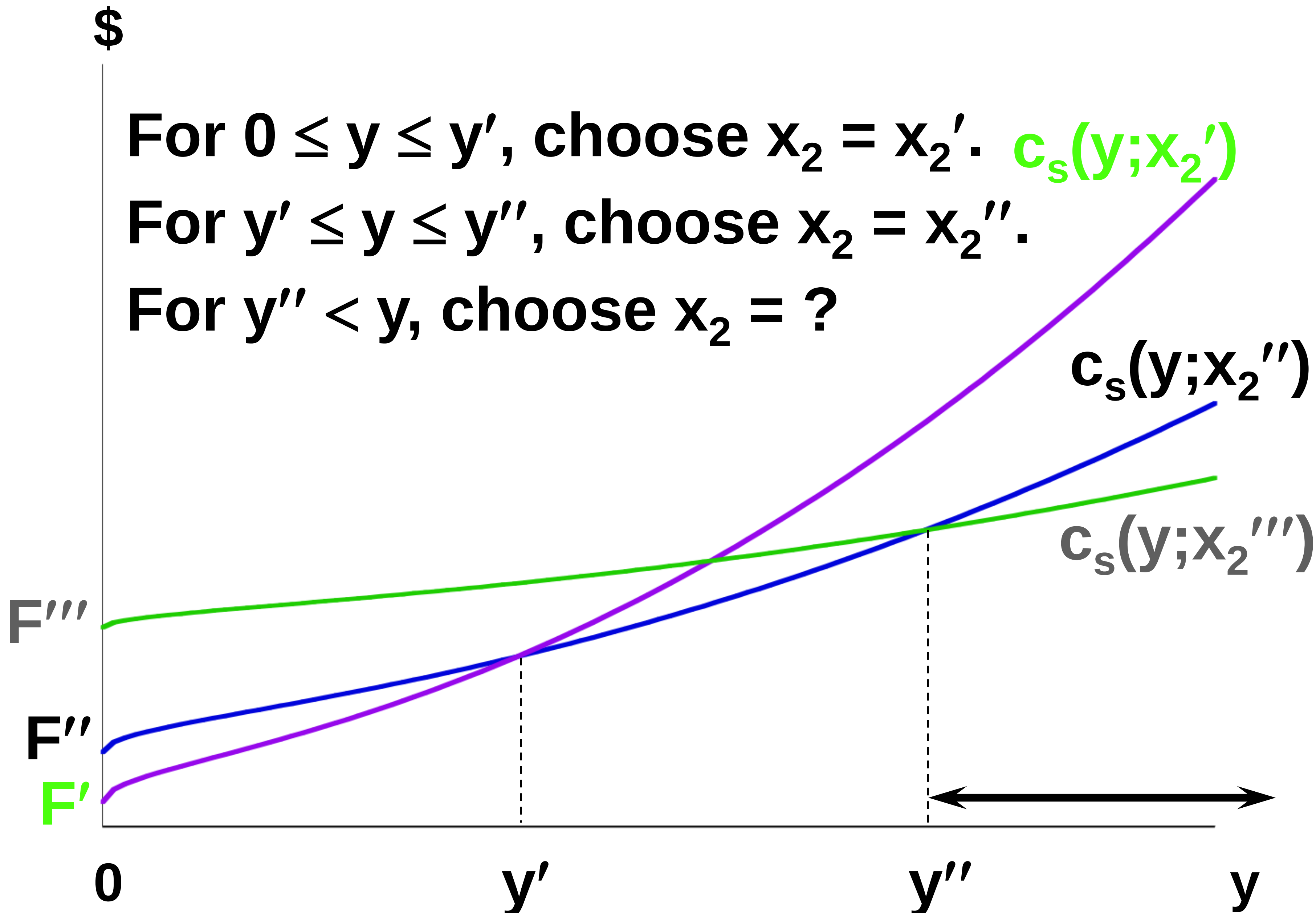
For $0 \leq y \leq y'$, choose x_2 = ?

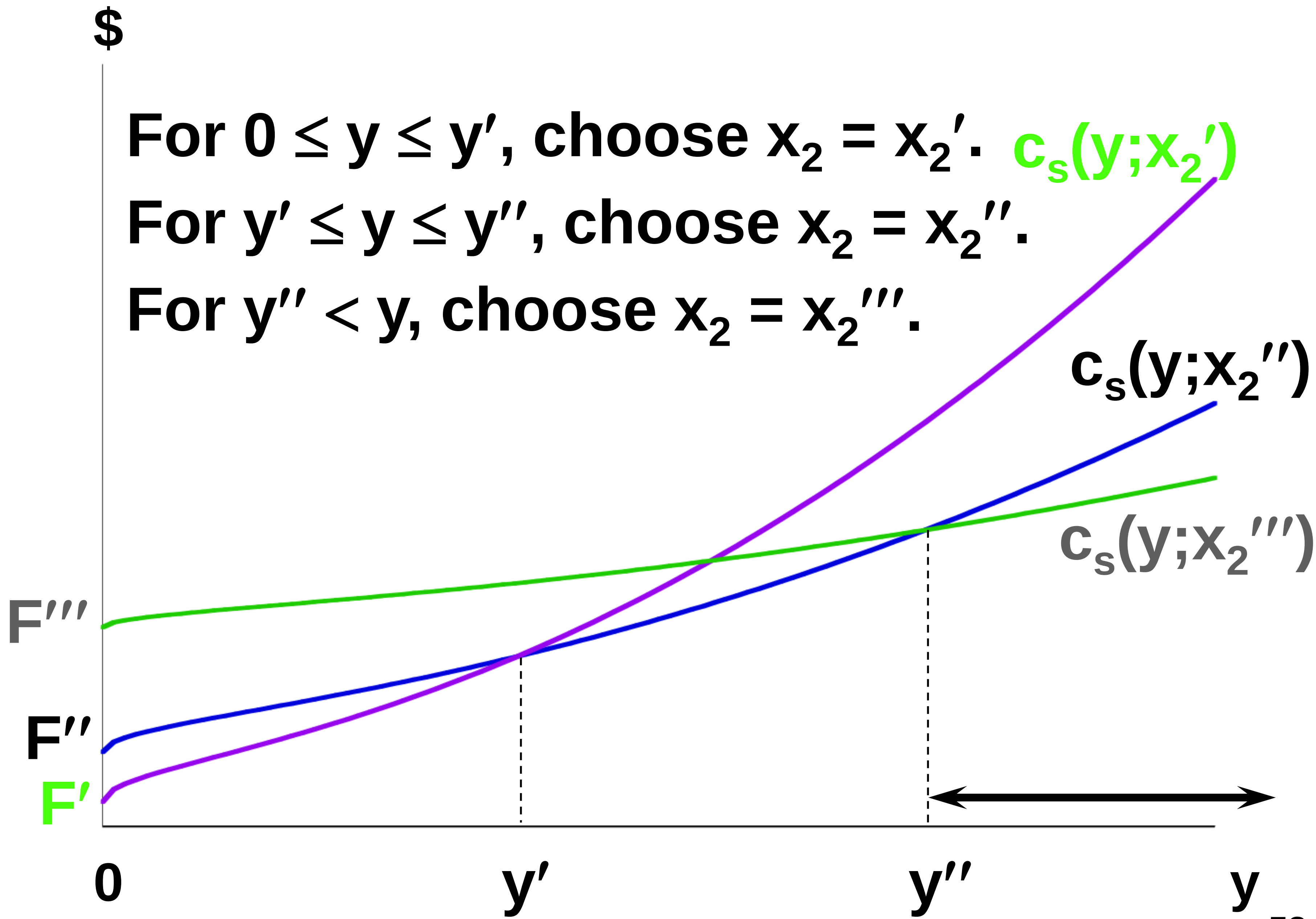


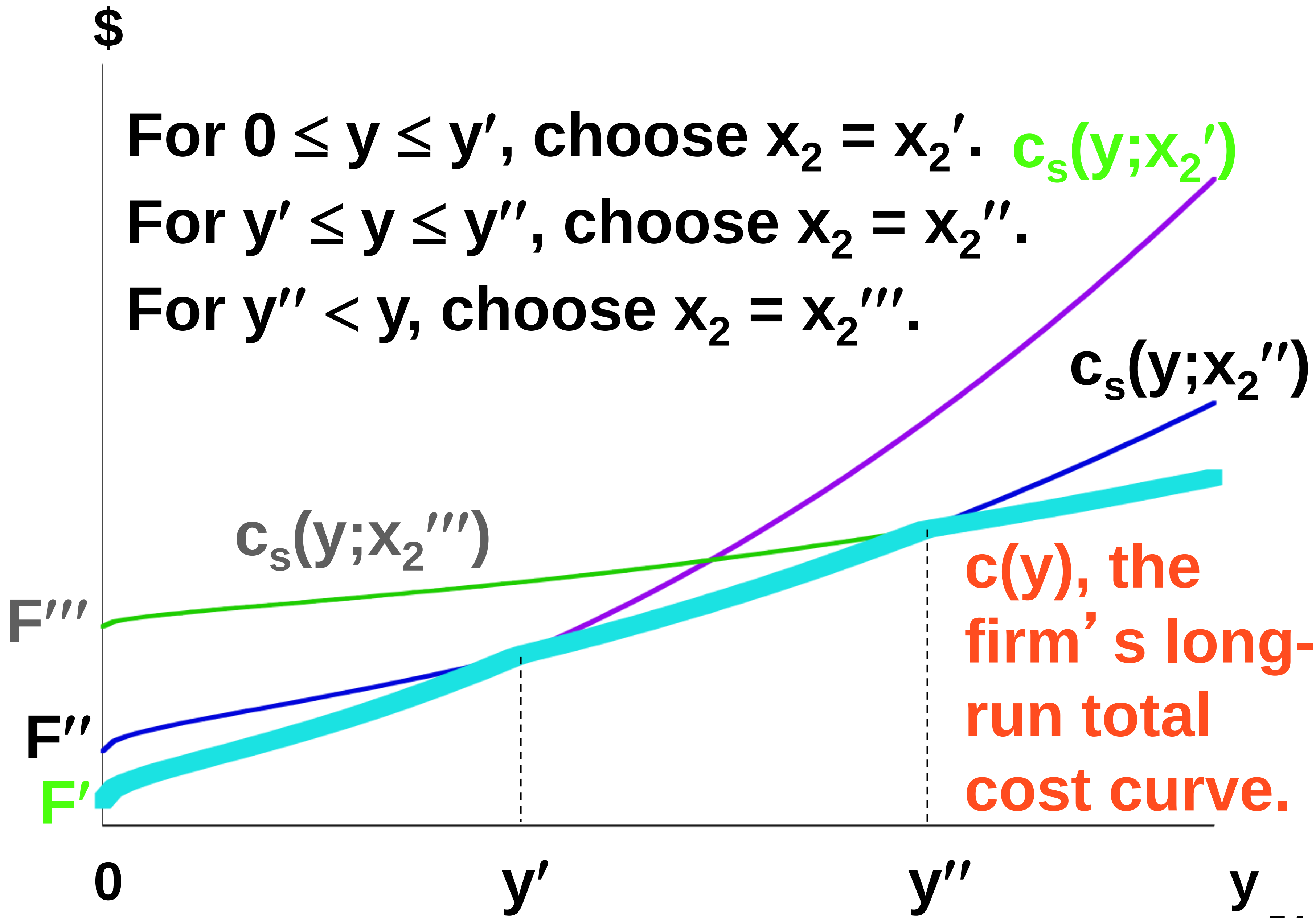

$
For 0 ≤ y ≤ y′, choose x2 = x2′.
cs(y;x2′)
cs(y;x2′′)
cs(y;x2′′′)
F′′′
F′′
F′
0
y′
y′′
y


$
For 0 ≤ y ≤ y′, choose x₂ = x₂′.
For y′ ≤ y ≤ y″, choose x₂ = ?
c_s(y;x_2′)
c_s(y;x_2″)
c_s(y;x_2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x₂ = x₂′.
For y′ ≤ y ≤ y″, choose x₂ = x₂″.
For y″ < y, choose x₂ = ?
$c_s(y;x_2')$
$c_s(y;x_2'')$
$c_s(y;x_2''')$
F‴
F″
F′
0
y′
y″
y


$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
For y″ < y, choose x2 = x2‴.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
F‴
F″
F′
0
y′
y″
y

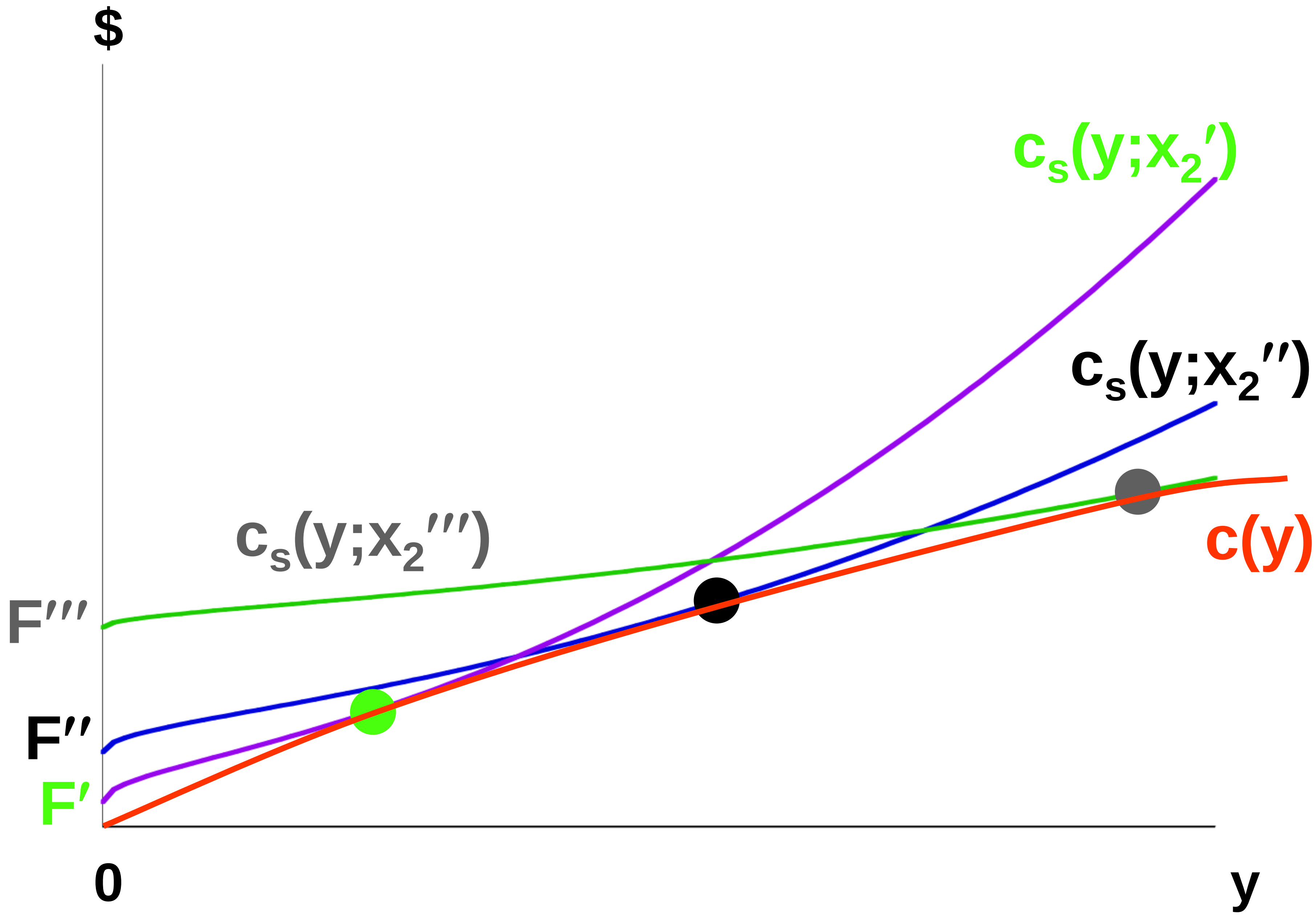

$
For 0 ≤ y ≤ y′, choose x2 = x2′.
For y′ ≤ y ≤ y″, choose x2 = x2″.
For y″ < y, choose x2 = x2‴.
cs(y;x2′)
cs(y;x2″)
cs(y;x2‴)
c(y), the firm’ s long-run total cost curve.
F‴
F″
F′
0
y′
y″
y

Short-Run & Long-Run Total Cost Curves

- **The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the lower envelope of the short-run total cost curves.**

Short-Run & Long-Run Total Cost Curves

- **If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.**


$
$c_s(y;x_2')$
$c_s(y;x_2'')$
$c_s(y;x_2''')$
c(y)
F'''
F''
F'
0
y

Short-Run & Long-Run Average Total Cost Curves

- **For any output level y, the long-run total cost curve always gives the lowest possible total production cost.**
- **Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.**
- **The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.**

Short-Run & Long-Run Average Total Cost Curves

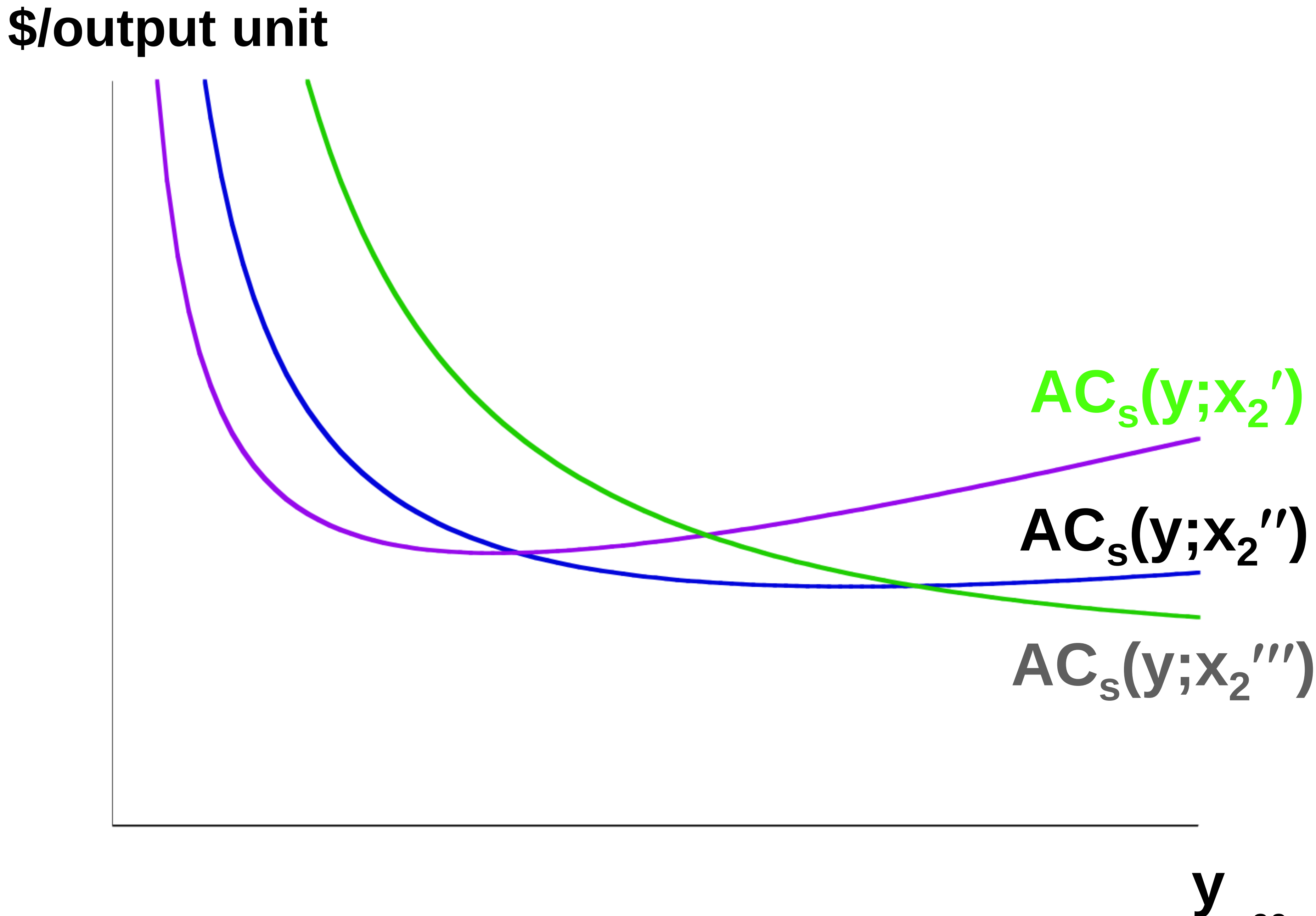
- **E.g. suppose again that the firm can be in one of just three short-runs;**

  $$x_2 = x_2'$$

  or $$x_2 = x_2'' \qquad (x_2' < x_2'' < x_2''')$$

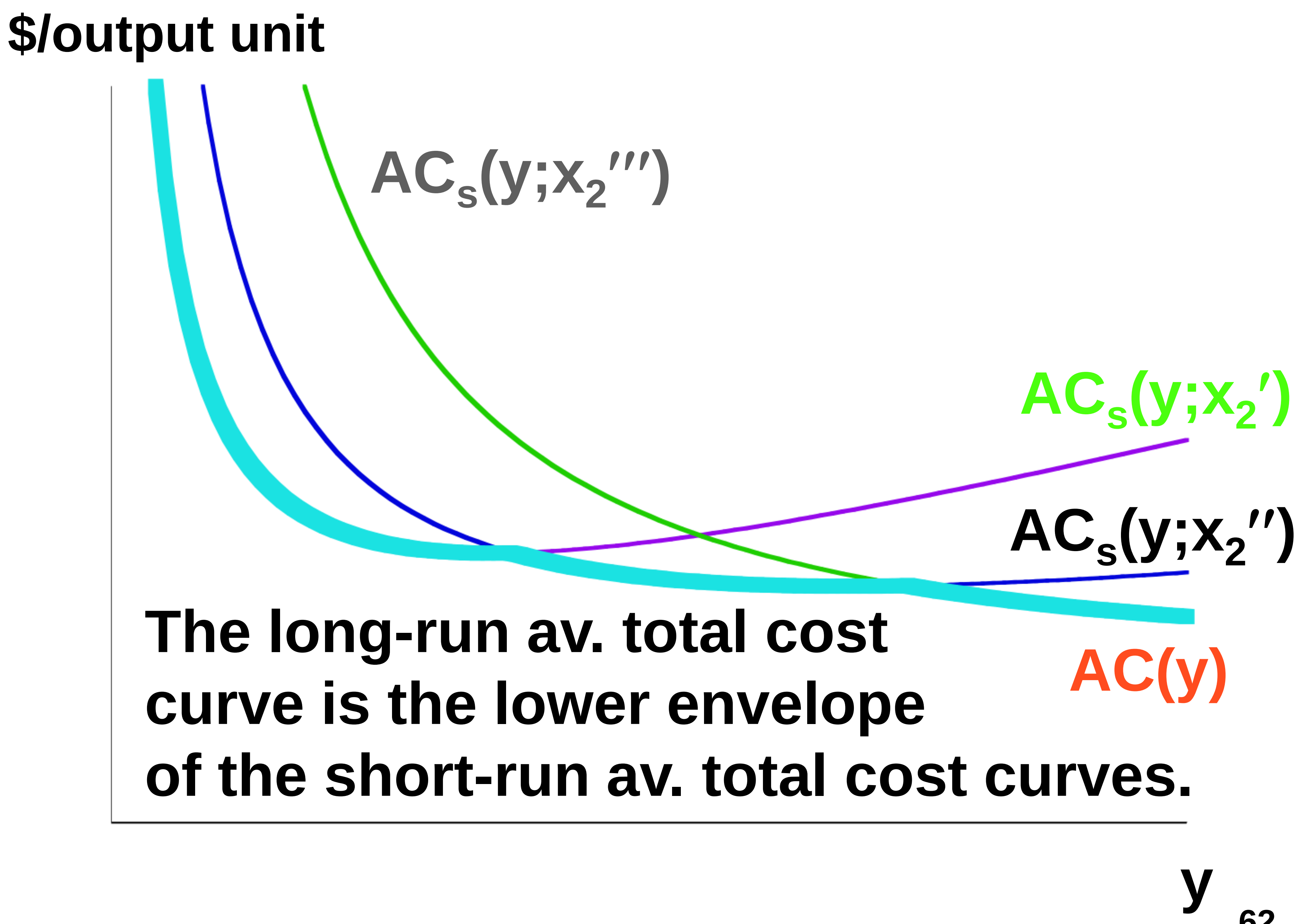
  or $$x_2 = x_2'''$$

  then the firm's three short-run average total cost curves are ...


$/output unit
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC_s(y;x_2''')$
y

Short-Run & Long-Run Average Total Cost Curves

- **The firm's long-run average total cost curve is the lower envelope of the short-run average total cost curves ...**



\$/output unit
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
AC(y)
The long-run av. total cost
curve is the lower envelope
of the short-run av. total cost curves.
y

Short-Run & Long-Run Marginal Cost Curves

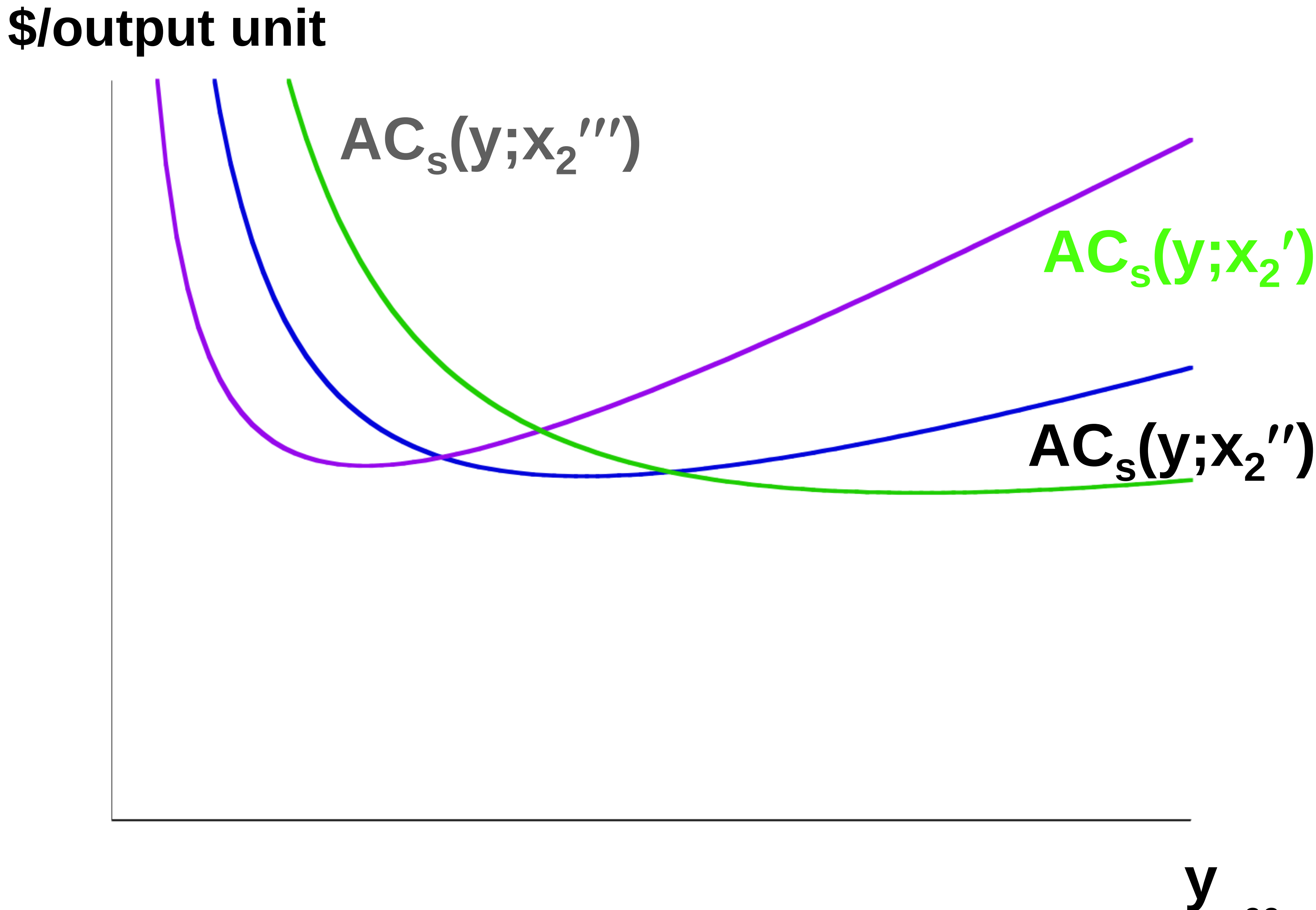
- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**

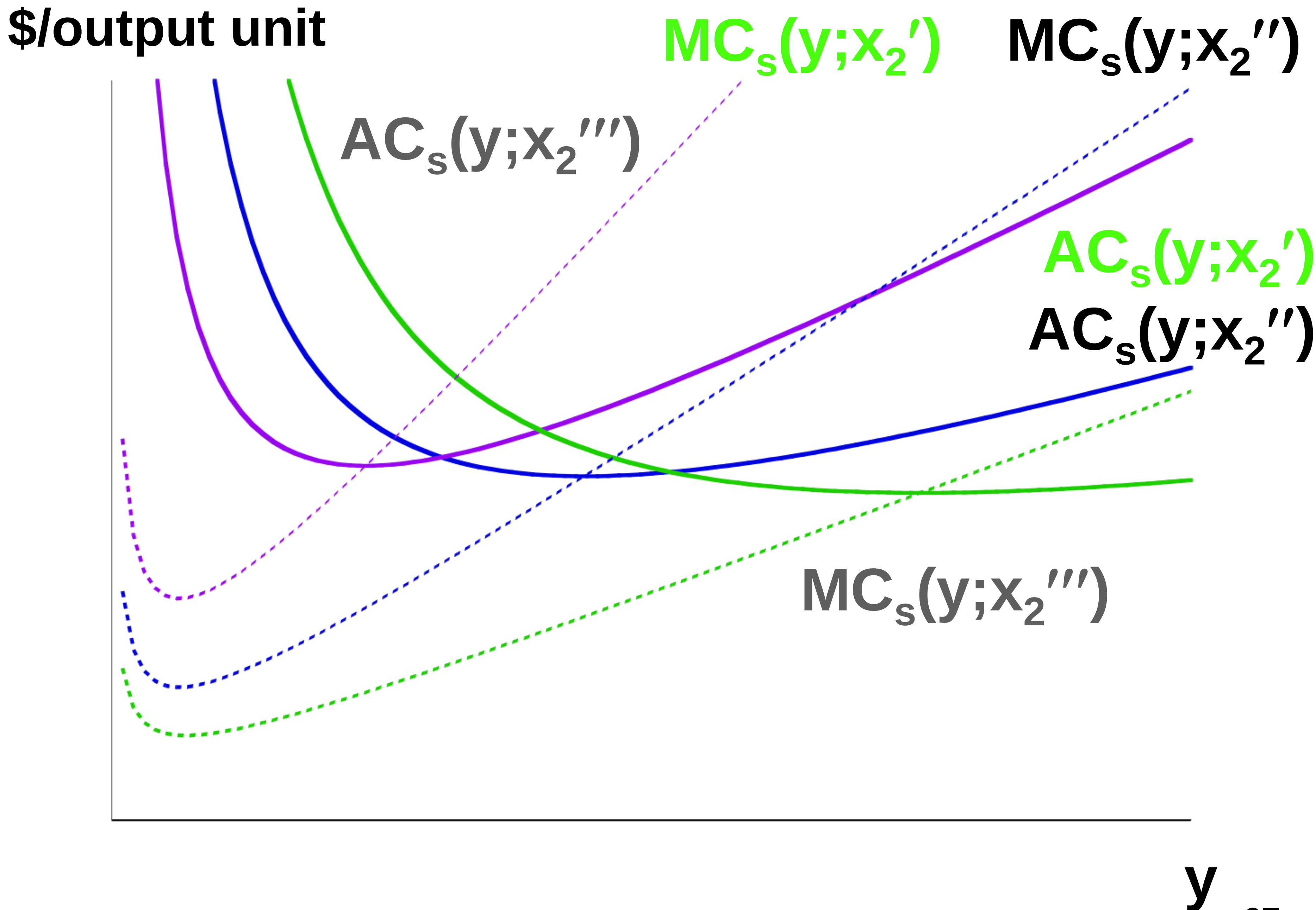
Short-Run & Long-Run Marginal Cost Curves

- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**
- **A: No.**

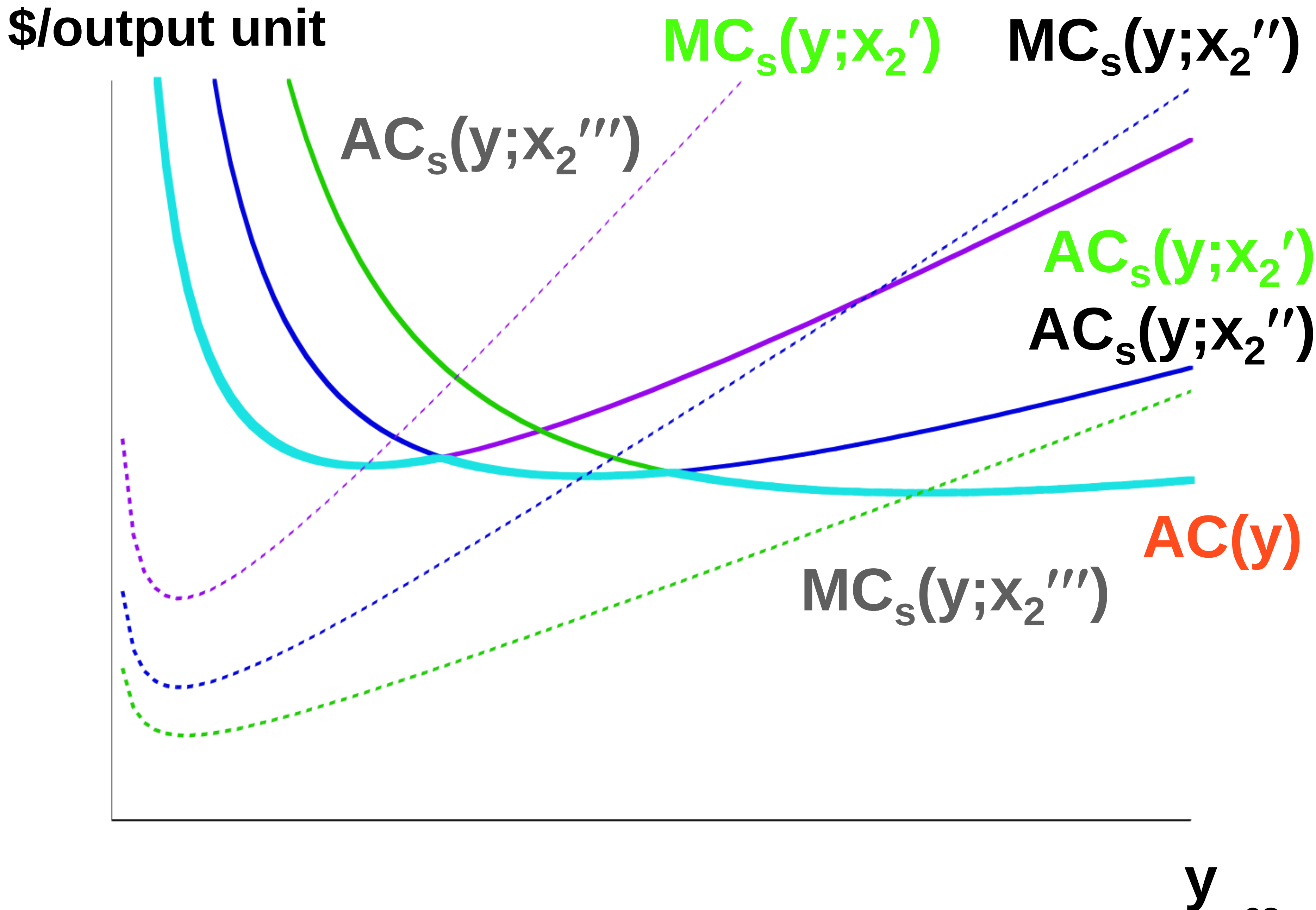
Short-Run & Long-Run Marginal Cost Curves

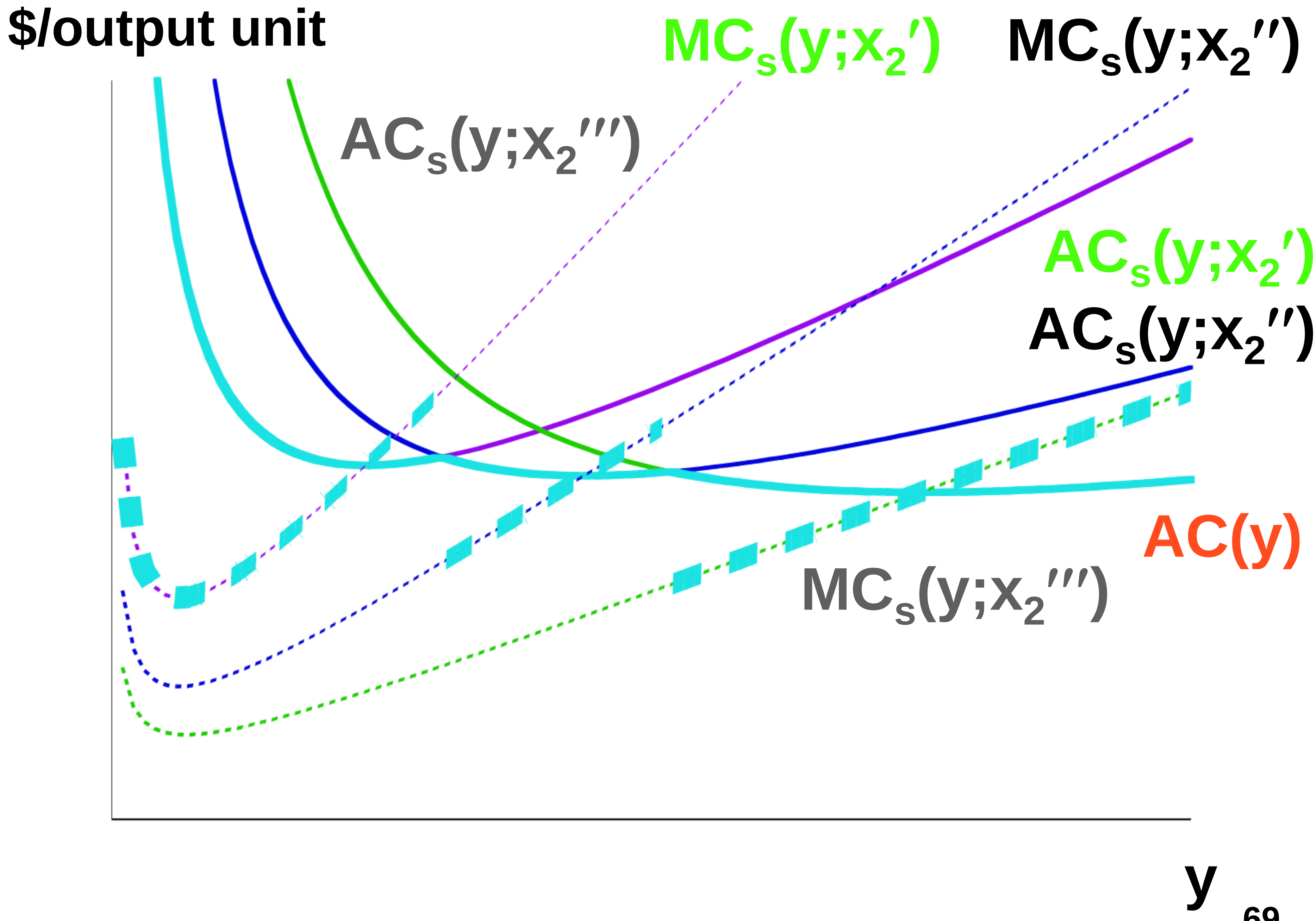
- **The firm's three short-run average total cost curves are ...**

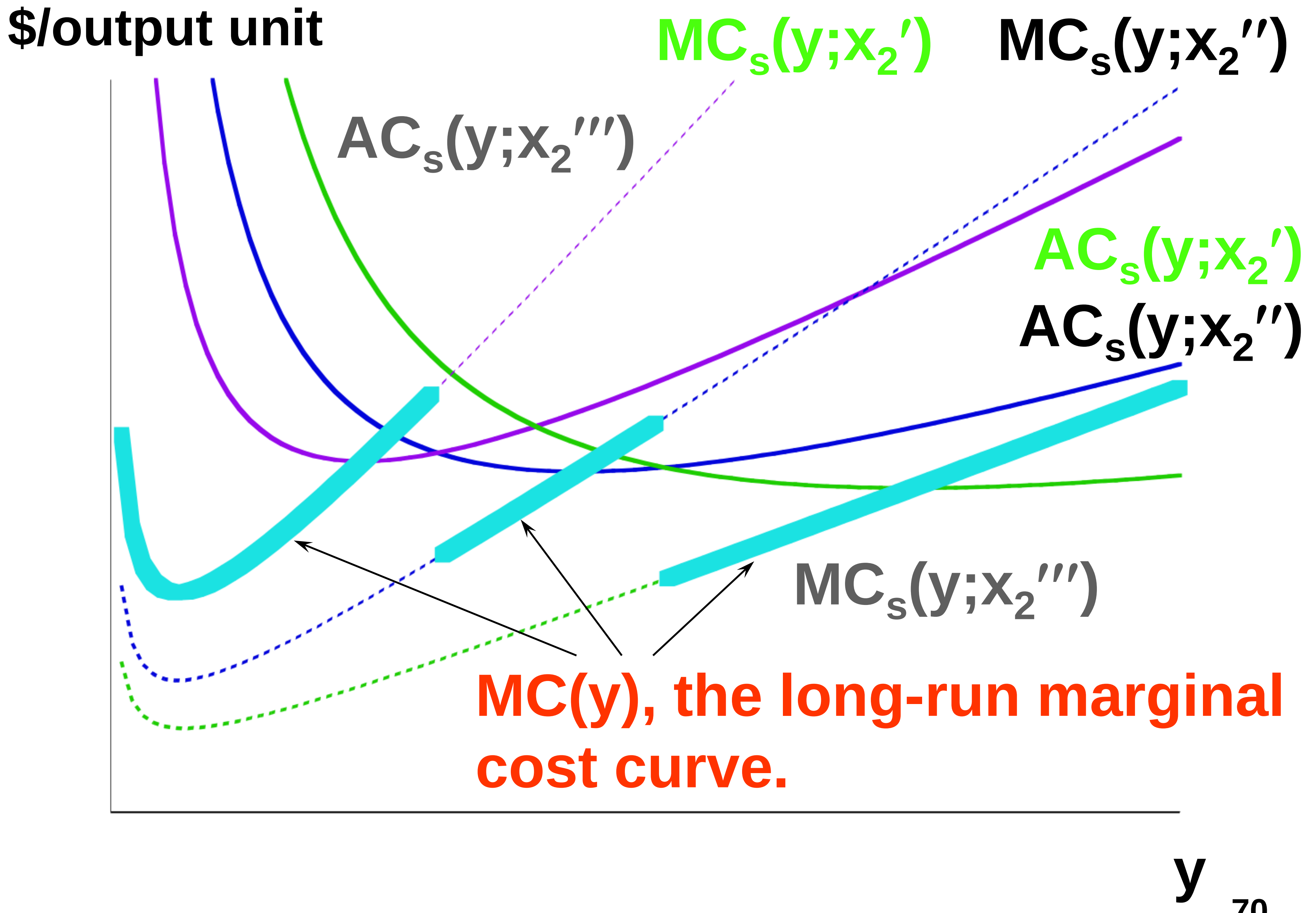

$/output unit
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
y



$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$MC_s(y;x_2''')$
y


$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC(y)$
$MC_s(y;x_2''')$
y

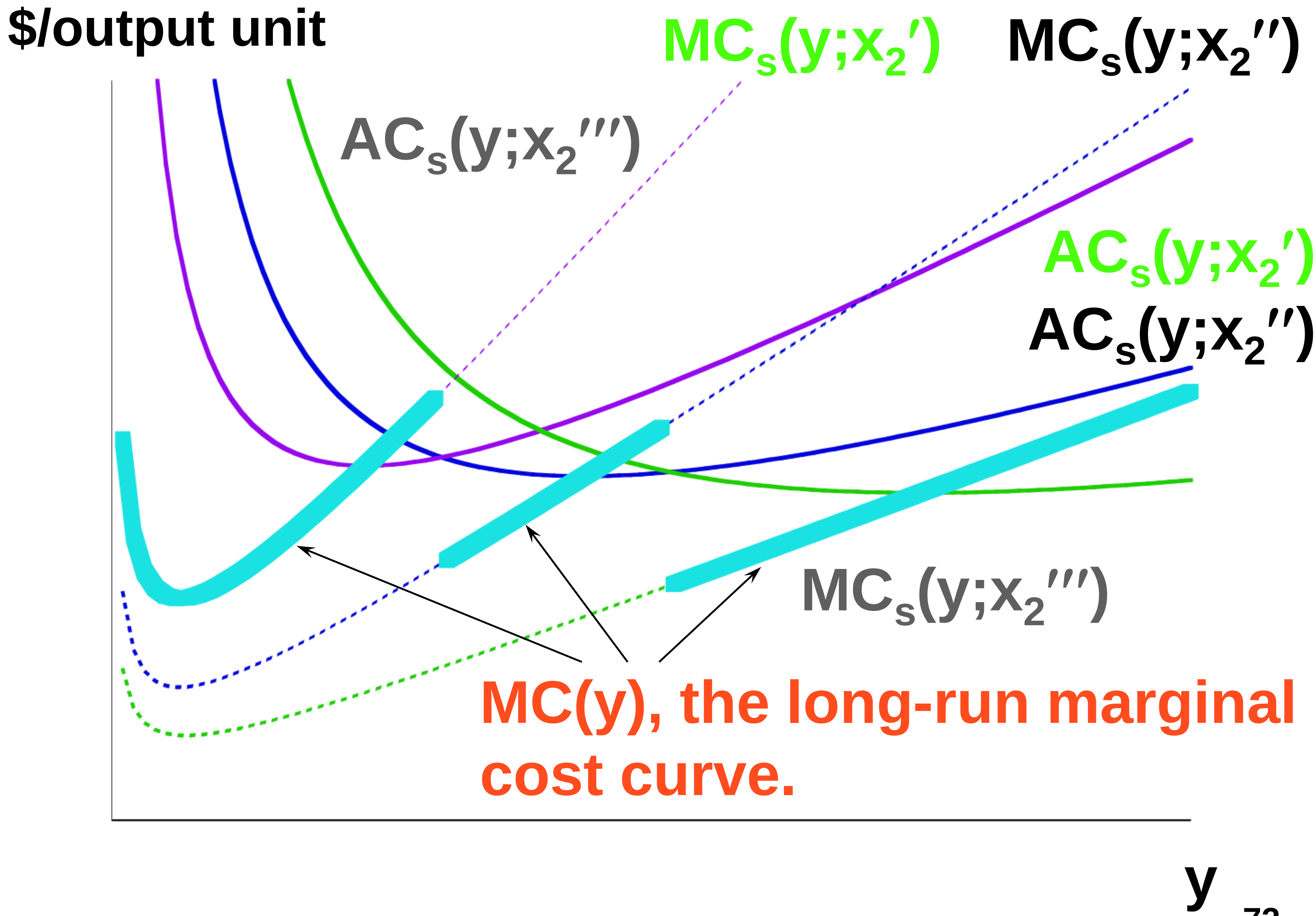

$/output unit
MCs(y;x2′)
MCs(y;x2″)
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
AC(y)
MCs(y;x2‴)
y



$/output unit
MCs(y;x2′)
MCs(y;x2″)
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
MCs(y;x2‴)
MC(y), the long-run marginal cost curve.
y

Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.**


$/output unit
MCs(y;x2′)
MCs(y;x2″)
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
MCs(y;x2‴)
MC(y), the long-run marginal cost curve.
y

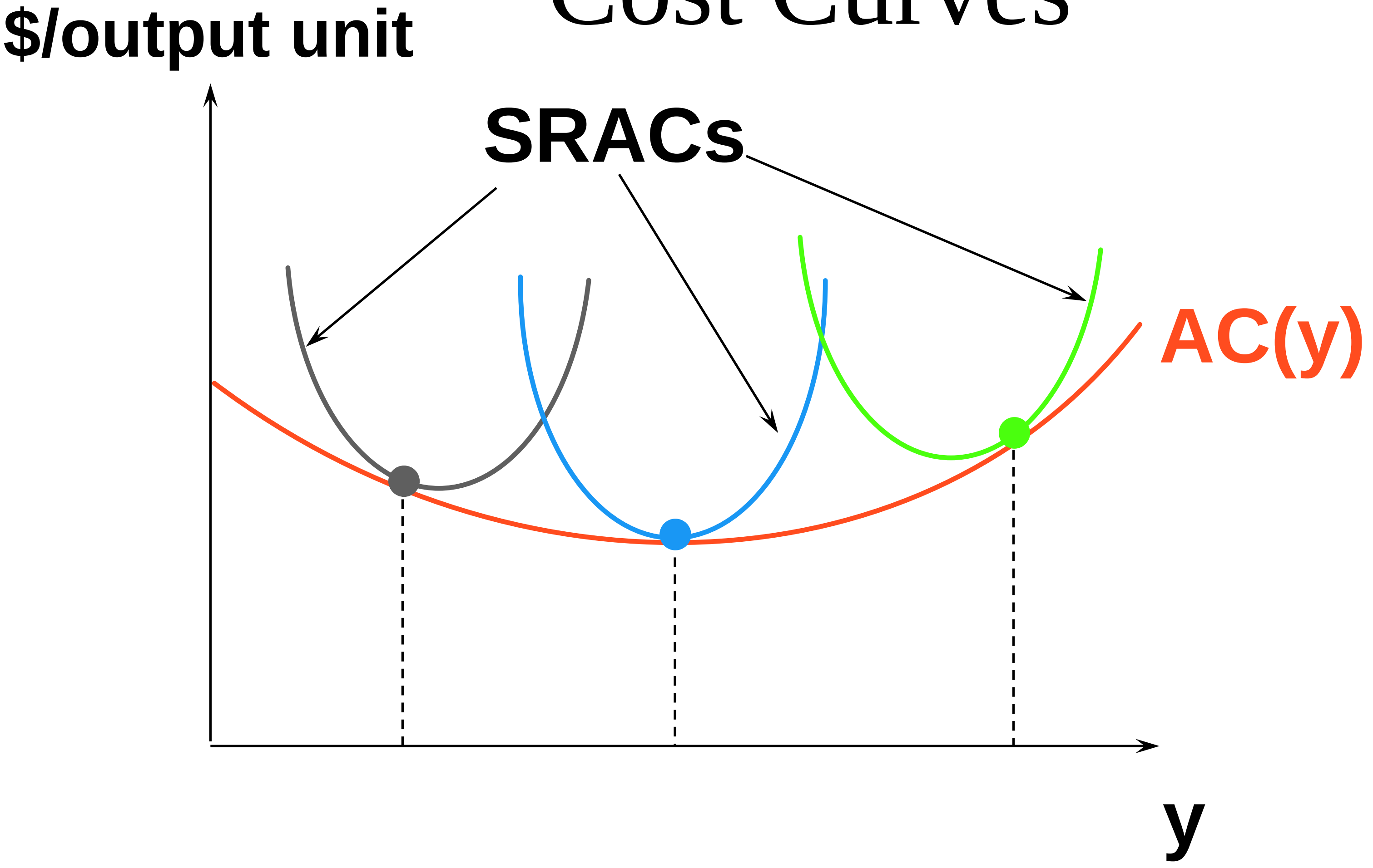
Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.**
- **This is always true, no matter how many and which short-run circumstances exist for the firm.**

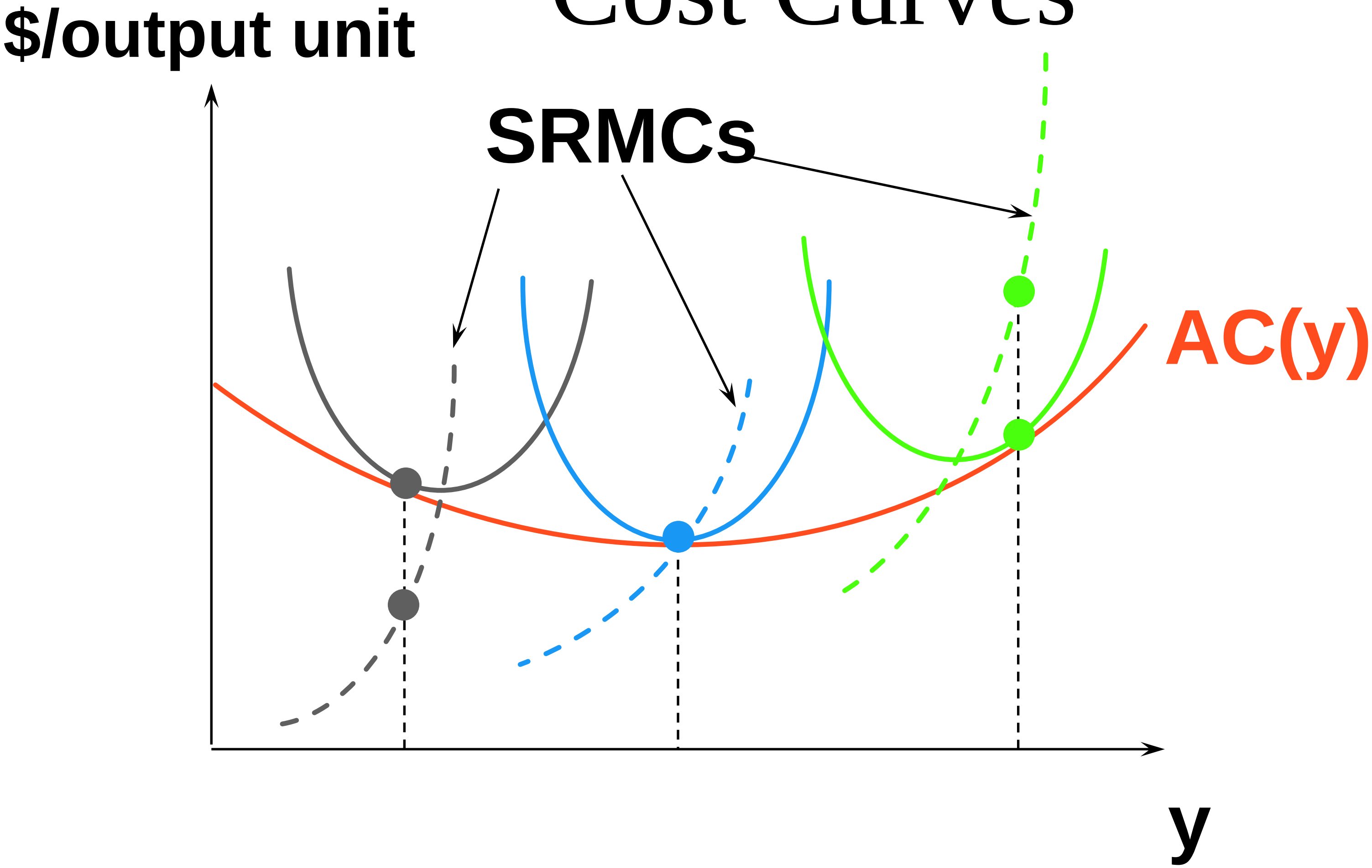
Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.**
- **So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...**

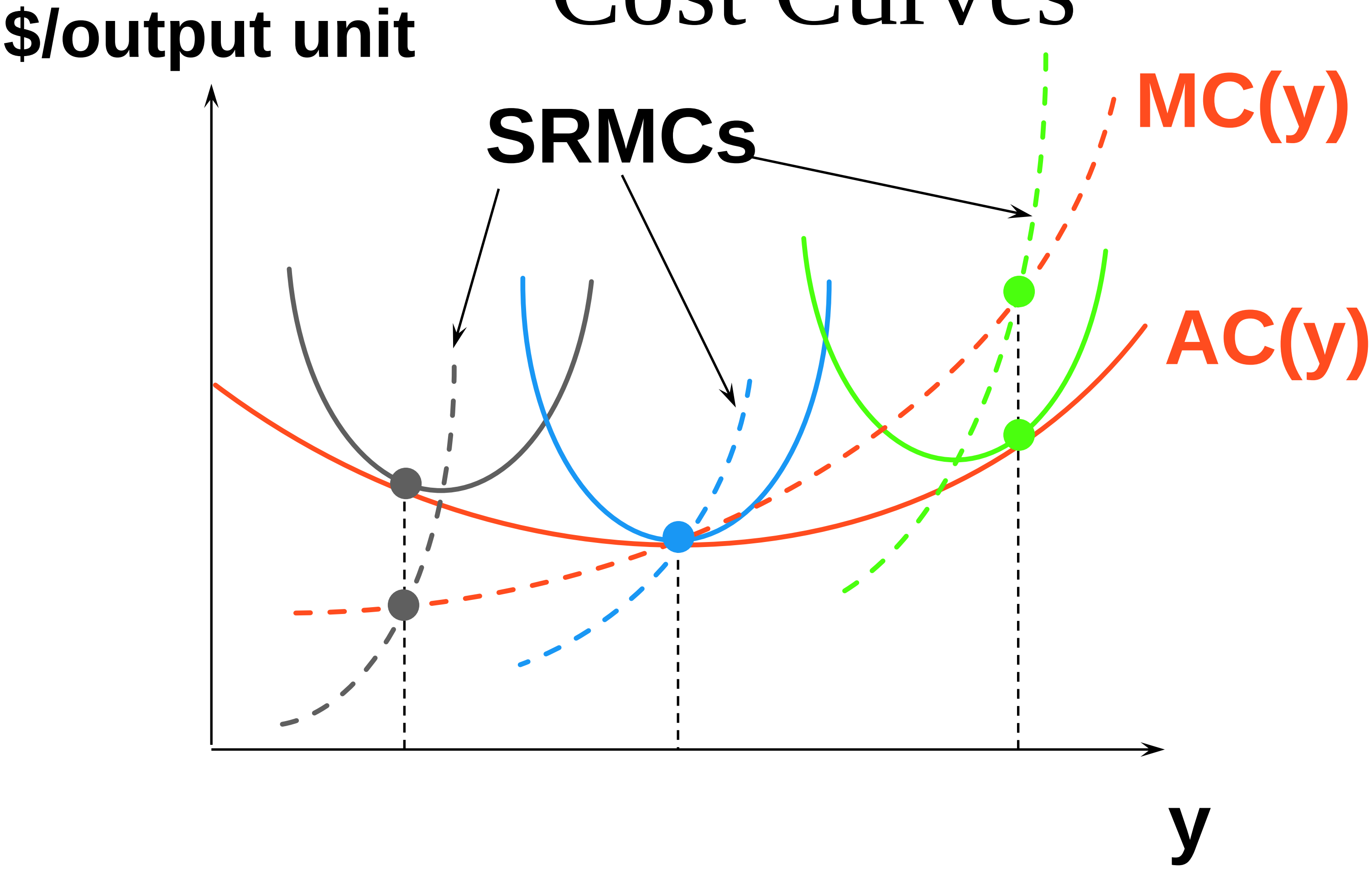
Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



- For each y > 0, the long-run MC equals the MC for the short-run chosen by the firm.