Microeconomic Theory I

Consumer preferences and utility

Stella Tsani

stsani@econ.uoa.gr

Lecture slides kindly offered by ΚΡΙΤΙΚΗ ©

Rationality in Economics

- **Behavioral Postulate:**
  A decisionmaker always chooses its most preferred alternative from its set of available alternatives.
- **So to model choice we must model decisionmakers' preferences.**

Preference Relations

- **Comparing two different consumption bundles, x and y:**
  - **strict preference: x is more preferred than is y.**
  - **weak preference: x is as at least as preferred as is y.**
  - **indifference: x is exactly as preferred as is y.**

Preference Relations

- **Strict preference, weak preference and indifference are all preference relations.**
- **Particularly, they are ordinal relations; *i.e.* they state only the order in which bundles are preferred.**

Preference Relations

- **$\succ$ denotes strict preference;**
  x $\succ$ y means that bundle x is preferred strictly to bundle y.

Preference Relations

- $\succ$ **denotes strict preference;**
  x $\succ$ y means bundle x is preferred strictly to bundle y.
- **~ denotes indifference; x ~ y means x and y are equally preferred.**

Preference Relations

- $\succ$ **denotes strict preference so** **x $\succ$ y means that bundle x is preferred strictly to bundle y.**
- **~ denotes indifference; x ~ y means x and y are equally preferred.**
- $\succsim$ **denotes weak preference;** **x $\succsim$ y means x is preferred at least as much as is y.**

Preference Relations

- **$x \succsim y$ and $y \succsim x$ imply $x \sim y$.**

Preference Relations

- **x $\succsim$ y and y $\succsim$ x imply x $\sim$ y.**
- **x $\succsim$ y and (not y $\succsim$ x) imply x $\succ$ y.**

Assumptions about Preference Relations

- **Completeness: For any two bundles x and y it is always possible to make the statement that either**

  $$x \succsim y$$

  or

  $$y \succsim x.$$

Assumptions about Preference Relations

- **Reflexivity: Any bundle x is always at least as preferred as itself; *i.e.***

$$\mathbf{x} \succsim \mathbf{x}.$$

Assumptions about Preference Relations

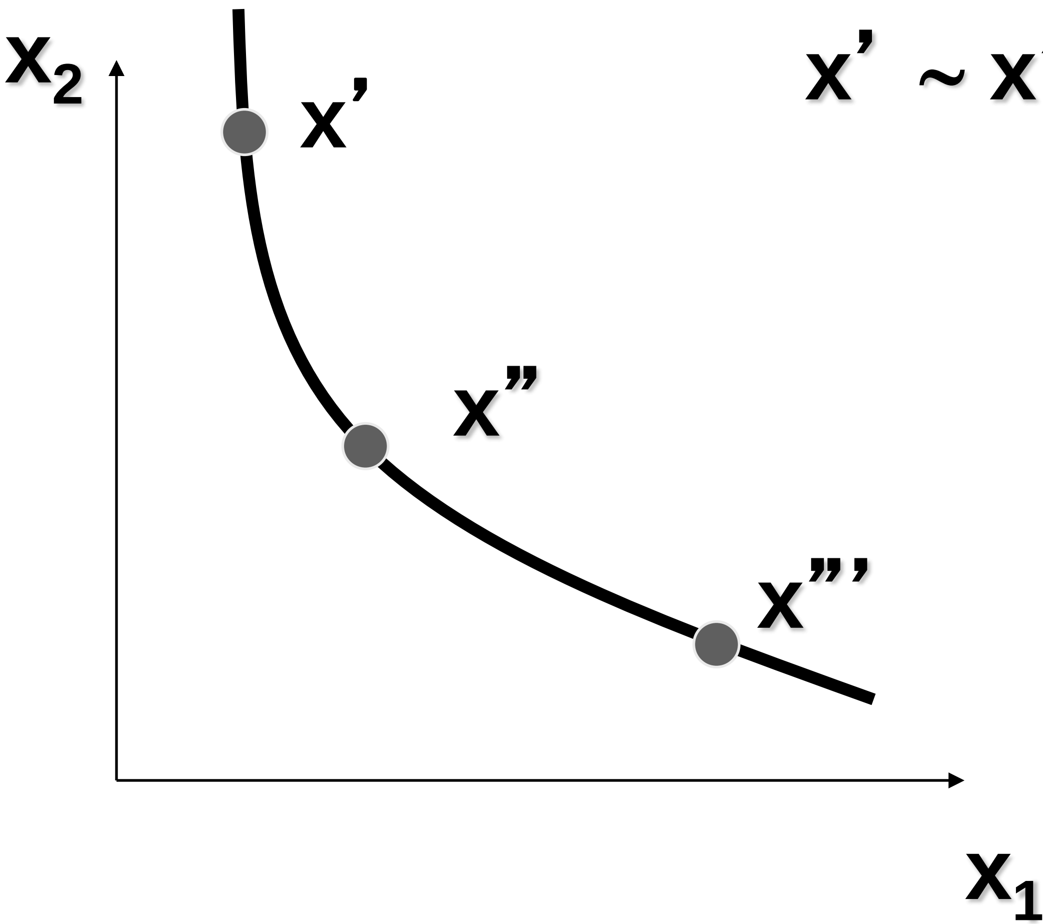
- **Transitivity: If**
  x is at least as preferred as y, and
  y is at least as preferred as z, then
  x is at least as preferred as z; *i.e.*

  $$x \succsim y \text{ and } y \succsim z \;\Rightarrow\; x \succsim z.$$

Indifference Curves

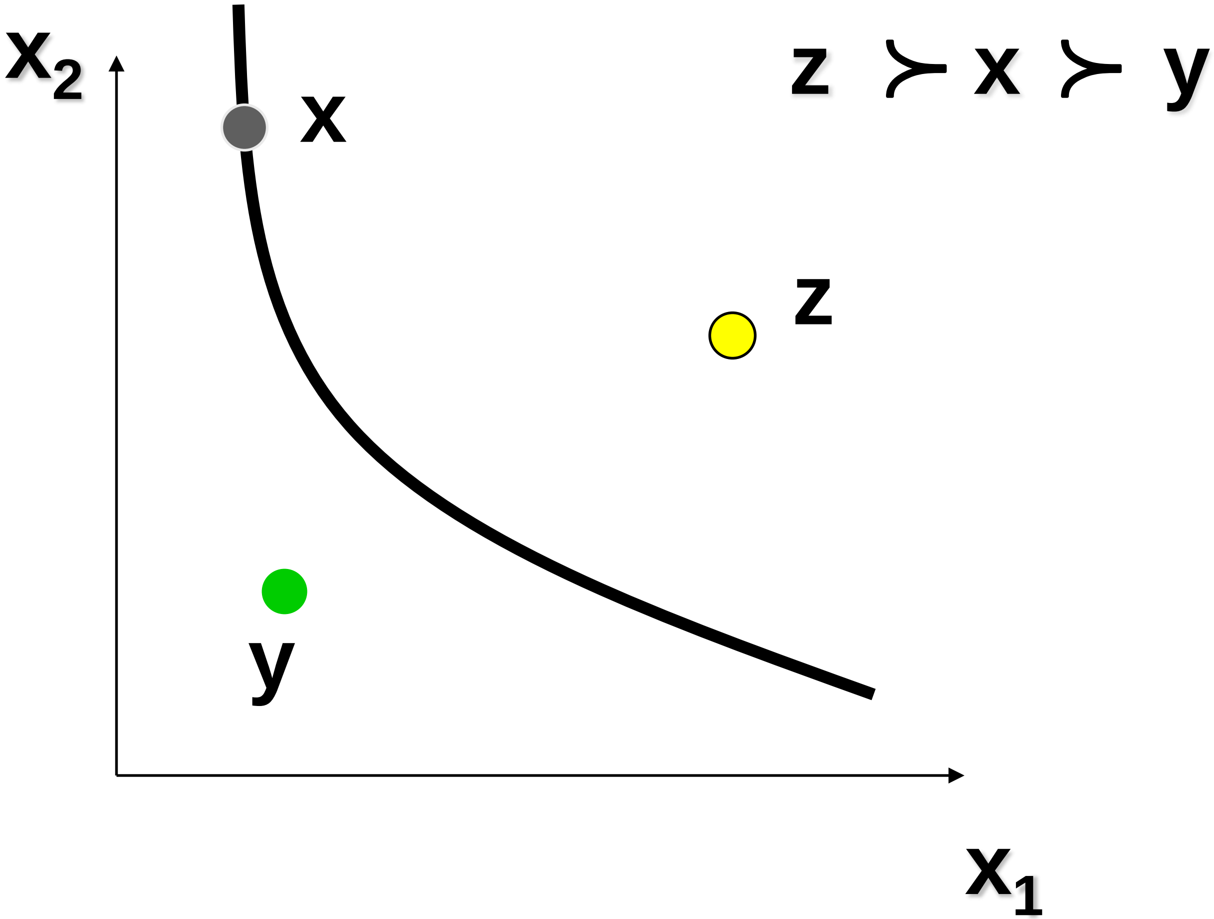
- **Take a reference bundle x'. The set of all bundles equally preferred to x' is the indifference curve containing x'; the set of all bundles y ~ x'.**
- **Since an indifference "curve" is not always a curve a better name might be an indifference "set".**

Indifference Curves

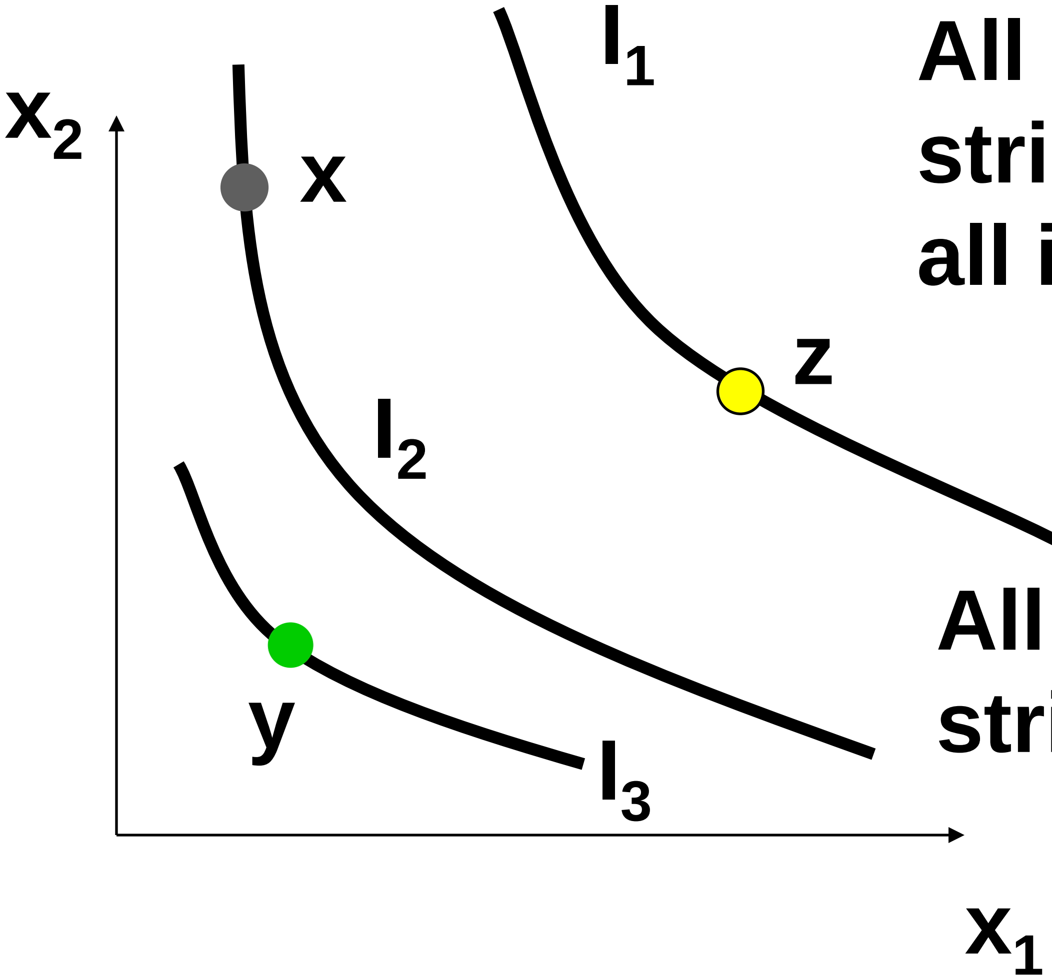


$x' \sim x'' \sim x'''$

Indifference Curves



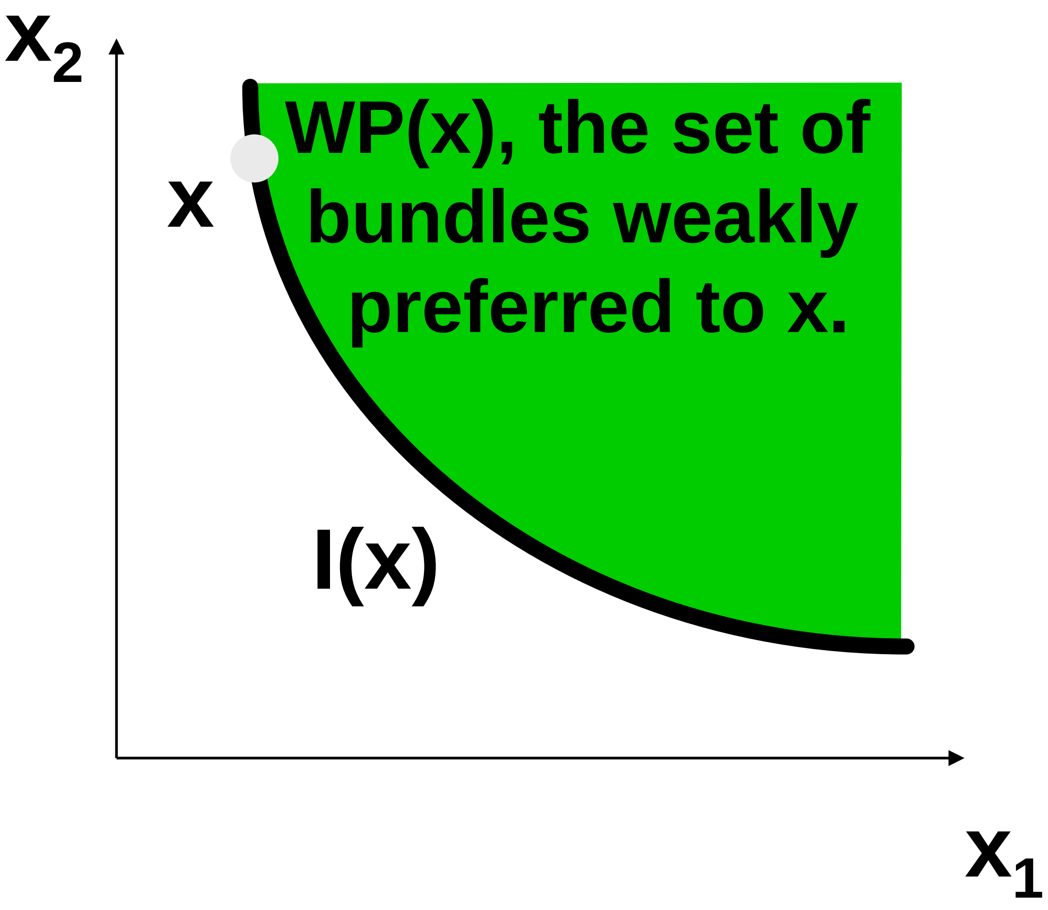
Indifference Curves



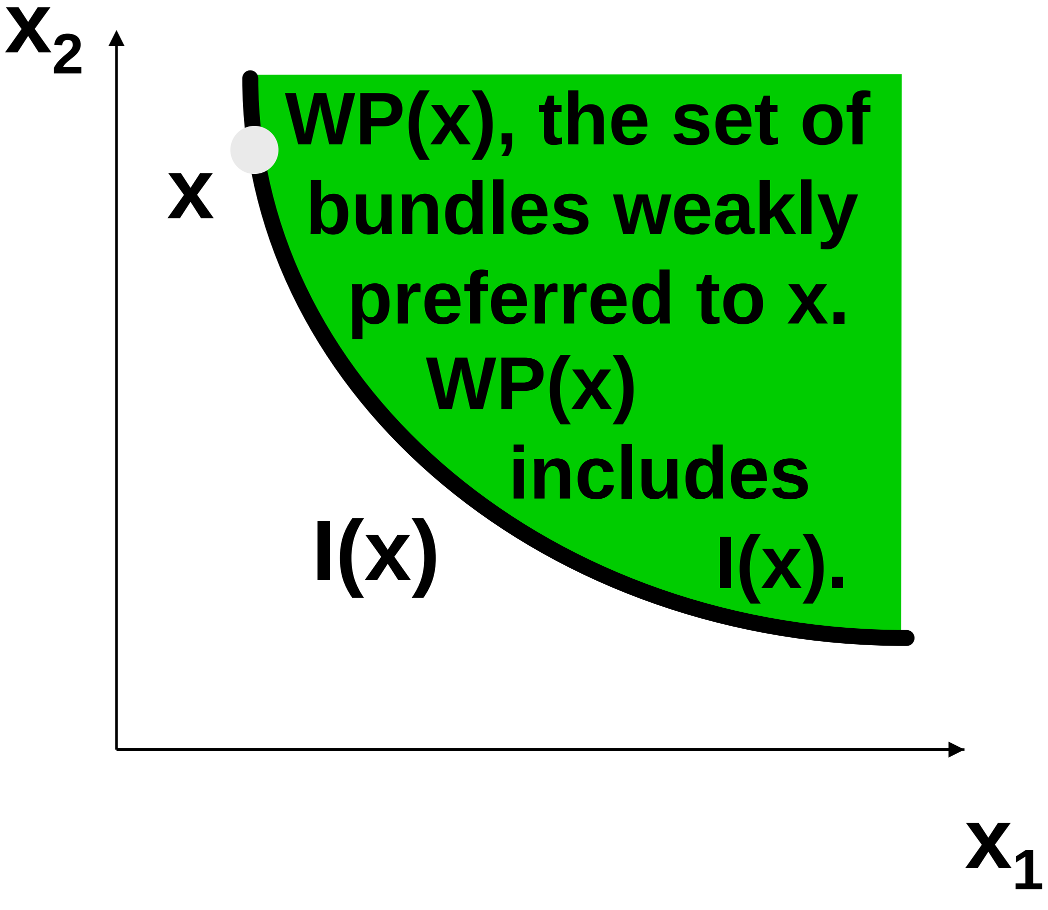
All bundles in I_1 are strictly preferred to all in I_2.

All bundles in I_2 are strictly preferred to all in I_3.

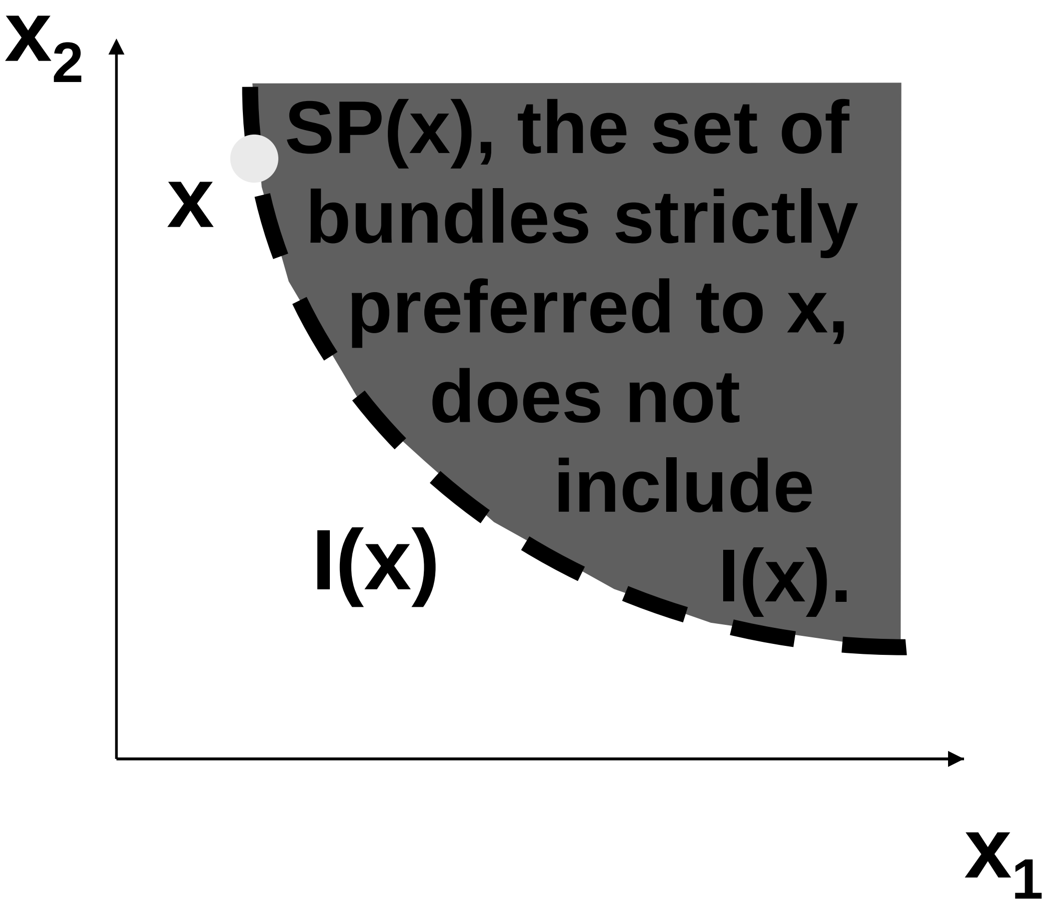
Indifference Curves



Indifference Curves

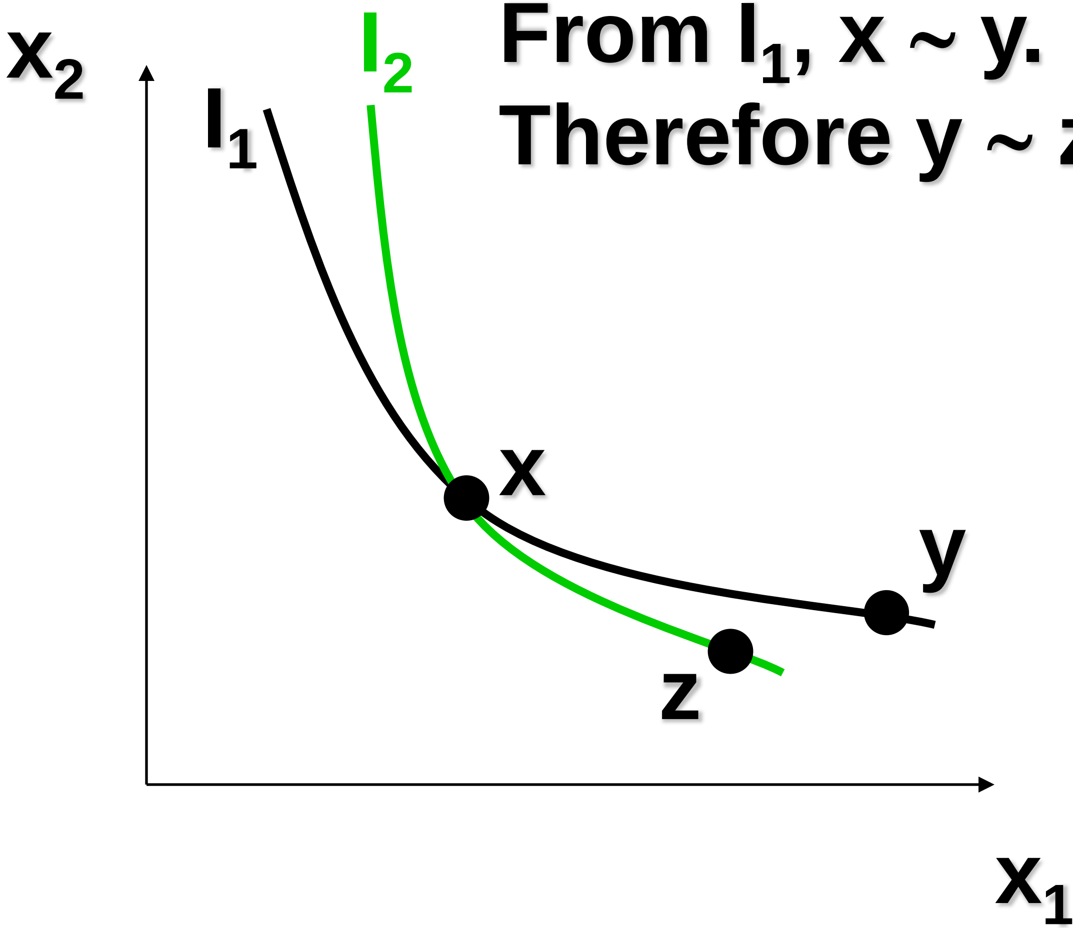


Indifference Curves



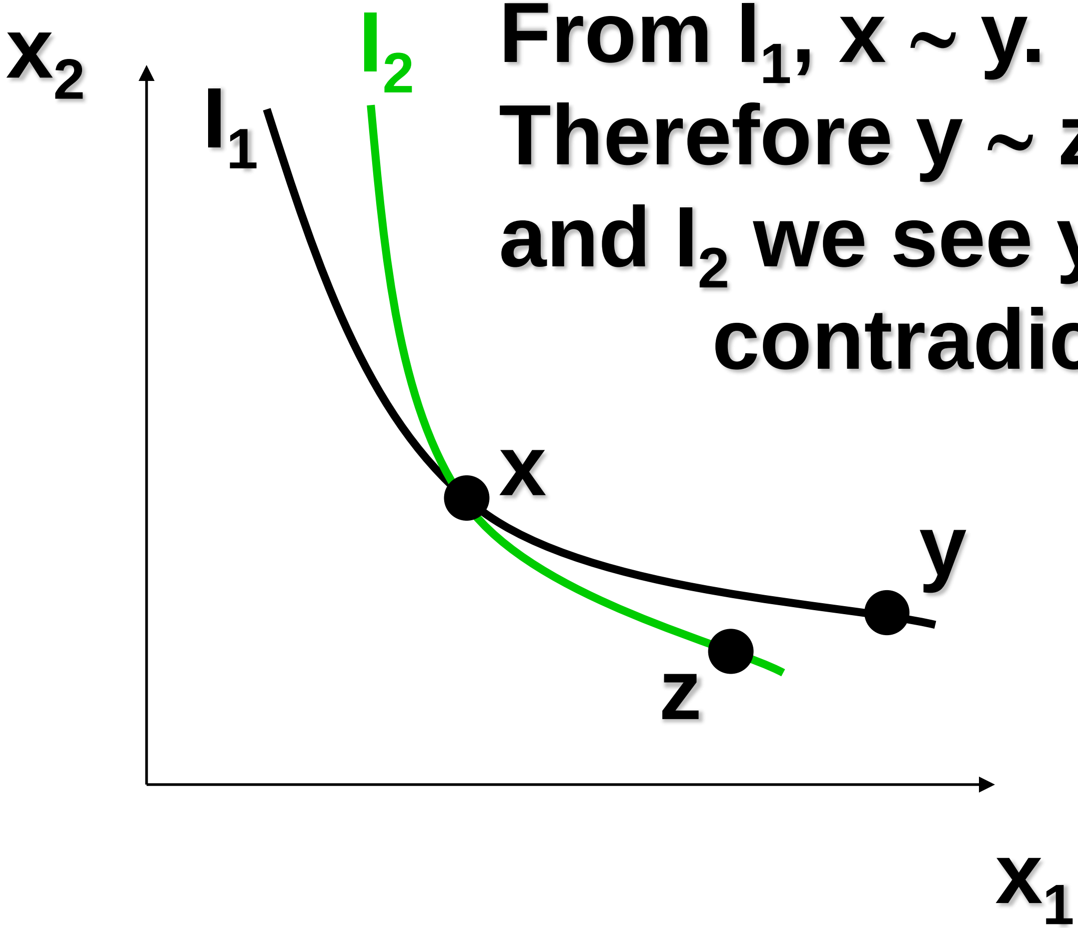
Indifference Curves Cannot Intersect

From I_1, x ~ y. From I_2, x ~ z. Therefore y ~ z.



Indifference Curves Cannot Intersect

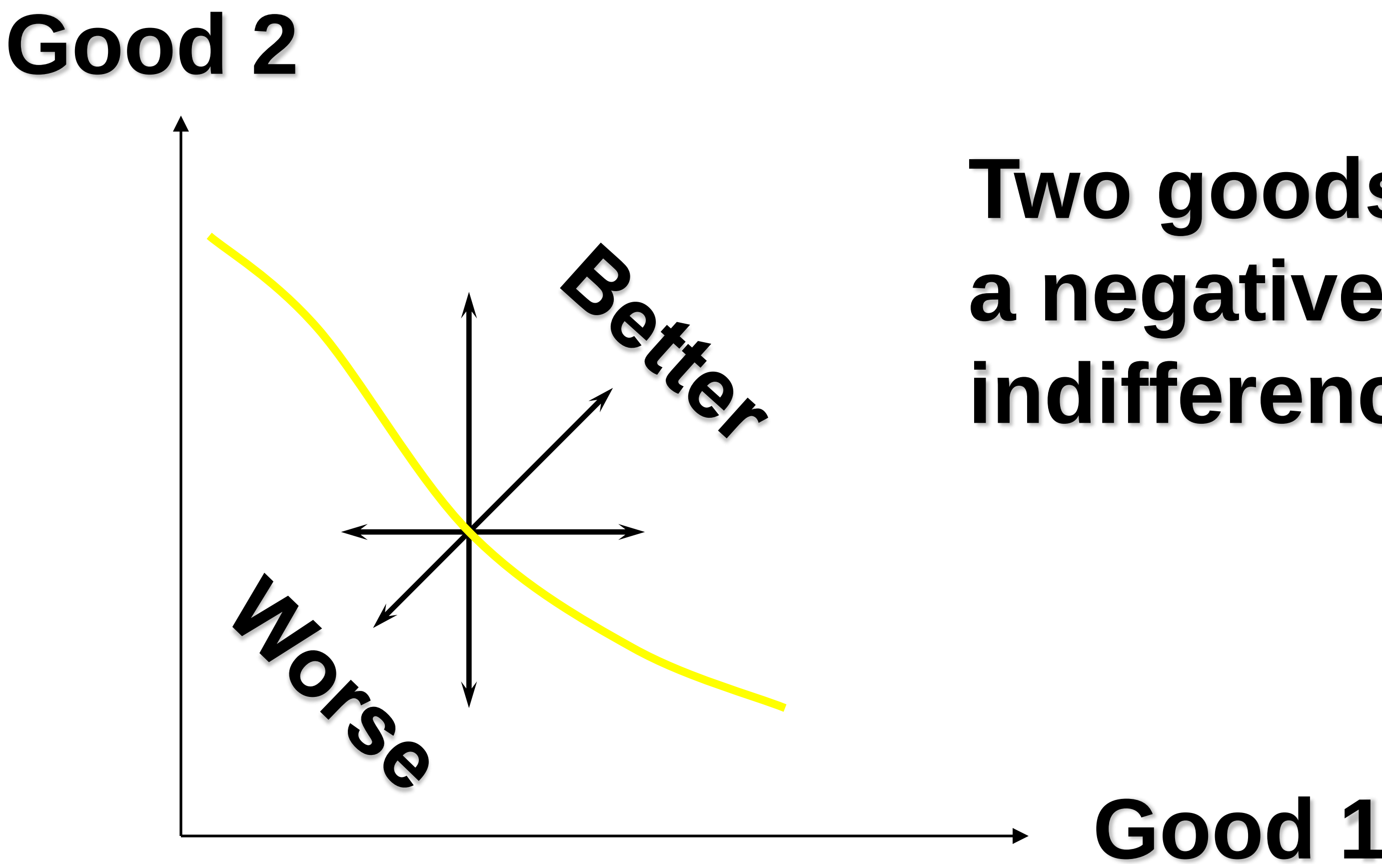
From I_1, $x \sim y$. From I_2, $x \sim z$. Therefore $y \sim z$. But from I_1 and I_2 we see $y \succ z$, a contradiction.



Slopes of Indifference Curves

- **When more of a commodity is always preferred, the commodity is a good.**
- **If every commodity is a good then indifference curves are negatively sloped.**

Slopes of Indifference Curves

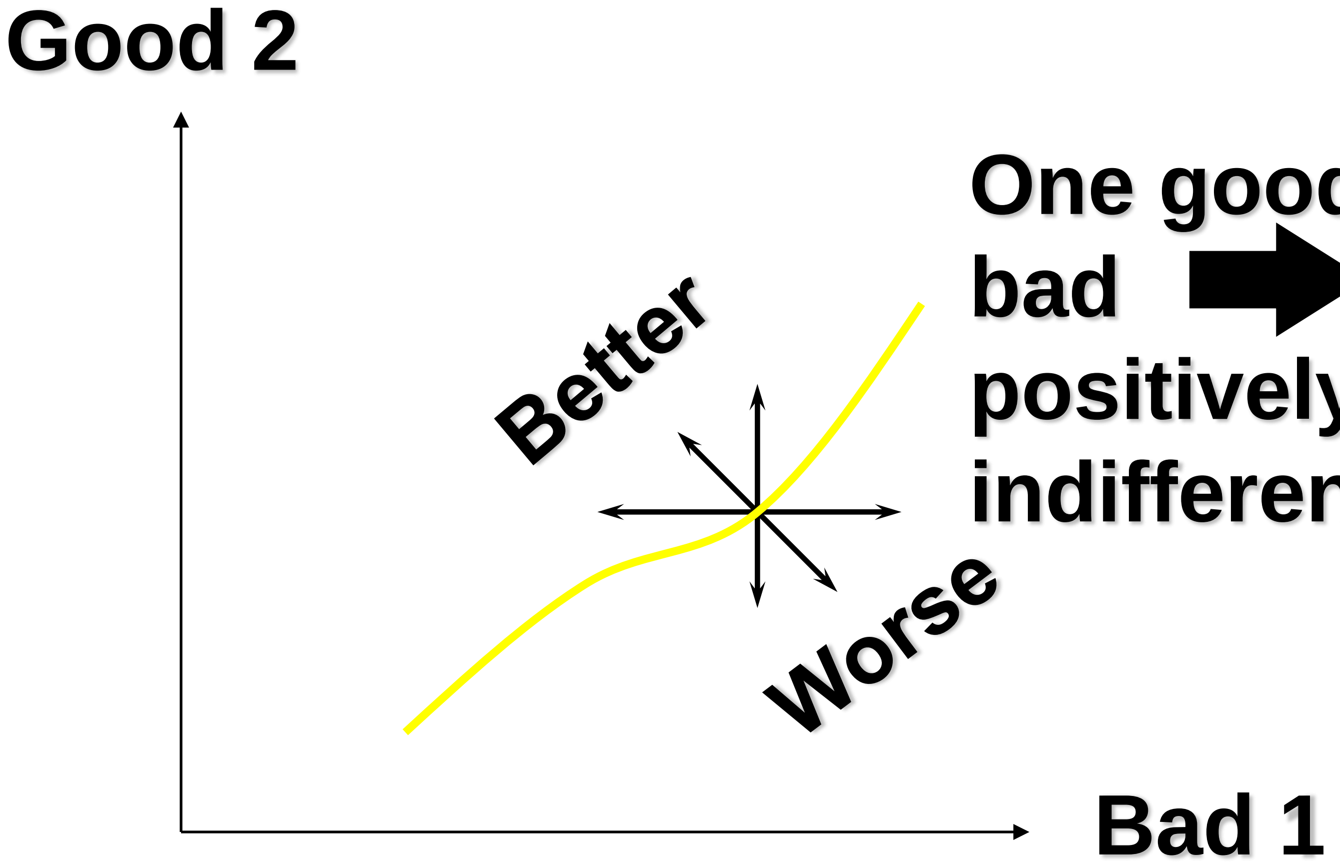


Two goods ➡ a negatively sloped indifference curve.

Slopes of Indifference Curves

- **If less of a commodity is always preferred then the commodity is a bad.**

Slopes of Indifference Curves

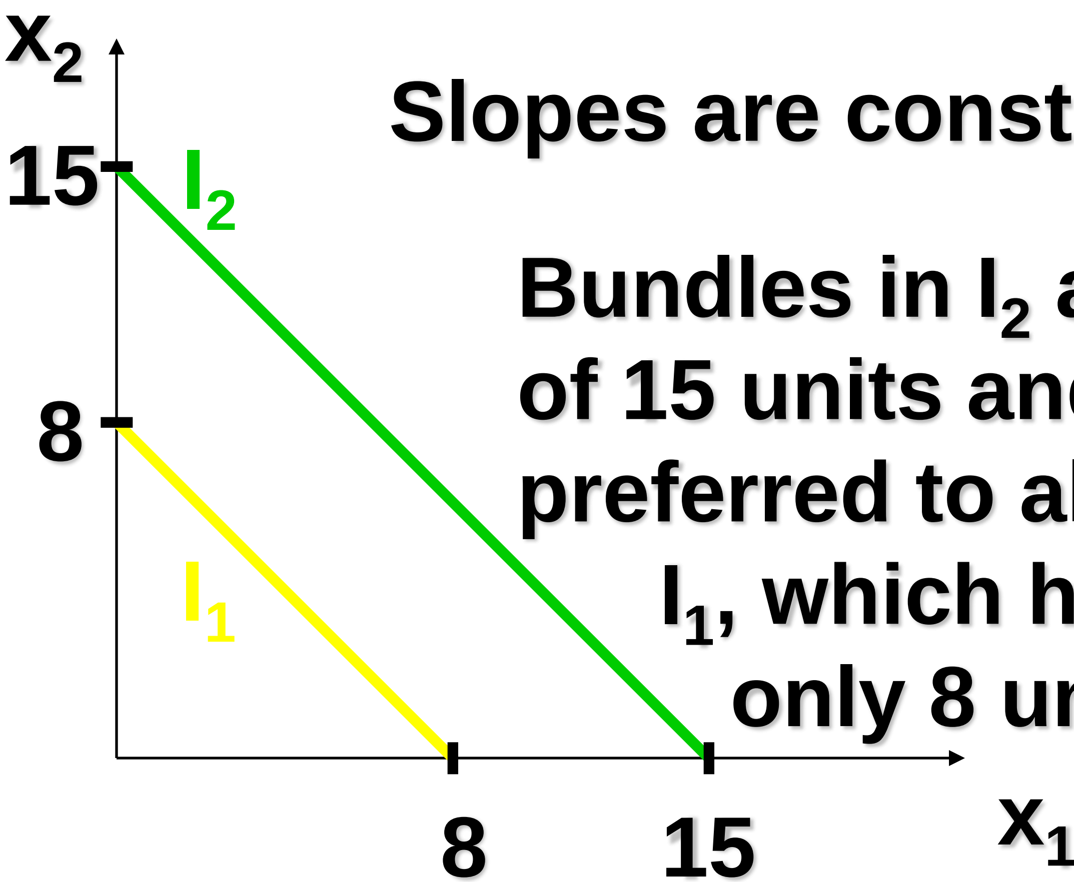


One good and one bad → a positively sloped indifference curve.

Extreme Cases of Indifference Curves; Perfect Substitutes

- **If a consumer always regards units of commodities 1 and 2 as equivalent, then the commodities are perfect substitutes and only the total amount of the two commodities in bundles determines their preference rank-order.**

Extreme Cases of Indifference Curves; Perfect Substitutes



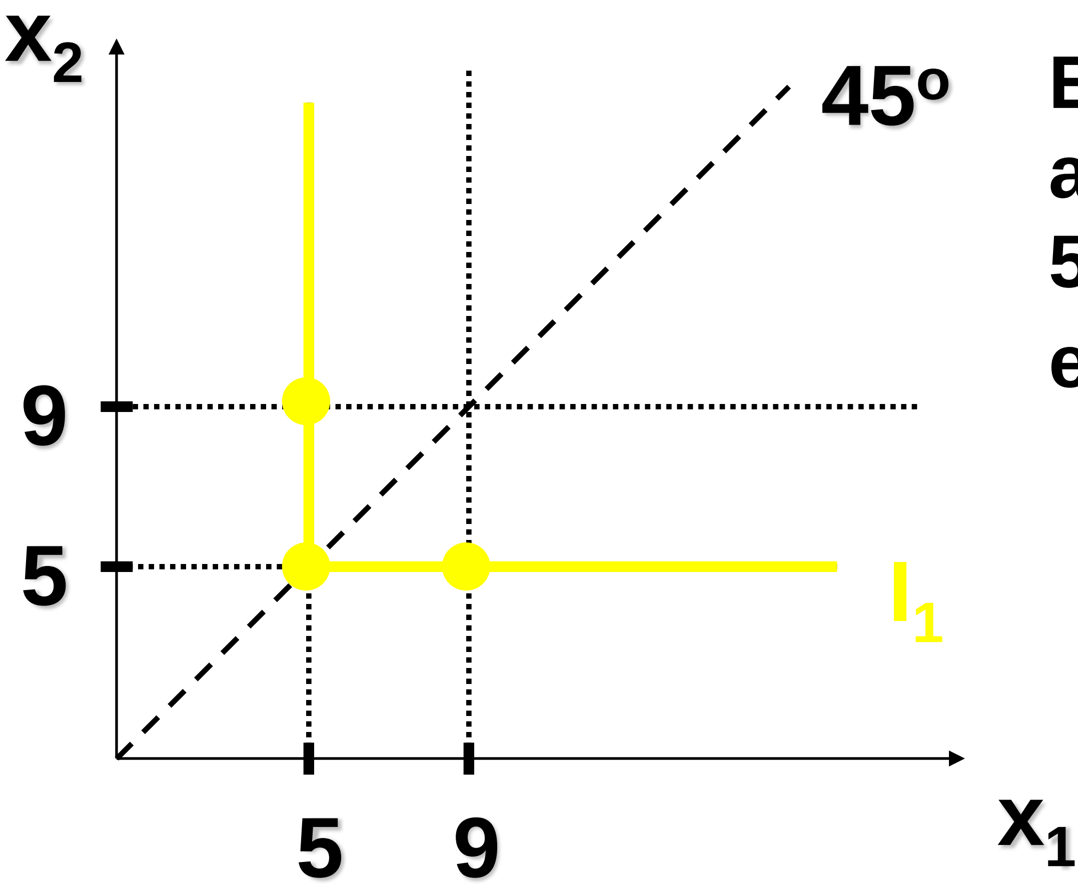
Slopes are constant at - 1.

Bundles in I_2 all have a total of 15 units and are strictly preferred to all bundles in I_1, which have a total of only 8 units in them.

Extreme Cases of Indifference Curves; Perfect Complements

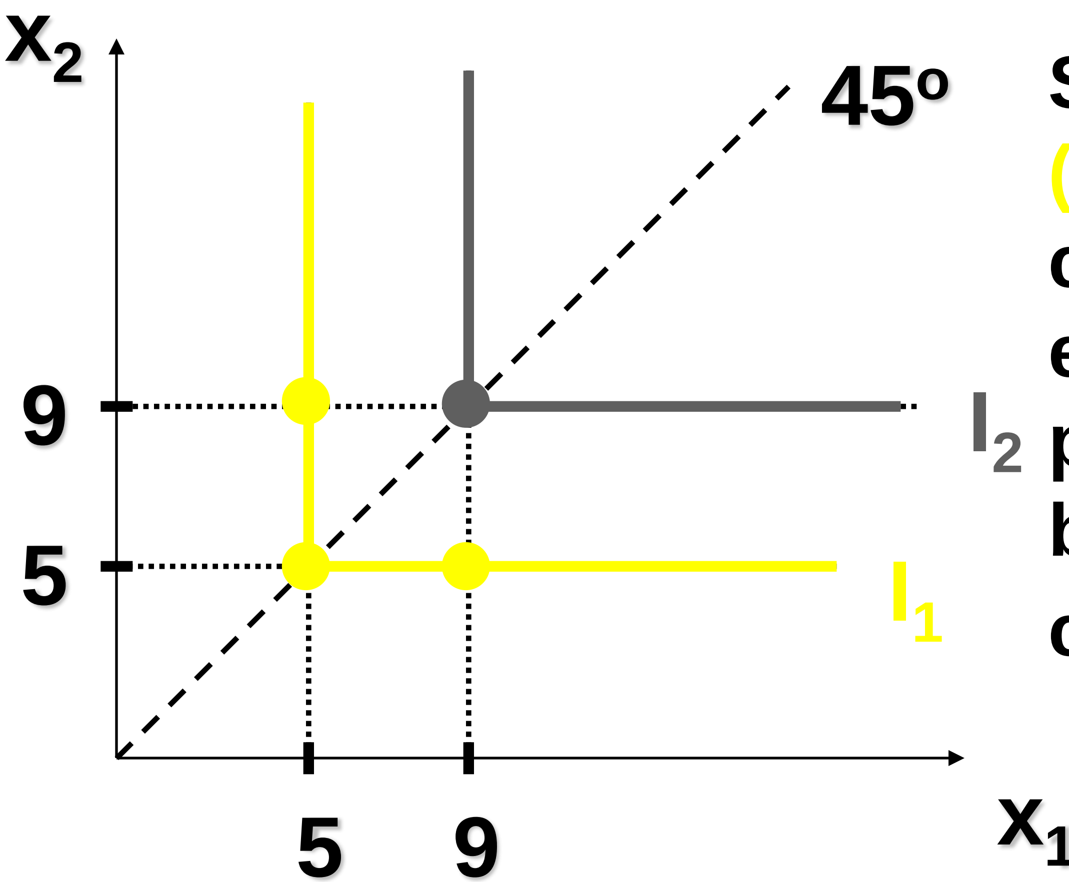
- **If a consumer always consumes commodities 1 and 2 in fixed proportion (e.g. one-to-one), then the commodities are perfect complements and only the number of pairs of units of the two commodities determines the preference rank-order of bundles.**

Extreme Cases of Indifference Curves; Perfect Complements



Each of (5,5), (5,9) and (9,5) contains 5 pairs so each is equally preferred.

Extreme Cases of Indifference Curves; Perfect Complements

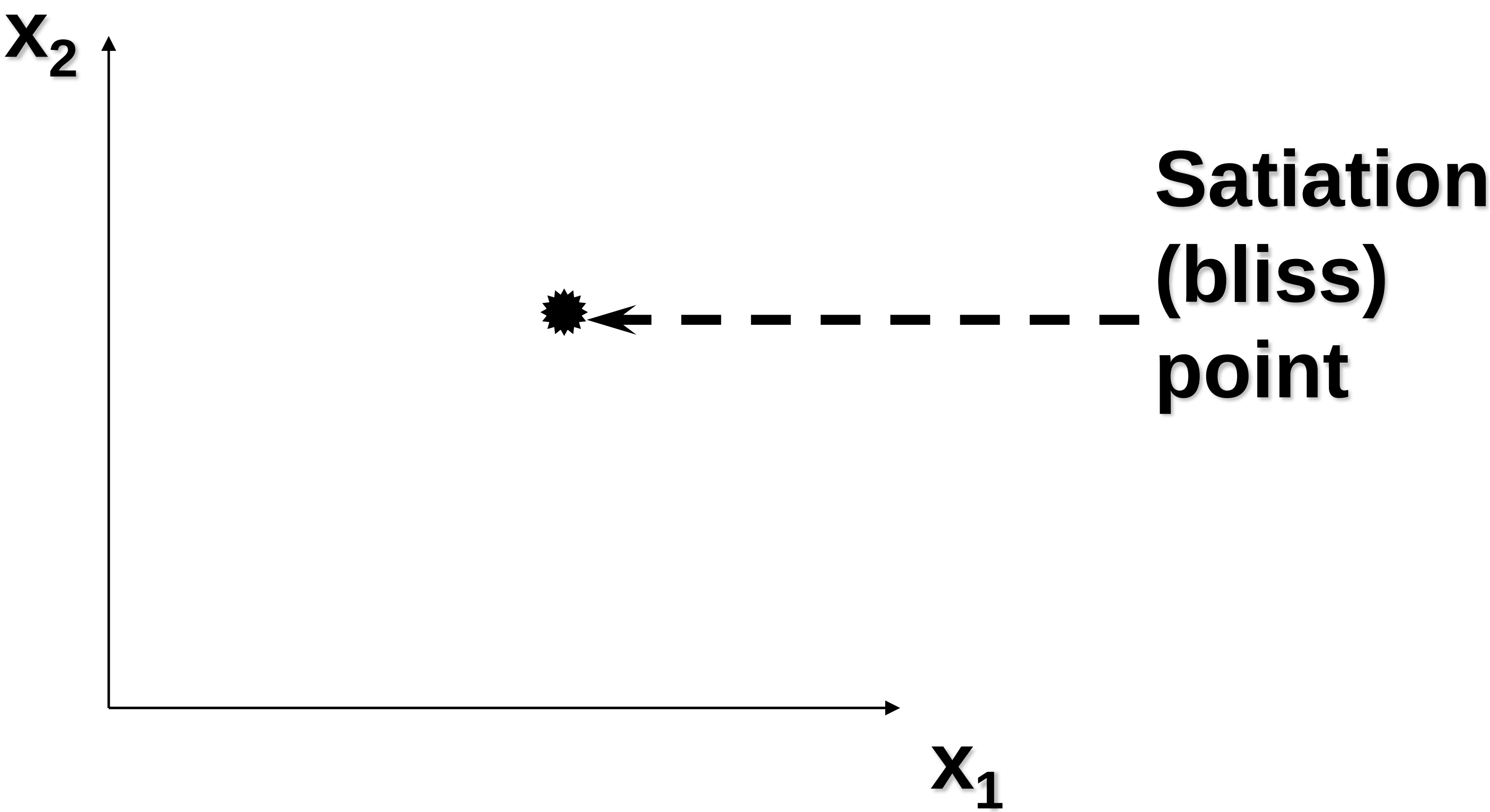


Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains 9 pairs.

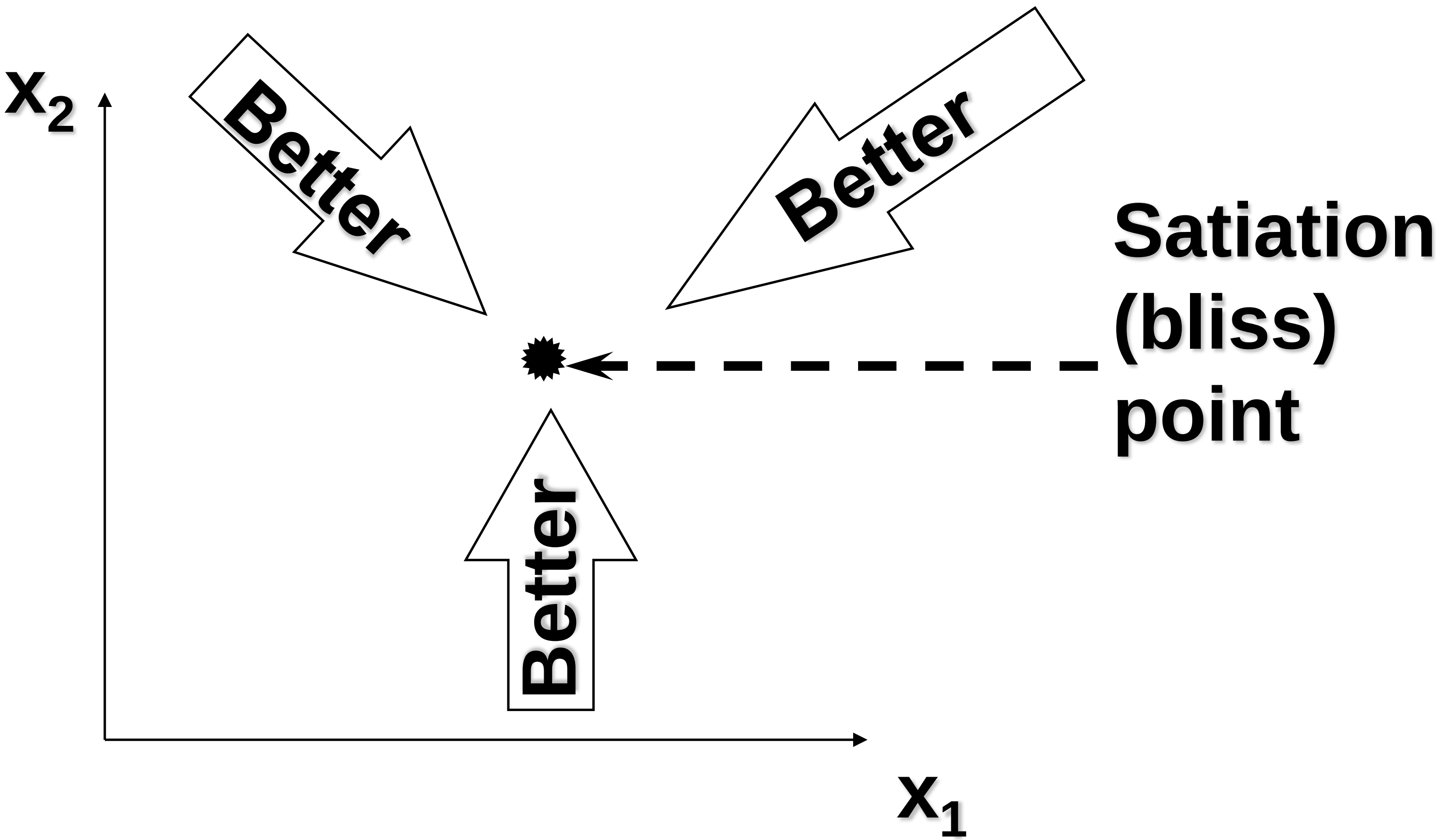
Preferences Exhibiting Satiation

- **A bundle strictly preferred to any other is a satiation point or a bliss point.**
- **What do indifference curves look like for preferences exhibiting satiation?**

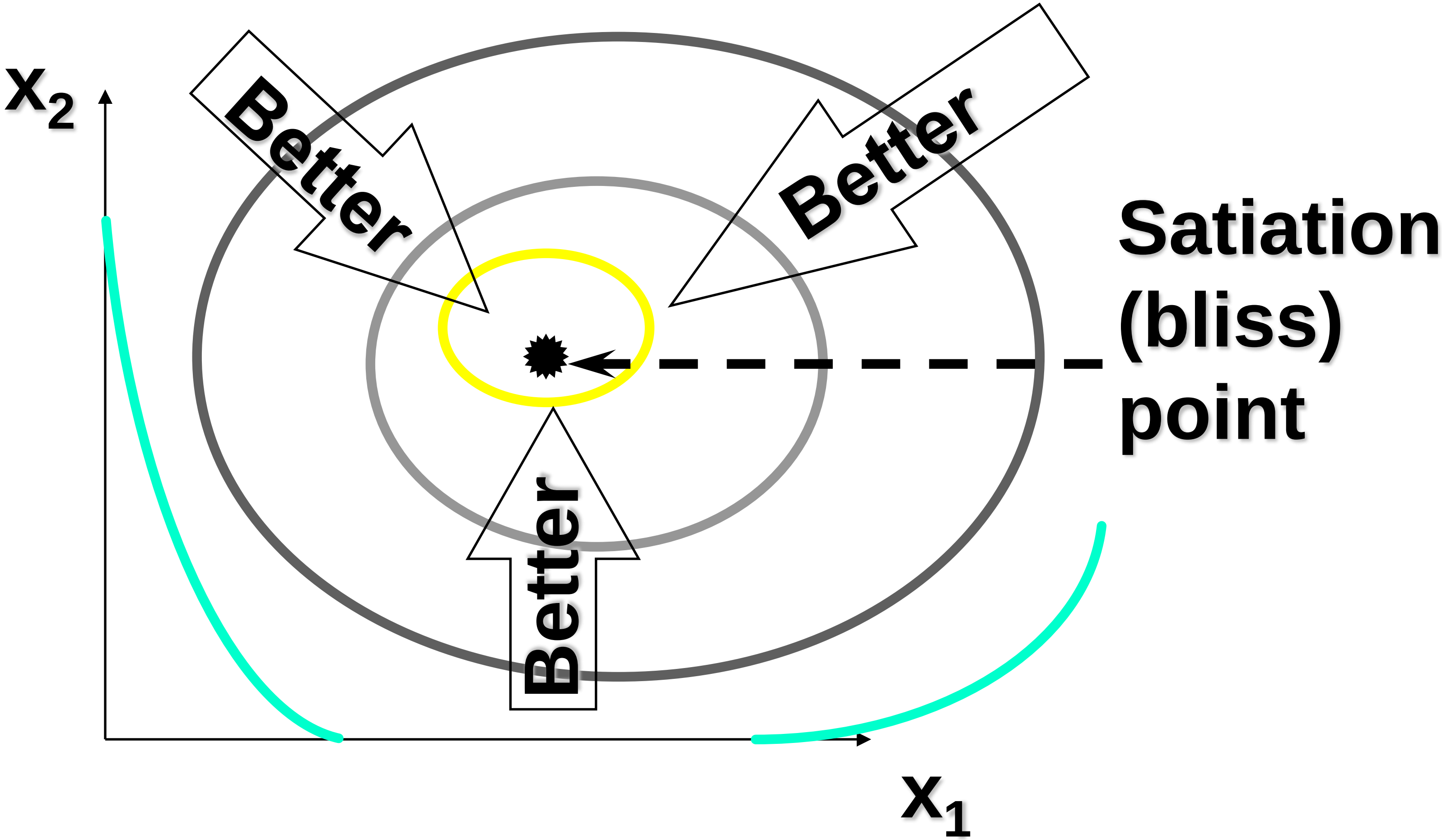
Indifference Curves Exhibiting Satiation



Indifference Curves Exhibiting Satiation



Indifference Curves Exhibiting Satiation



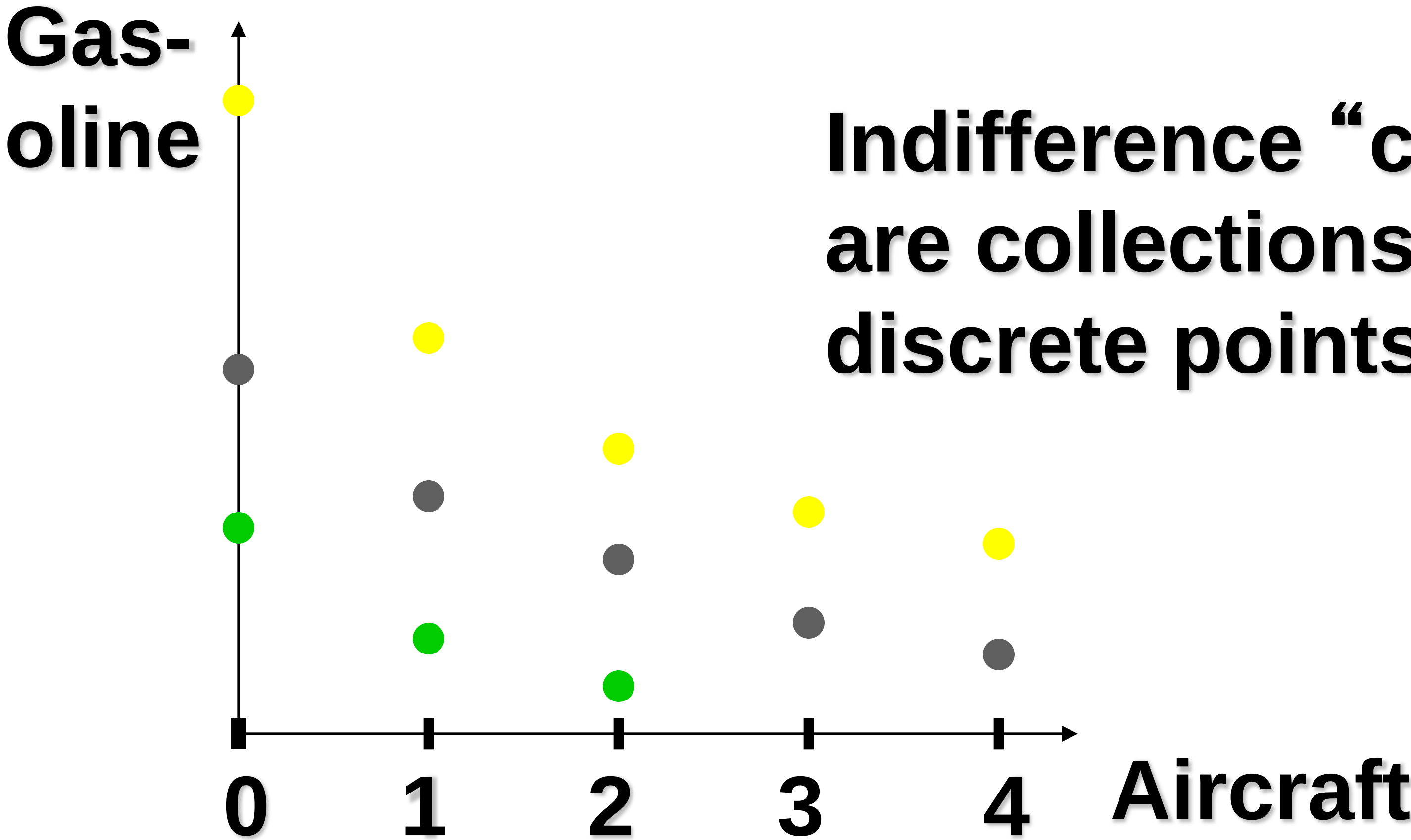
Indifference Curves for Discrete Commodities

- **A commodity is infinitely divisible if it can be acquired in any quantity; *e.g.* water or cheese.**
- **A commodity is discrete if it comes in unit lumps of 1, 2, 3, … and so on; *e.g.* aircraft, ships and refrigerators.**

Indifference Curves for Discrete Commodities

- **Suppose commodity 2 is an infinitely divisible good (gasoline) while commodity 1 is a discrete good (aircraft). What do indifference "curves" look like?**

Indifference Curves With a Discrete Good



Indifference "curves" are collections of discrete points.

Well-Behaved Preferences

- **A preference relation is "well-behaved" if it is**
  - **monotonic and convex.**
- **Monotonicity: More of any commodity is always preferred (*i.e.* no satiation and every commodity is a good).**

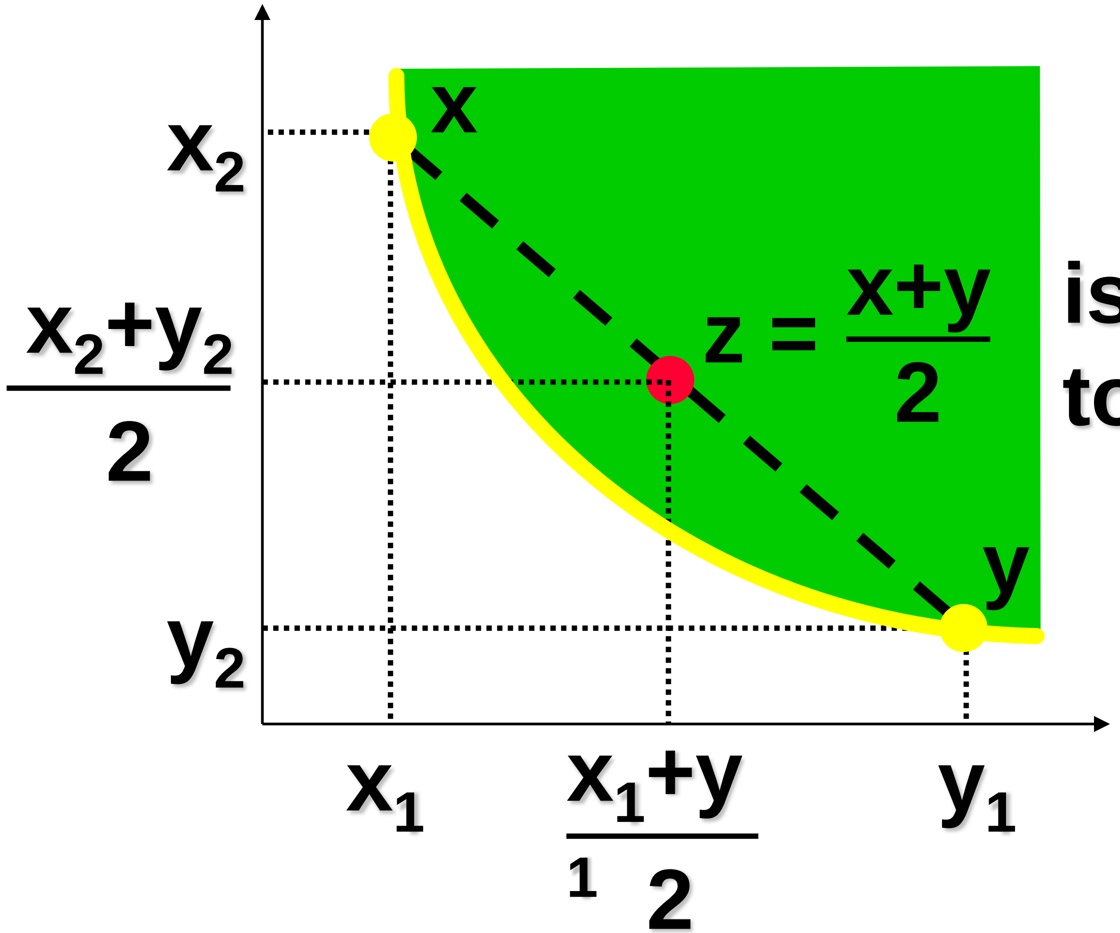
Well-Behaved Preferences

- **Convexity: Mixtures of bundles are (at least weakly) preferred to the bundles themselves. E.g., the 50-50 mixture of the bundles x and y is**

  z = (0.5)x + (0.5)y.

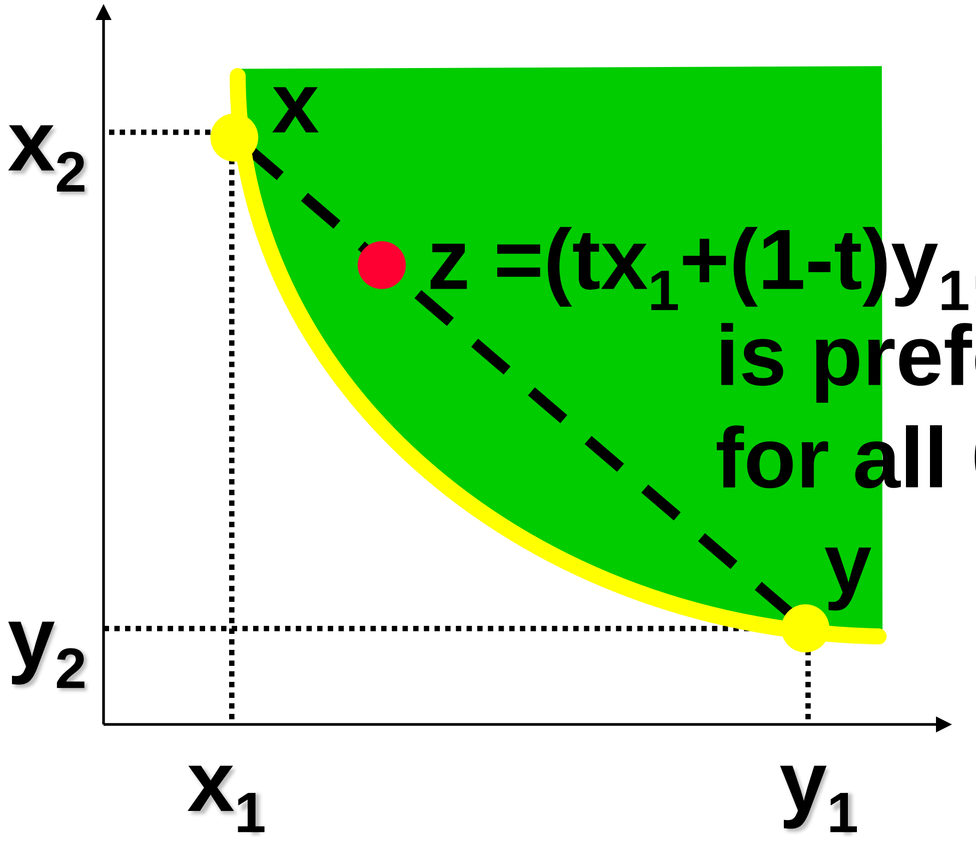
  z is at least as preferred as x or y.

Well-Behaved Preferences -- Convexity.



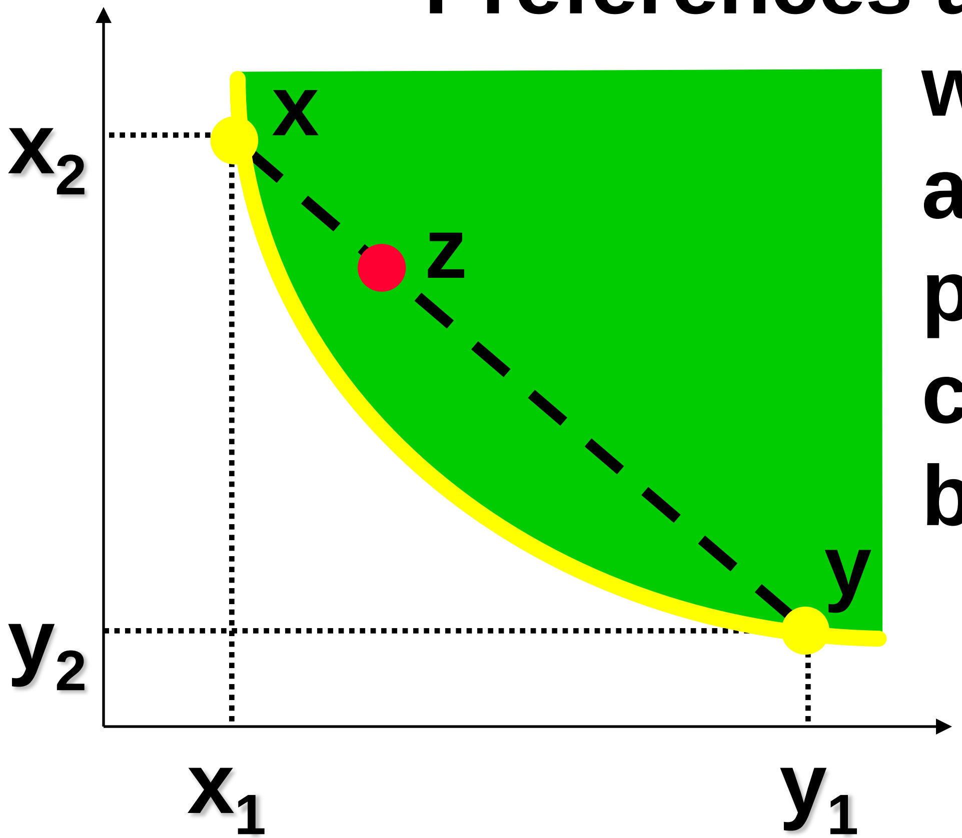
is strictly preferred to both x and y.

Well-Behaved Preferences -- Convexity.



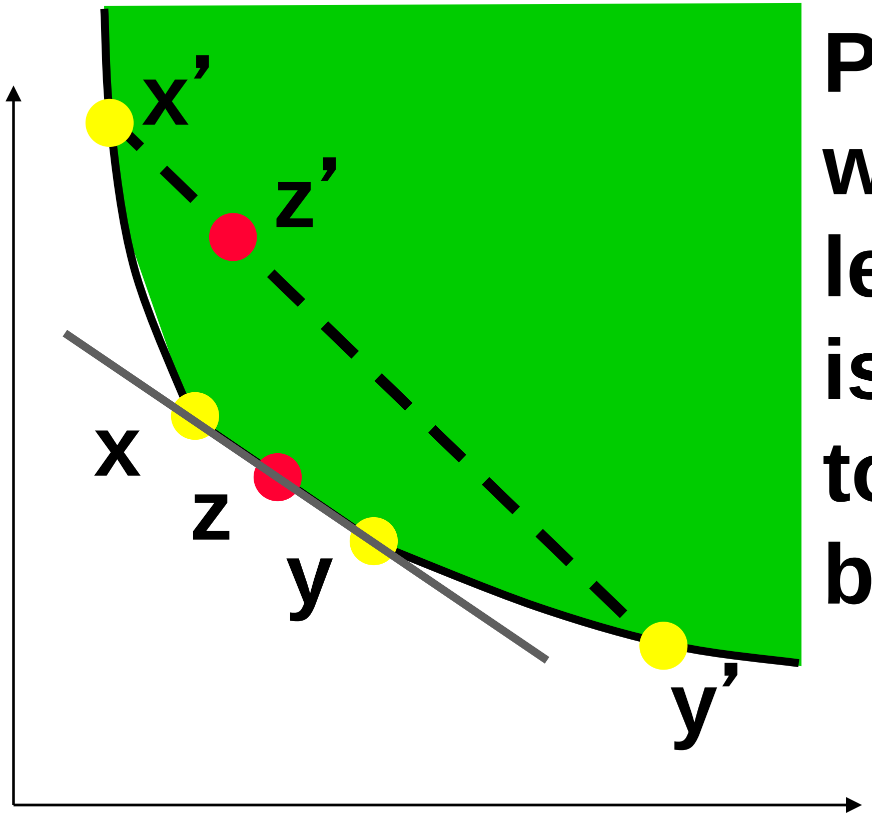
$z = (tx_1+(1-t)y_1,\ tx_2+(1-t)y_2)$ is preferred to x and y for all $0 < t < 1$.

Well-Behaved Preferences -- Convexity.



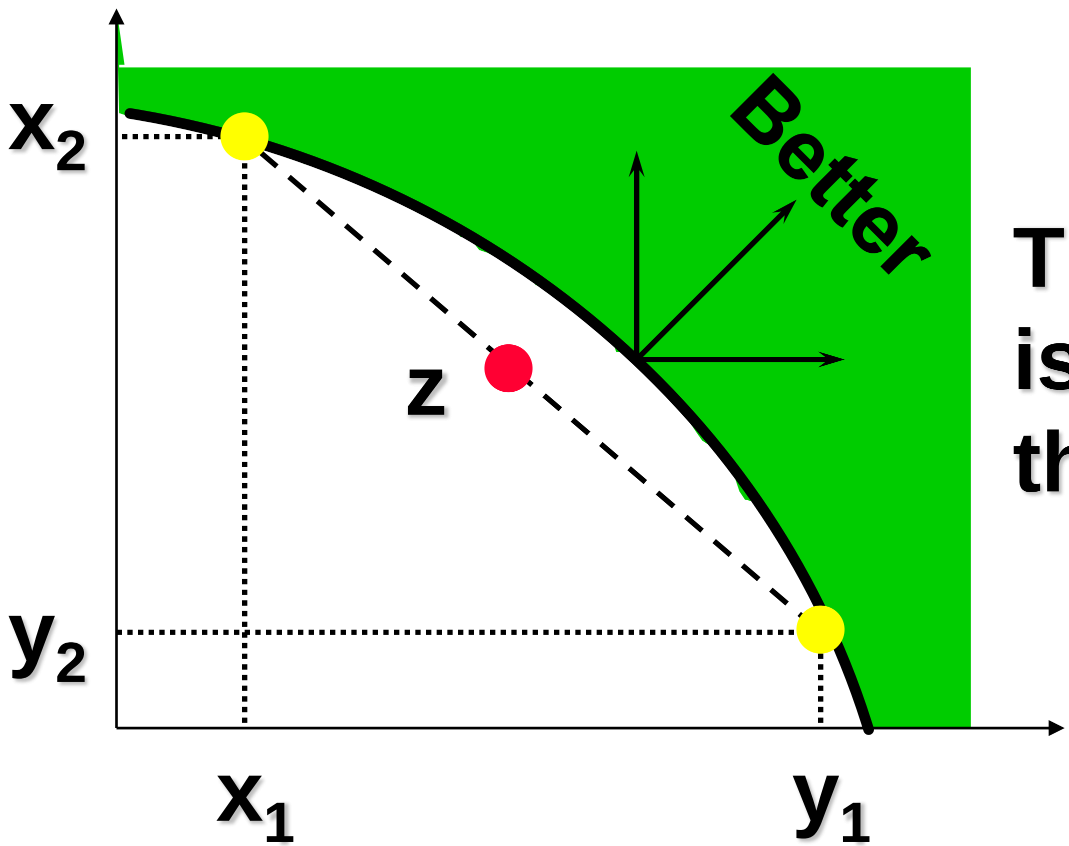
Preferences are strictly convex when all mixtures z are strictly preferred to their component bundles x and y.

Well-Behaved Preferences -- Weak Convexity.



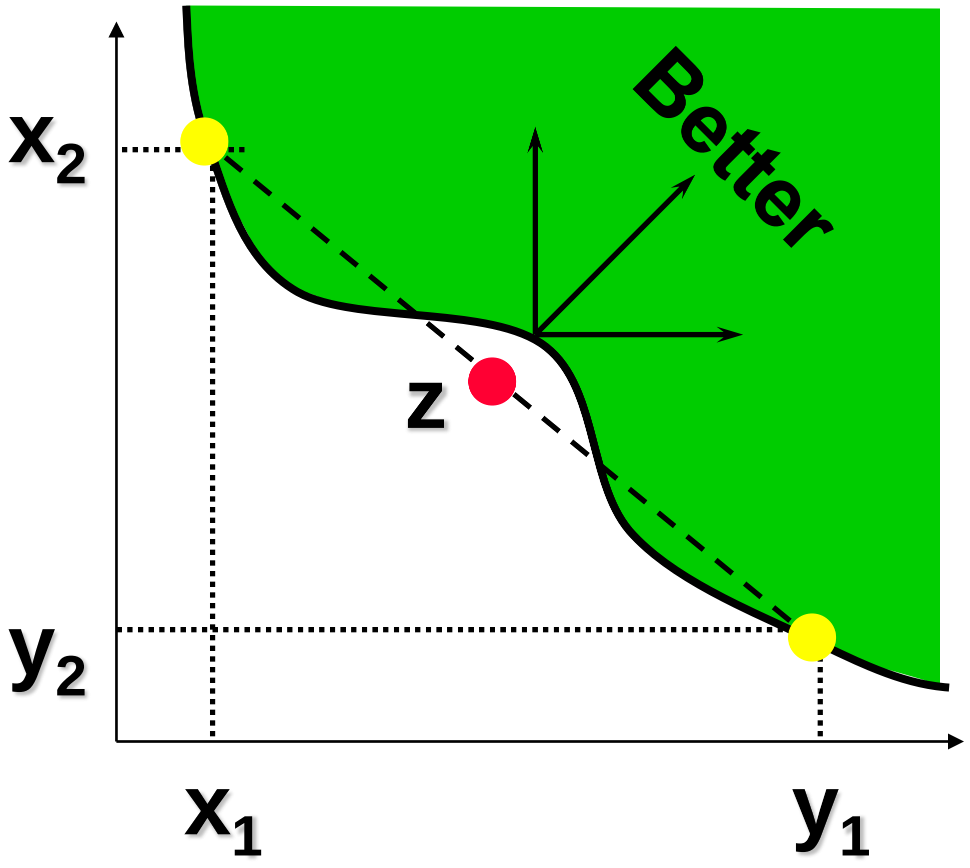
Preferences are weakly convex if at least one mixture z is equally preferred to a component bundle.

Non-Convex Preferences



The mixture z is less preferred than x or y.

More Non-Convex Preferences



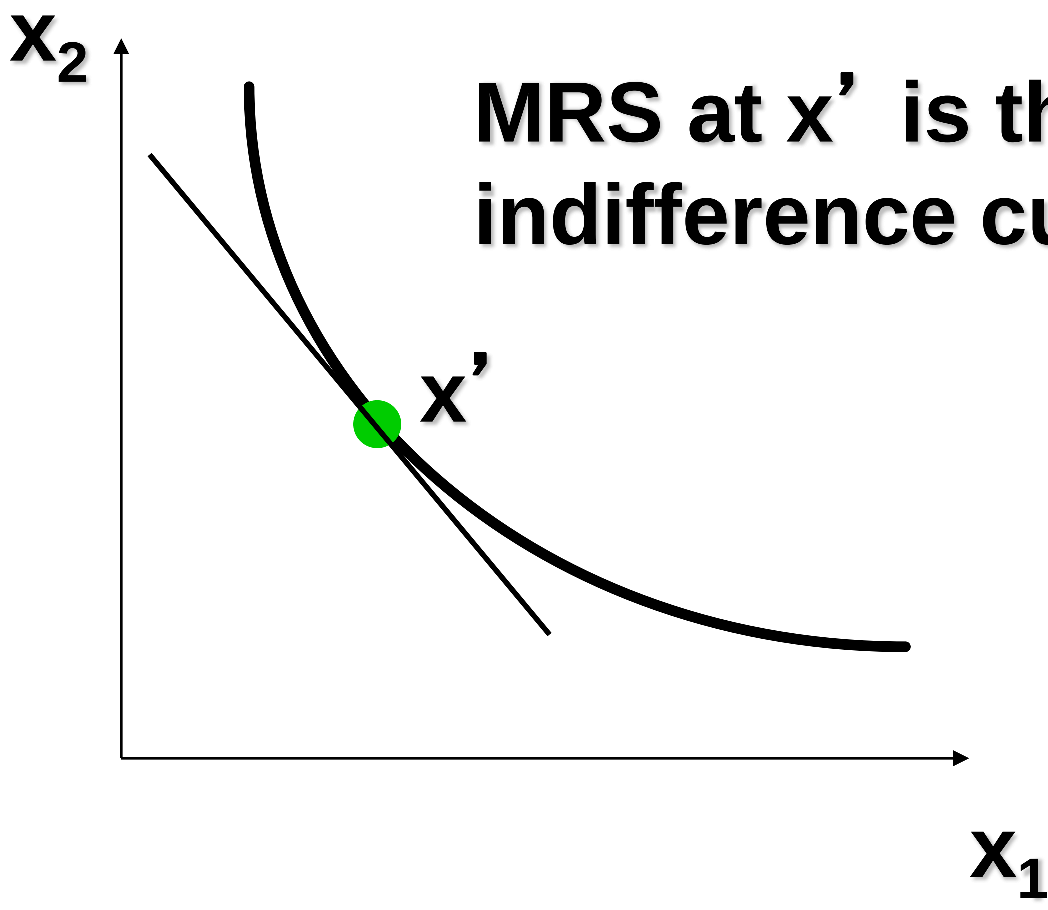
The mixture z is less preferred than x or y.

Slopes of Indifference Curves

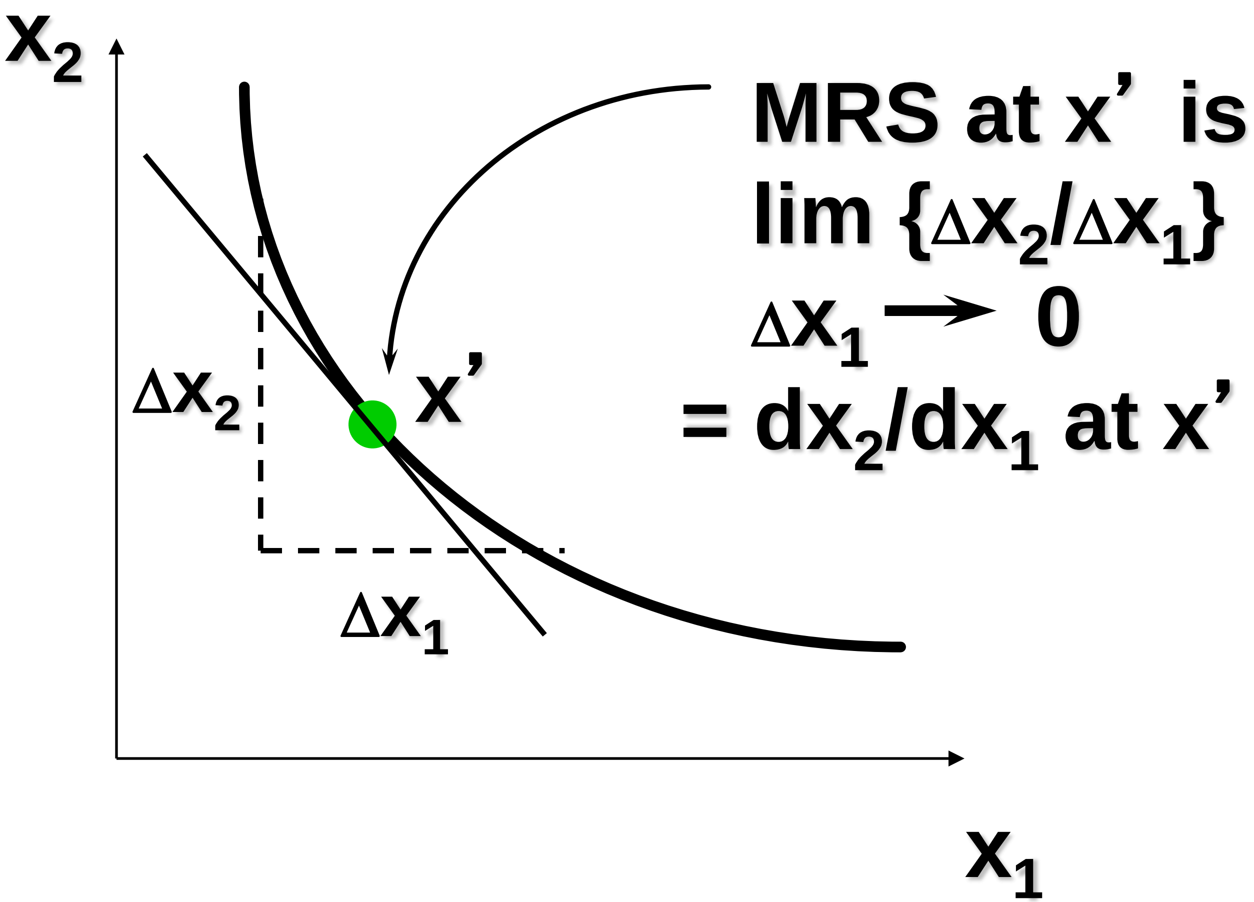
- **The slope of an indifference curve is its marginal rate-of-substitution (MRS).**
- **How can a MRS be calculated?**

Marginal Rate of Substitution

MRS at x' is the slope of the indifference curve at x'

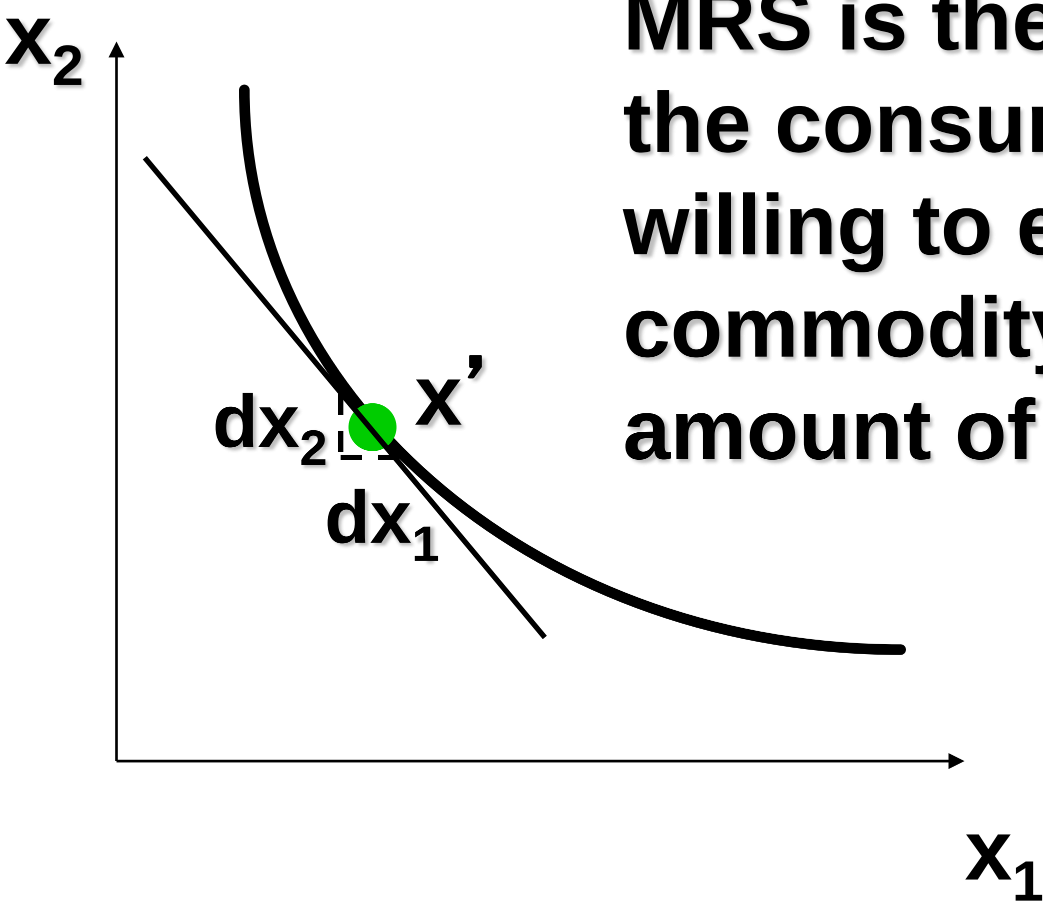


Marginal Rate of Substitution

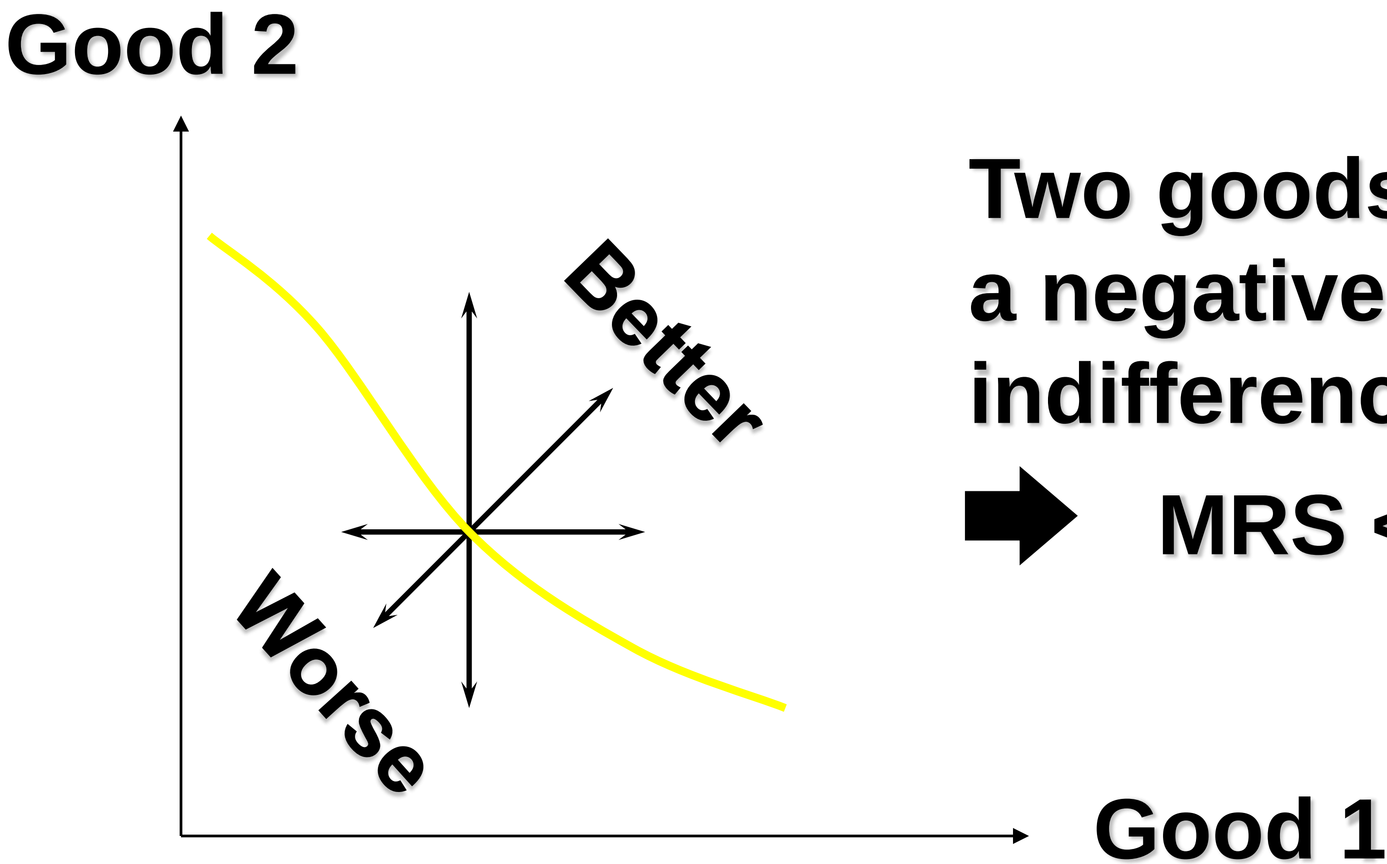


Marginal Rate of Substitution

dx_2 = MRS ´ dx_1 so, at x’ , MRS is the rate at which the consumer is only just willing to exchange commodity 2 for a small amount of commodity 1.



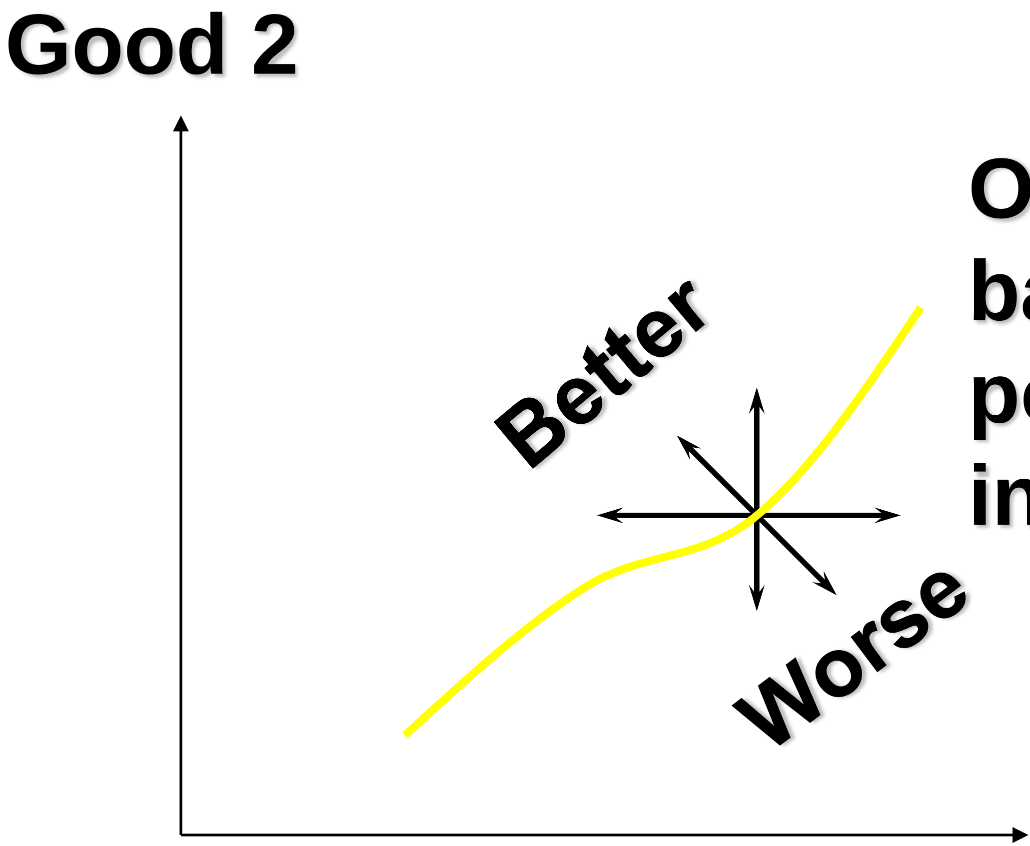
MRS & Ind. Curve Properties



Two goods ➡ a negatively sloped indifference curve

➡ MRS < 0.

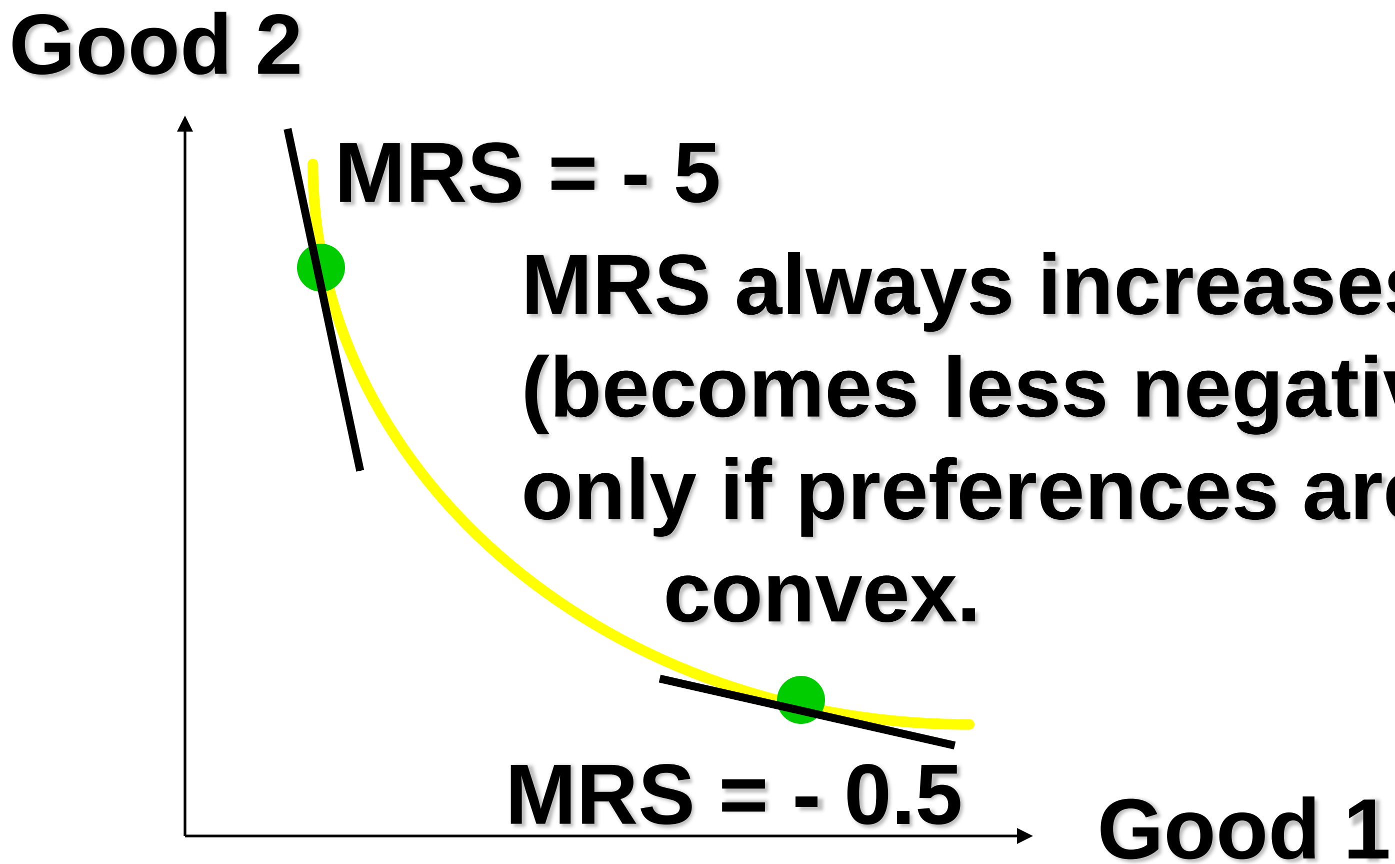
MRS & Ind. Curve Properties



One good and one bad → a positively sloped indifference curve

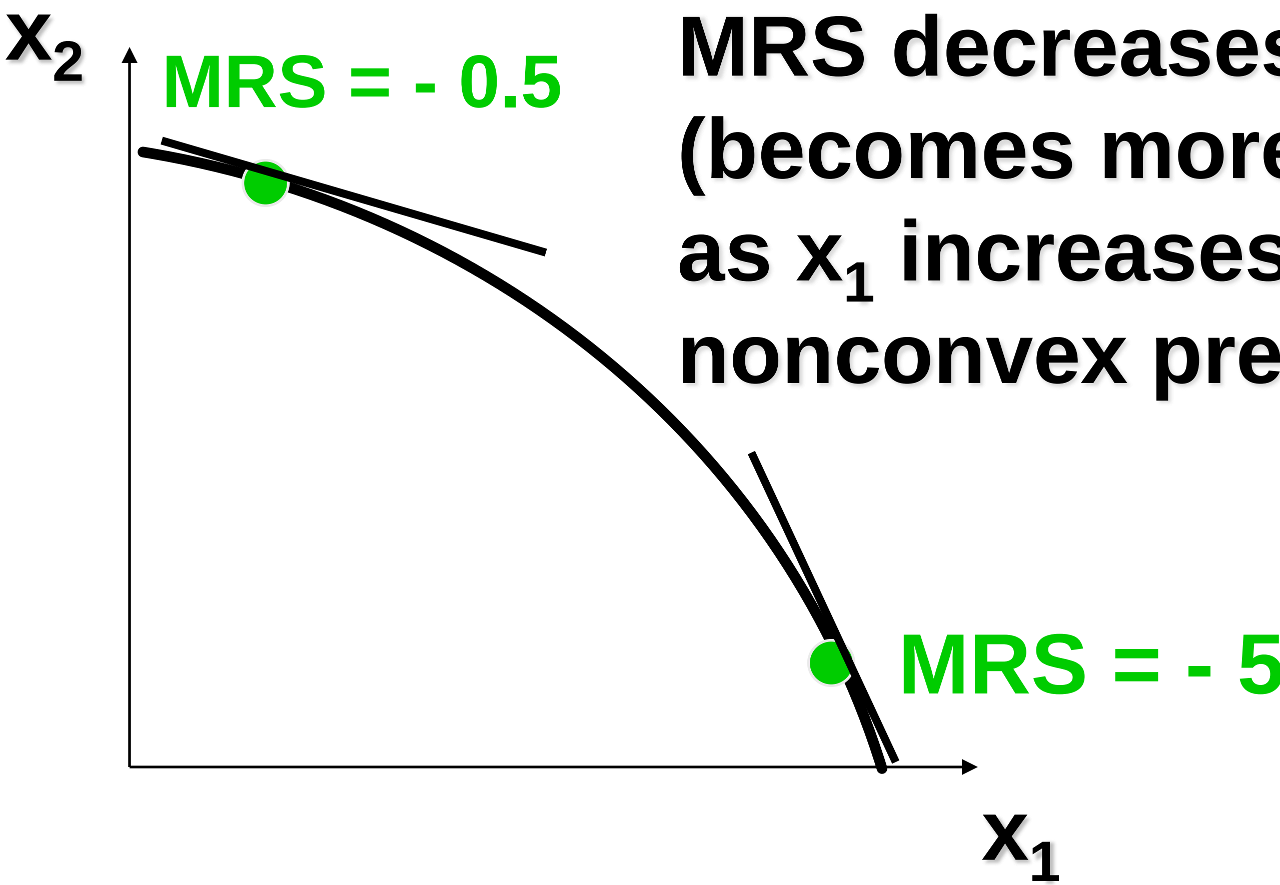
→ MRS > 0.

MRS & Ind. Curve Properties



MRS always increases with x_1 (becomes less negative) if and only if preferences are strictly convex.

MRS & Ind. Curve Properties

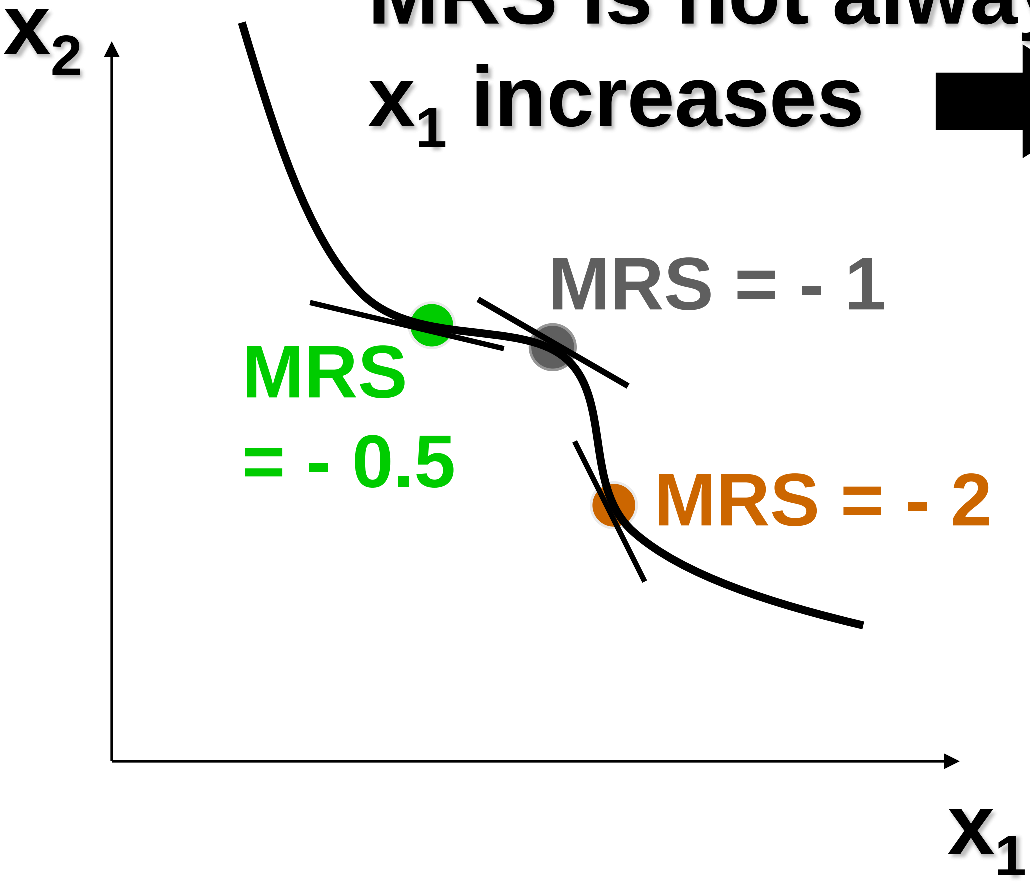


MRS decreases (becomes more negative) as x_1 increases

nonconvex preferences

MRS & Ind. Curve Properties

MRS is not always increasing as x_1 increases ➡ nonconvex preferences.



Preferences - A Reminder

- **x $\succ$ y: x is preferred strictly to y.**
- **x $\sim$ y: x and y are equally preferred.**
- **x $\succsim$ y: x is preferred at least as much as is y.**

Preferences - A Reminder

- **Completeness: For any two bundles x and y it is always possible to state either that**

  $$x \succsim y$$

  or that

  $$y \succsim x.$$

Preferences - A Reminder

- **Reflexivity: Any bundle x is always at least as preferred as itself; *i.e.***

$$\mathbf{x} \succsim \mathbf{x}.$$

Preferences - A Reminder

- **Transitivity: If**
  x is at least as preferred as y, and
  y is at least as preferred as z, then
  x is at least as preferred as z; *i.e.*

  $$x \succsim y \text{ and } y \succsim z \Rightarrow x \succsim z.$$

Utility Functions

- **A preference relation that is complete, reflexive, transitive and continuous can be represented by a continuous utility function.**
- **Continuity means that small changes to a consumption bundle cause only small changes to the preference level.**

Utility Functions

- **A utility function U(x) represents a preference relation $\succsim$ if and only if:**

$x' \succ x''$ $\Longleftrightarrow$ $U(x') > U(x'')$

$x' \prec x''$ $\Longleftrightarrow$ $U(x') < U(x'')$

$x' \sim x''$ $\Longleftrightarrow$ $U(x') = U(x'')$.

Utility Functions

- **Utility is an ordinal (i.e. ordering) concept.**
- ***E.g*. if U(x) = 6 and U(y) = 2 then bundle x is strictly preferred to bundle y. But x is not preferred three times as much as is y.**

Utility Functions & Indiff. Curves

- **Consider the bundles (4,1), (2,3) and (2,2).**
- **Suppose (2,3) $\succ$ (4,1) ~ (2,2).**
- **Assign to these bundles any numbers that preserve the preference ordering;**
  e.g. **U(2,3) = 6 > U(4,1) = U(2,2) = 4.**
- **Call these numbers utility levels.**

Utility Functions & Indiff. Curves

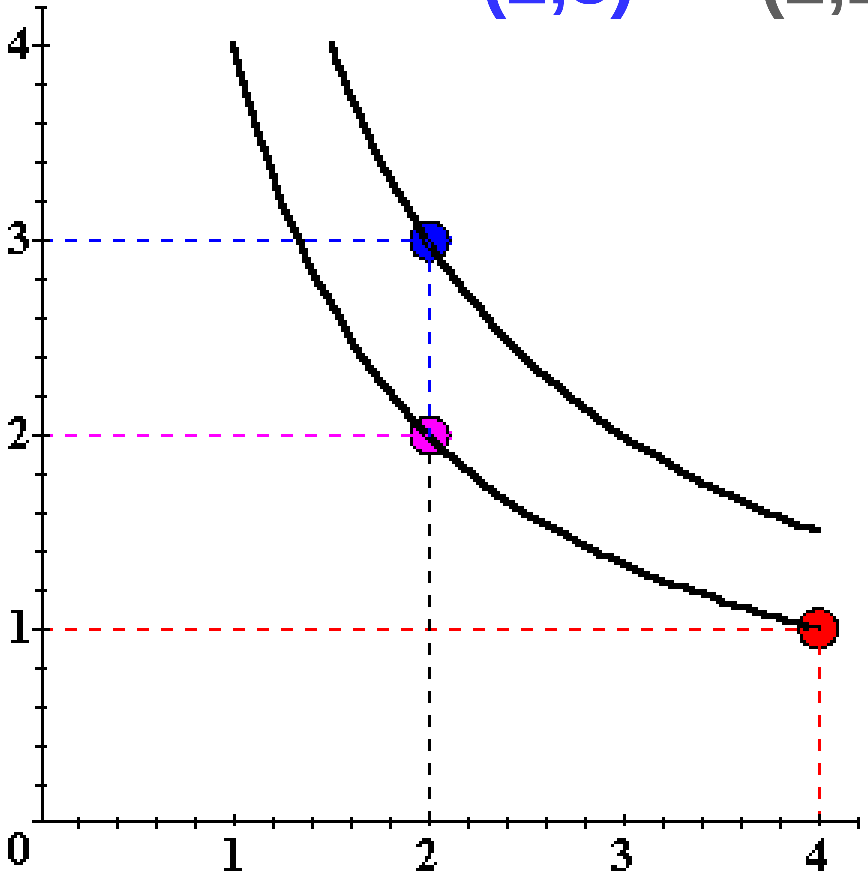
- **An indifference curve contains equally preferred bundles.**
- **Equal preference $\Rightarrow$ same utility level.**
- **Therefore, all bundles in an indifference curve have the same utility level.**

Utility Functions & Indiff. Curves

- **So the bundles (4,1) and (2,2) are in the indiff. curve with utility level U ≡ 4**
- **But the bundle (2,3) is in the indiff. curve with utility level U ≡ 6.**
- **On an indifference curve diagram, this preference information looks as follows:**

Utility Functions & Indiff. Curves

(2,3) (2,2) (4,1)

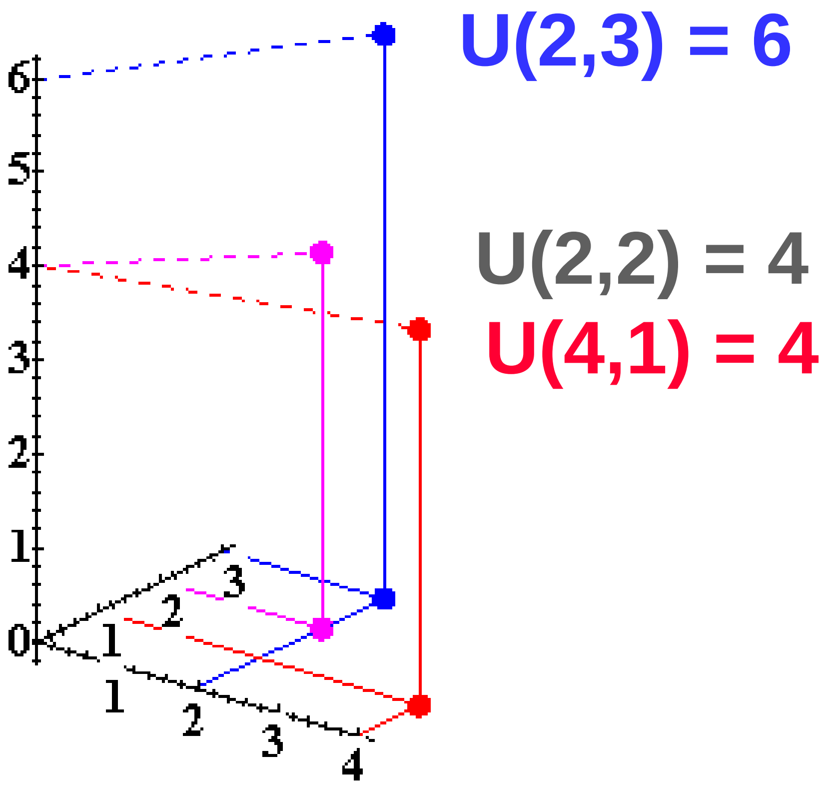


Utility Functions & Indiff. Curves

- **Another way to visualize this same information is to plot the utility level on a vertical axis.**

Utility Functions & Indiff. Curves

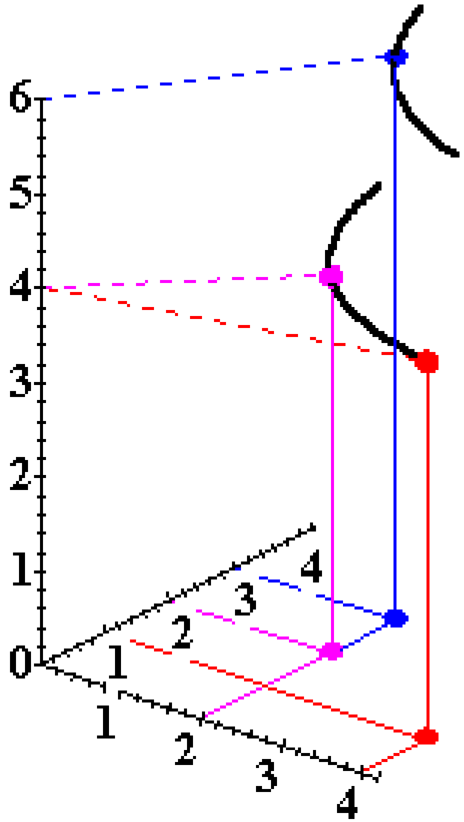
3D plot of consumption & utility levels for 3 bundles



Utility Functions & Indiff. Curves

- **This 3D visualization of preferences can be made more informative by adding into it the two indifference curves.**

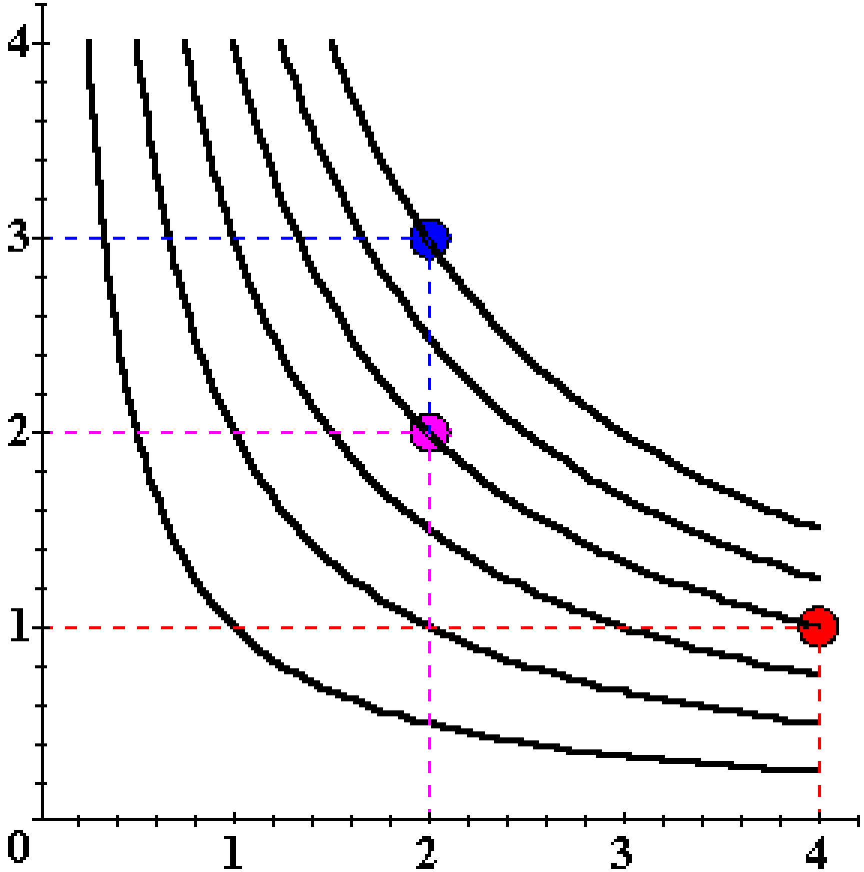
Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves

- **Comparing more bundles will create a larger collection of all indifference curves and a better description of the consumer's preferences.**

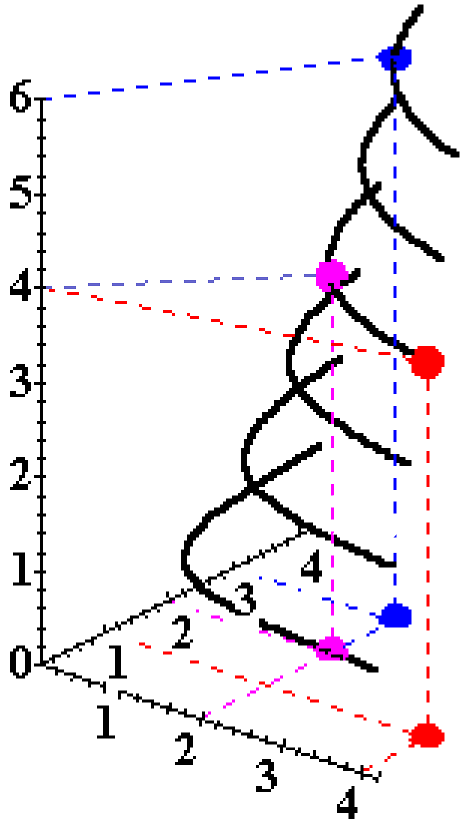
Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves

- **As before, this can be visualized in 3D by plotting each indifference curve at the height of its utility index.**

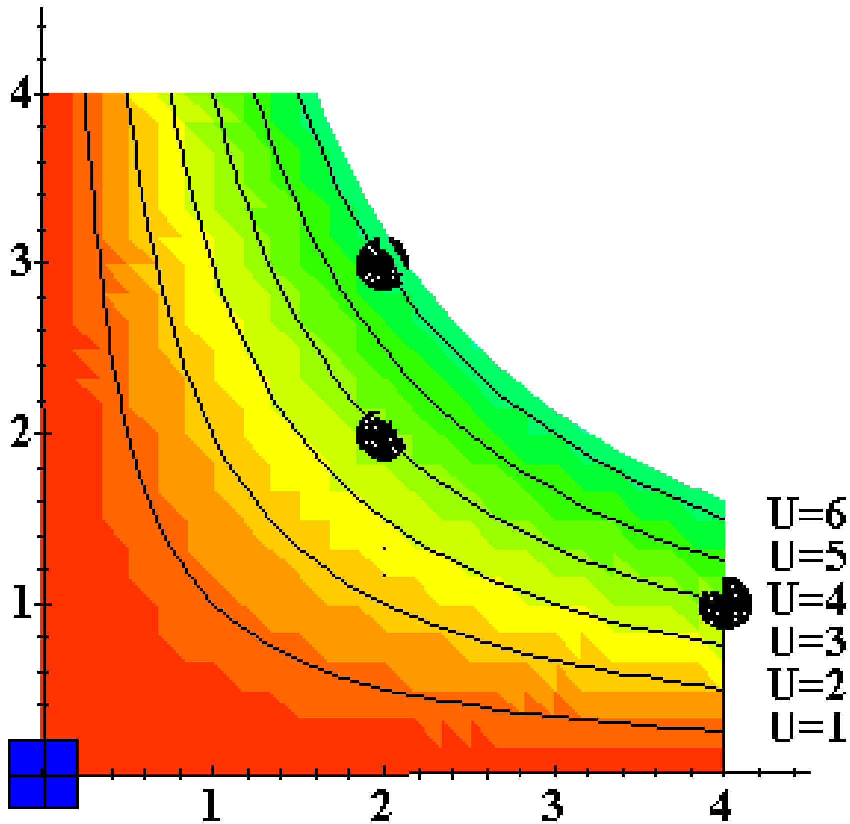
Utility Functions & Indiff. Curves



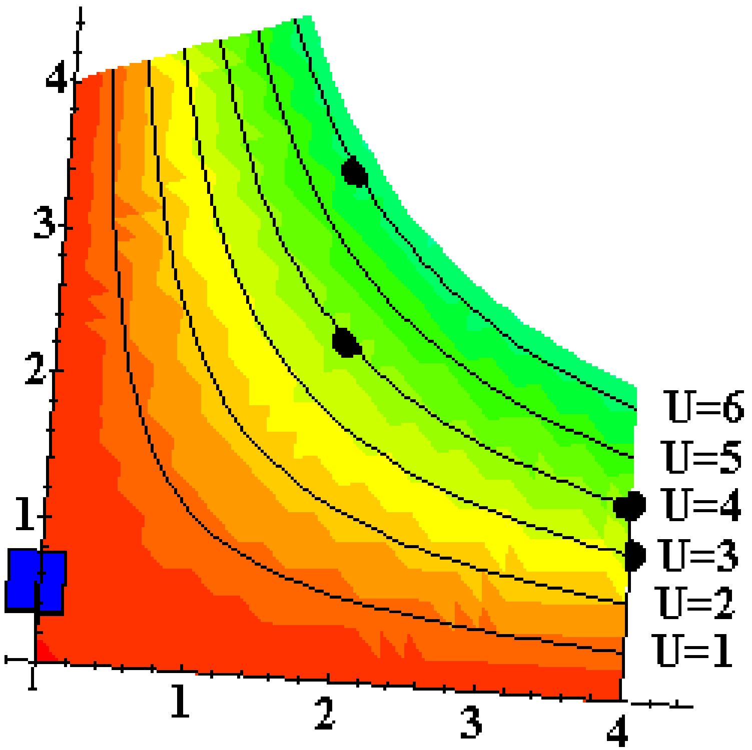
Utility Functions & Indiff. Curves

- **Comparing all possible consumption bundles gives the complete collection of the consumer's indifference curves, each with its assigned utility level.**
- **This complete collection of indifference curves completely represents the consumer's preferences.**

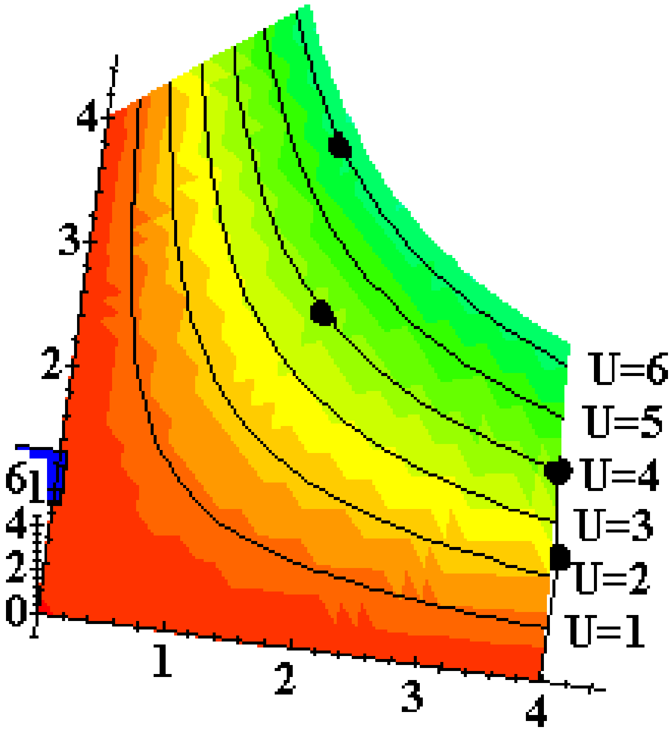
Utility Functions & Indiff. Curves



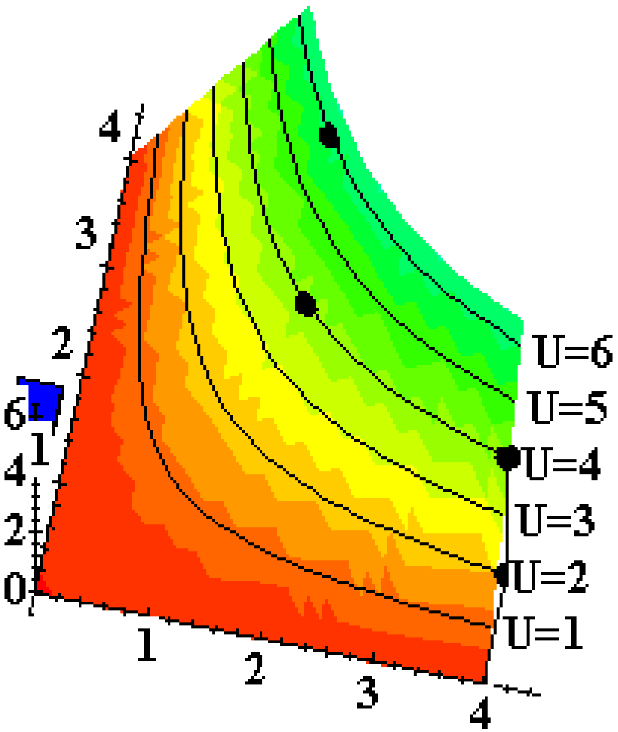
Utility Functions & Indiff. Curves



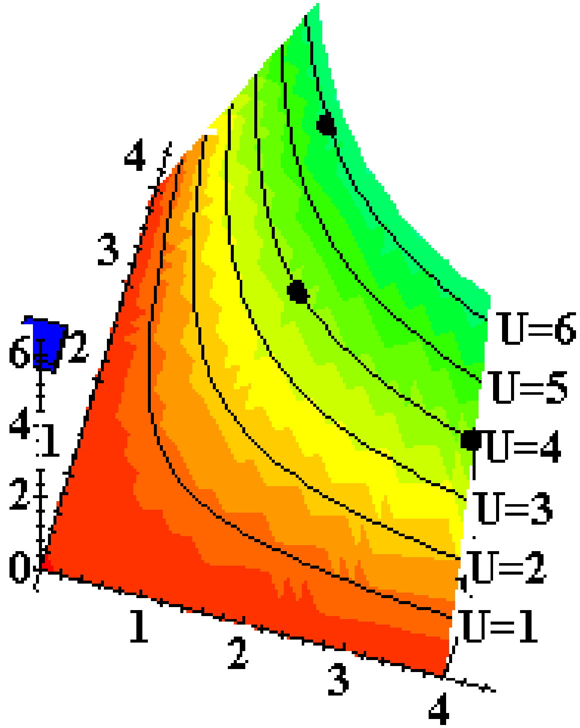
Utility Functions & Indiff. Curves



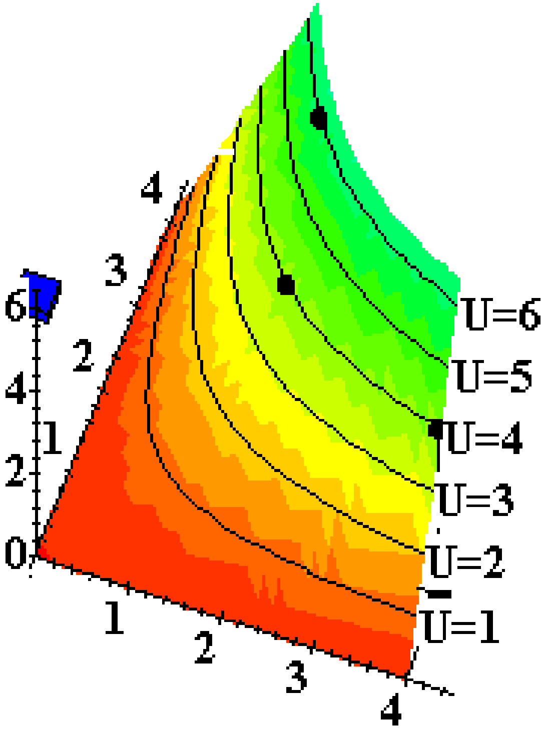
Utility Functions & Indiff. Curves



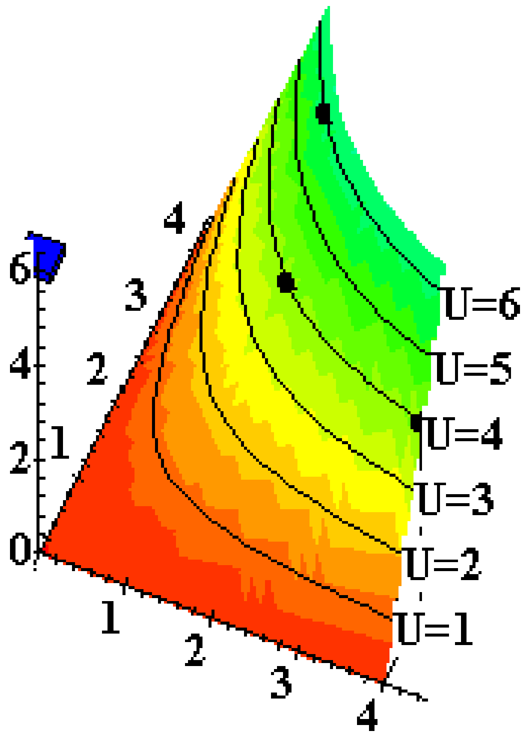
Utility Functions & Indiff. Curves



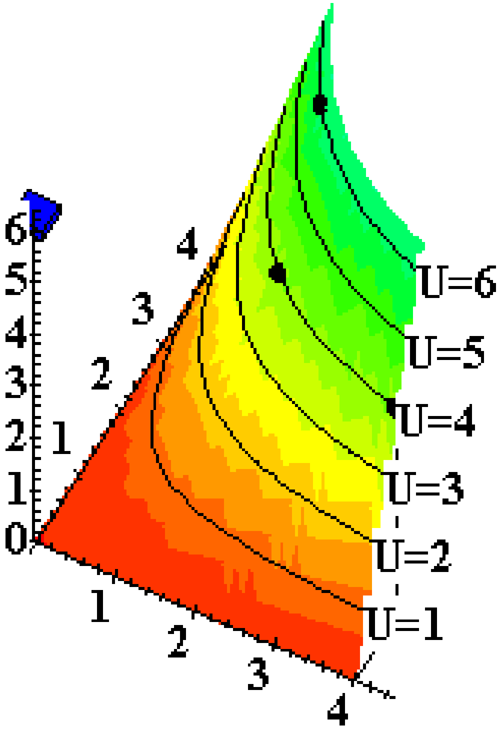
Utility Functions & Indiff. Curves



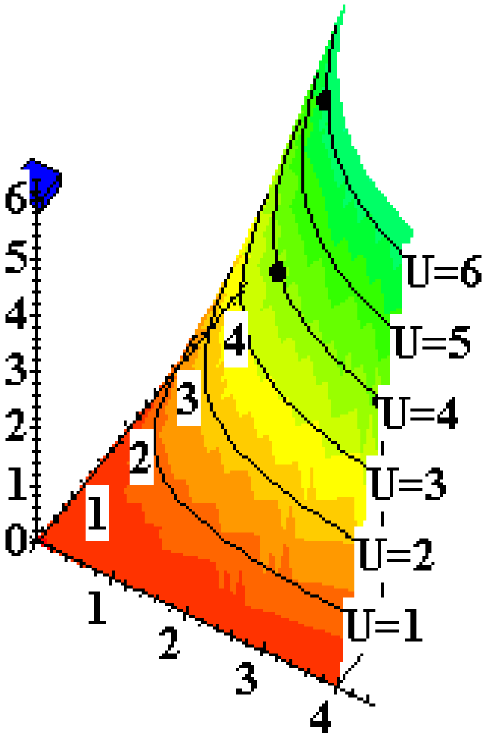
Utility Functions & Indiff. Curves



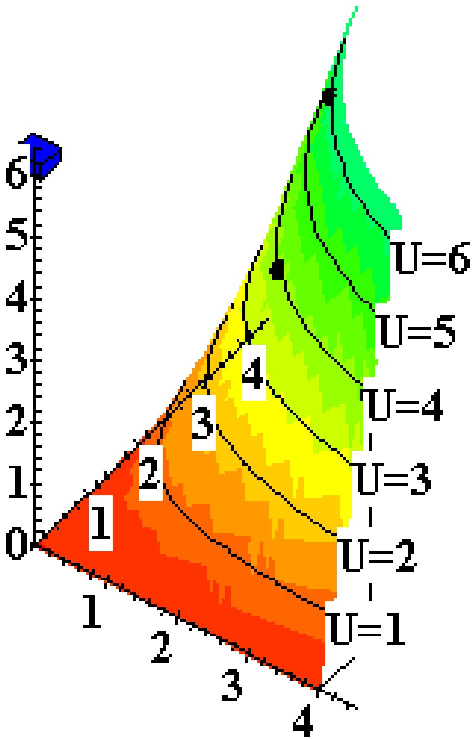
Utility Functions & Indiff. Curves



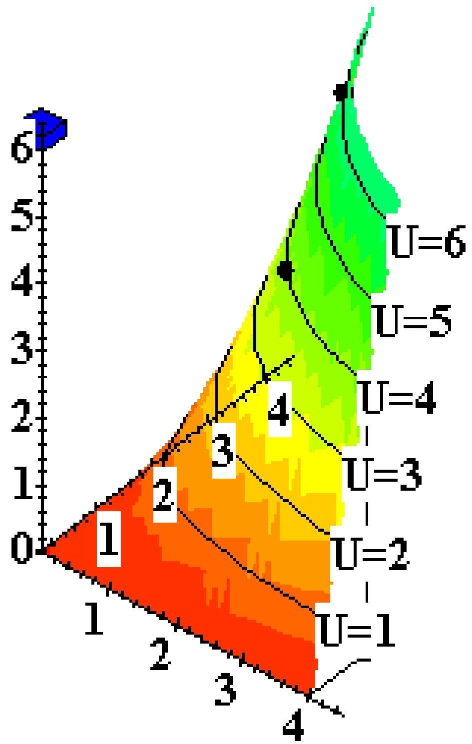
Utility Functions & Indiff. Curves



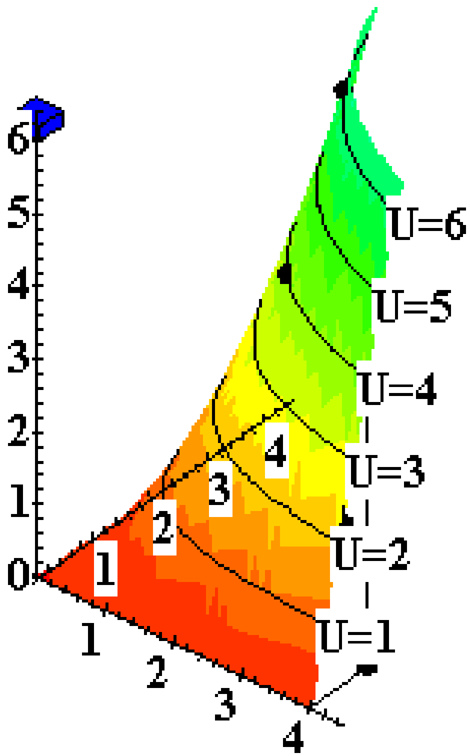
Utility Functions & Indiff. Curves



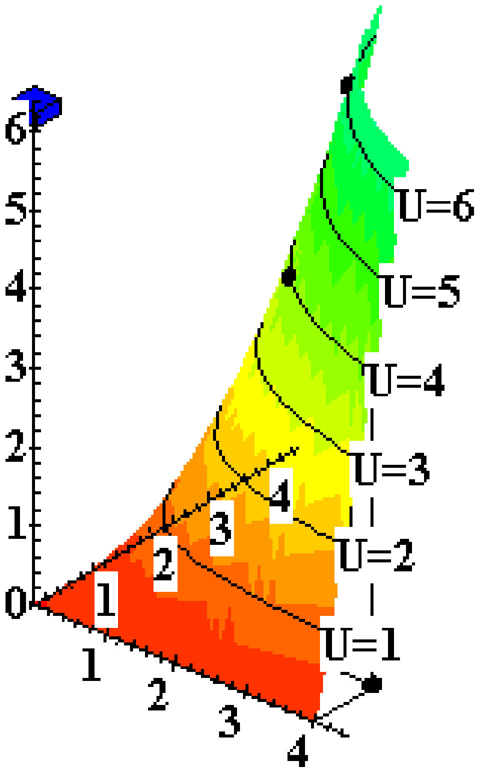
Utility Functions & Indiff. Curves



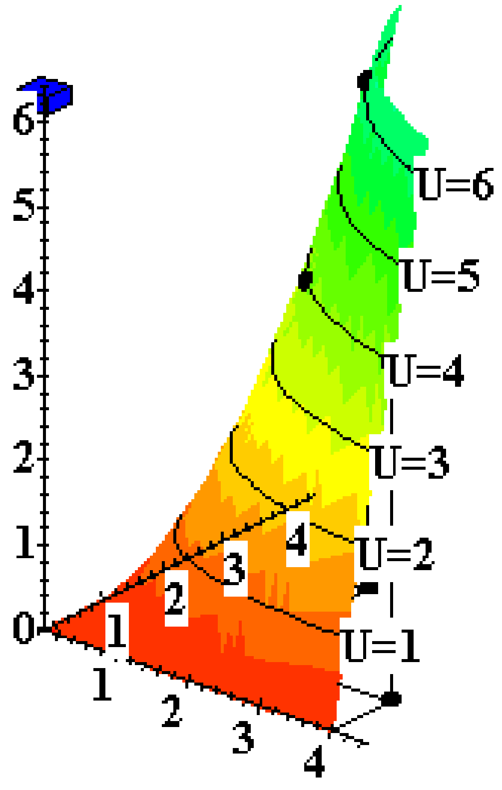
Utility Functions & Indiff. Curves



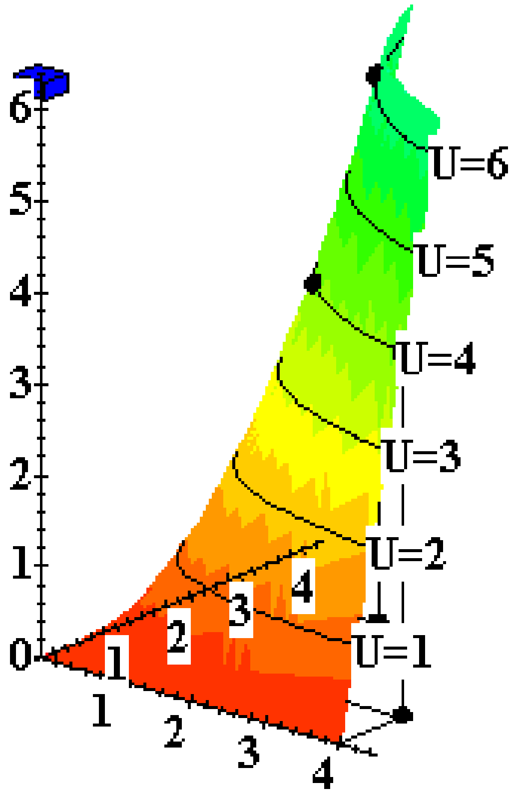
Utility Functions & Indiff. Curves



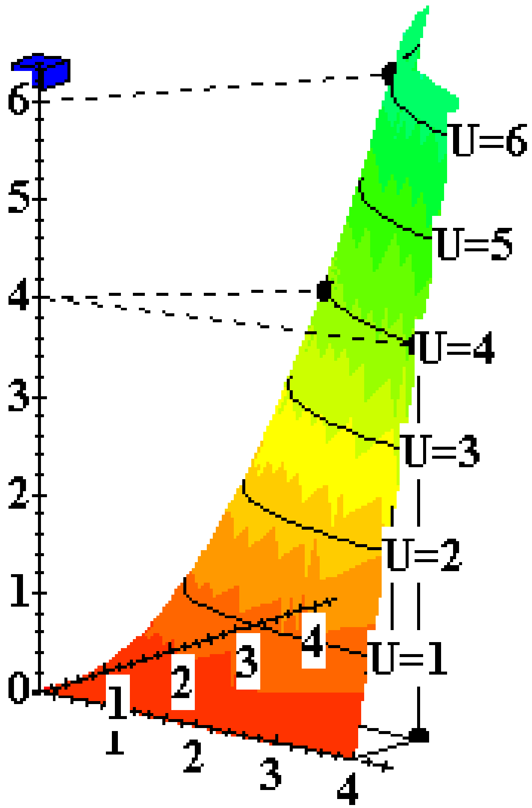
Utility Functions & Indiff. Curves

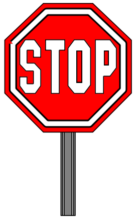


Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves





Utility Functions & Indiff. Curves

- **The collection of all indifference curves for a given preference relation is an indifference map.**
- **An indifference map is equivalent to a utility function; each is the other.**

Utility Functions

- **There is no unique utility function representation of a preference relation.**
- **Suppose $U(x_1,x_2) = x_1x_2$ represents a preference relation.**
- **Again consider the bundles (4,1), (2,3) and (2,2).**

Utility Functions

- **$U(x_1,x_2) = x_1x_2$, so**

  $U(2,3) = 6 > U(4,1) = U(2,2) = 4$;

  that is, $(2,3) \succ (4,1) \sim (2,2)$.

Utility Functions

- **$U(x_1,x_2) = x_1x_2$ ⟹ $(2,3) \succ (4,1) \sim (2,2)$.**
- **Define $V = U^2$.**

Utility Functions

- $U(x_1,x_2) = x_1x_2$ ⟹ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.
- Then $V(x_1,x_2) = x_1^2x_2^2$ and
  $V(2,3) = 36 > V(4,1) = V(2,2) = 16$
  so again
  $(2,3) \succ (4,1) \sim (2,2)$.
- V preserves the same order as U and so represents the same preferences.

Utility Functions

- **$U(x_1,x_2) = x_1x_2$ ⟹ $(2,3) \succ (4,1) \sim (2,2)$.**
- **Define W = 2U + 10.**

Utility Functions

- **$U(x_1,x_2) = x_1x_2$ ➡ $(2,3) \succ (4,1) \sim (2,2)$.**
- **Define W = 2U + 10.**
- **Then $W(x_1,x_2) = 2x_1x_2+10$ so $W(2,3) = 22 > W(4,1) = W(2,2) = 18$. Again, $(2,3) \succ (4,1) \sim (2,2)$.**
- **W preserves the same order as U and V and so represents the same preferences.**

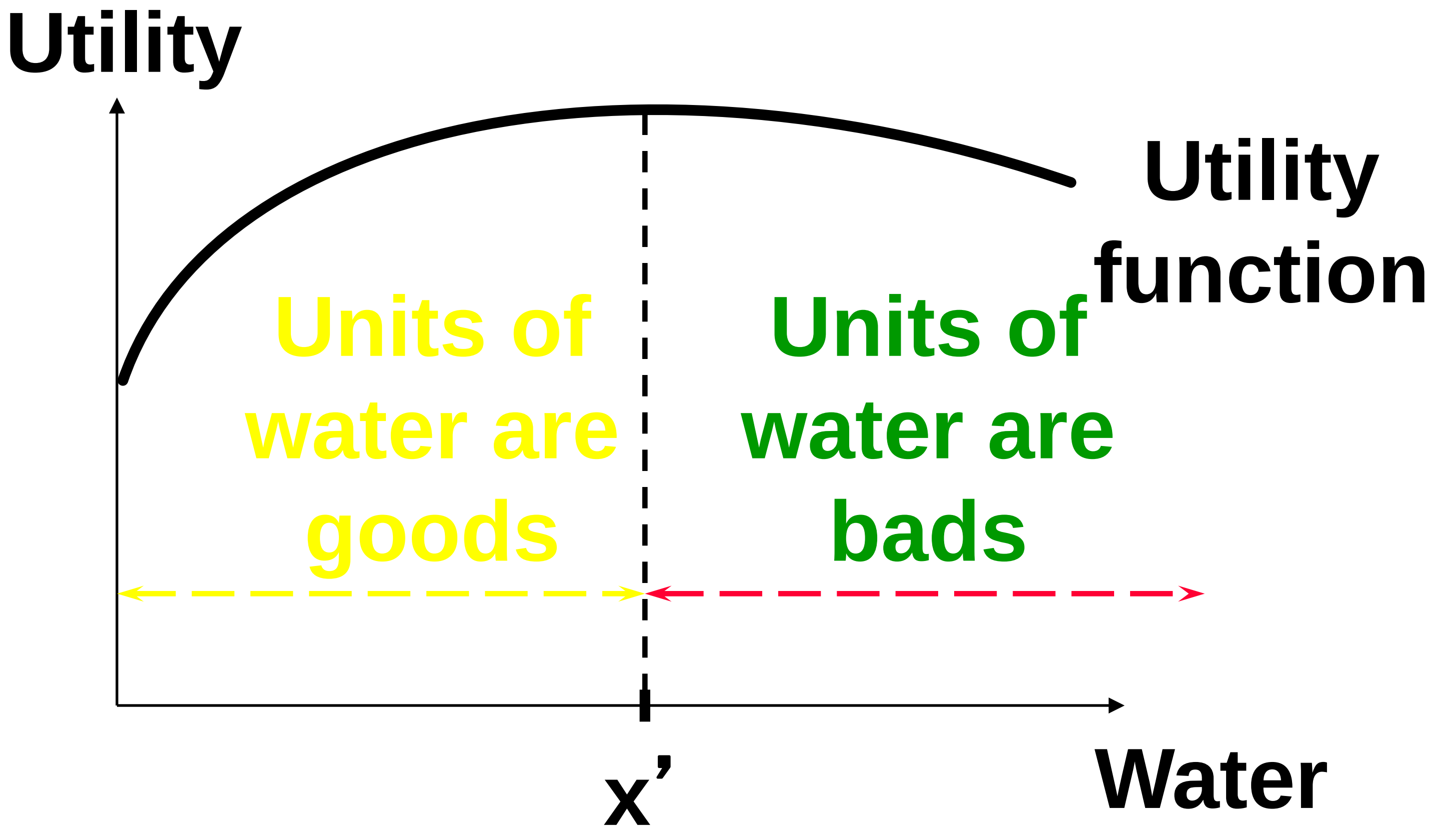
Utility Functions

- **If**
  - **U is a utility function that represents a preference relation $\succsim$ and**
  - **f is a strictly increasing function,**
- **then V = f(U) is also a utility function representing $\succsim$.**

Goods, Bads and Neutrals

- **A good is a commodity unit which increases utility (gives a more preferred bundle).**
- **A bad is a commodity unit which decreases utility (gives a less preferred bundle).**
- **A neutral is a commodity unit which does not change utility (gives an equally preferred bundle).**

Goods, Bads and Neutrals



Around x' units, a little extra water is a neutral.

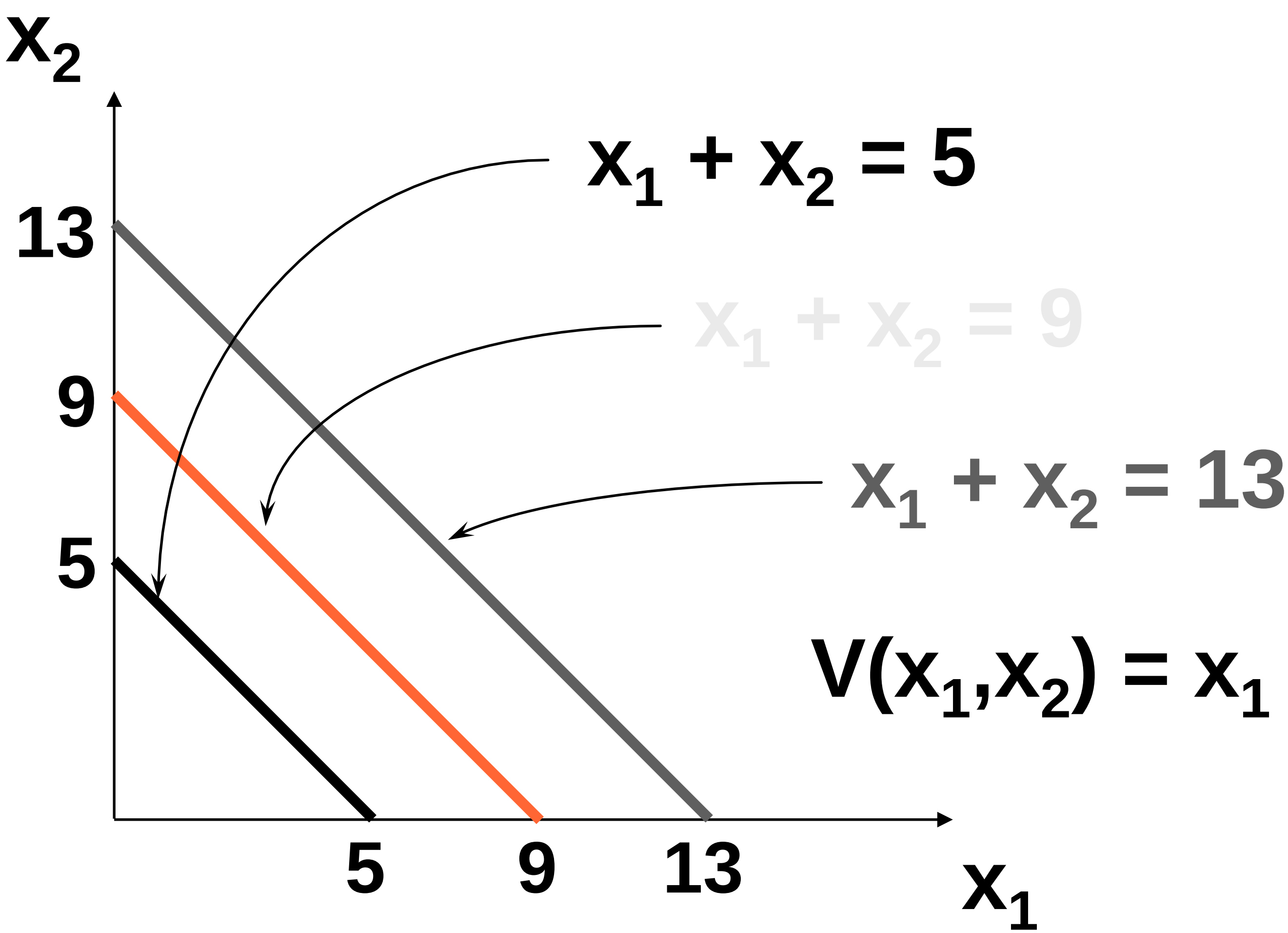
Some Other Utility Functions and Their Indifference Curves

- **Instead of $U(x_1,x_2) = x_1x_2$ consider**

$$V(x_1,x_2) = x_1 + x_2.$$

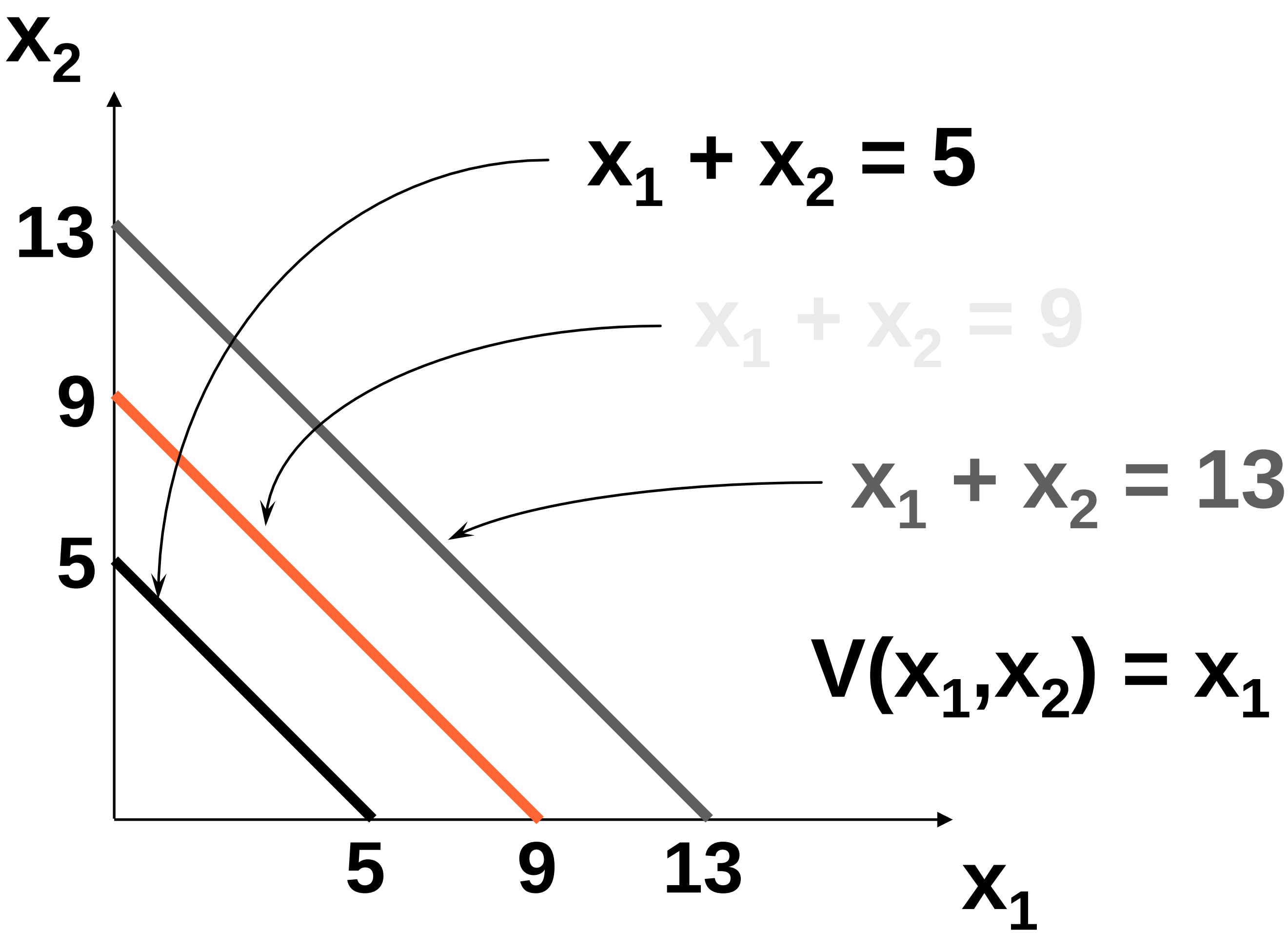
What do the indifference curves for this “perfect substitution” utility function look like?

Perfect Substitution Indifference Curves



$V(x_1,x_2) = x_1 + x_2.$

Perfect Substitution Indifference Curves



$$V(x_1,x_2) = x_1 + x_2.$$

All are linear and parallel.

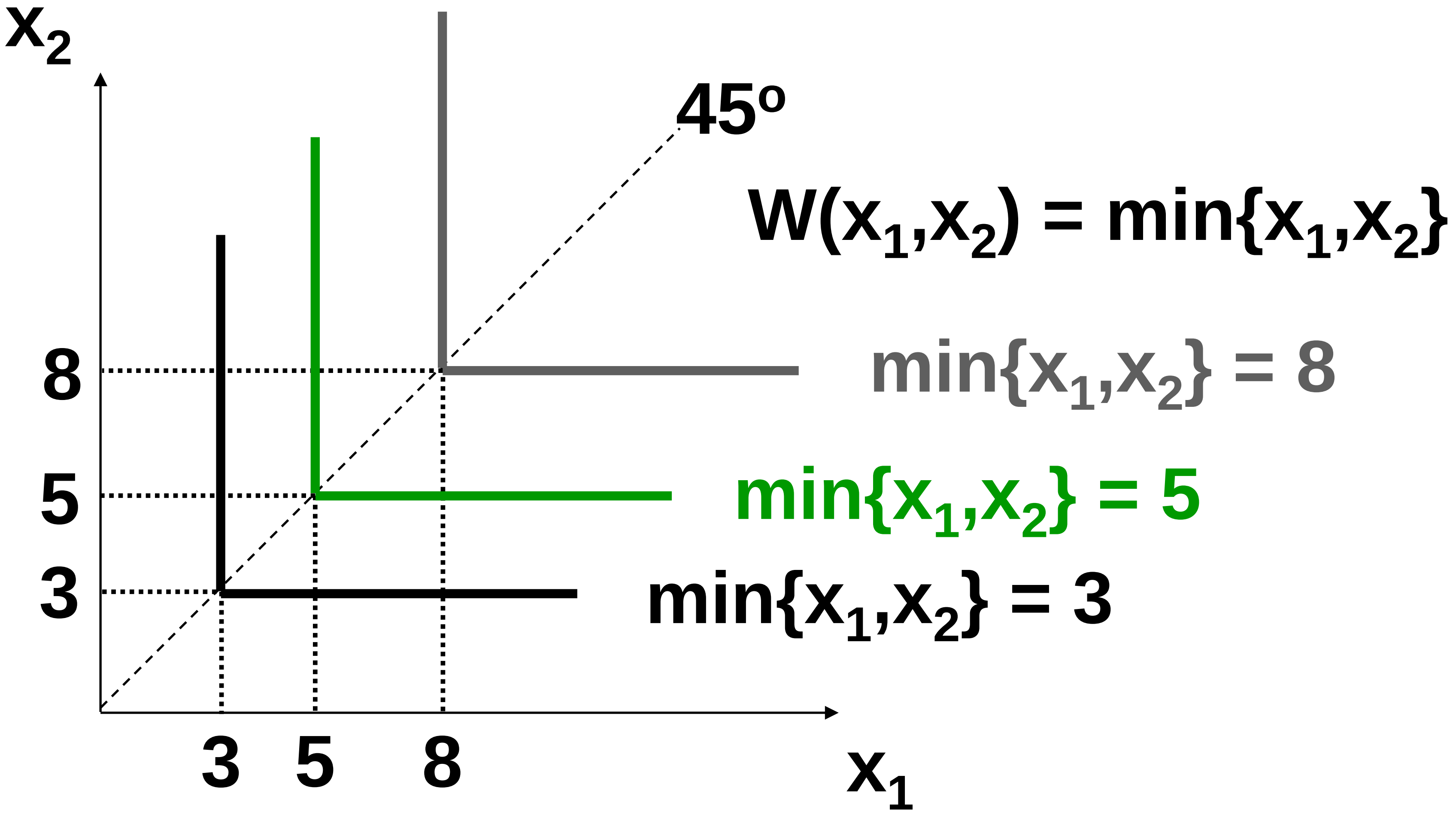
Some Other Utility Functions and Their Indifference Curves

- **Instead of $U(x_1,x_2) = x_1x_2$ or $V(x_1,x_2) = x_1 + x_2$, consider**

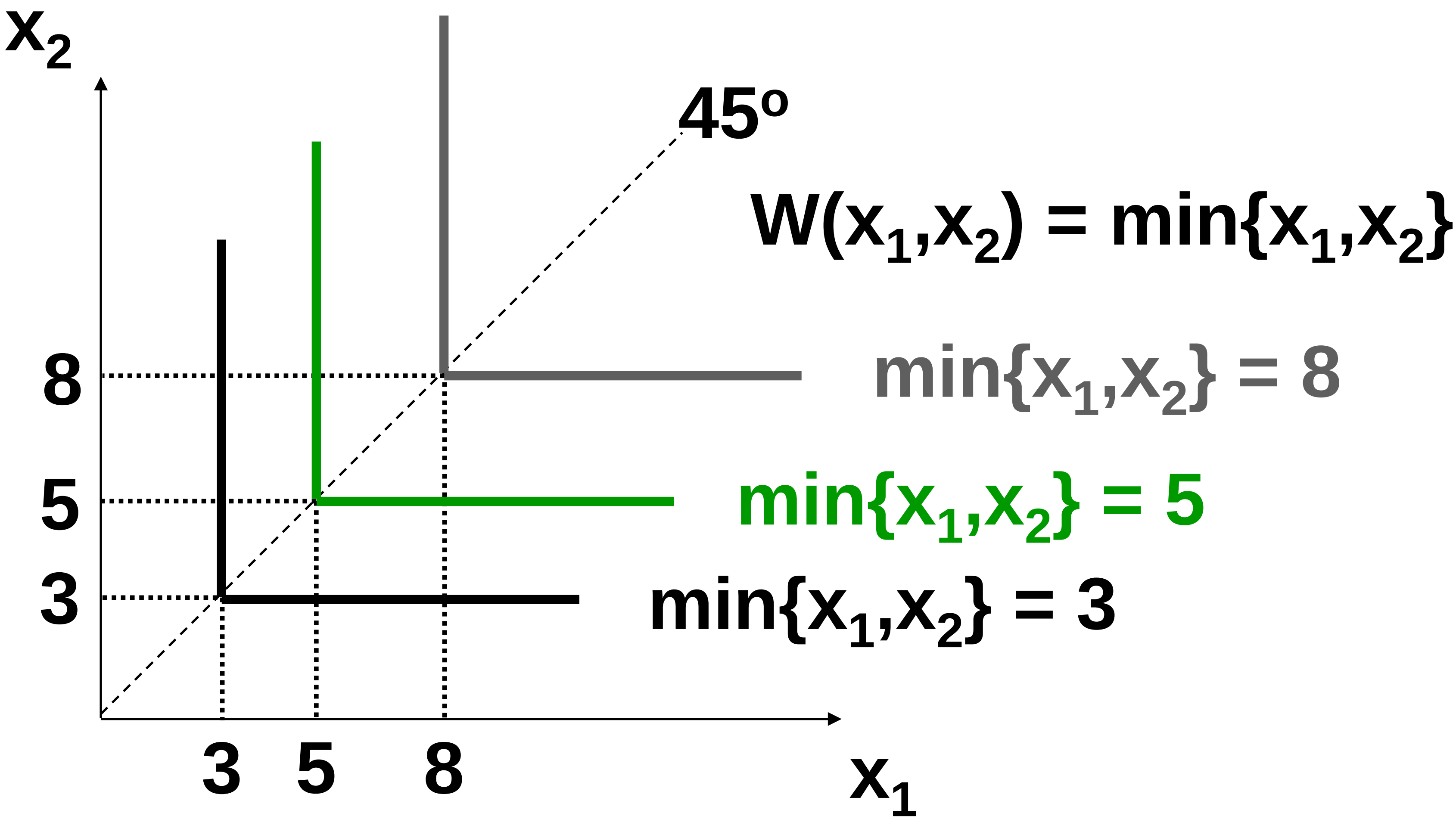
$$W(x_1,x_2) = \min\{x_1,x_2\}.$$

What do the indifference curves for this "perfect complementarity" utility function look like?

Perfect Complementarity Indifference Curves



Perfect Complementarity Indifference Curves



All are right-angled with vertices on a ray from the origin.

Some Other Utility Functions and Their Indifference Curves

- **A utility function of the form**

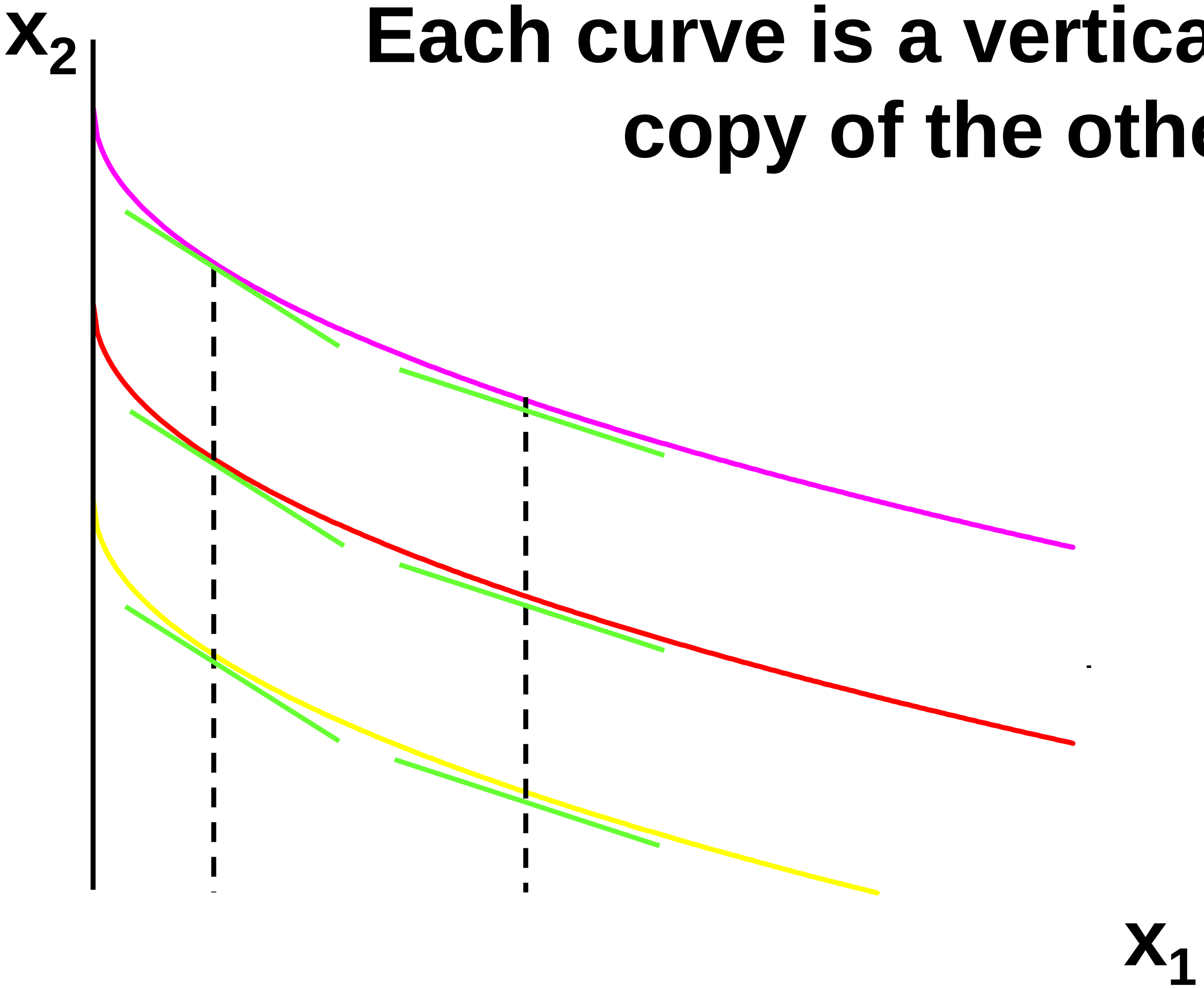
$$U(x_1,x_2) = f(x_1) + x_2$$

is linear in just x_2 and is called quasi-linear.

- ***E.g.*** **$U(x_1,x_2) = 2x_1^{1/2} + x_2$.**

Quasi-linear Indifference Curves

Each curve is a vertically shifted copy of the others.



Some Other Utility Functions and Their Indifference Curves

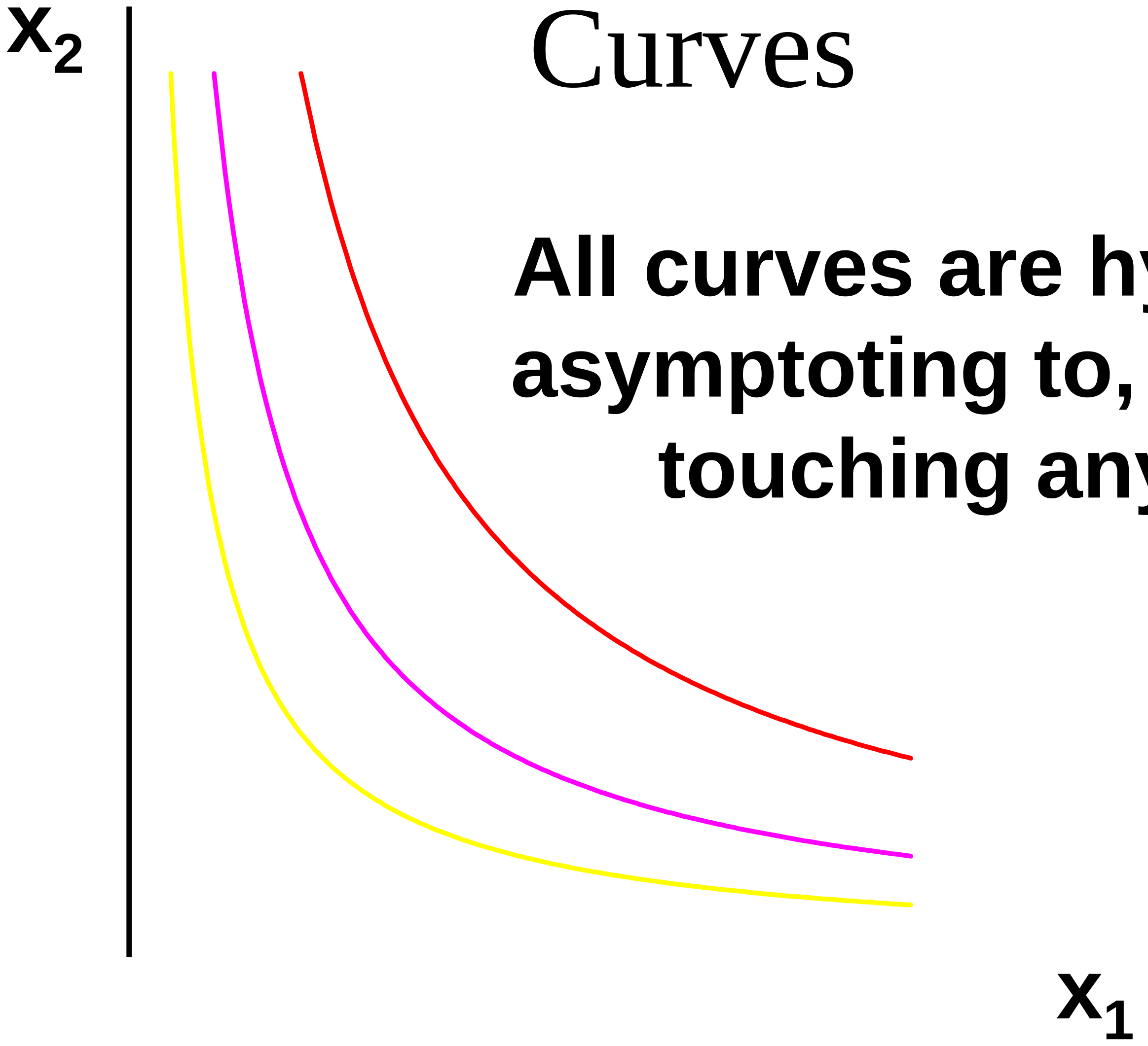
- **Any utility function of the form**

$$U(x_1,x_2) = x_1^a x_2^b$$

with a > 0 and b > 0 is called a Cobb-Douglas utility function.

- ***E.g.*** **$U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ (a = b = 1/2)**
  $V(x_1,x_2) = x_1 x_2^3$ (a = 1, b = 3)

Cobb-Douglas Indifference Curves



All curves are hyperbolic, asymptoting to, but never touching any axis.

Marginal Utilities

- **Marginal means "incremental".**
- **The marginal utility of commodity i is the rate-of-change of total utility as the quantity of commodity i consumed changes; *i.e.***

$$MU_i = \frac{\partial U}{\partial x_i}$$

Marginal Utilities

- *E.g.* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

Marginal Utilities

- *E.g.* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

Marginal Utilities

- *E.g.* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_2 = \frac{\partial U}{\partial x_2} = 2x_1^{1/2}x_2$$

Marginal Utilities

- *E.g.* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_2 = \frac{\partial U}{\partial x_2} = 2x_1^{1/2}x_2$$

Marginal Utilities

- *So,* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2$$

Marginal Utilities and Marginal Rates-of-Substitution

- **The general equation for an indifference curve is**

  $U(x_1,x_2) \equiv k$, a constant.

  Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

Marginal Utilities and Marginal Rates-of-Substitution

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

rearranged is

$$\frac{\partial U}{\partial x_2} dx_2 = -\frac{\partial U}{\partial x_1} dx_1$$

Marginal Utilities and Marginal Rates-of-Substitution

And

$$\frac{\partial U}{\partial x_2} dx_2 = -\frac{\partial U}{\partial x_1} dx_1$$

rearranged is

$$\frac{d\,x_2}{d\,x_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

This is the MRS.

Marg. Utilities & Marg. Rates-of-Substitution; An example

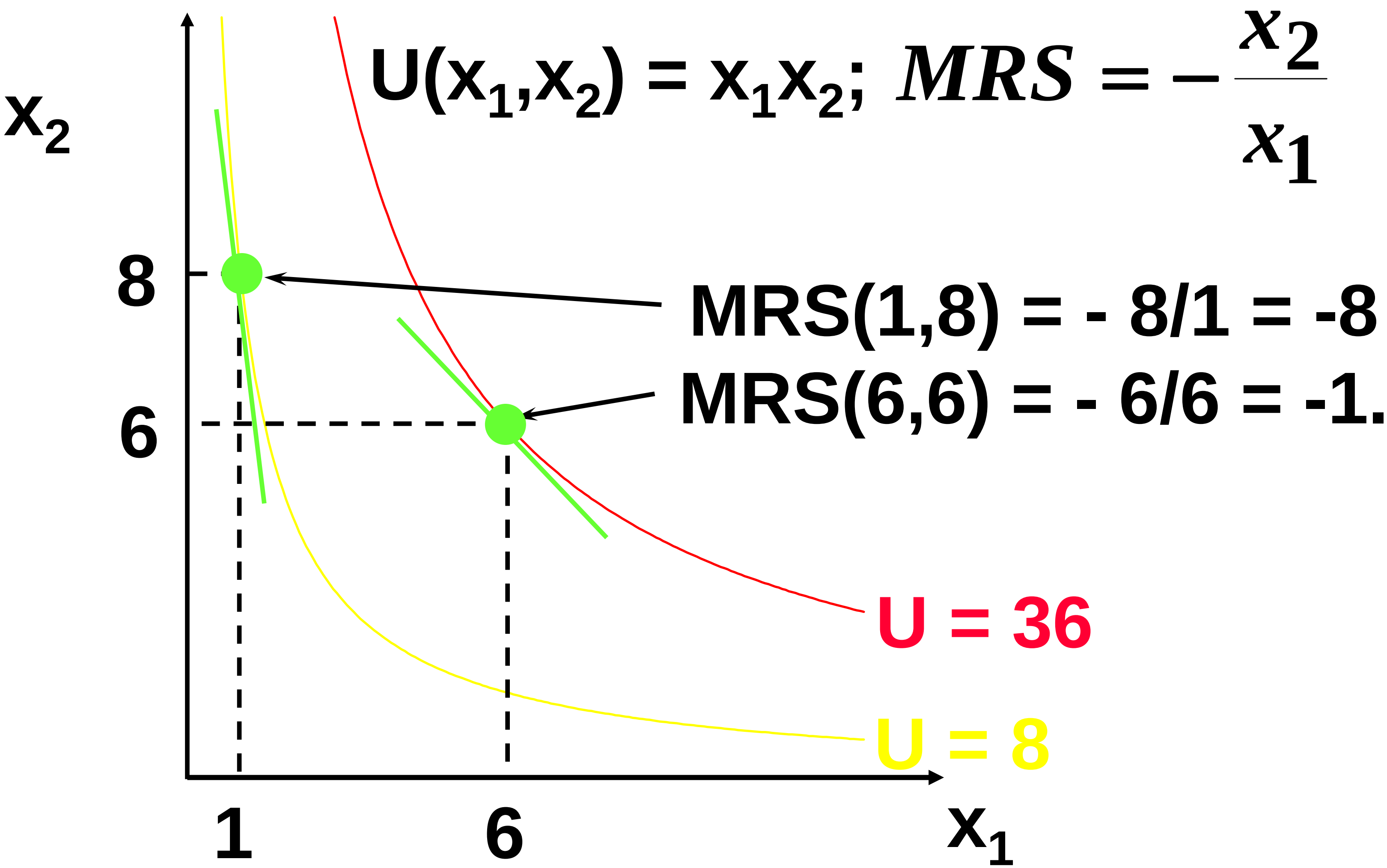
- **Suppose $U(x_1,x_2) = x_1x_2$. Then**

$$\frac{\partial U}{\partial x_1} = (1)(x_2) = x_2$$

$$\frac{\partial U}{\partial x_2} = (x_1)(1) = x_1$$

so $$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -\frac{x_2}{x_1}.$$

Marg. Utilities & Marg. Rates-of-Substitution; An example



Marg. Rates-of-Substitution for Quasi-linear Utility Functions

- A quasi-linear utility function is of the form $U(x_1,x_2) = f(x_1) + x_2$.

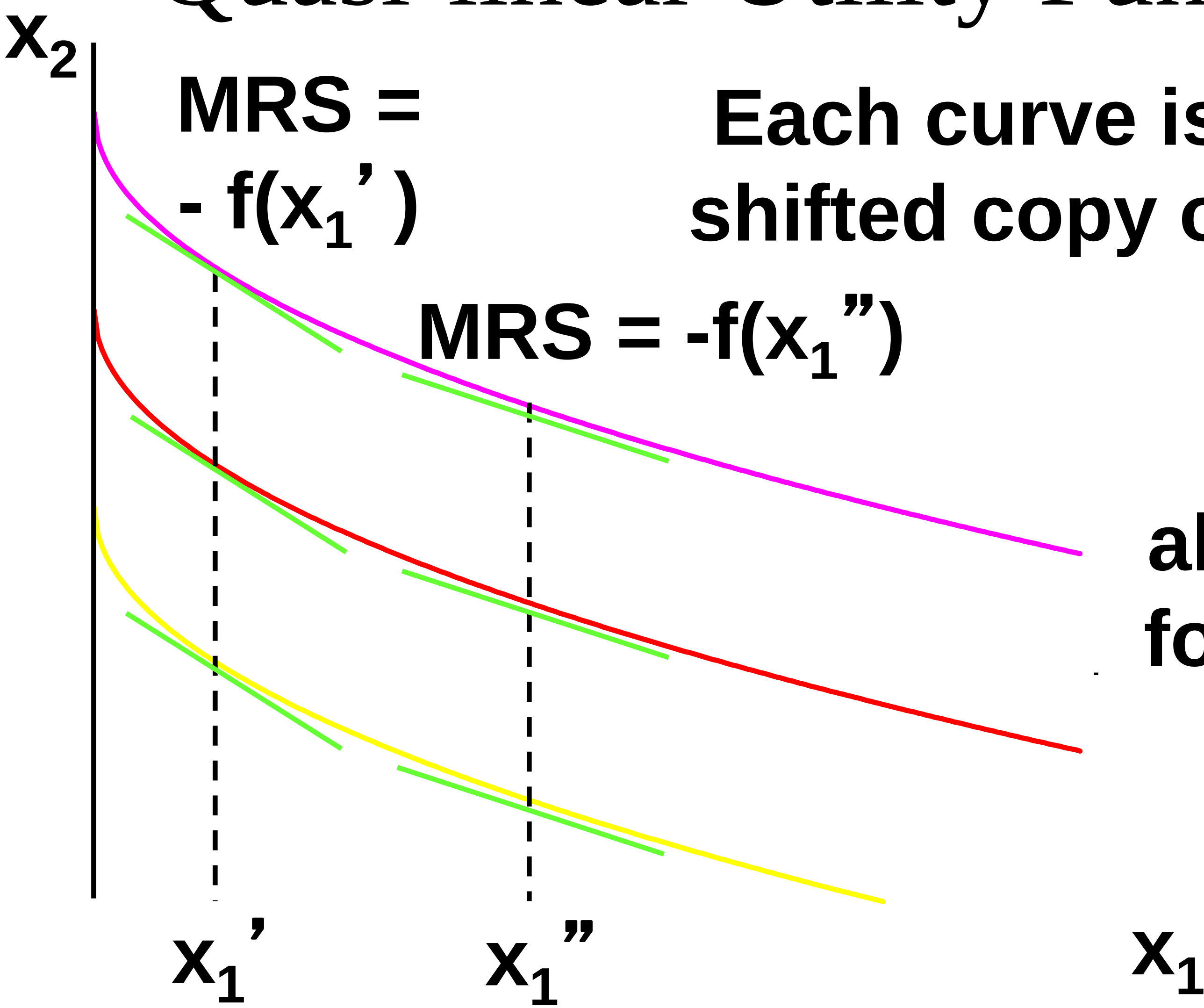
$$\frac{\partial U}{\partial x_1} = f'(x_1) \qquad \frac{\partial U}{\partial x_2} = 1$$

so $$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -f'(x_1).$$

Marg. Rates-of-Substitution for Quasi-linear Utility Functions

- **MRS = - f′ (x_1) does not depend upon x_2 so the slope of indifference curves for a quasi-linear utility function is constant along any line for which x_1 is constant. What does that make the indifference map for a quasi-linear utility function look like?**

Marg. Rates-of-Substitution for Quasi-linear Utility Functions



Each curve is a vertically shifted copy of the others.

MRS is a constant along any line for which x_1 is constant.

Monotonic Transformations & Marginal Rates-of-Substitution

- **Applying a monotonic transformation to a utility function representing a preference relation simply creates another utility function representing the same preference relation.**
- **What happens to marginal rates-of-substitution when a monotonic transformation is applied?**

Monotonic Transformations & Marginal Rates-of-Substitution

- **For $U(x_1,x_2) = x_1x_2$ the MRS = $-x_2/x_1$.**
- **Create $V = U^2$; *i.e.* $V(x_1,x_2) = x_1^2x_2^2$. What is the MRS for V?**

$$MRS = -\frac{\partial V/\partial x_1}{\partial V/\partial x_2} = -\frac{2x_1x_2^2}{2x_1^2x_2} = -\frac{x_2}{x_1}$$

which is the same as the MRS for U.

Monotonic Transformations & Marginal Rates-of-Substitution

- **More generally, if V = f(U) where f is a strictly increasing function, then**

$$MRS = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{f'(U) \times \partial U / \partial x_1}{f'(U) \times \partial U / \partial x_2}$$

$$= -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

So MRS is unchanged by a positive monotonic transformation.