Ask. 1

Δευτέρα, 1 Δεκεμβρίου 2025 12:11 μμ

(a) $\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1} = \sum_{\boxed{k=0}}^{\infty}\left(-\frac{3}{4}\right)^{k-1\boxed{+1}} = \sum_{k=0}^{\infty} 1\cdot\left(-\frac{3}{4}\right)^{k}$

είναι γεωμ. σειρά με $\alpha = 1$

κ' $r = -\frac{3}{4}$.

επειδή $|r| = \left|-\frac{3}{4}\right| < 1$, συγκλίνει

στο $\frac{\alpha}{1-r} = \frac{1}{1+\frac{3}{4}} = \frac{4}{7}$

(b) $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2} = \sum_{\boxed{k=0}}^{\infty}\left(\frac{2}{3}\right)^{k+2\boxed{+1}} = \sum_{k=0}^{\infty}\left(\frac{2}{3}\right)^{k+3}$

$= \sum_{k=0}^{\infty}\left(\frac{2}{3}\right)^{3}\cdot\left(\frac{2}{3}\right)^{k} = \sum_{k=0}^{\infty}\frac{8}{27}\cdot\left(\frac{2}{3}\right)^{k}$

Άρα γεωμ. σειρά με $\alpha = \frac{8}{27}$ κ' $r = \frac{2}{3}$

επειδή, $|r| = \left|\frac{2}{3}\right| < 1$, συγκλίνει

Επειδή, $|r| = \left|\frac{2}{3}\right| < 1$, συγκλίνει

στο $\frac{\alpha}{1-r} = \frac{\frac{8}{27}}{1-\frac{2}{3}} = \frac{3 \cdot 8}{27} = \frac{8}{9}$

(c) $\sum_{k=1}^{\infty}(-1)^{k-1}\frac{7}{6^{k-1}} = \sum_{k=1}^{\infty} 7 \cdot \frac{(-1)^{k-1}}{6^{k-1}} = \sum_{k=1}^{\infty} 7 \cdot \left(\frac{-1}{6}\right)^{k-1}$

$= \sum_{\boxed{k=0}}^{\infty} 7 \cdot \left(-\frac{1}{6}\right)^{k-1\boxed{+1}} = \sum_{k=0}^{\infty} 7 \cdot \left(-\frac{1}{6}\right)^{k}$

είναι γεωμ. σειρά με $\alpha = 7$, $r = -\frac{1}{6}$

Επειδή $|r| = \left|-\frac{1}{6}\right| < 1$ συγκλίνει

$\frac{\alpha}{1-r} = \frac{7}{1-\left(-\frac{1}{6}\right)} = 6$

(d) $\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^{k+1} = \sum_{\boxed{k=0}}^{\infty} \left(-\frac{3}{2}\right)^{k+1\boxed{+1}} = \sum_{k=0}^{\infty} \left(-\frac{3}{2}\right)^{k+2}$

$$= \sum_{k=0}^{\infty} \left(-\frac{3}{2}\right)^2 \cdot \left(-\frac{3}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \frac{9}{4} \left(-\frac{3}{2}\right)^k$$

Είναι Γεωμ. σειρά με $\alpha = \frac{9}{4}$, $r = -\frac{3}{2}$

Επειδή $|r| = \left|-\frac{3}{2}\right| > 1$, αποκλίνει.

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

$$\frac{1}{(k+2)(k+3)} = \frac{A}{k+2} + \frac{B}{k+3}$$

Α' τρόπος

Heaviside

• πολ/ζω με $k+2$

$$\frac{1}{k+3} = A + \frac{B\cdot(k+2)}{k+3}$$

Για $k=-2 \Rightarrow A = 1$

• πολλ/ζω με $k+3$

$$\frac{1}{k+2} = \frac{A\cdot(k+3)}{k+2} + B$$

Για $k=-3 \Rightarrow B=-1$

Β' τρόπος

$$\frac{1}{(k+2)(k+3)} = \frac{A}{k+2} + \frac{B}{k+3}$$

$$= \frac{A(k+3)}{(k+2)(k+3)} + \frac{B(k+2)}{(k+2)(k+3)}$$

$$= \frac{A(k+3)+B(k+2)}{(k+2)(k+3)}$$

$$= \frac{(A+B)k+3\cdot A+2B}{(k+2)(k+3)}$$

$$(\Rightarrow) \quad \frac{1}{(k+2)\cancel{(k+3)}} = \frac{(A+B)k+3\cdot A+2B}{(k+2)\cancel{(k+3)}}$$

$$\frac{1}{\cancel{(k+2)(k+3)}} = \frac{[illegible]}{\cancel{(k+2)(k+3)}}$$

$$\Rightarrow 1 = (A+B)k + 3A + 2B$$

$$\Rightarrow (A+B)k + (3A+2B-1) = 0$$

Άρα

$$\Rightarrow \begin{cases} A+B=0 \Rightarrow A=-B \\ 3A+2B-1=0 \Rightarrow -3B+2B-1=0 \end{cases}$$

$$\Rightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$$

Άρα

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)} = \sum_{k=1}^{\infty}\left(\frac{1}{k+2} - \frac{1}{k+3}\right)$$ τηλεσκ. σειρά

$$S_k = \left(\frac{1}{1+2} - \cancel{\frac{1}{1+3}}\right) + \left(\cancel{\frac{1}{2+2}} - \cancel{\frac{1}{2+3}}\right) + \left(\cancel{\frac{1}{3+2}} - \cancel{\frac{1}{3+3}}\right)$$

$$+ \left(\cancel{\frac{1}{4+2}} - \cdots - \cancel{\frac{1}{k}}\right) + \left(\cancel{\frac{1}{k}} - \cancel{\frac{1}{k+1}}\right) + \left(\cancel{\frac{1}{k+1}} - \cancel{\frac{1}{k+2}}\right) + \left(\cancel{\frac{1}{k+2}} - \frac{1}{k+3}\right)$$

$$= \frac{1}{1+2} - \frac{1}{k+3} = \frac{1}{3} - \frac{1}{k+3}$$

$$\lim_{k \to +\infty} S_k = \lim_{k \to +\infty} \frac{1}{3} - \underbrace{\frac{1}{k+3}}_{\to 0} = \frac{1}{3}$$

Άρα η σειρά συγκλίνει στο $\frac{1}{3}$.

(f) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{2^k} - \frac{1}{2^k \cdot 2}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{2^k} - \frac{1}{2^k} \cdot \frac{1}{2}\right)$

$$= \sum_{k=1}^{\infty} \frac{1}{2^k} \cdot \left(1 - \frac{1}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{2} \cdot \frac{1}{2^k}$$

$$= \sum^{\infty} \frac{1}{\ } \cdot \left(\frac{1}{\ }\right)^k - \sum^{\infty} \frac{1}{\ } \left(\frac{1}{\ }\right)^{k+1}$$

$$= \sum_{k=1}^{\infty} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{k} = \sum_{k=0}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{k+1}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{k} \cdot \frac{1}{2} = \sum_{k=0}^{\infty} \frac{1}{4}\left(\frac{1}{2}\right)^{k}$$

Γεωμ. σειρά με $\alpha = \frac{1}{4}$, $r = \frac{1}{2}$

Επειδή $|r| = \left|\frac{1}{2}\right| < 1$ συγκλίνει

στο $\frac{\alpha}{1-r} = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$

(g) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{9k^2 + 3k - 2} = *$

$\Delta = 3^2 - 4 \cdot 9 \cdot (-2) = 81$

$\rho_{1,2} = \frac{-3 \pm \sqrt{\Delta}}{2 \cdot 9} = \begin{cases} -\frac{2}{3} \\ \frac{1}{3} \end{cases}$

$$(*) = \sum_{k=1}^{\infty} \frac{1}{9\cdot\left(k+\frac{2}{3}\right)\left(k-\frac{1}{3}\right)} = \frac{1}{9}\sum_{k=1}^{\infty} \frac{1}{\left(k+\frac{2}{3}\right)\left(k-\frac{1}{3}\right)}$$

Άρα

$$\frac{1}{\left(k+\frac{2}{3}\right)\left(k-\frac{1}{3}\right)} = \frac{A}{k+\frac{2}{3}} + \frac{B}{k-\frac{1}{3}}$$

Heaviside

- πολ/ζω με $k-\frac{1}{3}$

$$\frac{1}{k+\frac{2}{3}} = \frac{A\cdot\left(k-\frac{1}{3}\right)}{k+\frac{2}{3}} + B$$

Για $k=\frac{1}{3}$ έχω $B=1$

- πολ/ζω με $k+\frac{2}{3}$

$$\frac{1}{} = A + \frac{B\left(k+\frac{2}{3}\right)}{}$$

$$\frac{1}{k-\frac{1}{3}} = A + \frac{B\left(k+\frac{2}{3}\right)}{k-\frac{1}{3}}$$

Για $k=-\frac{2}{3}$, έχουμε $A=-1$

Άρα

$$\frac{1}{9}\sum_{k=1}^{\infty}\frac{1}{\left(k+\frac{2}{3}\right)\left(k-\frac{1}{3}\right)} = \frac{1}{9}\cdot\sum_{k=1}^{\infty}\left(\frac{-1}{k+\frac{2}{3}}+\frac{1}{k-\frac{1}{3}}\right)$$

$*$

$$= \boxed{\frac{1}{9}}\ \boxed{\sum_{k=1}^{\infty}\left(\frac{1}{k-\frac{1}{3}}-\frac{1}{k+\frac{2}{3}}\right)}$$ τηλεσκ. σειρά

$$S_k = \left(\frac{1}{1-\frac{1}{3}}-\frac{1}{1+\frac{2}{3}}\right)+\left(\frac{1}{2-\frac{1}{3}}-\frac{1}{2+\frac{2}{3}}\right)+\dots$$

$$= \left(\frac{1}{\frac{2}{3}}-\cancel{\frac{1}{\frac{5}{3}}}\right)+\left(\cancel{\frac{1}{\frac{5}{3}}}-\cancel{\frac{1}{\frac{8}{3}}}\right)+$$

$$3 \quad \cancel{\ } \quad 3 \quad \cancel{\ } \quad \frac{ }{3} \quad \cancel{\ } \quad \frac{8}{3} \Big)$$

$$\ldots\ldots + \left(\cancel{\frac{1}{k-\frac{1}{3}}} - \frac{1}{k+\frac{2}{3}} \right)$$

$$= \frac{1}{\frac{2}{3}} - \frac{1}{k+2/3} = \frac{3}{2} - \frac{1}{k+\frac{2}{3}}$$

Άρα

$$\lim_{k \to +\infty} \frac{3}{2} - \cancel{\frac{1}{k+\frac{2}{3}}}^{\to 0} = \frac{3}{2}$$

Άρα η σειρά συγκλίνει

στο $\boxed{\frac{1}{9}} \cdot \frac{3}{2} = \frac{1}{6}$

(h) $\sum_{k=2}^{\infty} \frac{1}{k^2-1} = \sum_{k=2}^{\infty} \frac{1}{(k+1)(k-1)} = \sum_{\boxed{k=1}}^{\infty} \frac{1}{(k+1\boxed{+1})(k-1\boxed{+1})}$

∞

$$= \sum_{k=1}^{\infty} \frac{1}{(k+2)k}$$ Τμηλεοκ. σημεα.

$$\frac{1}{k(k+2)} = \frac{A}{k} + \frac{B}{k+2}$$

$$= \frac{A\cdot(k+2)+B\cdot k}{k(k+2)}$$

$$\Leftrightarrow \frac{1}{k(k+2)} = \frac{A(k+2)+Bk}{k(k+2)}$$

$$\Leftrightarrow 1 = A(k+2) + Bk$$

$$\Leftrightarrow 1 = (A+B)\cdot k + 2A$$

$$\Leftrightarrow (A+B)k + 2A - 1 = 0$$

$$\Rightarrow \begin{cases} 2A-1=0 \Rightarrow A=\frac{1}{2} \\ A+B=0 \Rightarrow B=-\frac{1}{2} \end{cases}$$

Άρα

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)k} = \sum_{k=1}^{\infty} \frac{\frac{1}{2}}{k} - \frac{\frac{1}{2}}{k+2}$$

$$S_k = \left(\boxed{\frac{\frac{1}{2}}{1}} - \cancel{\frac{\frac{1}{2}}{1+2}}\right) + \left(\boxed{\frac{\frac{1}{2}}{2}} - \cancel{\frac{\frac{1}{2}}{2+2}}\right)$$

$$+ \left(\cancel{\frac{\frac{1}{2}}{3}} - \cancel{\frac{\frac{1}{2}}{3+2}}\right) + \left(\cancel{\frac{\frac{1}{2}}{4}} - \cancel{\frac{\frac{1}{2}}{4+2}}\right)$$

$$+ \left(\cancel{\frac{\frac{1}{2}}{5}} - \frac{\frac{1}{2}}{5+2}\right) + \left(\cancel{\frac{\frac{1}{2}}{6}} - \frac{\frac{1}{2}}{6+2}\right)$$

$$\cdots\cdots$$

$$+ \left(\frac{\frac{1}{2}}{k-3} - \cancel{\frac{\frac{1}{2}}{k-1}}\right) + \left(\frac{\frac{1}{2}}{k-2} - \cancel{\frac{\frac{1}{2}}{k}}\right)$$

$$+\left(\cancel{\frac{\frac{1}{2}}{k-1}} - \boxed{\frac{\frac{1}{2}}{k+1}}\right) + \left(\cancel{\frac{\frac{1}{2}}{k}} - \boxed{\frac{\frac{1}{2}}{k+2}}\right)$$

$$= \frac{\frac{1}{2}}{1} + \frac{\frac{1}{2}}{2} - \frac{\frac{1}{2}}{k+1} - \frac{\frac{1}{2}}{k+2}$$

$$= \frac{3}{4} - \frac{\frac{1}{2}}{k+1} - \frac{\frac{1}{2}}{k+2}$$

Άρα

$$\lim_{k\to+\infty} \frac{3}{4} - \frac{\frac{1}{2}}{\cancel{k+1}}^{\to 0} - \frac{\frac{1}{2}}{\cancel{k+2}}^{\to 0} = \frac{3}{4}$$

Η σειρά συγκλίνει στο $\frac{3}{4}$

(i) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k-2} = \sum_{\boxed{k=1}}^{\infty} \frac{1}{k-2\ \boxed{+2}} = \sum_{k=1}^{\infty} \frac{1}{k}$

Η σειρά είναι αρμονική με $p=1$ άρα δεν ξέρουμε αν συγκλίνει ή αποκλίνει

$$\text{(k)} \sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}} = \sum_{k=1}^{\infty} \frac{4^{k-1}\cdot 4^{3}}{7^{k-1}} = \sum_{k=1}^{\infty} 4^3\cdot\left(\frac{4}{7}\right)^{k-1}$$

$$= \sum_{\boxed{k=0}}^{\infty} 4^3 \left(\frac{4}{7}\right)^{k-1\,\boxed{+1}} = \sum_{k=0}^{\infty} 4^3\cdot\left(\frac{4}{7}\right)^{k}$$

Γεωμ. σειρά με $\alpha = 4^3$, $r = \frac{4}{7}$

Επειδή $|r| = \left|\frac{4}{7}\right| < 1$, συγκλίνει

στο $\dfrac{\alpha}{1-r} = \dfrac{4^3}{1-\frac{4}{7}} = \dfrac{7}{3}\cdot 4^3$

$$\text{(l)} \sum_{k=1}^{\infty} 5^{3k}7^{1-k} = \sum_{k=1}^{\infty} 5^{3k}\cdot\frac{1}{7^{k-1}} = \sum_{k=1}^{\infty} \left(5^3\right)^k\cdot\frac{1}{7^{k-1}}$$

$$= \sum^{\infty} 5^3\cdot\left(5^3\right)^{k-1}\cdot\frac{1}{7^{k-1}}$$

$$= \sum_{k=1}^{\infty} 5^3 \cdot (5^3)^{k-1} \cdot \frac{1}{7^{k-1}}$$

$$= \sum_{k=1}^{\infty} 5^3 \left(\frac{5^3}{7}\right)^{k-1} = \sum_{\boxed{k=0}}^{\infty} 5^3 \left(\frac{5^3}{7}\right)^{k-1\boxed{+1}}$$

$$= \sum_{k=0}^{\infty} 5^3 \left(\frac{5^3}{7}\right)^{k}$$ Γεωμ. σειρά

με $\alpha = 5^3$, $r = \frac{5^3}{7}$

Επειδή $|r| = \left|\frac{5^3}{7}\right| > 1$, αποκλίνει