Notes 2_10_2025

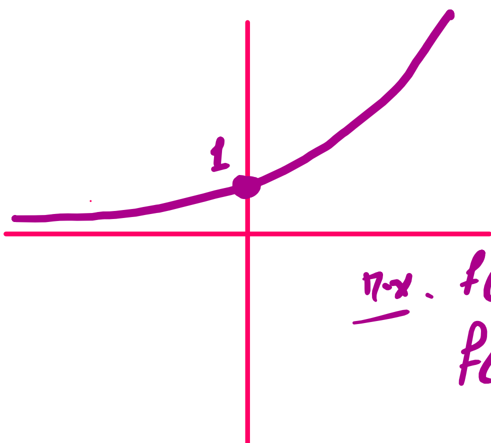
$f(x,y)$

$f'(x) = \frac{df}{dx}$

$f(x,y) = x^2 + y^2 + 4$

$f_x = \frac{\partial f}{\partial x} = 2x$ | $f_y = \frac{\partial f}{\partial y} = 2y$

Εκθετική – Λογαριθμική



$f(x) = \alpha^x$

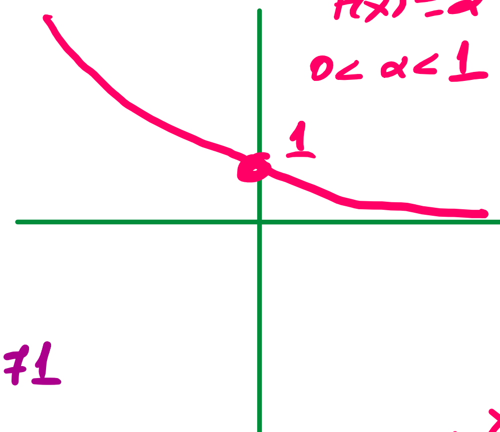
$\alpha > 1$

π.χ. $f(x) = 2^x$

$f(x) = e^x$, $e \approx 2.71$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to +\infty} f(x) = +\infty$



$f(x) = \alpha^x$

$0 < \alpha < 1$

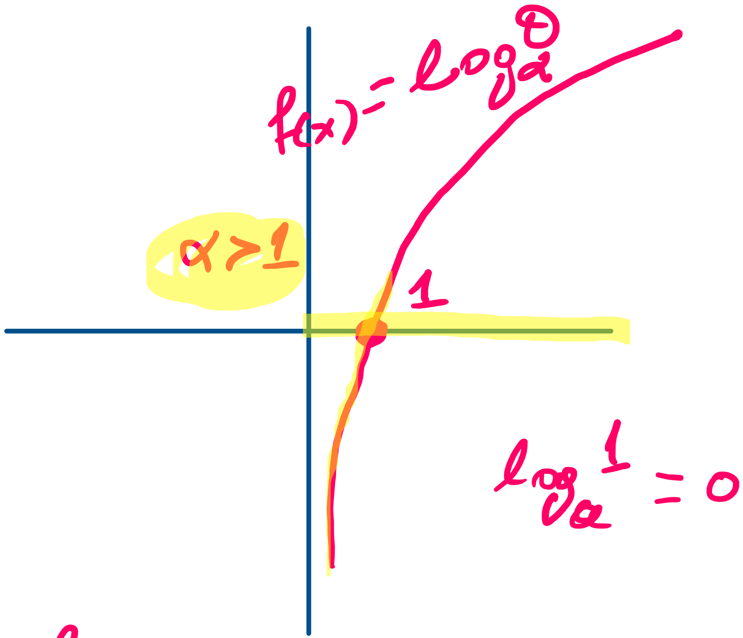
π.χ. $f(x) = \left(\frac{1}{2}\right)^x$

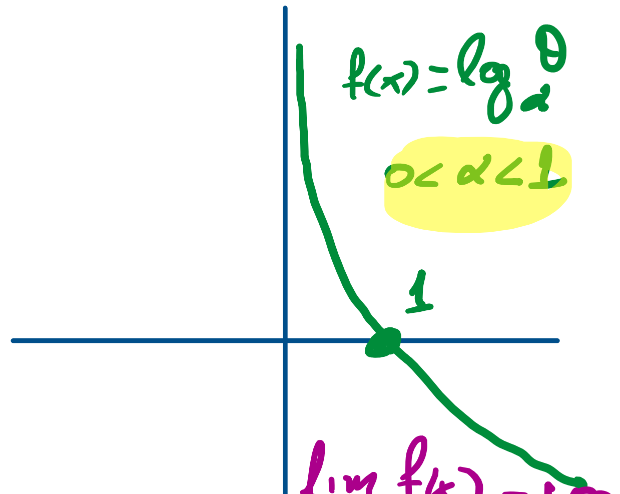
$g(x) = 3^{-x} = \frac{1}{3^x} = \left(\frac{1}{3}\right)^x$

$\lim_{x \to +\infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = +\infty$

$\boxed{\text{Λογαριθμική}}$





$\lim_{x\to 0^+} f(x) = +\infty$

$\lim_{x\to +\infty} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = -\infty$

$\lim_{x\to +\infty} f(x) = +\infty$

$f(x) = \log_\alpha \theta$, $\theta>0$

$\alpha = e \to \ln x = \log_e x$

Ιδιότητες:

$f(x) = e^x \qquad g(x) = \ln x$

$f(g(x)) = e^{\ln x} = x$

$g(f(x)) = \ln e^x = x \cdot \ln e = x \cdot 1 = x$

$\boxed{e^{\ln\theta} = \theta}$

$$\boxed{\begin{aligned} \ln(\theta_1\cdot\theta_2) &= \ln\theta_1 + \ln\theta_2 \\ \ln(\theta_1 : \theta_2) &= \ln\theta_1 - \ln\theta_2 \end{aligned}}$$

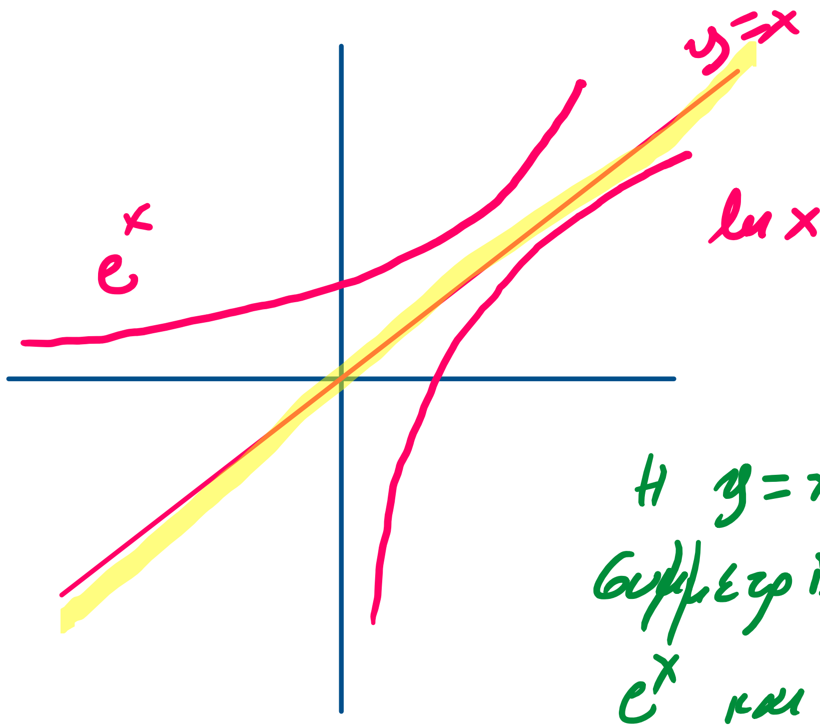
$\ln\theta^k \quad \ln\theta$

$\ln e = 1$

$$\boxed{\ln \theta^{\kappa} = \kappa \cdot \ln \theta}$$

$\ln e = 1$

$\ln 1 = 0$



Η $y=x$ είναι άξονας συμμετρίας της e^x και της $\ln x$.

►Άσκηση 2. **(Βλέπε ->Λυμένες ασκήσεις Κεφαλαίου V)** Να βρεθεί η παράγωγος των παρακάτω συναρτήσεων:

i. $f(x) = \dfrac{e^x}{3x^4}$ ***ii.*** $f(x) = \dfrac{e^{(8+x)^2}}{x^3}$

i) $f'(x) = \left(\dfrac{e^x}{3x^4}\right)' = \dfrac{(e^x)' \cdot 3x^4 - e^x \cdot (3x^4)'}{9x^8} =$

$= \dfrac{e^x \cdot 3x^4 - 12x^3 \cdot e^x}{9x^8} = \dfrac{3x^3 \cdot e^x (x-4)}{9x^8}$

ii) $f'(x) = \left(\dfrac{e^{(8+x)^2}}{x^3}\right)' = \dfrac{\left[e^{(8+x)^2}\right]' \cdot x^3 - e^{(8+x)^2} \cdot 3x^2}{x^6} =$

$= \dfrac{e^{(8+x)^2} \cdot 2(8+x) \cdot x^3 - e^{(8+x)^2} \cdot 3x^2}{x^6} =$

$= \dfrac{e^{(8+x)^2} \cdot x^2 \left[2x(8+x) - 3\right]}{x^6}$

$$= \frac{e \cdot x\,[2x(8+x)-3]}{x^6}$$

$(x^k)' = k \cdot x^{k-1}$, $(\alpha^x)' = \alpha^x \cdot \ln\alpha$, $h(x) = f(x)^{g(x)}$ π.χ. $f(x) = x^x$

$g(x) = (x^2+1)^{\sin x}$

ψίτονο

$$\boxed{h(x) = f(x)^{g(x)}}$$

Α' τρόπος:

Λογαριθμική παραγώγιση: $h'(x) = ?$

$h(x) = f(x)^{g(x)} \Rightarrow \ln h(x) = \ln f(x)^{g(x)} \Rightarrow$ $\boxed{\ln \theta^k = k \cdot \ln\theta}$

$\ln h(x) = g(x) \cdot \ln f(x) \Rightarrow$

$[\ln h(x)]' = [g(x) \cdot \ln f(x)]' \Rightarrow$

ρυθμός μεγέθυνσης

$$\boxed{[\ln f(x)]' = \frac{f'(x)}{f(x)}}$$

$\frac{1}{h(x)} \cdot h'(x) = g'(x) \cdot \ln f(x) + g(x) \frac{f'(x)}{f(x)}$

$$\boxed{h'(x) = h(x) \cdot \left[g'(x) \ln f(x) + \frac{g(x) f'(x)}{f(x)}\right]}$$

$(e^{x^2})' = e^{x^2} \cdot (x^2)'$

$\boxed{f(x) = \ln x}$, $f'(x) = \frac{1}{x}$

$\boxed{f(x) = \log_\alpha x}$

$\boxed{\alpha^x}$

►Άσκηση 19. (Βλέπε ->Λυμένες ασκήσεις Κεφαλαίου V)

Αν $x > 0$, να βρεθεί η παράγωγος της συνάρτησης $f(x) = x^x$

Λύση.

$$f(x) = x^x \Rightarrow \ln f(x) = \ln(x^x) \Rightarrow$$

$$\ln f(x) = x \cdot \ln x \Rightarrow$$

$$[\ln f(x)]' = (x \cdot \ln x)'$$

$$\frac{f'(x)}{f(x)} = \ln x + x \cdot \frac{1}{x} \Rightarrow$$

$$f'(x) = f(x) \cdot (\ln x + 1) \Rightarrow$$

$$\boxed{f'(x) = x^x \cdot (\ln x + 1)}$$

Β' τρόπος:

$$\boxed{e^{\ln \theta} = \theta} \rightsquigarrow \boxed{f(x) = e^{\ln f(x)}}$$

$$f(x) = \boxed{x^x} = e^{\ln \boxed{x^x}}$$

$$\boxed{x^x}$$

$$f(x) = e^{x \cdot \ln x}$$

$$f'(x) = \left(e^{x \cdot \ln x}\right)' = e^{x \cdot \ln x} \cdot (x \cdot \ln x)'$$

$$\boxed{x^x}$$

$$\boxed{f'(x) = \boxed{e^{x \cdot \ln x}} \cdot (\ln x + 1)}$$

Π.χ. $f(x) = (x^2+1)^{\sin x}$

$f(x) = e^{\ln (x^2+1)^{\sin x}} \Rightarrow f(x) = e^{\sin x \cdot \ln(x^2+1)}$

$f'(x) = (x^2+1)^{\sin x} \cdot \left(\sin x \cdot \ln(x^2+1)\right)' =$

$= (x^2+1)^{\sin x} \cdot \left(\cos x \cdot \ln(x^2+1) + \sin x \cdot \frac{2x}{x^2+1}\right)$