

Δ.Ε. Bernoulli

$$\alpha y'(x) + \beta y(x) = \gamma y^p(x)$$

$p \in \mathbb{R}$

$$xy' - 5x^2y = x^3 \sqrt{y}$$

$$5y' + 6y = 8y^2$$

$$y' - 2y = \sqrt{y}$$

$$y' + 6y = \frac{x}{y} = xy^{-1}$$

$$z = y^{1-p}$$

→ γραμμική
Δ.Ε.

~~$$y'(x) + y$$~~

$$y'(x) + xy(x) = xy^2(x)$$

$$p=2$$

$$z(x) = y^{1-2}(x) \Rightarrow \boxed{z = y^{-1}}$$

1^ο = πρόσ

$$z = y^{-1} \Rightarrow z = \frac{1}{y} \sim$$

$$\rightarrow y = \frac{1}{z}$$

$$\left(\frac{1}{z}\right)' + x\left(\frac{1}{z}\right) = x\left(\frac{1}{z}\right)^2$$

$$\frac{1' \cdot z - 1 \cdot z'}{z^2} + x\left(\frac{1}{z}\right) = \frac{x}{z^2}$$

$$-\frac{z'}{z^2} + x \cdot \frac{1}{z} = \frac{x}{z^2}$$

πολλαπλασιάζουμε με z^2

$$\boxed{-z' + x \cdot z = x}$$

2^{ος} υπόνοσ

$$z(x) = y^{-1}(x)$$

και παραχρησιμοποιούμε

$$z' = \boxed{-y^{-2}} y'$$

$$y' + x \cdot y = x y^2 \implies$$

$$-y^{-2} \cdot y' - x y^{-2} y = x(-y^{-2}) y^2$$

$$\underbrace{-y^{-2} y'} - x y^{-1} = -x$$

$$\boxed{z' - x \cdot \boxed{z} = -x}$$

1^η) αντίστροφο
σπαρτίλε

$$z'(x) - x z(x) = -x$$

2) ομογενής

$$z'(x) - x z(x) = 0 \implies$$

$$\frac{dz}{dx} - x z = 0 \implies \frac{dz}{dx} = x z \implies$$

$$\int \frac{dz}{z} = \int x dx \implies \ln|z| = \frac{x^2}{2} + C$$

$$z_{om}(x) = c e^{x^2/2}$$

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2) Μερίκι λύση, για να βρούμε $\varphi(x)$ είναι
 ώστε $z_m(x) = \varphi(x) \cdot z_{om}(x)$

$$(\varphi \cdot z_{om})' - x(\varphi \cdot z_{om}) = -x$$

$$\varphi' \cdot z_{om} + \varphi \cdot z_{om}' - x \varphi z_{om} = -x$$

$$\varphi' c e^{x^2/2} + \underbrace{\varphi \cdot c e^{x^2/2} \cdot x}_{\text{απαιτείται}} - \underbrace{x \varphi c e^{x^2/2}}_{\text{απαιτείται}} = -x$$

$$\varphi' c e^{x^2/2} = -x \Rightarrow \varphi' = \frac{1}{c} (-x e^{-x^2/2})$$

$$\varphi = \frac{1}{c} \int (-x) e^{-x^2/2} dx = \frac{1}{c} \int (e^{-x^2/2})' dx$$

από τον ορισμό

$$= \left(\frac{1}{c} e^{-x^2/2} \right)$$

$$z_m = \frac{1}{c} e^{-x^2/2} \cdot c e^{x^2/2} = 1$$

$$z(x) = c e^{x^2/2} + 1$$

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dpX lki $z(x) = y'(x) \Rightarrow$

$$y(x) = \frac{1}{z(x)} \Rightarrow$$

$$y(x) = \frac{1}{c e^{x^2/2} + 1}$$

$$z'(x) - x z(x) = -x$$

$$z(x) = 1 \leadsto (1)' - x \cdot 1 = -x \Rightarrow$$

$$0 - x = -x \quad \text{logica}$$

zuvu $z_m(x) = Ax + B$

$$(Ax+B)' - x(Ax+B) = -x$$

$$A \cdot \cancel{Ax^2} + \cancel{B}x = \cancel{-x} :$$

$$y_m(x) = 1$$

$$\begin{aligned} & \Rightarrow \\ & \left. \begin{aligned} A &= 0 \\ -A &= 0 \\ -B &= -1 \end{aligned} \right\} \begin{aligned} A &= 0 \\ B &= 1 \end{aligned} \end{aligned}$$

$$y'(x) + xy(x) = 6x\sqrt{y}$$

$$y' + xy = 6x y^{1/2}$$

$$\boxed{p = 1/2}$$

$$z = y^{1-1/2} \Rightarrow z = y^{1/2}$$

$$z = \sqrt{y} \Rightarrow y(x) = z^2(x)$$

$$(z^2(x))' + x z^2(x) = 6x \sqrt{z^2(x)}$$

$$2z \cdot z' + x z^2 = 6x z \quad \text{Dividi per } z(x)$$

$$\boxed{2z' + x \cdot z = 6x} \Rightarrow 2 \frac{dz}{dx} \quad \text{+ x.z = 0}$$

$$\Rightarrow 2 \frac{dz}{dx} = -x \cdot z \Rightarrow \int 2 \frac{dz}{z} = \int -x dx$$

$$2 \ln|z| = -\frac{x^2}{2} + C \Rightarrow \ln|z| = -\frac{x^2}{4} + C$$

$$\Rightarrow \boxed{z_{om}(x) = C \cdot e^{-x^2/4}}$$

$$z_{\text{trial}} \quad z_m = Ax + B$$

$$2(Ax+B)' + x(Ax+B) = 6x$$

$$2A + Ax^2 + Bx = 6x$$

$$2A = 0$$

$$A = 0$$

$$A = 0$$

$$B = 6$$

$$B = 6$$

$$z_m(x) = 6$$

$$\Rightarrow z(x) = C e^{-x^2/4} + 6$$

$$y(x) = z^2(x) \Rightarrow$$

$$y(x) = \left(C e^{-x^2/4} + 6 \right)^2$$

ΔE . Ανωτέρας Τάξεως

Γραμμικές 2^{ας} τάξεως

$$\alpha_2 y''(x) + \alpha_1 y'(x) + \alpha_0 y(x) = \phi(x)$$

$$\alpha_0, \alpha_1, \alpha_2 \in \mathbb{R}$$

$$5y'' - 6y' + 7y = 8e^x$$

$$y'' + y' - y = 4x + x^2$$

$$y(x) = y_{om}(x) + y_m(x)$$

$$\alpha_2 y''(x) + \alpha_1 y'(x) + \alpha_0 y(x) = 0$$

$$\alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0 = 0$$

⇓

$$\alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0 = 0$$

χαρακτηριστική εξίσωση

Λύνουμε την χαρ. και βρίσκουμε
 p_1, p_2

1) Αν $p_1, p_2 \in \mathbb{R}$ διαφορετικοί

$$y_{\text{om}}(x) = c_1 e^{p_1 x} + c_2 e^{p_2 x}$$

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$$(2) \quad p_1 = p_2 = p \in \mathbb{R}$$

$$y_{\text{om}}(x) = c_1 e^{px} + c_2 x e^{px}$$

$$(3) \quad p_1 = \alpha + bi, \quad p_2 = \alpha - bi$$

$\uparrow \quad \uparrow$
 $\alpha \quad b$

$$y_{\text{om}}(x) = c_1 e^{\alpha x} \cos bx + c_2 e^{\alpha x} \sin bx$$

$$y(x) = c_1 e^{5x} + c_2 e^{2x}$$

$$y(x) = c_1 e^{-5x} + c_2 e^{-2x}$$

$$y'' - 2y' - y = 0$$

$$\lambda^2 - 2\lambda - 1 = 0$$

$$\Delta = 4 - 4(-1) = 8$$

$$p_1 = \frac{2 - \sqrt{8}}{2} = 1 - \sqrt{2}$$

$$p_2 = \frac{2 + \sqrt{8}}{2} = 1 + \sqrt{2}$$

$$y(x) = C_1 e^{(1-\sqrt{2})x} + C_2 e^{(1+\sqrt{2})x}$$

$$y'' - y' - 2y = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\Delta = (-1)^2 - 4(-2) = 9$$

$$p_1 = \frac{1+3}{2} \rightarrow 2$$

$$p_2 = \frac{1-3}{2} \rightarrow -1$$

$$y(x) = C_1 e^{2x} + C_2 e^{-x}$$

$$y'' - 7y' = 0 \Rightarrow \lambda^2 - 7\lambda = 0$$

$$\Rightarrow \lambda(\lambda - 7) = 0$$

$$\Rightarrow p_1 = 0, p_2 = 7$$

$$y(x) = c_1 e^{0x} + c_2 e^{7x}$$

$$\boxed{y'' = 0} \rightarrow \lambda^2 = 0 \rightarrow \begin{matrix} 0 \\ 0 \end{matrix}$$

is ex

$$y_{\text{com}}(x) = c_1 e^{0x} + c_2 x e^{0x} =$$

$$= c_1 + c_2 x$$

$$y'' + 4y' + 5y = 0$$

$$\lambda^2 + 4\lambda + 5 = 0 \quad \Delta = 16 - 20 = -4$$

$$\begin{aligned} p_1 &= \frac{-4 + \sqrt{-4}}{2} = \frac{-4 + \sqrt{(-1) \cdot 4}}{2} = \\ &= \frac{-4 + \sqrt{-1} \cdot \sqrt{4}}{2} = \frac{-4 + i \cdot 2}{2} = \boxed{-2 + i} \end{aligned}$$

$$\begin{aligned} p_2 &= \frac{-4 - \sqrt{-4}}{2} = \frac{-4 - \sqrt{(-1) \cdot 4}}{2} = \boxed{-2 - i} \\ &\quad \boxed{\alpha = -2} \quad \boxed{b = 1} \end{aligned}$$

$$y(x) = c_1 e^{-2x} \cos x + c_2 e^{-2x} \sin x$$

$$y(\infty) = 0 + 0 = 0$$

$$y'' + 4y = 0$$

$$\lambda^2 + 4 = 0$$

$$\Rightarrow \lambda^2 = -4 \Rightarrow \lambda = \pm \sqrt{-4}$$

$$\lambda = \pm \sqrt{(-1) \cdot 4} = \pm \sqrt{-1} \cdot \sqrt{4} = \pm i 2$$

$$\rho_1 = 2i, \quad \rho_2 = -2i$$

$$a = 0, \quad b = 2$$

$$y_{\text{om}}(x) = c_1 e^{0x} \eta_1 2x + c_2 e^{0x} \omega 2x$$

$\Downarrow$

$$y_{\text{om}}(x) = c_1 \eta_1 2x + c_2 \omega 2x$$