Cowell Exercises - Answers

which implies

$$\lambda q = \left[q \left[\frac{w_1}{\alpha_1} \right]^{\alpha_1} \left[\frac{w_2}{\alpha_2} \right]^{\alpha_2} \right]^{\frac{1}{\alpha_1 + \alpha_2}} \tag{2.8}$$

So, using (2.7) and (2.8), the corresponding cost function is

$$\begin{aligned} C(\mathbf{w}, q) &= w_1 z_1^* + w_2 z_2^* \\ &= [\alpha_1 + \alpha_2] \left[q \left[\frac{w_1}{\alpha_1} \right]^{\alpha_1} \left[\frac{w_2}{\alpha_2} \right]^{\alpha_2} \right]^{\frac{1}{\alpha_1 + \alpha_2}}. \end{aligned}$$

4. Using the production functions we have, for any $t > 0$:

$$\phi(t\mathbf{z}) = [tz_1]^{\alpha_1} [tz_2]^{\alpha_2} = t^{\alpha_1 + \alpha_2} \phi(\mathbf{z}).$$

Therefore we have DRTS/CRTS/IRTS according as $\alpha_1 + \alpha_2 \lesseqqgtr 1$. If we look at average cost as a function of q we find that AC is increasing/constant/decreasing in q according as $\alpha_1 + \alpha_2 \lesseqqgtr 1$.

5. Using (2.7) and (2.8) conditional demand functions are

$$H^1(\mathbf{w}, q) = \left[q \left[\frac{\alpha_1 w_2}{\alpha_2 w_1} \right]^{\alpha_2} \right]^{\frac{1}{\alpha_1 + \alpha_2}}$$

$$H^2(\mathbf{w}, q) = \left[q \left[\frac{\alpha_2 w_1}{\alpha_1 w_2} \right]^{\alpha_1} \right]^{\frac{1}{\alpha_1 + \alpha_2}}$$

and are smooth with respect to input prices.

1

$$\left[\frac{\alpha_1}{w_1} \lambda q \right]^{\alpha_1} \left[\frac{\alpha_2}{w_2} \lambda q \right]^{\alpha_2} = q$$

2

$$\left. \begin{aligned} z_1^* &= \frac{\alpha_1}{w_1} \lambda q \\ z_2^* &= \frac{\alpha_2}{w_2} \lambda q \end{aligned} \right\}.$$

3

$$\left. \begin{aligned} z_1^* &= \frac{\alpha_1}{w_1} \lambda q \\ z_2^* &= \frac{\alpha_2}{w_2} \lambda q \end{aligned} \right\}.$$

Cowell Exercises - Answers

3. The coordinates of the corner A are $(\alpha_1 q, \alpha_2 q)$ and, given $\mathbf{w}$, this immediately yields the minimised cost.

$$C(\mathbf{w}, q) = w_1\alpha_1 q + w_2\alpha_2 q.$$

The methods in Exercise 2.4 since the Lagrangean is not differentiable at the corner.

4. Conditional demand is constant if all prices are positive

$$\begin{aligned} H^1(\mathbf{w}, q) &= \alpha_1 q \\ H^2(\mathbf{w}, q) &= \alpha_2 q. \end{aligned}$$

5. Given the linear case

$$q = \alpha_1 z_1 + \alpha_2 z_2$$

- Isoquants are as in Figure 2.9.
- It is obvious that the solution will be either at the corner $(q/\alpha_1, 0)$ if $w_1/w_2 < \alpha_1/\alpha_2$ or at the corner $(0, q/\alpha_2)$ if $w_1/w_2 > \alpha_1/\alpha_2$, or otherwise anywhere on the isoquant
- This immediately shows us that minimised cost must be.

$$C(\mathbf{w}, q) = q \min\left\{\frac{w_1}{\alpha_1}, \frac{w_2}{\alpha_2}\right\}$$

- So conditional demand can be multivalued:

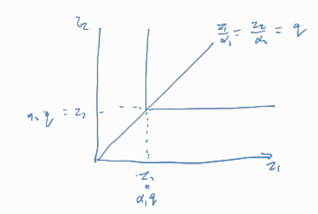
$$H^1(\mathbf{w}, q) = \begin{cases} \frac{q}{\alpha_1} & \text{if } \frac{w_1}{w_2} < \frac{\alpha_1}{\alpha_2} \\ z_1^* \in \left[0, \frac{q}{\alpha_1}\right] & \text{if } \frac{w_1}{w_2} = \frac{\alpha_1}{\alpha_2} \\ 0 & \text{if } \frac{w_1}{w_2} > \frac{\alpha_1}{\alpha_2} \end{cases}$$

$$H^2(\mathbf{w}, q) = \begin{cases} 0 & \text{if } \frac{w_1}{w_2} < \frac{\alpha_1}{\alpha_2} \\ z_2^* \in \left[0, \frac{q}{\alpha_2}\right] & \text{if } \frac{w_1}{w_2} = \frac{\alpha_1}{\alpha_2} \\ \frac{q}{\alpha_2} & \text{if } \frac{w_1}{w_2} > \frac{\alpha_1}{\alpha_2} \end{cases}$$

- Case 3 is a test to see if you are awake: the isoquants are not convex to the origin: an experiment with a straight-edge to simulate an isocost line will show that it is almost like case 2 – the solution will be either at the corner $(\sqrt{q/\alpha_1}, 0)$ if $w_1/w_2 < \sqrt{\alpha_1/\alpha_2}$ or at the corner $(0, \sqrt{q/\alpha_2})$ if $w_1/w_2 > \sqrt{\alpha_1/\alpha_2}$ (but nowhere else). So the cost function is :

$$C(\mathbf{w}, q) = \min\left\{w_1\sqrt{\frac{q}{\alpha_1}}, w_2\sqrt{q/\alpha_2}\right\}.$$

1



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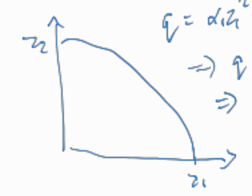

$q = \alpha_1 z_1^2 + \alpha_2 \cdot z_2^2$

$\Rightarrow q = \alpha_1 z_1^2 + 0$

$\Rightarrow z_1 = \sqrt{q/\alpha_1}$

3

z_1 if $w_1\sqrt{q/\alpha_1} < w_2\sqrt{q/\alpha_2}$

$\Rightarrow \frac{w_1}{w_2} < \sqrt{\frac{\alpha_1}{\alpha_2}}$

Exercise 2.8 *For any homothetic production function show that the cost function must be expressible in the form*

$$C(\mathbf{w}, q) = a(\mathbf{w})\, b(q).$$

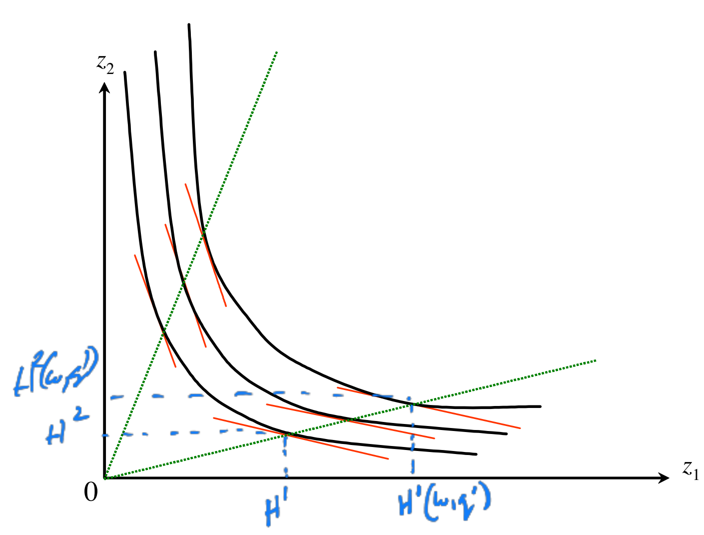

Figure 2.11: Homotheticity: expansion path

Outline Answer

From the definition of homotheticity, the isoquants must look like Figure 2.11; interpreting the tangents as isocost lines it is clear from the figure that the expansion paths are rays through the origin. So, if $H^i(\mathbf{w}, q)$ is the demand for input i conditional on output q, the optimal input ratio

$$\frac{H^i(\mathbf{w}, q)}{H^j(\mathbf{w}, q)} = \frac{H^i(w,q')}{H^j(w,q')}$$

must be independent of q and so we must have

$$\frac{H^i(\mathbf{w}, q)}{H^i(\mathbf{w}, q')} = \frac{H^j(\mathbf{w}, q)}{H^j(\mathbf{w}, q')}$$

for any q, q'. For this to true it is clear that the ratio $H^i(\mathbf{w}, q)/H^i(\mathbf{w}, q')$ must be independent of $\mathbf{w}$. Setting $q' = 1$ we therefore have

$$\boxed{\frac{H^1(\mathbf{w}, q)}{H^1(\mathbf{w}, 1)} = \frac{H^2(\mathbf{w}, q)}{H^2(\mathbf{w}, 1)}} = \dots = \frac{H^m(\mathbf{w}, q)}{H^m(\mathbf{w}, 1)} = b(q)$$

and so

$$H^i(\mathbf{w}, q) = b(q) H^i(\mathbf{w}, 1).$$

e.g. C-D (homogeneous)

$\ell(w,r,q) = \left(\frac{r}{w}\right)^{\frac{1}{2}} \cdot \left(\frac{q}{20}\right)^{3/4}$

$$\frac{\left(\frac{r}{w}\right)^{\frac{1}{2}} \cdot \left(\frac{q}{20}\right)^{3/4}}{\left(\frac{r}{w}\right)^{\frac{1}{2}} \cdot \left(\frac{1}{20}\right)^{3/4}} = \frac{\left(\frac{w}{r}\right)^{\frac{1}{2}} \cdot \left(\frac{q}{20}\right)^{3/4}}{\left(\frac{w}{r}\right)^{\frac{1}{2}} \left(\frac{1}{20}\right)^{3/4}} = q$$

$\Rightarrow H^1(w,q) = H^1(w,1) \cdot q$

Exercise 2.9 *Consider the production function*

$$q = \left[\alpha_1 z_1^{-1} + \alpha_2 z_2^{-1} + \alpha_3 z_3^{-1}\right]^{-1}$$

1. *Find the long-run cost function and sketch the long-run and short-run marginal and average cost curves and comment on their form.*
2. *Suppose input 3 is fixed in the short run. Repeat the analysis for the short-run case.*
3. *What is the elasticity of supply in the short and the long run?*

Outline Answer

1. The production function is clearly homogeneous of degree 1 in all inputs – i.e. in the long run we have constant returns to scale. But CRTS implies constant average cost. So

$$\text{LRMC} = \text{LRAC} = \text{constant}$$

LRMC = LRAC

Their graphs will be an identical straight line.

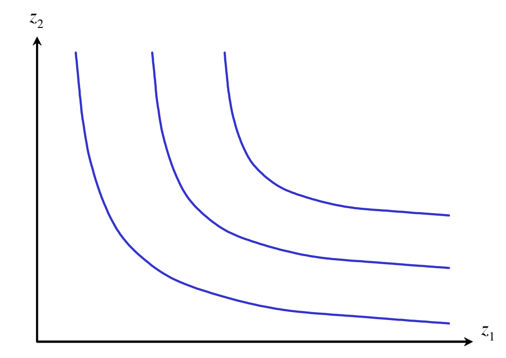

Figure 2.12: Isoquants do not touch the axes

2. In the short run $z_3 = \bar{z}_3$ so we can write the problem as the following Lagrangean

$$\hat{\mathcal{L}}(\mathbf{z}, \hat{\lambda}) = w_1 z_1 + w_2 z_2 + \hat{\lambda}\left[q - \left[\alpha_1 z_1^{-1} + \alpha_2 z_2^{-1} + \alpha_3 \bar{z}_3^{-1}\right]^{-1}\right]; \quad (2.13)$$

or, using a transformation of the constraint to make the manipulation easier, we can use the Lagrangean

$$\mathcal{L}(\mathbf{z}, \lambda) = w_1 z_1 + w_2 z_2 + \lambda\left[\alpha_1 z_1^{-1} + \alpha_2 z_2^{-1} - k\right] \quad (2.14)$$

where λ is the Lagrange multiplier for the transformed constraint and

$$k := q^{-1} - \alpha_3 \bar{z}_3^{-1}. \quad (2.15)$$

1

Στην άσκηση 2.9 δεν καταλαβαίνω πως προέκυψε ο τύπος του z_2 και ο τύπος 2.20. Θα επιθυμούσα να λύσουμε συνολικά τις 2.8 και

2

$$\frac{1}{\alpha_1(t z_1)^{-1} + \alpha_2(t z_2)^{-1} + \alpha_3(t z_3)^{-1}} = \frac{1}{t^{-1}(\quad)}$$

$$= t \cdot q$$

3

$$\frac{1}{\frac{1}{z_1} + \frac{1}{z_2}} = q$$

4

$q = [\quad]^{-1}$

$\Rightarrow q^{-1} = [\quad]$ same constraint new λ.

Note that the isoquant is

$$z_2 = \frac{\alpha_2}{k - \alpha_1 z_1^{-1}}.$$

From the Figure 2.12 it is clear that the isoquants do not touch the axes and so we will have an interior solution. The first-order conditions are

$$w_i - \lambda \alpha_i z_i^{-2} = 0,\ i = 1, 2 \tag{2.16}$$

which imply

$$z_i = \sqrt{\frac{\lambda \alpha_i}{w_i}},\ i = 1, 2 \tag{2.17}$$

To find the conditional demand function we need to solve for λ. Using the production function and equations (2.15), (2.17) we get

$$k = \sum_{j=1}^{2} \alpha_j \left[\frac{\lambda \alpha_j}{w_j}\right]^{-1/2} \tag{2.18}$$

from which we find

$$\sqrt{\lambda} = \frac{b}{k} \tag{2.19}$$

where

$$b := \sqrt{\alpha_1 w_1} + \sqrt{\alpha_2 w_2}.$$

Substituting from (2.19) into (2.17) we get minimised cost as

$$\begin{aligned} \tilde{C}(\mathbf{w}, q; \bar{z}_3) &= \sum_{i=1}^{2} w_i z_i^* + w_3 \bar{z}_3 && (2.20)\\ &= \frac{b^2}{k} + w_3 \bar{z}_3 && (2.21)\\ &= \frac{q b^2}{1 - \alpha_3 \bar{z}_3^{-1} q} + w_3 \bar{z}_3. && (2.22) \end{aligned}$$

Marginal cost is

$$\frac{b^2}{\left[1 - \alpha_3 \bar{z}_3^{-1} q\right]^2} \tag{2.23}$$

and average cost is

$$\frac{b^2}{1 - \alpha_3 \bar{z}_3^{-1} q} + \frac{w_3 \bar{z}_3}{q}. \tag{2.24}$$

Let $\underline{q}$ be the value of q for which MC=AC in (2.23) and (2.24) – at the minimum of AC in Figure 2.13 – and let $\underline{p}$ be the corresponding minimum value of AC. Then, using p =MC in (2.23) for $p \geq \underline{p}$ the short-run supply curve is given by

$$q^* = S(\mathbf{w}, p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 0 \text{ or } \underline{q} & \text{if } p = \underline{p} \\ q = \frac{\bar{z}_3}{\alpha_3}\left[1 - \frac{b}{\sqrt{p}}\right] & \text{if } p > \underline{p} \end{cases}$$

1

$$k := q^{-1} - \alpha_3 \bar{z}_3^{-1}. \quad (2.15)$$

$q = (\alpha_1 z_1^{-1} + \alpha_2 z_2^{-1} + \alpha_3 z_3^{-1})^{-1}$

$k = \alpha_1 z_1^{-1} + \alpha_2 z_2^{-1} + \cancel{\alpha_3 \bar{z}_3^{-1}} - \cancel{\alpha_3 \bar{z}_3^{-1}}$

$= \alpha_1 \left[\frac{\lambda a_1}{w_1}\right]^{-\frac{1}{2}} + \alpha_2 \left[\frac{\lambda a_2}{w_2}\right]^{-\frac{1}{2}}$

2

$k = a_1 \left[\frac{\lambda a_1}{w_1}\right]^{-\frac{1}{2}} + a_2 \left[\frac{\lambda a_2}{w_2}\right]^{-\frac{1}{2}}$

$= \lambda^{-\frac{1}{2}} \left(\sqrt{a_1 w_1} + \sqrt{a_2 w_2}\right] \Rightarrow \sqrt{\lambda} = \frac{\sqrt{a w_1} + \sqrt{a_2 w_2}}{k}$

3

from which we find

$$\sqrt{\lambda} = \frac{b}{k} \tag{2.19}$$

where

$$b := \sqrt{\alpha_1 w_1} + \sqrt{\alpha_2 w_2}.$$

Substituting from (2.19) into (2.17) we get minimised cost as

$$z_i = \sqrt{\frac{\lambda \alpha_i}{w_i}},\ i = 1, 2 \tag{2.17}$$

$z_1 = \frac{b}{k} \cdot \frac{\alpha_1}{w_1}, \quad z_2 = \frac{b}{k} \cdot \frac{\alpha_2}{w_2}$

$\Rightarrow \quad w_1 \frac{b}{k} \sqrt{\frac{\alpha_1}{w_1}} + w_2 \cdot \frac{b}{k} \sqrt{\frac{\alpha_2}{w_2}} + w_3 \bar{z}_3$

$= \frac{b}{k} \cdot \sqrt{\alpha_1 w_1} + \frac{b}{k} \sqrt{\alpha_2 w_2} + w_3 \bar{z}_3$

$= \frac{b}{k} (\quad) + w_3 \cdot \bar{z}_3$

$= \frac{b^2}{k} + w_3 \cdot \bar{z}_3$

4

$\frac{b^2}{1 - \alpha_3 \bar{z}_3^{-1} q} + \frac{q b^2 \alpha_3 \bar{z}_3^{-1}}{(1 - \alpha_3 \bar{z}_3^{-1} q)^2} =$

$= \frac{b^2 (1 - \alpha_3 \bar{z}_3^{-1} q)}{(1 - a_3 \bar{z}_3^{-1} q)^2} + \frac{q b^2 \alpha_3 \bar{z}_3^{-1} q}{(1 - \alpha_3 \bar{z}_3^{-1} q)^2}$

$= \frac{b^2}{(1 - \alpha_3 \bar{z}_3^{-1} q)^2}$

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3. Differentiating the last line in the previous formula we get

$$\frac{d\ln q}{d\ln p} = \frac{p}{q}\frac{dq}{dp} = \frac{1}{2}\frac{1}{\sqrt{p}/b - 1} > 0$$

Note that the elasticity decreases with b. In the long run the supply curve coincides with the MC,AC curves and so has infinite elasticity.

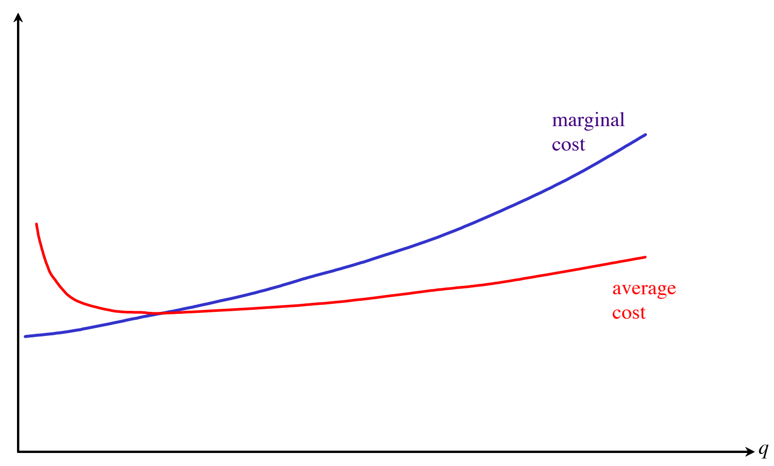


Figure 2.13: Short-run marginal and average cost

$$q = \frac{\bar{z}_3}{\alpha_3}\left[1 - \frac{b}{\sqrt{p}}\right]$$

$$\frac{dq}{dp} = \frac{1}{2}\frac{\bar{z}_3}{\alpha_3}\cdot\frac{b}{p^{-3/2}}$$

$$p \cdot \frac{dq}{dp} = \frac{1}{2}\frac{\bar{z}_3}{\alpha_3}\frac{b}{\sqrt{p}}$$

$$\frac{p}{q}\frac{dq}{dp} = \frac{\frac{1}{2}\cancel{\frac{\bar{z}_3}{\alpha_3}}\frac{b}{\sqrt{p}}}{\cancel{\frac{\bar{z}_3}{\alpha_3}}\cdot\left[1 - \frac{b}{\sqrt{p}}\right]}$$

$$= \frac{\frac{1}{2}\frac{b}{\sqrt{p}}}{\left[1 - \frac{b}{\sqrt{p}}\right]}\cdot\frac{\frac{\sqrt{p}}{b}}{\frac{\sqrt{p}}{b}} = \frac{1}{2}\frac{1}{\sqrt{p}/b - 1}$$

Exercise 3.3 *A firm has the cost function*

$$F_0 + \frac{1}{2}aq_i^2$$

where q_i is the output of a single homogenous good and F_0 and a are positive numbers.

1. *Find the firm's supply relationship between output and price p; explain carefully what happens at the minimum-average-cost point $\underline{p} := \sqrt{2aF_0}$.*
2. *In a market of a thousand consumers the demand curve for the commodity is given by*
$$p = A - bq$$
where q is total quantity demanded and A and b are positive parameters. If the market is served by a single price-taking firm with the cost structure in part 1 explain why there is a unique equilibrium if $b \leq a\left[A/\underline{p} - 1\right]$ and no equilibrium otherwise.
3. *Now assume that there is a large number N of firms, each with the above cost function: find the relationship between average supply by the N firms and price and compare the answer with that of part 1. What happens as $N \to \infty$?*
4. *Assume that the size of the market is also increased by a factor N but that the demand per thousand consumers remains as in part 2 above. Show that as N gets large there will be a determinate market equilibrium price and output level.*

Outline Answer

1. Given the cost function
$$F_0 + \frac{1}{2}aq_i^2$$
marginal cost is aq_i and average cost is $F_0/q_i + \frac{1}{2}aq_i$. Marginal cost intersects average cost where
$$aq_i = F_0/q_i + \frac{1}{2}aq_i$$
i.e. where output is
$$\underline{q} := \sqrt{2F_0/a} \tag{3.9}$$
and marginal cost is
$$\underline{p} := \sqrt{2aF_0} \tag{3.10}$$
For $p > \underline{p}$ the supply curve is identical to the marginal cost curve $q_i = p/a$; for $p < \underline{p}$ the firm supplies 0 to the market; at $p = \underline{p}$ the firm supplies either 0 or $\underline{q}$. There is no price which will induce a supply in the interior of the interval $\left(0, \underline{q}\right)$. Summarising, firm i's optimal output is given by
$$q_i^* = S(p) := \begin{cases} p/a, \text{ if } p > \underline{p} \\ q \in \{0, \underline{q}\} \text{ if } p = \underline{p} \\ 0, \text{ if } p < \underline{p} \end{cases} \tag{3.11}$$

Στην άσκηση 3.3 γιατί S(p)=p/a στην πρώτη περίπτωση και το ερώτημα 4.

$p = MC \Rightarrow p = a \cdot q \Rightarrow q = p/a$

$C = F_0 + \frac{1}{2}aq_i^2 \Rightarrow MC = aq_i$

2. The equilibrium, if it exists, is found where supply=demand at a given price. This would imply

$$\frac{p}{a} = \frac{A-p}{b}$$
$$p = \frac{aA}{a+b}$$

which would, in turn, imply an equilibrium quantity

$$q = \frac{A}{a+b}$$

but it can only be valid if $\frac{A}{a+b} \geq \underline{q}$. Noting that $\underline{q} = \underline{p}/a$ this condition is equivalent to $a\left[\frac{A}{\underline{p}} - 1\right] \geq b$.

3. If there are N such firms, each firm responds to price as in (3.11), and so the average output $\bar{q} := \frac{1}{N}\sum_{i=1}^{N} q_i^*$ is given by

$$\bar{q} = \begin{cases} p/a, \text{ if } p > \underline{p} \\ q \in J(\underline{q}) \text{ if } p = \underline{p} \\ 0, \text{ if } p < \underline{p} \end{cases} \tag{3.12}$$

where $J(\underline{q}) := \{\frac{i}{N}\underline{q} : i = 0, 1, ..., N\}$. As $N \to \infty$ the set $J(\underline{q})$ becomes dense in $[0, \underline{q}]$, and so we have the average supply relationship:

$$\bar{q} = \begin{cases} p/a, \text{ if } p > \underline{p} \\ q \in [0, \underline{q}] \text{ if } p = \underline{p} \\ 0, \text{ if } p < \underline{p} \end{cases} \tag{3.13}$$

4. Given that in the limit the average supply curve is continuous and of the piecewise linear form (3.13), and that the demand curve is a downward-sloping straight line, there must be a unique market equilibrium. The equilibrium will be found at $\left(\underline{p}, \frac{A-\underline{p}}{b}\right)$ which, using (3.10) is $\left(\sqrt{2aF_0}, \frac{A-\sqrt{2aF_0}}{b}\right)$. Using (3.9) this can be written $(\underline{p}, \beta\underline{q})$ where

$$\beta := \frac{A-\underline{p}}{b\underline{p}/a}$$

In the equilibrium a proportion β of the firms produce $\underline{q}$ and $1-\beta$ of the firms produce 0.

1

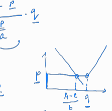
$S(q) = \frac{p}{a}$
q_s

$p = A - bq$
$q_D = \frac{A-p}{b}$

2

$\frac{A - \sqrt{2aF_0}}{b} = \frac{A-\underline{p}}{b} \quad \underline{q} \cdot \frac{1}{\underline{q}} = \frac{1}{\underline{p}/a}$

$= \underbrace{\frac{A-\underline{p}}{b\frac{\underline{p}}{a}}}_{\beta} \cdot \underline{q}$



Total quantity $\frac{A-\underline{p}}{b}$

divide this by $\underline{q}$ to find the proportion of firms producing $\underline{q}$ while $1-\beta$ will producing zero.

Exercise 3.4 *A firm has a fixed cost F_0 and marginal costs*

$$c = a + bq$$

where q is output.

1. *If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? If the competitive price were above this level, find the amount of output q^* that the firm would produce.*

2. *If the firm is actually a monopolist and the inverse demand function is*

$$p = A - \frac{1}{2}Bq$$

(where $A > a$ and $B > 0$) find the expression for the firm's marginal revenue in terms of output. Illustrate the optimum in a diagram and show that the firm will produce

$$q^{**} := \frac{A-a}{b+B}$$

What is the price charged p^{**} and the marginal cost c^{**} at this output level? Compare q^{**} and q^*.

3. *The government decides to regulate the monopoly. The regulator has the power to control the price by setting a ceiling $p_{\max}$. Plot the average and marginal revenue curves that would then face the monopolist. Use these to show:*

   (a) *If $p_{\max} > p^{**}$ the firm's output and price remain unchanged at q^{**} and p^{**}*

   (b) *If $p_{\max} < c^{**}$ the firm's output will fall below q^{**}.*

   (c) *Otherwise output will rise above q^{**}.*

Outline Answer

1. Total costs are

$$F_0 + aq + \frac{1}{2}bq^2$$

So average costs are

$$\frac{F_0}{q} + a + \frac{1}{2}bq$$

which are a minimum at

$$\underline{q} = \sqrt{2\frac{F_0}{b}} \tag{3.14}$$

where average costs are

$$\sqrt{2bF_0} + a \tag{3.15}$$

Marginal and average costs are illustrated in Figure 3.1: notice that MC is linear and that AC has the typical U-shape if $F_0 > 0$. For a price above the level (3.15) the first-order condition for maximum profits is given by

$$p = a + bq$$

1

$$\frac{F_0}{\sqrt{2F_0/b}} + a + \frac{1}{2}b\sqrt{2F_0/b}$$
$$= \sqrt{\tfrac{1}{2}\cdot F_0\cdot b} + \sqrt{\tfrac{1}{2}\cdot F_0\cdot b} + a$$
$$= 2\sqrt{\tfrac{1}{2}F_0 b} + a = \sqrt{\tfrac{4}{2}F_0 b} + a = \sqrt{2bF_0} + a$$

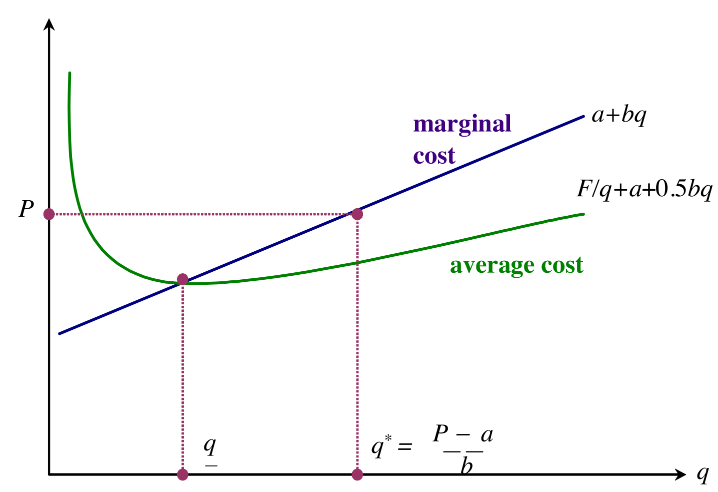


Figure 3.1: Perfect competition

from which we find

$$q^* := \frac{p-a}{b}$$

– see figure 3.1.

2. If the firm is a monopolist marginal revenue is

$$\frac{\partial}{\partial q}\left[Aq - \frac{1}{2}Bq^2\right] = A - Bq$$

Hence the first-order condition for the monopolist is

$$A - Bq = a + bq \tag{3.16}$$

from which the solution q^{**} follows. Substituting for q^{**} we also get

MC = MR

$$c^{**} = A - Bq^{**} = \frac{Ab + Ba}{B + b} \tag{3.17}$$

$$p^{**} = A - \frac{1}{2}Bq^{**} = c^{**} + \frac{1}{2}B\frac{A-a}{b+B} \tag{3.18}$$

– see figure 3.2.

3. Consider how the introduction of a price ceiling will affect average revenue. Clearly we now have

$$\text{AR}(q) = \left\{ \begin{array}{l} p_{\max} \text{ if } q \le q_0 \\ A - \frac{1}{2}Bq \text{ if } q \ge q_0 \end{array} \right\} \tag{3.19}$$

where $q_0 := 2\left[A - p_{\max}\right]/B$: average revenue is a continuous function of q but has a kink at q_0. From this we may derive marginal revenue which is

$$\text{MR}(q) = \left\{ \begin{array}{l} p_{\max} \text{ if } q < q_0 \\ A - Bq \text{ if } q > q_0 \end{array} \right\} \tag{3.20}$$

1

$Bq + bq = A - a \Rightarrow q = \frac{A-a}{B+b}$

2

$p = A - \frac{1}{2}Bq$ (inverse demand)

$p_{max} = A - \frac{1}{2}Bq \Rightarrow q = 2[A - p_{max}]/B$

Chapter 10

Strategic Behaviour

Exercise 10.1 *Table 10.1 is the strategic form representation of a simultaneous move game in which strategies are actions.*

	s_1^b	s_2^b	s_3^b
s_1^a	0, 2	3, 1	4, 3
s_2^a	2, 4	0, 3	3, 2
s_3^a	1, 1	2, 0	2, 1

Table 10.1: Elimination and equilibrium

1. *Is there a dominant strategy for either of the two agents?*
2. *Which strategies can always be eliminated because they are dominated?*
3. *Which strategies can be eliminated if it is common knowledge that both players are rational?*
4. *What are the Nash equilibria in pure strategies?*

Outline Answer:

1. No player has a dominant strategy.
2. Both s_3^a and s_2^b can be eliminated as individually irrational.
3. With common knowledge of rationality we can eliminate the dominated strategies: s_3^a and s_2^b.
4. The Nash Equilibria in pure strategies are (s_2^a, s_1^b) and (s_1^a, s_3^b)

Στην άσκηση 10.1 γιατί στο ερώτημα 4 δεν είναι πιθανό σημείο ισορροπίας το σημείο 3,2 αντί για το 2,4.

WHY NOT 3,2 INSTEAD OF 2,4

$$\left.\begin{array}{l}\text{IF } s_2^a \text{ best response for b is } s_1^b \\ \text{IF } s_1^b \text{ '' '' '' a '' } s_2^a\end{array}\right\} \text{NASH}$$