Chapter 3

The Firm and the Market

> ... the struggle for survival tends to make those organisations prevail, which are best fitted to thrive in their environment, but not necessarily those best fitted to benefit their environment, unless it happens that they are duly rewarded for all the benefits which they confer, whether direct or indirect. – Alfred Marshall, *Principles of Economics*, 8th edition, pages 596,597

3.1 Introduction

Chapter 2 considered the economic problem of the firm in splendid isolation. The firm received signals (prices of inputs, prices of outputs) from the outside world and responded blindly with perfectly calculated optimal quantities. The demand for inputs and the supply of output pertained only to the behaviour of this single economic actor.

It is now time to extend this to consider more fully the rôle of the firm in the market. We could perhaps go a stage further and characterise the market as "the industry", although this arguably sidesteps the issue because the definition of the industry presupposes the definition of specific commodities. To pursue this route we need to examine the joint effect of several firms responding to price signals together. What we shall not be doing at this stage of the argument is to consider the possibility of strategic game-theoretic interplay amongst firms; this needs new analytical tools and so comes after the discussion in chapter 10.

We extend our discussion of the firm by introducing three further developments:

- We consider the market equilibrium of many independent price-taking firms producing either an identical product or closely related products.

- We look at problems raised by interactions amongst firms in their production process.

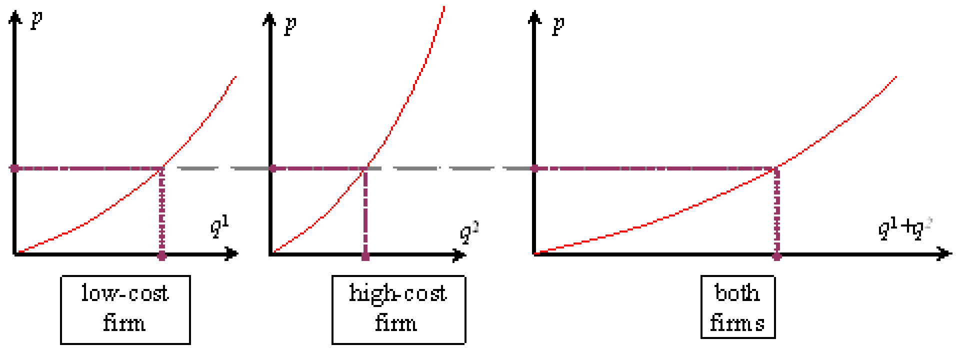

Figure 3.1: A market with two firms

- We extend the price-taking paradigm to analyse situations where the firm can control market prices to some extent. What are these? One of the simplest cases – but in some ways a rather unusual one – is that discussed in section 3.6 where there is but a single firm in the market. However this special case of monopoly provides a useful general framework of analysis into which other forms of "monopolistic competition" can be fitted (see section 3.7).

We shall build upon the analysis of the individual competitive firm's supply function, as discussed on page 30 above, and we will briefly examine difficulties in the concept of market equilibrium. The crucial assumption that we shall make is that each firm faces determinate demand conditions: either they take known market prices as given or they face a known demand function such as (3.7).

3.2 The market supply curve

How is the overall supply of product to the market related to the story about the supply of the individual firm sketched in section 2.3.1 of chapter 2?

We begin with an overly simplified version of the supply curve. Suppose we have a market with just two potential producers – low-cost firm 1 and high-cost firm 2 each of which has zero fixed costs and rising marginal costs. Let us write q^f for the amount of the single, homogeneous output produced by firm f (for the moment f can take just the values 1 or 2). The supply curve for each firm is equal to the marginal cost curve – see the first two panels in Figure 3.1. To construct the supply curve to the market (on the assumption that both firms continue to act as price takers) pick a price on the vertical axis; read off the value of q^1 from the first panel, the value of q^2 from the second panel; in the third panel plot $q^1 + q^2$ at that price; continuing in this way for all other prices you get the market supply curve depicted in the third panel. clearly the aggregation

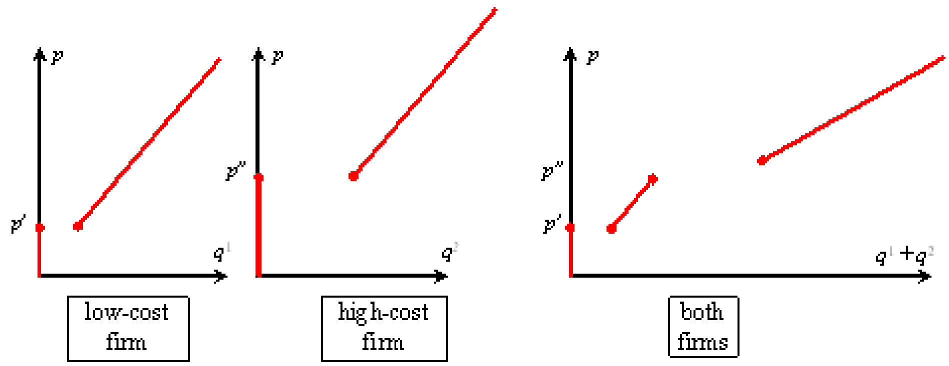

Figure 3.2: Another market with two firms

of individual supply curves involves a kind of "horizontal sum" process.

However, there are at least three features of this story that strike one immediately as unsatisfactory: (1) the fact that each firm just carries on as a price taker even though it (presumably) knows that there is just one other firm in the market; (2) the fixed number of firms and (3) the fact that each firm's supply curve is rather different from that which we sketched in chapter 2. Point 1 is a big one and going to be dealt with in chapter 10; point 2 comes up later in this chapter (section 3.5). But point 3 is dealt with right away.

The problem is that we have assumed away a feature of the supply function that is evident in Figure 2.12. So, instead of the case in Figure 3.1, imagine a case where the two firms have different fixed costs and marginal costs that rise everywhere at the same rate. The situation is now as in Figure 3.2. Consider what happens as the price of good 1 output rises from 0. Initially only firm 1 is in the market for prices in the range $p' \leq p < p''$ (left-hand panel). Once the price hits p'' firm 2 enters the market (second panel): the combined behaviour of the two firms is depicted in the third panel. Notice the following features of Figure 3.2.

- Even though each firm's supply curve has the same slope, the aggregate supply curve is flatter – in our example it is exactly half the slope. (This feature was already present in the earlier case)
- There is a discontinuity in aggregate supply as each firm enters the market.

A discontinuous supply curve in the aggregate might seem to be rather problematic – how do you find the equilibrium in one market if the demand curve goes through one of the "holes" in the supply curve? This situation is illustrated in Figure 3.3. Here it appears that there is no market equilibrium at all: above price p'' the market will supply more than consumers demand of the product, below p'' there will be the reverse problem (at a given price p people want to consume more than is being produced); and exactly at p'' it is not self-evident

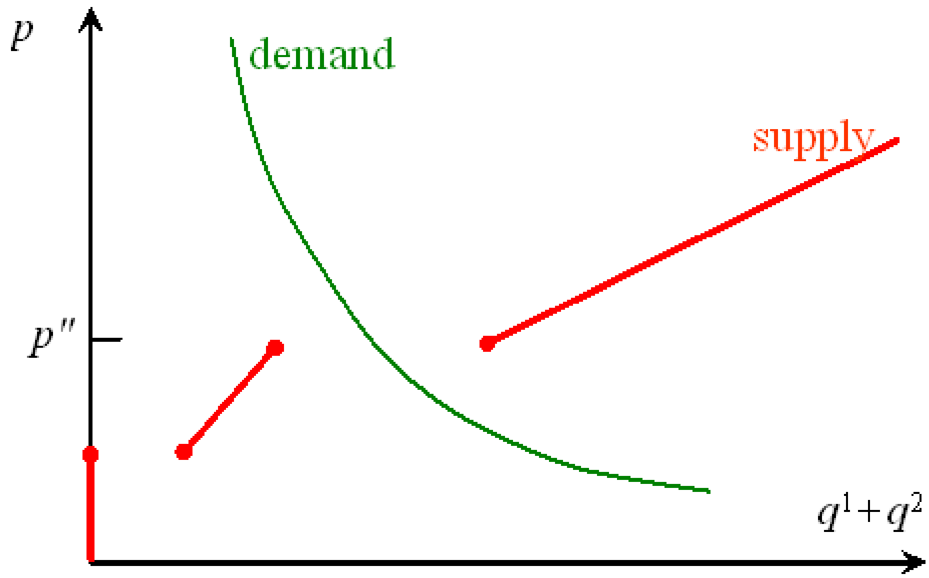

Figure 3.3: Absence of market equilibrium

what will happen; given the way that the demand curve has been drawn you will never get an exact match between demand and supply.

These simple exercise suggests a number of directions in which the analysis of the firm in the market might be pursued.

- *Market size and equilibrium.* We shall investigate how the problem of the existence of equilibrium depends on the number of firms in the market.

- *Interactions amongst firms.* We have assumed that each firm's supply curve is in effect independent of any other firm's actions. How would such interactions affect aggregate market behaviour?

- *The number of firms.* We have supposed that there was some arbitrarily given number of firms n_f in the market – as though there were just n_f licences for potential producers. In principle we ought to allow for the possibility that new firms can set up in business, in which case n_f becomes *endogenous.*

- *Product Differentiation.*We have supposed that for every commodity $i = 1, 2, ..., n$ there is a large number of firms supplying the market with indistinguishable units of that commodity. In reality there may be only a few suppliers of any one narrowly-defined commodity type although there is still effective competition amongst firms because of substitution in consumption amongst the product types. Instead of supplying identical

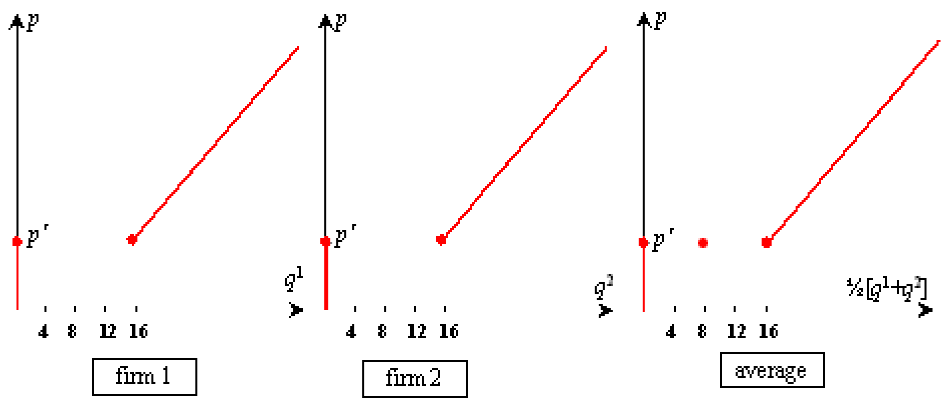

Figure 3.4: Average supply of two identical firms

packets of tea to the market, firms may sell packets that are distinguished by brand-name, or they may sell them in locations that distinguish them as being particularly convenient for particular groups of consumers.

Let us deal with each of these issues in turn.

3.3 Large numbers and the supply curve

Actually, this problem of nonexistence may not be such a problem in practice. To see why consider again the second example of section 3.2 where each firm had a straight-line marginal cost curve. Take firm 1 as a standard case and imagine the effect of there being potentially many small firms just like firm 1: if there were a huge number of firms waiting in the wings which would enter the market as p hit p' what would the aggregate supply curve look like?To answer this question consider first of all a market in which there are just two identical firms. Suppose that each firm has the supply curve illustrated in either of the first two panels of Figure 3.4. Using the notation of section 3.2 the equation of either firm's supply curve is given by:[1]

$$q^f = \begin{cases} 0, & \text{if } p < p' \\ 16 + \alpha[p - p'], & \text{if } p \geq p' \end{cases} \tag{3.1}$$

Clearly for $p > p'$ total output is given by

$$q^1 + q^2 = 32 + 2\alpha[p - p'] \tag{3.2}$$

and so for $p > p'$ *average* output is given by

$$\frac{1}{2}[q^1 + q^2] = 16 + \alpha[p - p'] \tag{3.3}$$

[1] Write down a cost function consistent with this supply curve.

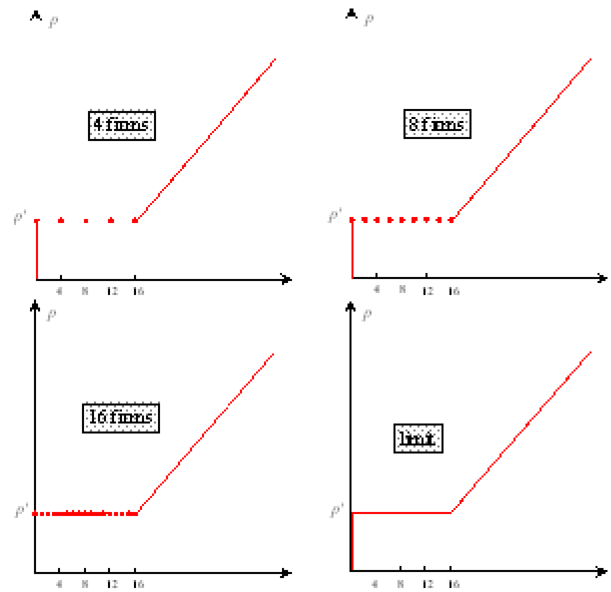

Figure 3.5: Average supply of lots of firms

Obviously for $p < p'$ total – and hence average – output is zero. But what happens exactly at $p = p'$? Clearly either we must have either $(q^1 = 0, q^2 = 0)$ or $(q^1 = 0, q^2 = 16)$, or $(q^1 = 16, q^2 = 0)$ or $(q^1 = 16, q^2 = 16)$. In other words total output could have the value 0, 16 or 32, so of course average output has the value 0, 8 or 16. Notice that the average supply in the market is almost like that for each firm, but there is an additional "blob" at $q = 8$.

We can extend this idea to a market with more firms. We do this by considering more replications. This is illustrated in Figure 3.5. Notice that in the top left hand panel where there are four firms, there are three intermediate blobs. The top right-hand panel and the bottom left-hand panel display the result of two more replications of the firms in the market – to 8 firms and 16 firms respectively.[2] So we can see that in the limit this large number of small firms looks indistinguishable from a market incorporating firms each of which has a continuous supply curve, as illustrated in the bottom right-hand panel of Figure 3.5.

So, if we can appeal to a regularity condition – in our example a large number of small, similar firms in the market – the elementary diagram incorporating a continuous supply curve is a valid approach for the analysis of market equilibrium. Fortunately this regularity condition can be generalised, but the principle

[2] If there are n_f identical firms, how many blobs will there be? Use this argument to show why in the limit the average supply curve of the industry looks as though it is continuous.

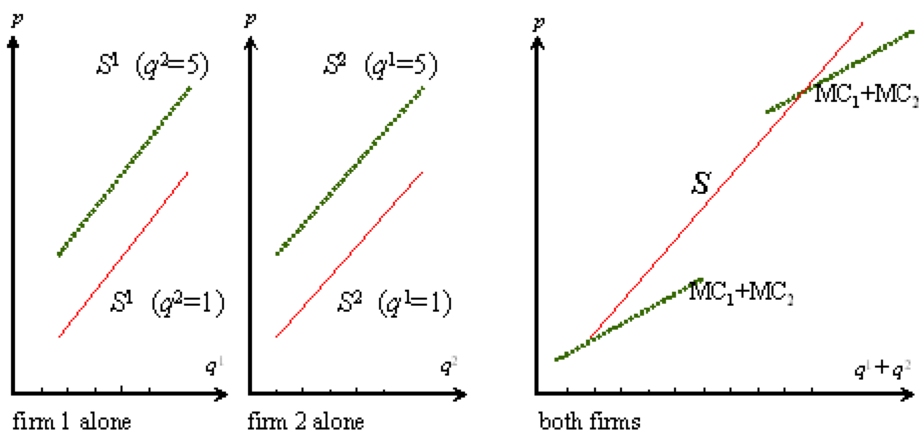

Figure 3.6: Industry supply with negative externality

of “large numbers, small firms” remains.

3.4 Interaction amongst firms

All of the preceding analysis has been predicated on the basis that each firm’s production possibilities are independent of every other firm’s production decisions.However, we also need to take into account the possibility of technological interactions between firms – interactions that do not occur through conventional market mechanisms. One firm’s choice of outputs and inputs affects the others’ technological possibilities. This interaction could be in either of two directions: *negative externalities* whereby the increase in the output by one firm – a polluter perhaps – raises the marginal costs of other firms, and *positive externalities* whereby the increase in output by one firm – perhaps a firm that undertakes the general training of workers in an area – lowers the marginal costs of other firms.

Consider a negative externality in the case of two identical firms. If one firm increases its output, the other firm’s marginal costs are pushed up. So the position of either firm’s supply curve depends on the other’s output decision. This is illustrated in Figure 3.6. Suppose that market price is such that each firm wants to supply one unit of output: the firm’s supply curve is as shown by the solid line in each of the first two panels. Then market demand rises: the price goes up and each firm expands output, let us say to five units. Because of the negative externality each firm’s expansion pushes up marginal costs of the other firm – see the firm supply curves drawn as broken lines. When we draw in the supply curve for the market notice that the slope is steeper than would have been the case had there been no externality (in the third panel compare

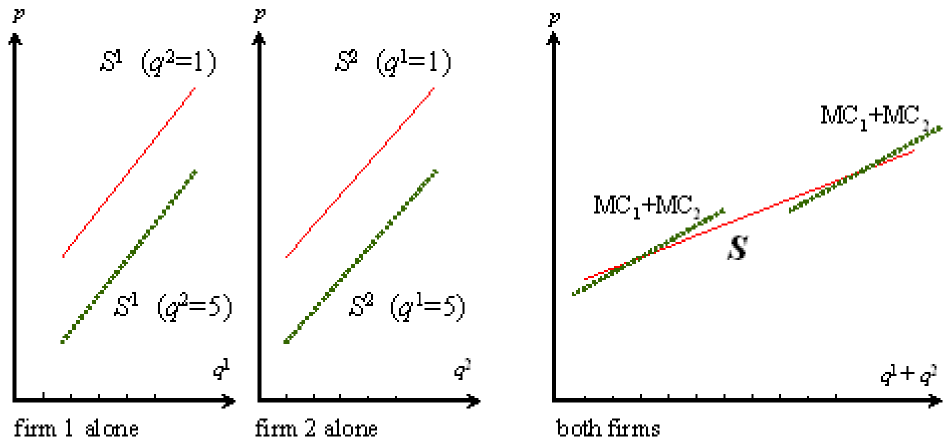

Figure 3.7: Industry supply with positive externality

the supply curve S with the two broken lines).

We can also consider the effect of a positive externality simply by interchanging the labels in each part of the above figure. In this case, as each firm expands output, the other firm's marginal costs fall. Again we can run through the same story of what happens as market demand rises, but now the firm's supply curves shift the other way. If you do this notice that, for this particular case, the aggregate supply curve is less steeply upward-sloping than that for either firm – see Figure 3.7. However the resulting market supply curve could be horizontal or even be forward-falling.[3]

3.5 The size of the industry

In the elementary examples of constructing market supply curves from the behavioural response of individual firms (sections 3.2 and 3.3) we made the unwarranted assumption that there was a known, fixed, number of firms n_f: Rather than just *assuming* that there are 2, 4, 8, 16,... firms we need to examine the economic principle that will determine the size of the industry.

Again we work within the context of price-taking firms. If all firms are earning positive profits, as depicted by the shaded area in Figure 3.8, then it is clear why this fixed-n_f approach to constructing the analysis of the supply-and-demand equilibrium in the market will not do. The reason is that other new firms may be able to set up and make a profit. If so, then presumably they will try to do this. How many firms will do so? How will the number of firms n_f be

[3] Suppose each firm's individual supply curve is upward sloping but that the market curve is forward-falling. Explain what happens as market demand increases.

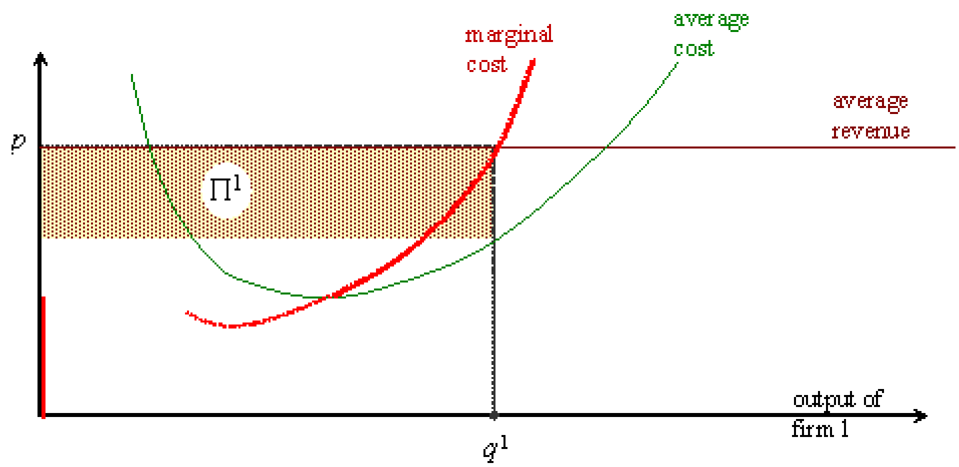

Figure 3.8: Temporary equilibrium of one firm

determined?

We can answer this by extending the elementary argument of the last paragraph. Let the firms be numbered in the order in which they would enter the industry, $1, 2, ..., N, ...$ and suppose the number of firms currently in the industry is n_f. Let q^N be the profit-maximising output for firm N in a price-taking equilibrium (in other words the optimal output and inputs given market prices as we considered for the single competitive firm on page 26). Allow n_f gradually to increase: 1,2,3...: output price p will fall if the market demand curve is downward sloping.[4] If there is a value N such that

$$\Pi^N(q^N) \geq 0 \tag{3.4}$$

and

$$\Pi^{N+1}(q^{N+1}) < 0 \tag{3.5}$$

then $n_f = N$ must represent an equilibrium number of firms.[5] In this full equilibrium we will find that the "marginal firm" is in the situation as depicted in Figure 3.9: profit is zero since the firm is producing where

$$p = MC = AC \tag{3.6}$$

Thus in the full market equilibrium the behaviour of each firm is determined by the standard "price=marginal cost" rule, and the number of firms is solved by a zero-profit condition.

[4] Explain what will happen to input prices if factors are not in perfectly elastic supply.
[5] Provide a one-line argument to explain why this is so.

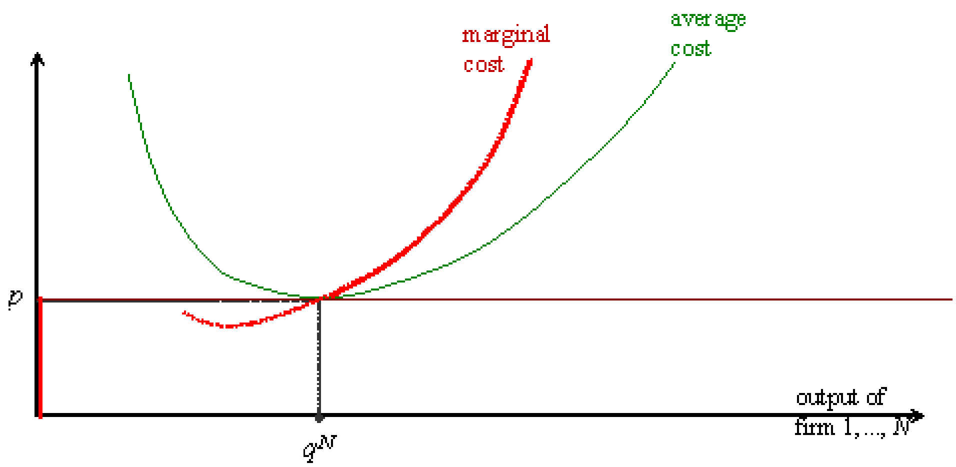

Figure 3.9: Equilibrium of the marginal competitive firm

3.6 Price-setting

So far we have assumed that the firm just accepts all prices as parametrically given. This seems reasonable if the firm has no market power, but it would be interesting to see how the optimisation problem would change were the firm in a position to make a price. We will look at three straightforward developments of the basic model of the firm to examine the effect on the firm's behaviour of having market power.

3.6.1 Simple monopoly

We begin with a case that is easy and unrealistic, but that forms is a very useful starting point. We shall assume that the markets for all inputs are competitive as before: so we can be sure that the derivation of the cost function will go through in the just the same manner as we did it originally. The only effective change to our model is that we shall assume that the product price is a determinate function of output.[6] In other words there is an *inverse demand curve* for output given by:

$$p = p(q). \tag{3.7}$$

This gives the "price that the market will bear". It is useful to introduce the *product demand elasticity* η (a negative number):

$$\eta := \frac{d\log q}{d\log p} = \frac{p(q)}{qp_q(q)}. \tag{3.8}$$

[6] Under what circumstances in the industry would this specification be insufficient? What other information about the market or about the"rules of the game" might be required in order for the firm to determine the price of p at which it can sell its output?

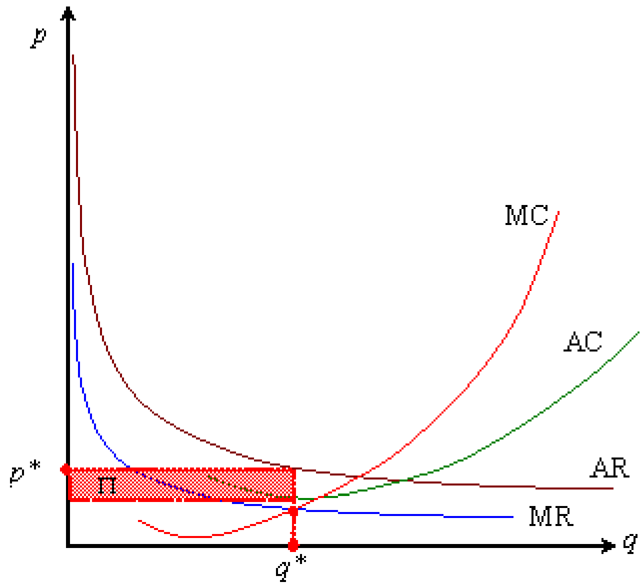

Figure 3.10: Equilibrium of the monopolist

This encapsulates important information for the monopolist: what is the percentage change in the price that the market will bear given a 1-percent change in the volume of output unloaded on to the market?

Profits may now be written as the expression

$$p(q)q - C(\mathbf{w}, q). \tag{3.9}$$

The first-order condition for a maximum is:

$$p(q) + p_q(q)q = C_q(\mathbf{w}, q), \tag{3.10}$$

or, in plain language

$$\boxed{\begin{array}{c}\text{marginal}\\ \text{revenue}\end{array}} = \boxed{\begin{array}{c}\text{marginal}\\ \text{cost}\end{array}}$$

Solving equation (3.10) for q determines the monopolist's optimal output – see the point q^* in Figure 3.10 where the AR (average revenue curve) is the demand curve and MR is marginal revenue.[7] But what of the price?

[7] (a) The average revenue and marginal curves have been drawn of the case where η is a constant. Write down explicit formulae for these curves in this special case. (b) Now suppose that market price given by the relationship $p = a - bq$. Draw the AR and MR curves.

Condition (3.10) can be expressed in another way that illuminates the rational behaviour of the monopolist. A rearrangement of (3.10) gives:[8]

$$p = \frac{C_q(\mathbf{w}, q)}{1 + 1/\eta}. \tag{3.11}$$

The denominator in (3.11) is smaller than 1. So this means that the price-maker uses his market power to force price above marginal cost.

A simple interpretation of (3.11) is that the monopolist uses information about the *shape* of the market demand curve (captured in the parameter η) and not just the price to determine how much of the product to supply to the market. So, in the sense of Definition 2.7 there is no determinate supply curve for the monopolistic firm . Nevertheless there is a determinate solution to the monopolist's problem.

3.6.2 Discriminating monopolist

However, this is just one narrow interpretation of market power. What if the monopolist had yet more power? Suppose for example that the firm could effectively divide the market and sells in two separated markets with prices p^1, p^2 determined as follows

$$\begin{aligned} p^1 &= p^1\left(q^1\right) \\ p^2 &= p^2\left(q^2\right). \end{aligned}$$

where q^1 and q^2 are the amounts delivered to each market and total output is $q = q^1 + q^2$. Profits are now:

$$p^1\left(q^1\right)q^1 + p^2\left(q^2\right)q^2 - C(\mathbf{w}, q) \tag{3.12}$$

To find a maximum we need the following pair of expressions

$$p^i_q\left(q^i\right)q^i + p^i\left(q^i\right) - C_q(\mathbf{w}, q),\ i = 1, 2 \tag{3.13}$$

The outcome of the profit-maximisation problem is one of two types: a solution where the monopolist sells in one market only[9] and, more interestingly, the case where the monopolist sells in both markets and (3.13) yields

$$p^1_q\left(q^1\right)q^1 + p^1\left(q^1\right) = p^2_q\left(q^2\right)q^2 + p^2\left(q^2\right) = C_q(\mathbf{w}, q).$$

or, if η^1 and η^2 are the demand elasticities in the two markets:

$$p^1\left[1 + \frac{1}{\eta^1}\right] = p^2\left[1 + \frac{1}{\eta^2}\right] = C_q(\mathbf{w}, q). \tag{3.14}$$

It is clear that profits are higher[10] than in the case of the simple monopolist and – from (3.14) – that if $\eta^1 < \eta^2 < -1$ then $p^2 > p^1$. We have the intuitively reasonable result that if the monopolistic firm can split the market then it will charge the higher price in the submarket that has the less elastic demand.

[8] For this condition to be meaningful we must have $\eta < -1$. Explain what happens if this condition is violated. Hint: plot (3.9) on a graph and think about what happens as $q \to 0$.

[9] Write down the condition that must be satisfied in this case, derived from (3.13).

[10] Provide an intuitive argument to show that this is true.

3.6.3 Entry fee

Could the monopolist do more – perhaps exercise market power by setting an entry fee for the market? Here is a quick and easy approach to the problem.

One way of interpreting the demand curve (AR) in Figure 3.10 is that the height of the curve $p(x)$ at any output level x gives the consumer's willingness to pay for an extra unit of output given that x units have already been supplied; if this is above the current market price then the consumer is enjoying a "surplus" – the willingness to pay minus the price. Given that an amount q is actually being supplied to the market and that the price is $p(q)$, the total amount of this surplus is given by the expression

$$\int_0^q p(x)\,dx - p(q)\,q, \tag{3.15}$$

the large shaded area in Figure 3.11.

The concept of consumer's surplus is discussed further in chapter 4 (page 90); we use it here to give some extra leverage to the monopolist. Suppose the firm were able to charge an entry fee F_0 to the market in order to capture the consumer's surplus. Then in addition to the conventional profits term $p(q)\,q - C_q(\mathbf{w}, q)$ (the shaded rectangle in Figure 3.11) has the fee revenue F_0 equal to (3.15) so that in this case total profits are

$$\begin{aligned} & \left[\int_0^q p(x)\,dx - p(q)\,q\right] + p(q)\,q - C(\mathbf{w}, q) \\ = & \int_0^q p(x)\,dx - C(\mathbf{w}, q) \end{aligned}$$

Differentiating with respect to q the FOC for this problem is just

$$p(q) - C_q(\mathbf{w}, q) = 0 \tag{3.16}$$

so that we have the nice result that in this case the monopolist sets price equal to marginal cost – see Figure 3.11. Here the firm uses a *two-part tariff* (p, F_0) to charge for its provision of the good.[11]

Example 3.1 *The monopoly-with-entry-fee model has been applied to Disneyland (Oi 1971). Here the marginal cost of some individual entertainments is effectively zero so that the entry fee is set in such a way as to capture the consumer surplus and the rides are then free of charge.*

This model raises further, deeper issues that will be discussed in chapter 11 (page 332).

[11] What type of goods could be carged for in this way?

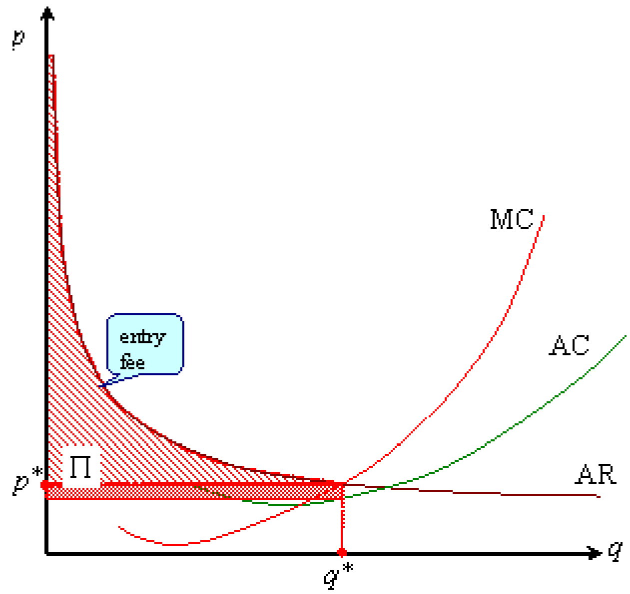

Figure 3.11: Monopolistic market with an entry fee

3.7 Product variety

What if the firms are not all making an identical product? If there is effective product differentiation, then individual firms act as *quasi monopolists* (with downward sloping demand curves instead of facing a given market price). The form of the equilibrium, however, is fairly similar to the homogeneous product case. We need to set out the analogue to perfectly competitive equilibrium in which we discussed the determination of the number of firms: in effect an equilibrium under product differentiation.

Because each firm may have a local monopoly, its behaviour will be different from that discussed in section 3.5. In order to analyse this let us first of all take the situation where the market contains a fixed number of firms. Each firm will make quasi-monopolistic profits (as shown in Figure 3.12), the size of which will depend on the degree of market power that it enjoys through the effective product differentiation which "ties" a section of the market to it.But as we saw in section 3.5 which dealt with homogeneous goods, the fixed-number assumption will not do. If all firms are making positive profits then other firms making products that are differentiated (perhaps only slightly differentiated) will enter the market in the hope of capturing some of these profits. Now, if any new firm enters the market, this will affect the AR and MR curves of other firms: the extent to which this happens will depend on the extent to which the new firm's product is perceived to be a close substitute for the outputs of other firms. The equilibrium is a form of "monopolistic competition"; for the

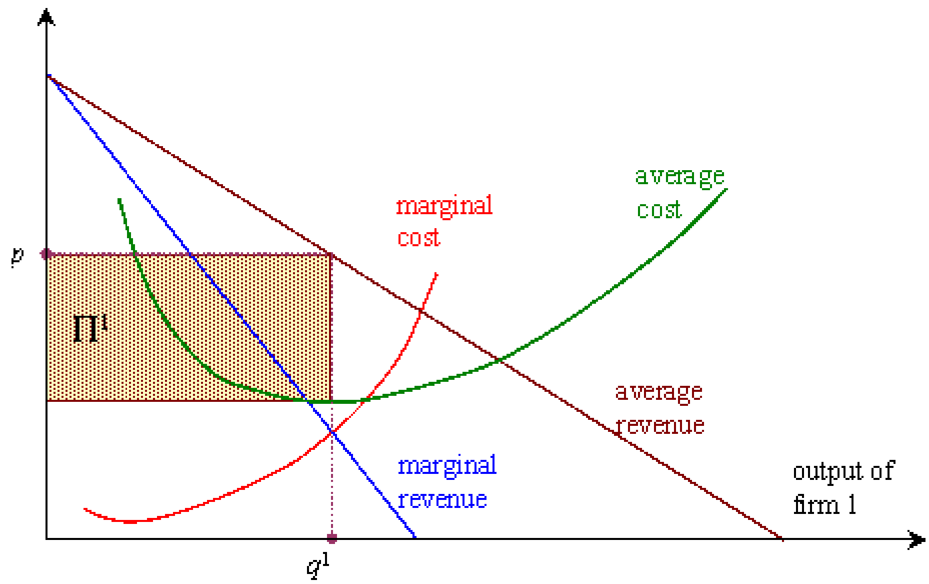

Figure 3.12: Equilibrium for the local monopolist

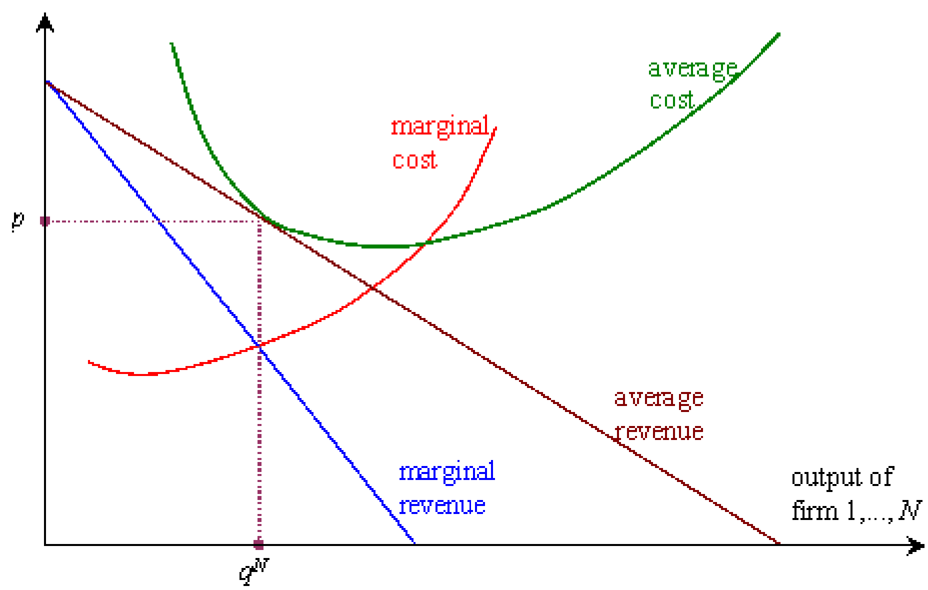

Figure 3.13: The marginal firm in monopolistic competition

marginal firm, the situation is as in Figure 3.13. It makes zero profits but faces a downward-sloping demand curve.

Example 3.2 *What makes one type of good "close" to another in monopolistic competition? One might expect competition amongst firms to be localised in that people are loyal to brands and do not regard products from other firms as perfect substitutes. But how could you identify this localised competition empirically? Schmalensee (1985) shows how to do this in the case of the breakfast cereal industry.*

3.8 Summary

Extending the analysis of a firm in isolation to the mass of firms in the market is fairly straightforward as long as we make a key assumption about the economic environment in which they operate. Each firm faces a determinate demand curve for its product or, in the case where there is product variety, a determinate pattern of demand curves for the various products. On this assumption we can then move on from the approach of chapter 2 and find straightforward, interpretable conditions for firms' equilibrium behaviour. A marginal condition determines the equilibrium output for each firm and a condition on market demand and average costs determines how many firms will be present in the market.

The question of what happens when there is no determinate demand curve is a deep one and will be addressed after we have thought anew about firms' interaction and equilibrium.

3.9 Reading notes

The classic reference on monopolistic competition is Chamberlin (1933); see also Dixit and Stiglitz (1977) that is used as a basis for Exercise 3.2. Various types of discriminating monopoly are treated by Pigou (1952) chapter 17.

3.10 Exercises

3.1 *(The phenomenon of "natural monopoly") Consider an industry in which all the potential member firms have the same cost function C. Suppose it is true that for some level of output $\overline{q}$ and for any nonnegative outputs q, q' of two such firms such that $q + q' \leq \overline{q}$ the cost function satisfies the "subadditivity" property*

$$C(\mathbf{w}, q + q') < C(\mathbf{w}, q) + C(\mathbf{w}, q').$$

1. *Show that this implies that for all integers $N > 1$*

$$C(\mathbf{w}, q) < NC\left(\mathbf{w}, \frac{q}{N}\right), \text{ for } 0 \leq q \leq \overline{q}$$

2. *What must average and marginal curves look like in this case?*

3. *May one conclude that a monopoly must be more efficient in producing this good?*

3.2 *In a particular industry there are* n *profit-maximising firms each producing a single good. The costs for firm* i *are*

$$C_0 + cq_i$$

where C_0 *and* c *are parameters and* q_i *is the output of firm* i*. The goods are not regarded as being exactly identical by the consumers and the inverse demand function for firm* i *is given by*

$$p_i = \frac{Aq_i^{\alpha-1}}{\sum_{j=1}^{n} q_j^{\alpha}}$$

where α *measures the degree of substitutability of the firms' products,* $0 < \alpha \leq 1$*.*

1. *Assuming that each firm takes the output of all the other firms as given, write down the first-order conditions yielding firm 1's output conditional on the outputs* $q_2, ..., q_n$*. Hence, using the symmetry of the equilibrium, show that in equilibrium the optimal output for any firm is*

$$q_i^* = \frac{A\alpha\,[n-1]}{n^2 c}$$

and that the elasticity of demand for firm i *is*

$$\frac{n}{n - n\alpha + \alpha}$$

2. *Consider the case* $\alpha = 1$*. What phenomenon does this represent? Show that the equilibrium number of firms in the industry is less than or equal to* $\sqrt{\frac{A}{C_0}}$*.*

3.3 *A firm has the cost function*

$$F_0 + \frac{1}{2}aq_i^2$$

where q_i *is the output of a single homogenous good and* F_0 *and* a *are positive numbers.*

1. *Find the firm's supply relationship between output and price* p*; explain carefully what happens at the minimum-average-cost point* $\underline{p} := \sqrt{2aF_0}$*.*

2. *In a market of a thousand consumers the demand curve for the commodity is given by*
$$p = A - bq$$
where q is total quantity demanded and A and b are positive parameters. If the market is served by a single price-taking firm with the cost structure in part 1 explain why there is a unique equilibrium if $b \leq a\left[A/\underline{p} - 1\right]$ and no equilibrium otherwise.

3. *Now assume that there is a large number N of firms, each with the above cost function: find the relationship between average supply by the N firms and price and compare the answer with that of part 1. What happens as $N \to \infty$?*

4. *Assume that the size of the market is also increased by a factor N but that the demand per thousand consumers remains as in part 2 above. Show that as N gets large there will be a determinate market equilibrium price and output level.*

3.4 *A firm has a fixed cost F_0 and marginal costs*
$$c = a + bq$$
where q is output.

1. *If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? If the competitive price were above this level, find the amount of output q^* that the firm would produce.*

2. *If the firm is actually a monopolist and the inverse demand function is*
$$p = A - \frac{1}{2}Bq$$
(where $A > a$ and $B > 0$) find the expression for the firm's marginal revenue in terms of output. Illustrate the optimum in a diagram and show that the firm will produce
$$q^{**} := \frac{A - a}{b + B}$$
What is the price charged p^{**} and the marginal cost c^{**} at this output level? Compare q^{**} and q^*.

3. *The government decides to regulate the monopoly. The regulator has the power to control the price by setting a ceiling $p_{\max}$. Plot the average and marginal revenue curves that would then face the monopolist. Use these to show:*
   - (a) *If $p_{\max} > p^{**}$ the firm's output and price remain unchanged at q^{**} and p^{**}*

(b) If $p_{\max} < c^{**}$ *the firm's output will fall below* q^{**}*.*

(c) Otherwise output will rise above q^{**}*.*

3.5 *A monopolist has the cost function*

$$C(q) = 100 + 6q + \frac{1}{2}[q]^2$$

1. *If the demand function is given by*

$$q = 24 - \frac{1}{4}p$$

   calculate the output-price combination which maximises profits.

2. *Assume that it becomes possible to sell in a separate second market with demand determined by*

$$q = 84 - \frac{3}{4}p.$$

   Calculate the prices which will be set in the two markets and the change in total output and profits from case 1.

3. *Now suppose that the firm still has access to both markets, but is prevented from discriminating between them. What will be the result?*