ΠΜΣ «Οικονομική Επιστήμη»
ΕΚΠΑ Τμήμα Οικονομικών Επιστημών
Microeconomic Theory – Producer Theory January 2020
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Problem Set 3

1. Consider a duopoly with identical firms. The cost function for firm f is

$$C_0 + cq^f \ , f = 1,2$$

The inverse demand function is

$$\beta_0 - \beta q$$

where C_0, c, β_0 and β are all positive numbers and total output is given by

$$q \ = q^1 + q^2$$

(a) Find the isoprofit contour and the reaction function for firm 2.
(b) Find the Cournot-Nash equilibrium for the industry and illustrate it in q 1 ; q2 -space.
(c) Find the joint-profit maximising solution for the industry and illustrate it on the same diagram.
(d) If firm 1 acts as leader and firm 2 as a follower find the Stackelberg solution.
(e) Draw the set of payoff possibilities and plot the payoffs for cases 2-4 and for the case where there is a monopoly.

2. You are given the following payoffs associated with two pure strategies of each of two players (a, b) in a simultaneous move game.

	Player b		
Player a		s_1^b	s_2^b
	s_1^a	3,5	10,0
	s_2^a	6,2	6,4

(a) Are there any dominant strategies in this game? Explain.
(b) Are there any Nash equilibria in this game? Explain.
(c) How would you describe this game? Can you think of any real world examples?
(d) Find the mixed-strategy equilibrium.
(e) Show the mixed-strategy equilibrium in the space of probabilities. Explain.
(f) Show an extensive form of this simultaneous move game. Explain.

3. Consider a sequential-move bargaining game between Player 1 (proposer) and Player 2 (responder). Player 1 makes a take-it-or-leave-it offer to Player 2, specifying an amount $s = \left\{0, \frac{1}{2}v, v\right\}$ out of an initial surplus v, i.e., no share of the pie, half of the pie, or all of the pie. If Player 2 accepts such a distribution Player 2 receives the offer s, while Player 1 keeps the remaining surplus $v - s$. If Player 2 rejects, both players get a zero payoff.

(a) Describe the strategy space for every player.
(b) Provide the normal-form representation of this bargaining game.
(c) Does any player have strictly dominated pure strategies?
(d) Does any player have strictly dominated mixed strategies?

4. Consider the following payoff matrix depicting a simultaneous-move game between players 1 and 2.

		Player 2		
		d	e	f
Player 1	A	$\underline{8}, 0$	$8, 1$	$8, \underline{8}$
	B	$\underline{8}, 0$	$6, \underline{8}$	$\underline{9}, 1$
	C	$6, \underline{8}$	$\underline{9}, 1$	$8, 1$

(a) What equilibrium prediction can you find using Iterated Deletion of Strictly Dominated Strategies (IDSDS)?
(b) Can you identify one or more Nash equilibria in pure strategies?
(c) Can you find a mixed strategy Nash equilibrium?

5. Consider the following simultaneous-move game with payoff matrix

		Player 2	
		A	B
Player 1	A	$u_1(A,A), u_2(A,A)$	$u_1(A,B), u_2(A,B)$
	B	$u_1(B,A), u_2(B,A)$	$u_1(B,B), u_2(B,B)$

In addition, assume that $u_i(A,A) = uj\ (A,A)$ and $u_i(B,B) = u_j\ (B,B)$, and that $u_i(A,B) = u_j\ (B,A)$ for every player $i = \{1,2\}$ and $j \neq i$. Find players' strictly dominated strategies and the Nash equilibria of the game (allowing for both pure and mixed strategies) in the following settings. Interpret and relate your results to common games.

(a) $u_i(A,A) > u_i(B,A) and\ u_i(A,B) > u_i(B,B)$ for every player i.
(b) $u_i(A,A) > u_i(B,A) and\ u_i(A,B) < u_i(B,B)$ for every player i.
(c) $u_i(A,A) < u_i(B,A) and\ u_i(A,B) < u_i(B,B)$ for every player i.
(d) $u_i(A,A) < u_i(B,A) and\ u_i(A,B) > u_i(B,B)$ for every player i.