ΠΜΣ «Οικονομική Επιστήμη»
ΕΚΠΑ Τμήμα Οικονομικών Επιστημών
Microeconomic Theory – Producer Theory January 2020
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**Problem Set 2– Key**

*1. A firm has a fixed cost* $F_0$ *and marginal costs*

$$c = a + bq$$

*where* $q$ *is output.*

(a) *If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? If the competitive price were above this level, find the amount of output* $q^*$ *that the firm would produce.*

The Total Costs are

$$F_0 + aq + \frac{1}{2}bq^2$$

And thus the average costs are

$$\frac{F_0}{q} + a + \frac{1}{2}bq$$

which minimum is $\bar{\bar{q}} = \sqrt{2\frac{F_0}{b}}$ and for that output, the average costs are $\sqrt{2bF_0} + a$

For a price above the level $\sqrt{2bF_0} + a$ the first-order condition for maximum profits is given by

$$p = a + bq$$

from which we find

$$q^* := \frac{p-a}{b}$$

(b) *If the firm is actually a monopolist and the inverse demand function is*

$$p = A - \frac{1}{2}Bq$$

*(where* $A > a$ *and* $B > 0$*) find the expression for the firm's marginal revenue in terms of output. Illustrate the optimum in a diagram and show that the firm will produce*

$$q^{**} := \frac{A-a}{b+B}$$

*What is the price charged* $p^{**}$ *and the marginal cost* $c^{**}$ *at this output level? Compare* $q^{**}$ *and* $q^*$*.*

If the firm is a monopolist marginal revenue is

$$\frac{\vartheta}{\vartheta q}\left[Aq - \frac{1}{2}Bq^2\right] = A - Bq$$

hence the FOC for the monopolist is

$$A - Bq = a + bq$$

from which the solution $q^{**}$ follows. Substituting for $q^{**}$ we also

$$c^{**} = A - Bq^{**} = \frac{Ab + Ba}{B + b}$$

$$p^{**} = A - \frac{1}{2}Bq^{**} = c^{**} + \frac{1}{2}B\frac{A-a}{b+B}$$

(c) *The government decides to regulate the monopoly. The regulator has the power to control the price by setting a ceiling $p_{max}$. Plot the average and marginal revenue curves that would then face the monopolist. Use these to show:*

a. *If $p_{max} > p^{**}$ the firm's output and price remain unchanged at $q^{**}$ and $p^{**}$*
b. *If $p_{max} < c^{**}$ the firm's output will fall below $q^{**}$*
c. *Otherwise output will rise above $q^{**}$.*

Consider how the introduction of a price ceiling will affect average revenue. Clearly we now have,

$$AR(q) = \begin{Bmatrix} p_{max} \; if \; q \leq q_0 \\ A - \frac{1}{2}Bq \; if \; q \geq q_0 \end{Bmatrix}$$

where $q_0 := 2[A - p_{max}]/B$ max] average revenue is a continuous function of $q$ but has a kink at $q_0$. Now we may derive the marginal revenue, that is

$$MR(q) = \begin{Bmatrix} p_{max} \; if \; q \leq q_0 \\ A - Bq \; if \; q \geq q_0 \end{Bmatrix}$$

*2. A monopolist has the cost function*

$$C(q) = 100 + 6q + \frac{1}{2}[q]^2$$

(a) *If the demand function is given by*

$$q = 24 - \frac{1}{4}p$$

*calculate the output-price combination which maximises profits*

Maximizing the simple monopolistís profits,

$$\Pi_0 = (94 - 4q)q - \left(100 + 6q + \frac{1}{2}[q]^2\right)$$

which with respect to q yields optimum output of $q_0 = 10$. Hence $p_0 = 56$ and $\Pi_0 = 350$.

(b) *Assume that it becomes possible to sell in a separate second market with demand determined by*

$$q = 84 - \frac{3}{4}p$$

*Calculate the prices which will be set in the two markets and the change in total output and profits from case (a).*

The new problem is to choose $q_1, q_2$ so as to maximise the function

$$\Pi_{12} = (94 - 4q_1)q_1 + \left(112 - \frac{4}{3}q_2\right)q_2 - \left(100 + 6q_1 + 6q_2 + \frac{1}{2}[q_1 + q_2]^2\right)$$

The FOC yield

$$9q_1 + q_2 = 90$$
$$q_1 + \frac{11}{3}q_2 = 106$$

Thus we have $\begin{matrix} q_1 = 7 & p_1 = 68 \\ q_2 = 27 & p_2 = 76 \end{matrix}$ and $\Pi_{12} = 1646$

(c) *Now suppose that the firm still has access to both markets, but is prevented from discriminating between them. What will be the result?*

If we abandon discrimination, a uniform price $\hat{p}$ must be charged. If $\hat{p} < 112$, nothing is sold to either market. If $112 > \hat{p} > 96$ only market 2 is served. If $96 > \hat{p}$ both markets are served and the demand curve is $\hat{q} = 108 - p$. Clearly this is the relevant region. Maximising simple monopoly profits we find $\hat{q} = 34, \hat{p} = 74$ and $\widehat{\Pi} = 1634$.
Hence the total output is identical to that under discrimination, $p_1 < \hat{p} < p_2$ and $\Pi_{12} > \widehat{\Pi}$. These results are quite general.

*3. Suppose that a firm owns two plants, each producing the same good. Every plant j's average cost is given by*

$$AC_j(q_j) = a + \beta_j q_j \text{ for } q_j \geq 0, \text{ where } j = \{1,2\}$$

*where coefficient $\beta_j$ may differ from plant to plant, i.e. if $\beta_1 > \beta_2$ plant 2 is more efficient than plant 1 since its average costs increase less rapidly in output. Assume that you are asked to determine the cost-minimizing distribution of aggregate output $q = q_1 + q_2$, among the two plants (i.e., for a given aggregate output q, how much q1 to produce in plant 1 and how much q2 to produce in plant 2.) For simplicity, consider that aggregate output q satisfies $q < \frac{a}{\max_j |\beta_j|}$. (You will be using this condition in part b.)*

(a) *If $\beta_j > 0$ for every plant j, how should output be located among the two plants?*

The cost-minimization problem in which we find the optimal combination of output $q_1$ and $q_2$ that minimizes the total cost of production across plants is

$$\min_{q_1,q_2} TC_1(q_1) + TC_2(q_2)$$
$$s.t.\ \ q_1 + q_2 = q$$

or equivalently, the profit maximization problem in which firms choose the optimal combination of output $q_1$ and $q_2$ that maximizes the total profits across all plants is

$$\max_{q_1,q_2} \underbrace{pq_1 - TC_1(q_1)}_{\pi_1} + \underbrace{pq_2 - TC_2(q_2)}_{\pi_2}$$
$$s.t.\ \ q_1 + q_2 = q$$

Given that $AC_j(q_j) = a + \beta_j q_j$, we derive that $TC_j(q_j) = (a + \beta_j q_j)q_j$. In this way the above PMP yields,

$$\max_{q_1, q_2} pq_1 - (a + \beta_1 q_1)q_1 + pq_2 - (a + \beta_2 q_2)q_2$$

$$s.t.\ q_1 + q_2 = q$$

Using the FOC, they yield

$$p - a - 2\beta_1 q_1 = p - a - 2\beta_2 q_2$$

by rearranging and replacing into the constraint we get

$$q_1 + \underbrace{\frac{\beta_1}{\beta_2} q_1}_{q_2} = q$$

and solving for $q_1$ entails the cost-minimizing production in plant 1,

$$q_1\left(1 + \frac{\beta_1}{\beta_2}\right) = q\ , thus\ q_1 = \frac{\beta_2}{\beta_1 + \beta_2} q$$

And by symmetry we have,

$$q_2 = \frac{\beta_1}{\beta_1 + \beta_2} q$$

*(b) If $\beta_j < 0$ for every plant j, how should output be located among the two plants?*

First, note that $\beta_j < 0$ implies that the average cost $AC_j(q_j) = a + \beta_j q_j$ is decreasing in output. Hence, it is cost-minimizing to concentrate all production on the plant with the smallest $\beta_j < 0$ (the most negative $\beta_j$) because average costs (and total costs) are minimized by doing so.

*(c) If $\beta_j > 0$ for for some plants and $\beta_i < 0$ for others?*

Similarly as in part (b), the firm now faces some plants with increasing average costs (those with $\beta_j > 0$) and some plants with decreasing average costs (those with $\beta_j < 0$). Hence, it is cost-minimizing to concentrate all production on the plant/s with the smallest $\beta_j < 0$, since it benefits from the most rapidly decreasing average costs.

In a firm in which both plants exhibit decreasing average costs, but $\beta_2 < \beta_1 < 0$ , implying that it is beneficial for the firm to concentrate all output in plant 2. In addition, note that the average cost in plant 1 is positive for all $q_1$as long as $\alpha - \beta_1 q_1 > 0$ , or $q_1 < \frac{\alpha}{\beta_1}$, where $\frac{\alpha}{\beta_1}$ represents the horizontal intercept of $AC_1$. Similarly for firm 2, where $AC_2$ for all $q_2$as long as $q_2 < \frac{\alpha}{\beta_2}$, where $\frac{\alpha}{\beta_2}$ represents the horizontal intercept of $AC_2$. Hence, the original condition $q < \frac{a}{\max_j |\beta_j|}$ is equivalent to $q < \min_j \frac{a}{|\beta_j|}$

*4. A firm has a fixed cost of €400 and a total variable costs* $= 20q + 0.25\,q^2$ *where* $q$ *is output.*

(a) *If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? How much output* $q^*$ *would it produce at this price? What is the perfectly competitive firm's supply curve?*
(b) *If the firm is actually a monopolist and the inverse demand function is* $p = 170 - q$. *What is the price charged* $p^{**}$ *and the marginal cost* $c^{**}$ *at this ouput. Illustrate the monopoly optimum in a diagram.*
(c) *The government decides to regulate the monopoly. The government can set a ceiling of* $p_{max}$. *In a separate duplicate graph of b plot the average and marginal revenue curves that would face the monopolist, explaining how output will react to different price ceilings relative to* $c^{**}$ *and* $p^{**}$.
(d) *Linking to diagram in (b) provide a diagramatic exposition of monopolistic competition and explain.*

**The key for this exercise is on this directory: https://eclass.uoa.gr/modules/document/index.php?course=ECON258&openDir=/54ce0fc5DNfu