Problem Set 1

1. Consider a firm whose production function f(z) exhibits constant returns to scale. Show that its cost function can be expressed as $c(w, q) = q \cdot c(w, 1)$, i.e., the cost per unit times the number of units produced.

2. Suppose a firm's production function has the Cobb-Douglas form

$$q = z_1^{a_1} z_2^{a_2}$$

where z_1 and z_2 are inputs, q is output and a_1, a_2 are positive parameters.

   (a) Draw the isoquants. Do they touch the axes?
   (b) What is the elasticity of substitution in this case?
   (c) Using the Lagrangean method find the cost-minimising values of the inputs and the cost function.
   (d) Under what circumstances will the production function exhibit (a) decreasing (b) constant (c) increasing returns to scale? Explain this using first the production function and then the cost function.
   (e) Find the conditional demand curve for input 1.

3. Consider the following profit function that has been obtained from a technology that uses a single input, z:

$$\pi(p, w) = p^2 w^a$$

where p is the output price, w is the input price and a is a parameter value.

   (a) Check if the profit function satisfies homogeneity of degree one jointly in both p and w. In particular, determine for which values of this property is satisfied.
   (b) Assuming the value of a for which the profit function satisfies homogeneity of degree one, check if the profit function $\pi(p, w)$ satisfies the following properties: (1) non-decreasing in output price p, (2) non-increasing in input prices w, and (3) convex in prices p and w.
   (c) Calculate the supply function of the firm, $q(p, w)$, and its demand for inputs, $z(p, w)$

4. For the production function

$$q = z_1^{1/4} z_2^{1/4}$$

   (a) Find the conditional demand functions for z1 and z2.
   (b) Find the cost function.
   (c) Find the supply function.
   (d) Find the input demand (Marshallian) function for $z1$. Briefly explain other ways of deriving the demand function.
   (e) Find the short run supply function when $\bar{z}_2 = 16$ ($z_2^{1/4} = 2$). Will this firm always supply at a positive price? Explain.