Prerequisites

Almost essential
Monopoly

Useful, but optional
Game Theory: Strategy and Equilibrium

Duopoly

MICROECONOMICS
Principles and Analysis
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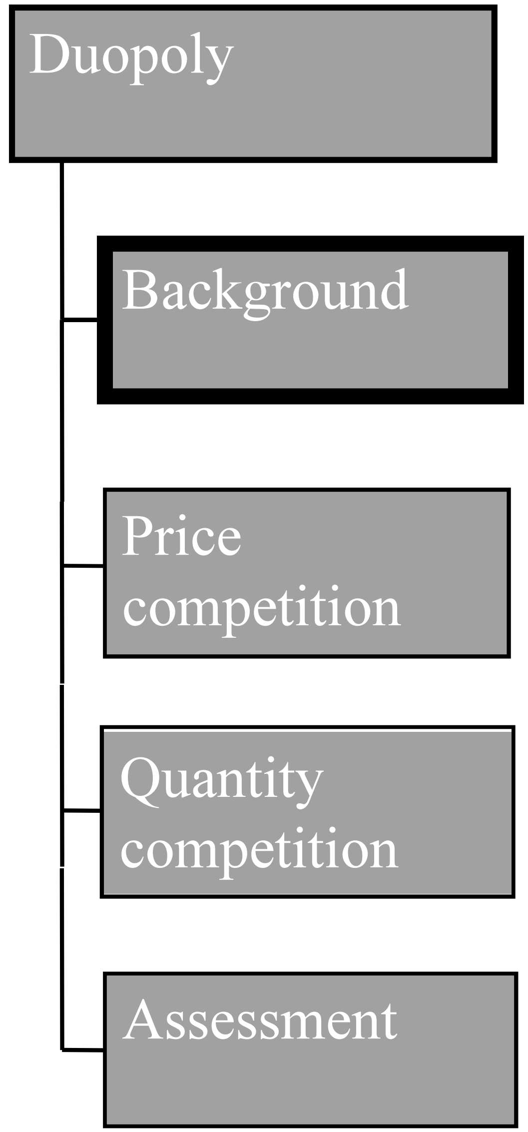
Overview...



How the basic elements of the firm and of game theory are used.

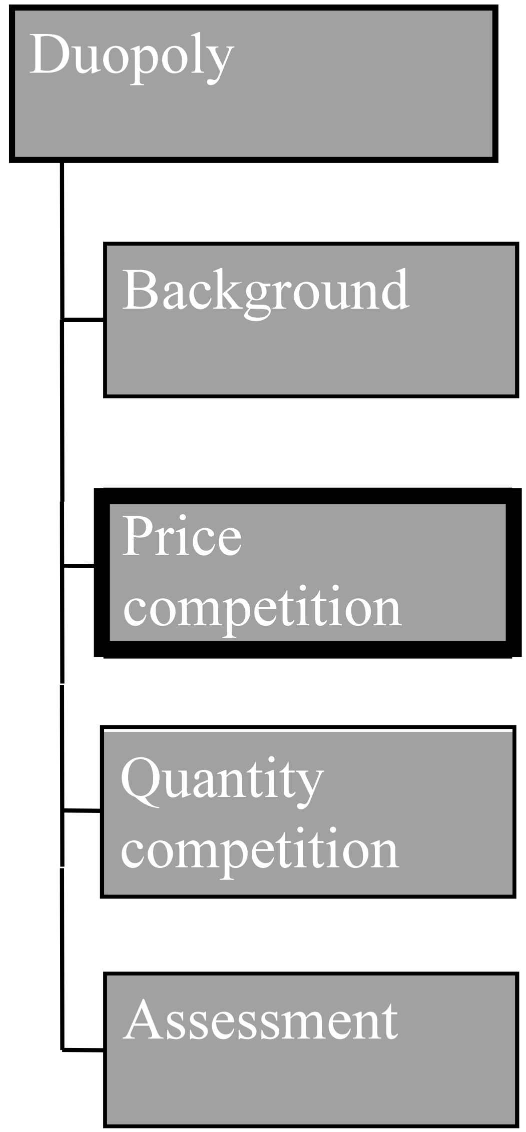
Basic ingredients

- Two firms:
  - Issue of entry is not considered.
  - But monopoly could be a special limiting case.
- Profit maximisation.
- Quantities or prices?
  - There's nothing within the model to determine which "weapon" is used.
  - It's determined *a priori.*
  - Highlights artificiality of the approach.
- Simple market situation:
  - There is a known demand curve.
  - Single, homogeneous product.

Reaction

- We deal with “competition amongst the few”.
- Each actor has to take into account what others do.
- A simple way to do this: *the reaction function*.
- Based on the idea of “best response”.
  - We can extend this idea…
  - In the case where more than one possible reaction to a particular action.
  - It is then known as a reaction *correspondence*.
- We will see how this works:
  - Where reaction is in terms of prices.
  - Where reaction is in terms of quantities.

Overview...



Introduction to a simple simultaneous move price-setting problem.

Competing by price

- There is a market for a single, homogeneous good.
- Firms announce prices.
- Each firm does not know the other's announcement when making its own.
- Total output is determined by demand.
  - Determinate market demand curve
  - Known to the firms.
- Division of output amongst the firms determined by market "rules."
- Let's take a specific model with a clear-cut solution…

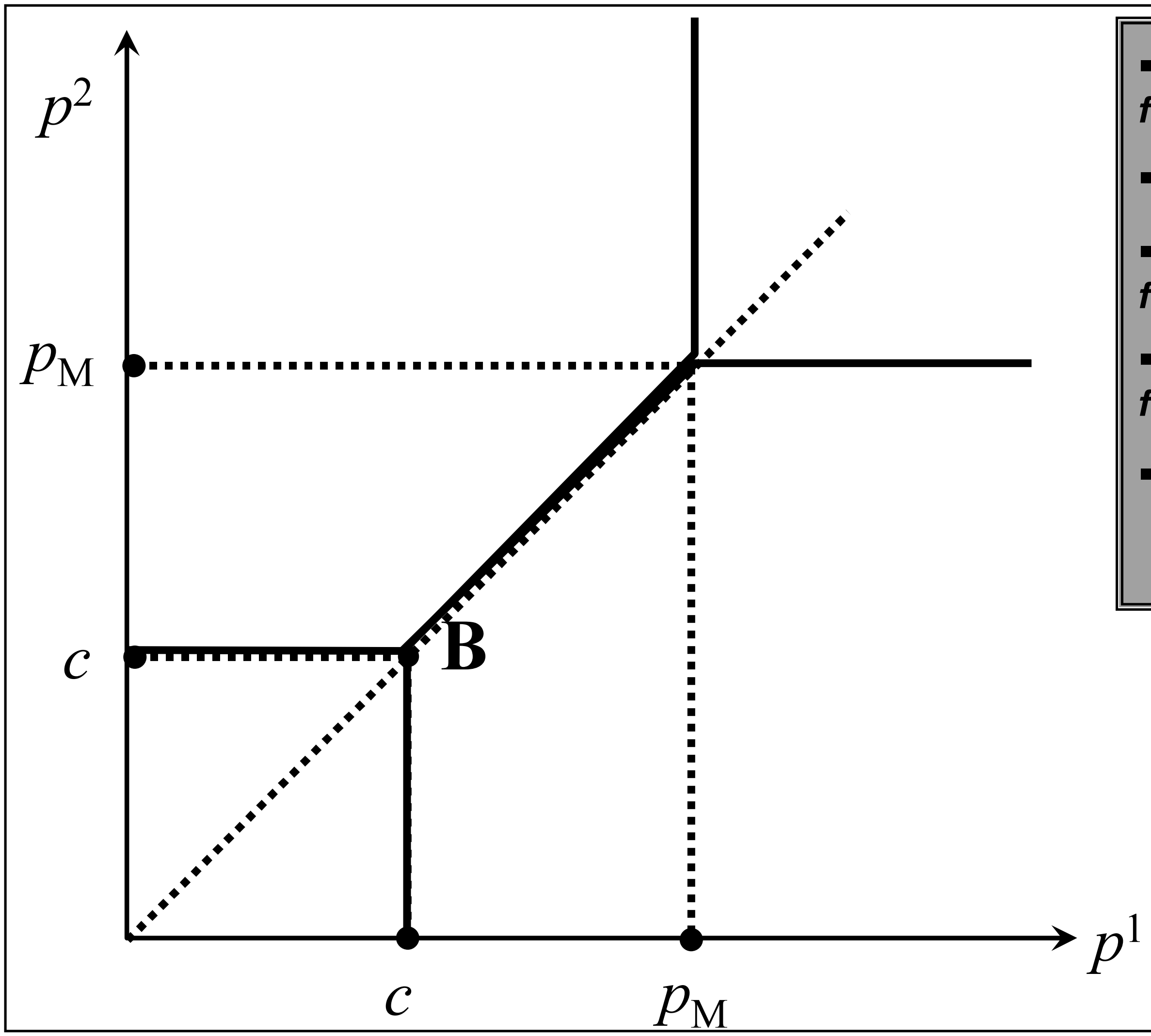
Bertrand – basic set-up

- Two firms can potentially supply the market.
- Each firm: zero fixed cost, constant marginal cost c.
- If one firm alone supplied the market it would charge monopoly price $p_M > c$.
- If both firms are present they announce prices.
- The outcome of these announcements:
  - If $p^1 < p^2$ firm 1 captures the whole market.
  - If $p^1 > p^2$ firm 2 captures the whole market.
  - If $p^1 = p^2$ the firms supply equal amounts to the market.
- What will be the equilibrium price?

Bertrand – best response?

- Consider firm 1's response to firm 2
- If firm 2 foolishly sets a price p^2 above p_M then it sells zero output.
  - Firm 1 can safely set monopoly price p_M .
- If firm 2 sets p^2 above c but less than or equal to p_M then firm 1 can "undercut" and capture the market.
  - Firm 1 sets $p^1 = p^2 - \delta$, where $\delta > 0$.
  - Firm 1's profit always increases if δ is made smaller…
  - …but to capture the market the discount δ must be positive!
  - So strictly speaking there's no *best* response for firm 1.
- If firm 2 sets price equal to c then firm 1 cannot undercut
  - Firm 1 also sets price equal to c .
- If firm 2 sets a price below c it would make a loss.
  - Firm 1 would be crazy to match this price.
  - If firm 1 sets $p^1 = c$ at least it won't make a loss.
- Let's look at the diagram…

Bertrand model – equilibrium

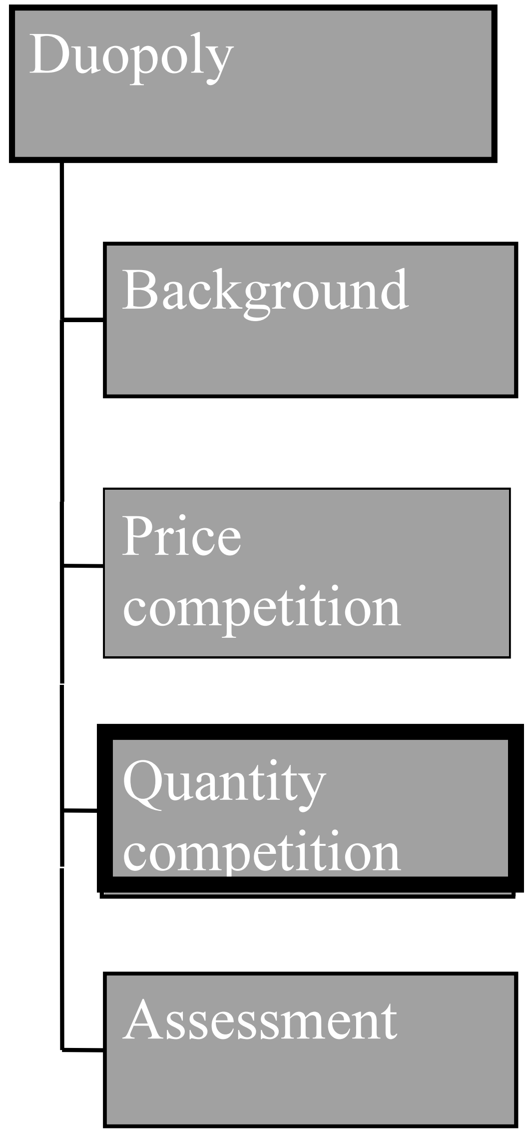


- *Marginal cost for each firm*
- *Monopoly price level*
- *Firm 1's reaction function*
- *Firm 2's reaction function*
- *Bertrand equilibrium*

Bertrand – assessment

- Using "natural tools" – prices.
- Yields a remarkable conclusion.
- Mimics the outcome of perfect competition
  - Price = MC.
- But it is based on a special case.
- Neglects some important practical features
  - Fixed costs.
  - Product diversity
  - Capacity constraints.
- Outcome of price-competition models usually very sensitive to these.

Overview...



- **Collusion**
- The Cournot model
- Leader-Follower

The link with monopoly and an introduction to two simple "competitive" paradigms.

quantity models

- Now take *output quantity* as the firms' choice variable.
- Price is determined by the market once total quantity is known:
  - An auctioneer?
- Three important possibilities:

1. Collusion:
   - Competition is an illusion.
   - Monopoly by another name.
   - But a useful reference point for other cases
2. Simultaneous-move competing in quantities:
   - Complementary approach to the Bertrand-price model.
3. Leader-follower (sequential) competing in quantities.

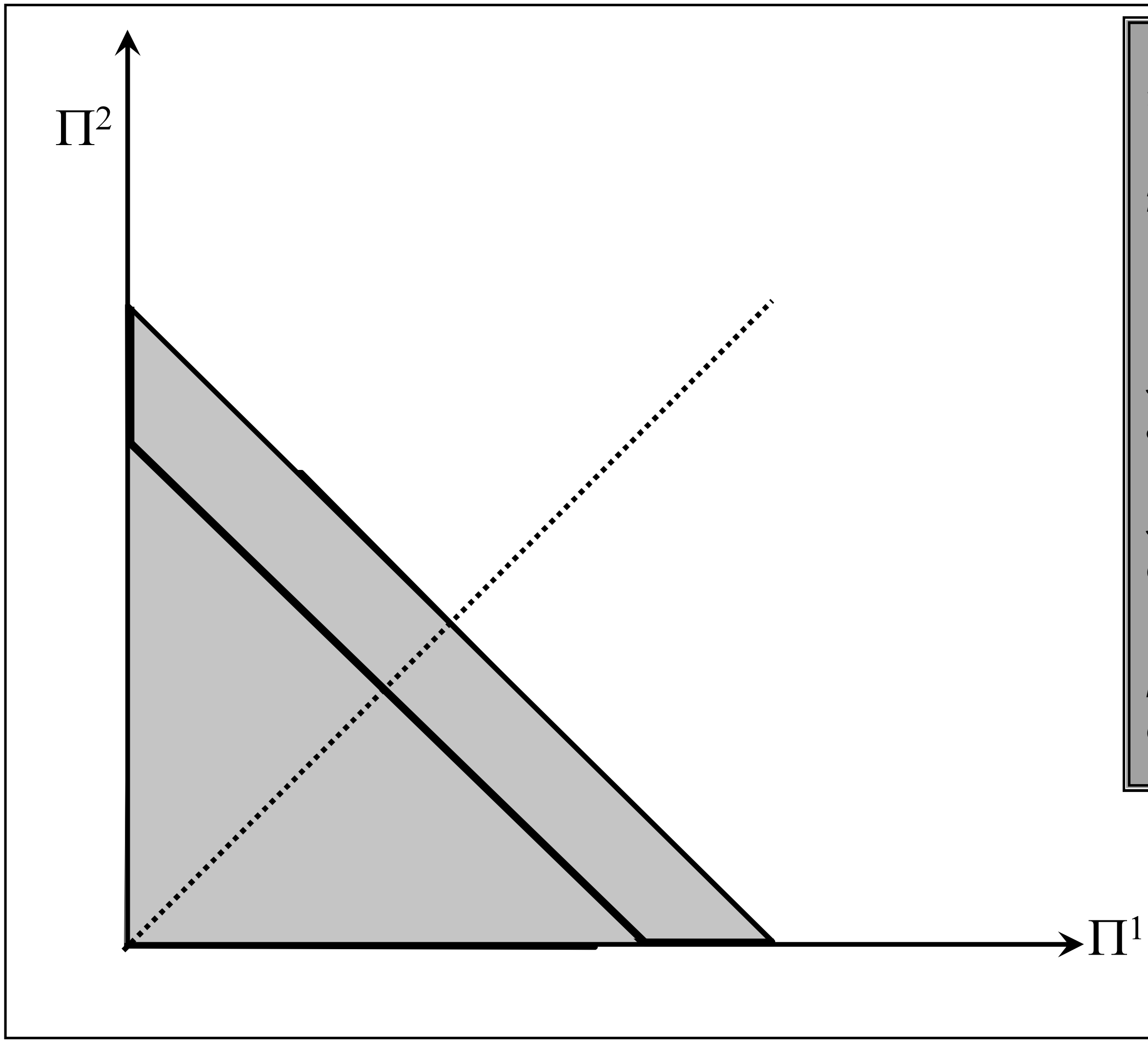
Collusion – basic set-up

- Two firms agree to maximise joint profits.
- This is what they can make by acting as though they were a single firm.
  - Essentially a monopoly with two plants.
- They also agree on a rule for dividing the profits.
  - Could be (but need not be) equal shares.
- In principle these two issues are separate.

The profit frontier

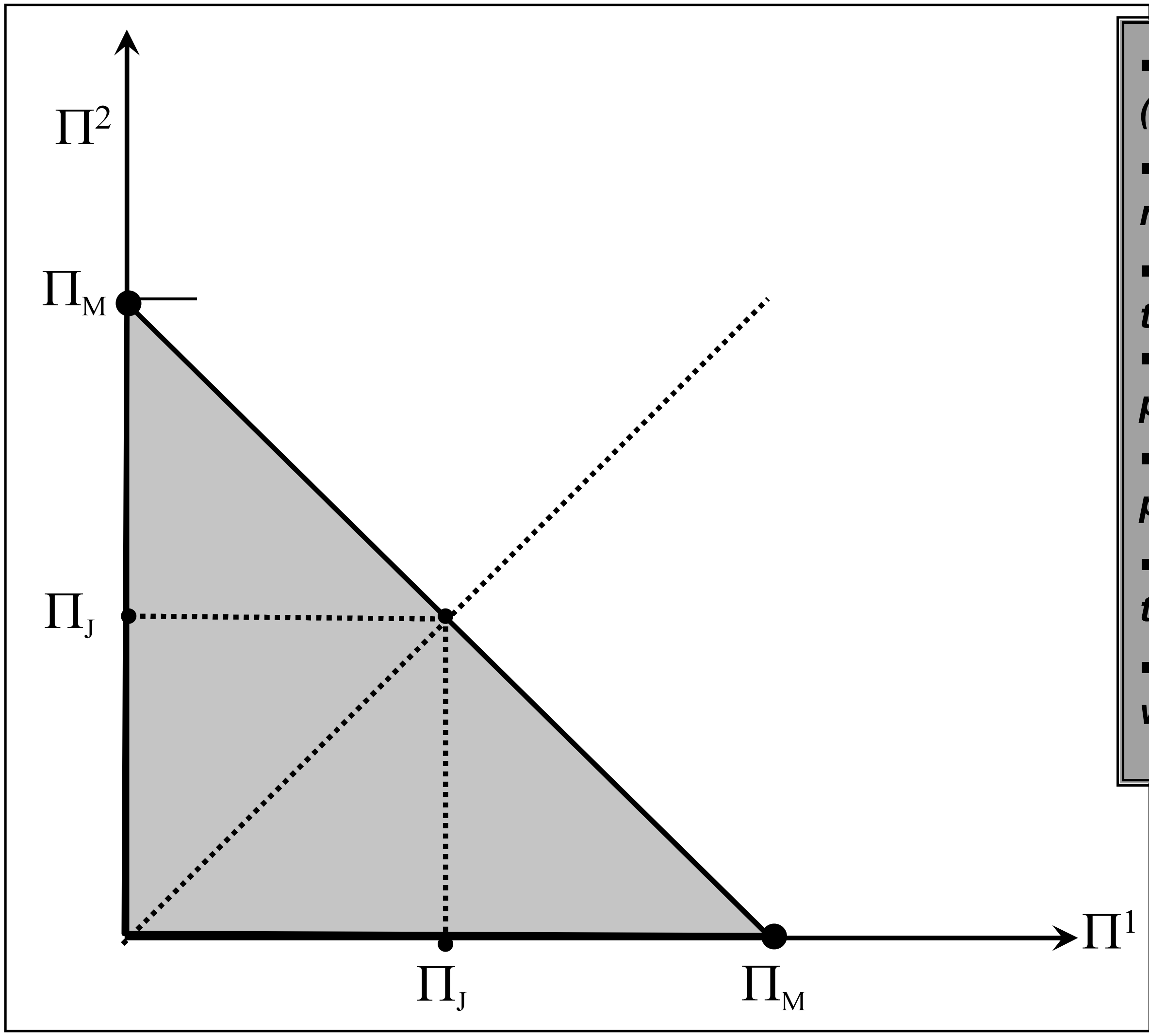
- To show what is possible for the firms…
- …draw the *profit frontier*.
- Show the possible combination of profits for the two firms
  - given demand conditions
  - given cost function
- Start with the case where cash transfers between the firms are not possible

Frontier – non-transferable profits



- *Suppose profits can't be transferred between firms*
- *Take case of identical firms*
- *Constant returns to scale*
- *Decreasing returns to scale in each firm (1): MC always rising*
- *Decreasing returns to scale in each firm (2): capacity constraints*
- *Increasing returns to scale in each firm (fixed cost and constant marginal cost)*

Frontier – transferable profits



- ***Increasing returns to scale (without transfers)***
- ***Now suppose firms can make "side-payments"***
- ***So profits <u>can</u> be transferred between firms***
- ***Profits if everything were produced by firm 1***
- ***Profits if everything were produced by firm 2***
- ***The profit frontier if transfers are possible***
- ***Joint-profit maximisation with equal shares***

- *Cash transfers "convexify" the set of attainable profits.*

Collusion – simple model

- Take the special case of the "linear" model where marginal costs are identical: $c^1 = c^2 = c$.
- Will both firms produce a positive output?
  - If unlimited output is possible then only one firm needs to incur the fixed cost…
  - …in other words a true monopoly.
  - But if there are capacity constraints then both firms may need to produce.
  - Both firms incur fixed costs.
- We examine both cases – capacity constraints first.

Collusion: capacity constraints

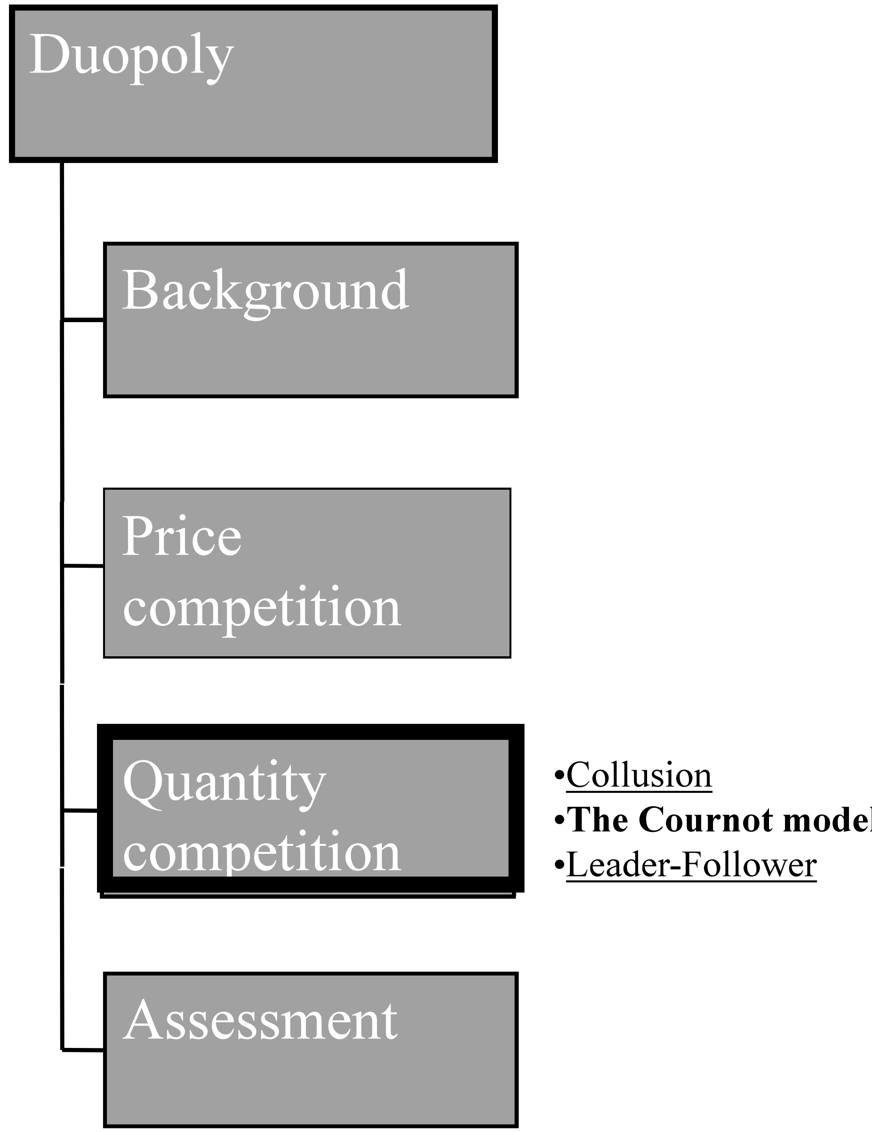
- If both firms are active total profit is
  $[a - bq]\, q \ - \ [C_0^1 + C_0^2 + cq]$
- Maximising this, we get the FOC:
  $a - 2bq - c = 0.$
- Which gives equilibrium quantity and price:
  $$q = \frac{a - c}{2b}; \qquad p = \frac{a + c}{2}.$$
- So maximised profits are:
  $$\Pi_M = \frac{[a - c]^2}{4b} \ - \ [C_0^1 + C_0^2].$$
- Now assume the firms are identical: $C_0^1 = C_0^2 = C_0$.
- Given equal division of profits each firm's payoff is
  $$\Pi_J = \frac{[a - c]^2}{8b} \ - \ C_0.$$

Collusion: no capacity constraints

- With no capacity limits and constant marginal costs…
- …there seems to be no reason for both firms to be active.
- Only need to incur *one* lot of fixed costs C_0.
  - C_0 is the smaller of the two firms' fixed costs.
  - Previous analysis only needs slight tweaking.
- Modify formula for Π_J by replacing C_0 with ½C_0.
- But is the division of the profits still implementable?

Overview...

Simultaneous move "competition" in quantities



Cournot – basic set-up

- Two firms.
  - Assumed to be profit-maximisers
  - Each is fully described by its cost function.
- Price of output determined by demand.
  - Determinate market demand curve
  - Known to both firms.
- Each chooses the quantity of output.
  - Single homogeneous output.
  - Neither firm *knows* the other's decision when making its own.
- Each firm makes an *assumption* about the other's decision
  - Firm 1 assumes firm 2's output to be given number.
  - Likewise for firm 2.
- How do we find an equilibrium?

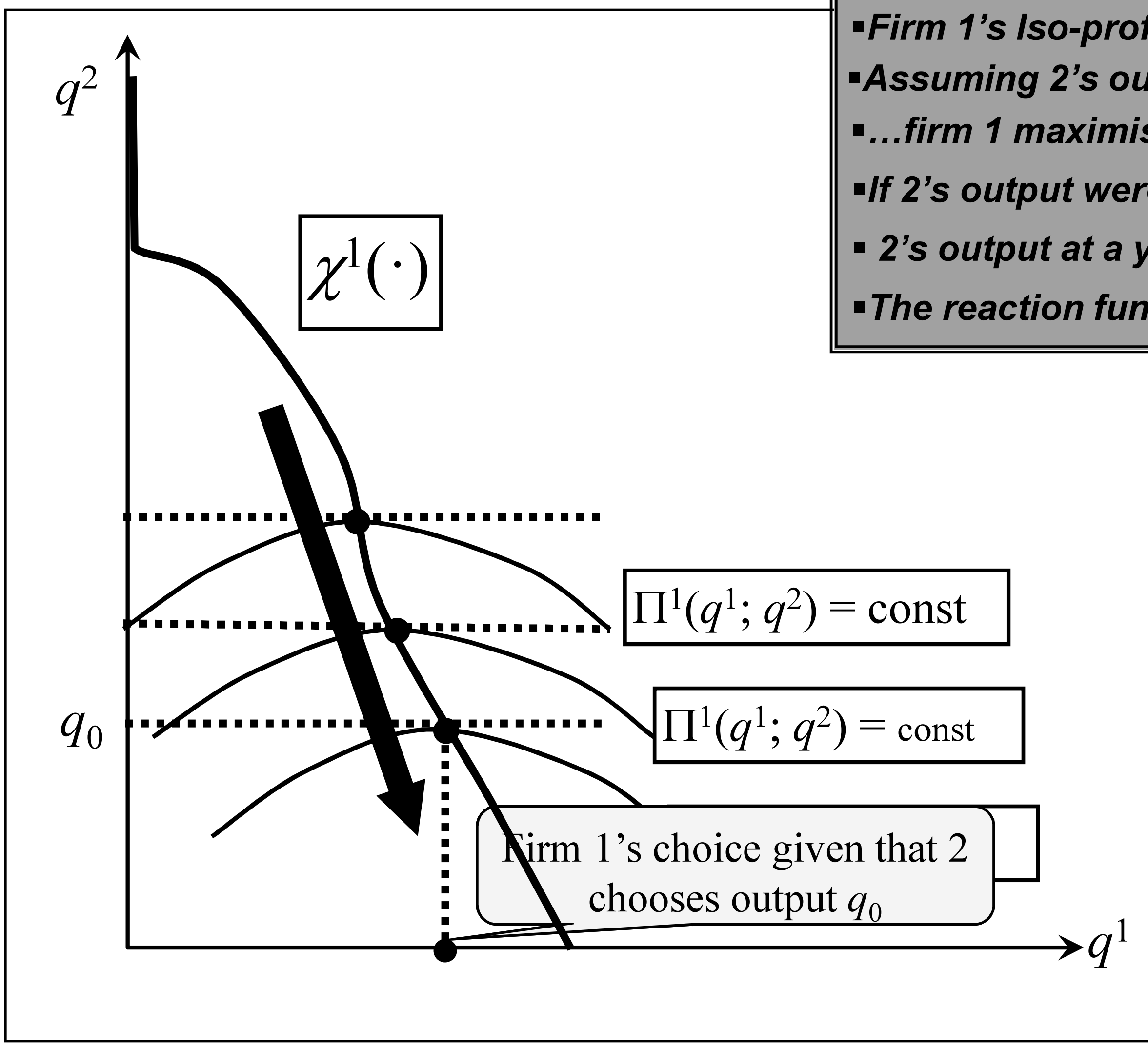
Cournot – model setup

- Two firms labelled $f = 1,2$
- Firm f produces output q^f.
- So total output is:
  - $q = q^1 + q^2$
- Market price is given by:
  - $p = p(q)$
- Firm f has cost function $C^f(\cdot)$.
- So profit for firm f is:
  - $p(q)\, q^f - C^f(q^f)$
- Each firm's profit depends on the other firm's output
  - (because p depends on total q).

Cournot – firm's maximisation

- Firm 1's problem is to choose q^1 so as to maximise
  $\Pi^1(q^1; q^2) := p\,(q^1 + q^2)\, q^1 - C^1(q^1)$
- Differentiate Π^1 to find FOC:
  $$\frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = p_q(q^1 + q^2)\, q^1 + p(q^1 + q^2) - C_q^1(q^1)$$
- For an interior solution this is zero.
- Solving, we find q^1 as a function of q^2 .
- This gives us 1's *reaction function*, χ^1 :
  $q^1 = \chi^1(q^2)$
- Let's look at it graphically…

Cournot – the reaction function

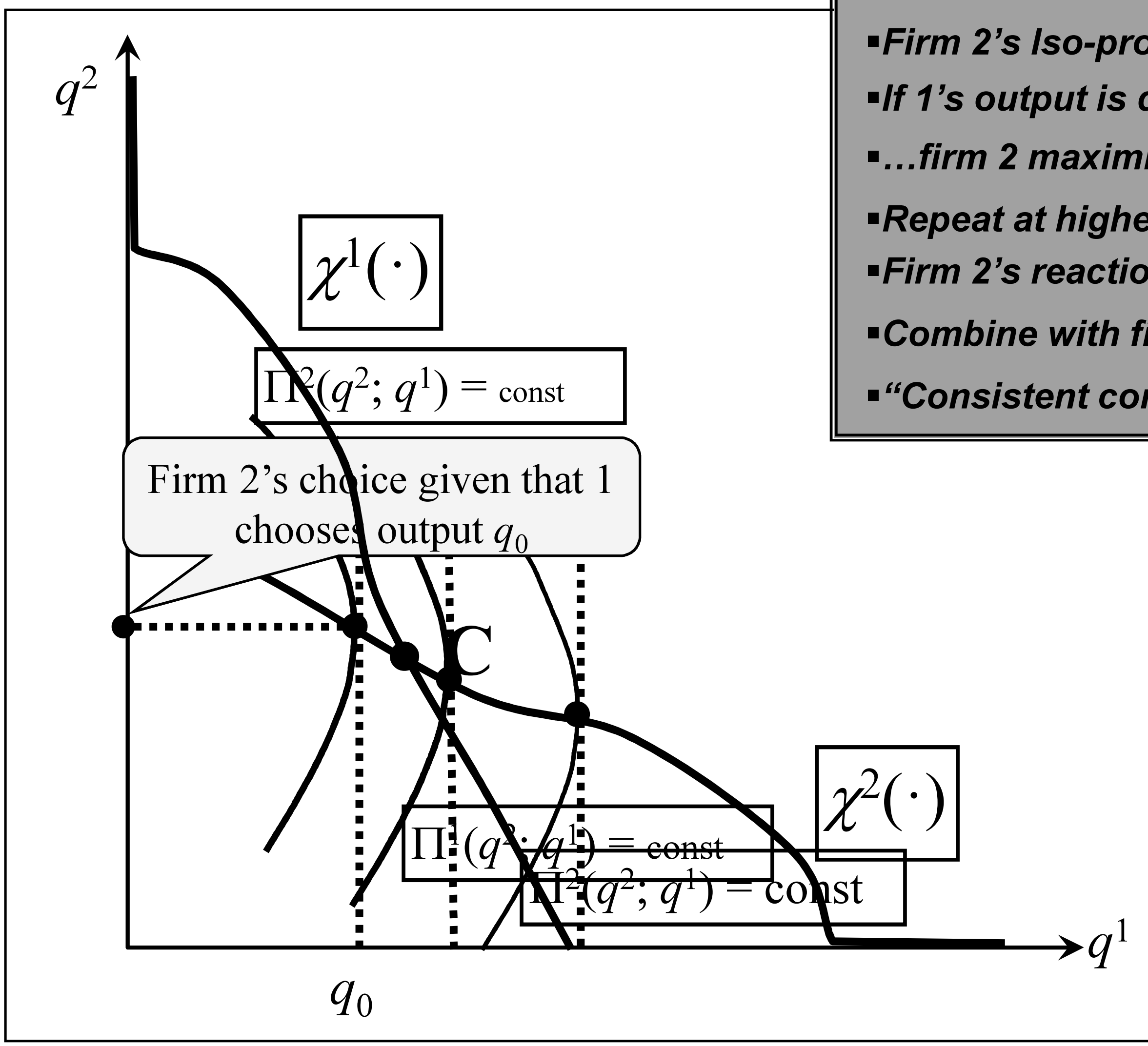


- *Firm 1's Iso-profit curves*
- *Assuming 2's output constant at q_0 …*
- *…firm 1 maximises profit*
- *If 2's output were constant at a higher level*
- *2's output at a yet higher level*
- *The reaction function*

Cournot – solving the model

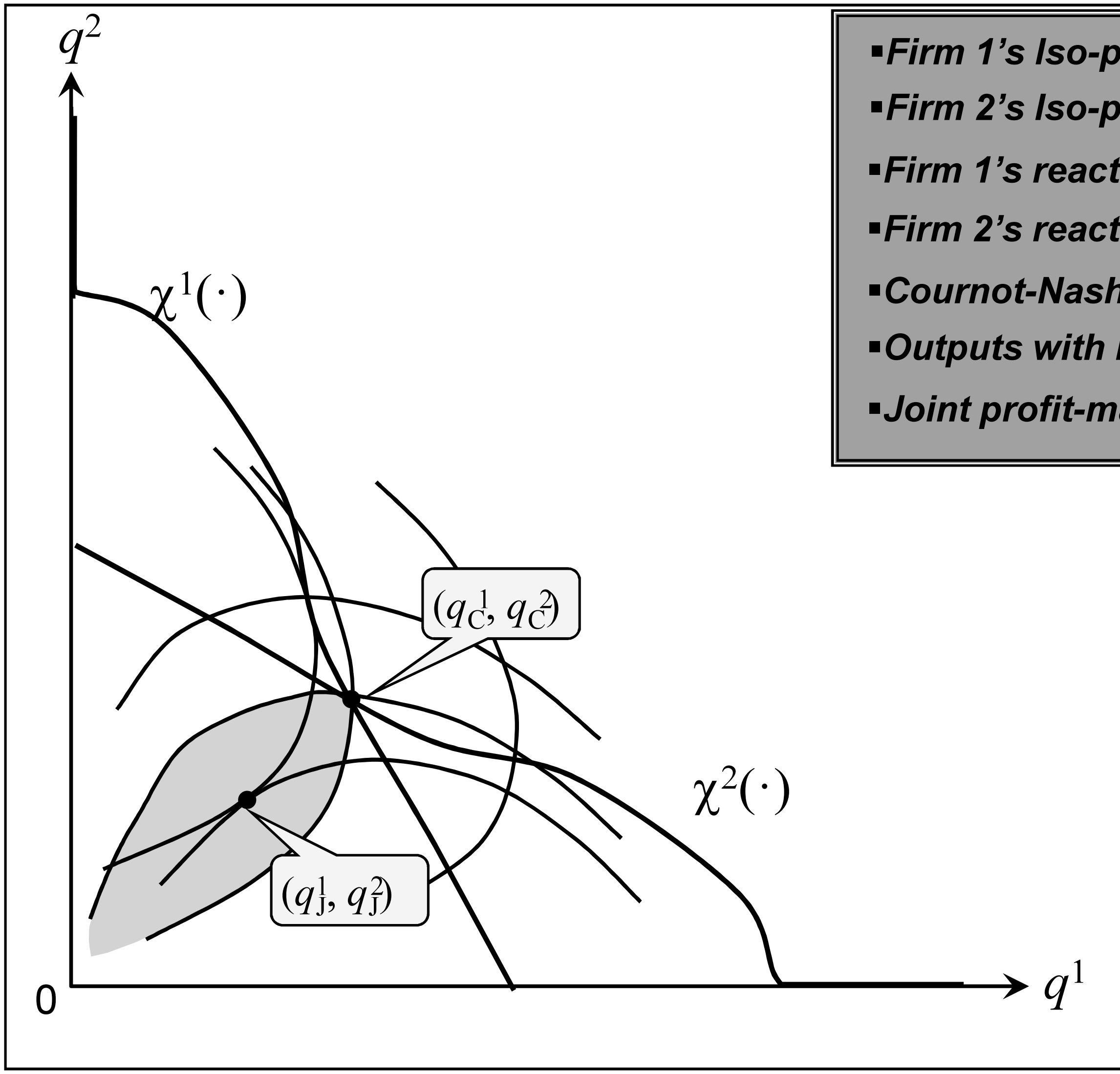
- $\chi^1(\cdot)$ encapsulates profit-maximisation by firm 1.
- Gives firm's reaction 1 to a fixed output level of the competitor firm:
  - $q^1 = \chi^1 (q^2)$
- Of course firm 2's problem is solved in the same way.
- We get q^2 as a function of q^1 :
  - $q^2 = \chi^2 (q^1)$
- Treat the above as a pair of simultaneous equations.
- Solution is a pair of numbers (q_C^1 , q_C^2).
  - So we have $q_C^1 = \chi^1(\chi^2(q_C^1))$ for firm 1…
  - … and $q_C^2 = \chi^2(\chi^1(q_C^2))$ for firm 2.
- This gives the *Cournot-Nash equilibrium* outputs.

Cournot-Nash equilibrium (1)



- ***Firm 2's Iso-profit curves***
- ***If 1's output is q_0 …***
- ***…firm 2 maximises profit***
- ***Repeat at higher levels of 1's output***
- ***Firm 2's reaction function***
- ***Combine with firm 's reaction function***
- ***"Consistent conjectures"***

Cournot-Nash equilibrium (2)



- *Firm 1's Iso-profit curves*
- *Firm 2's Iso-profit curves*
- *Firm 1's reaction function*
- *Firm 2's reaction function*
- *Cournot-Nash equilibrium*
- *Outputs with higher profits for both firms*
- *Joint profit-maximising solution*

The Cournot-Nash equilibrium

- Why "Cournot-Nash" ?
- It is the general form of Cournot's (1838) solution.
- But it also is the Nash equilibrium of a simple quantity game:
  - The players are the two firms.
  - Moves are simultaneous.
  - Strategies are actions – the choice of output levels.
  - The functions give the best-response of each firm to the other's strategy (action).
- To see more, take a simplified example…

Cournot – a "linear" example

- Take the case where the inverse demand function is:

  $p = \beta_0 - \beta q$

- And the cost function for f is given by:

  $C^f(q^f) = C_0^f + c^f q^f$

- So profits for firm f are:

  $[\beta_0 - \beta q\,]\, q^f - [C_0^f + c^f q^f]$

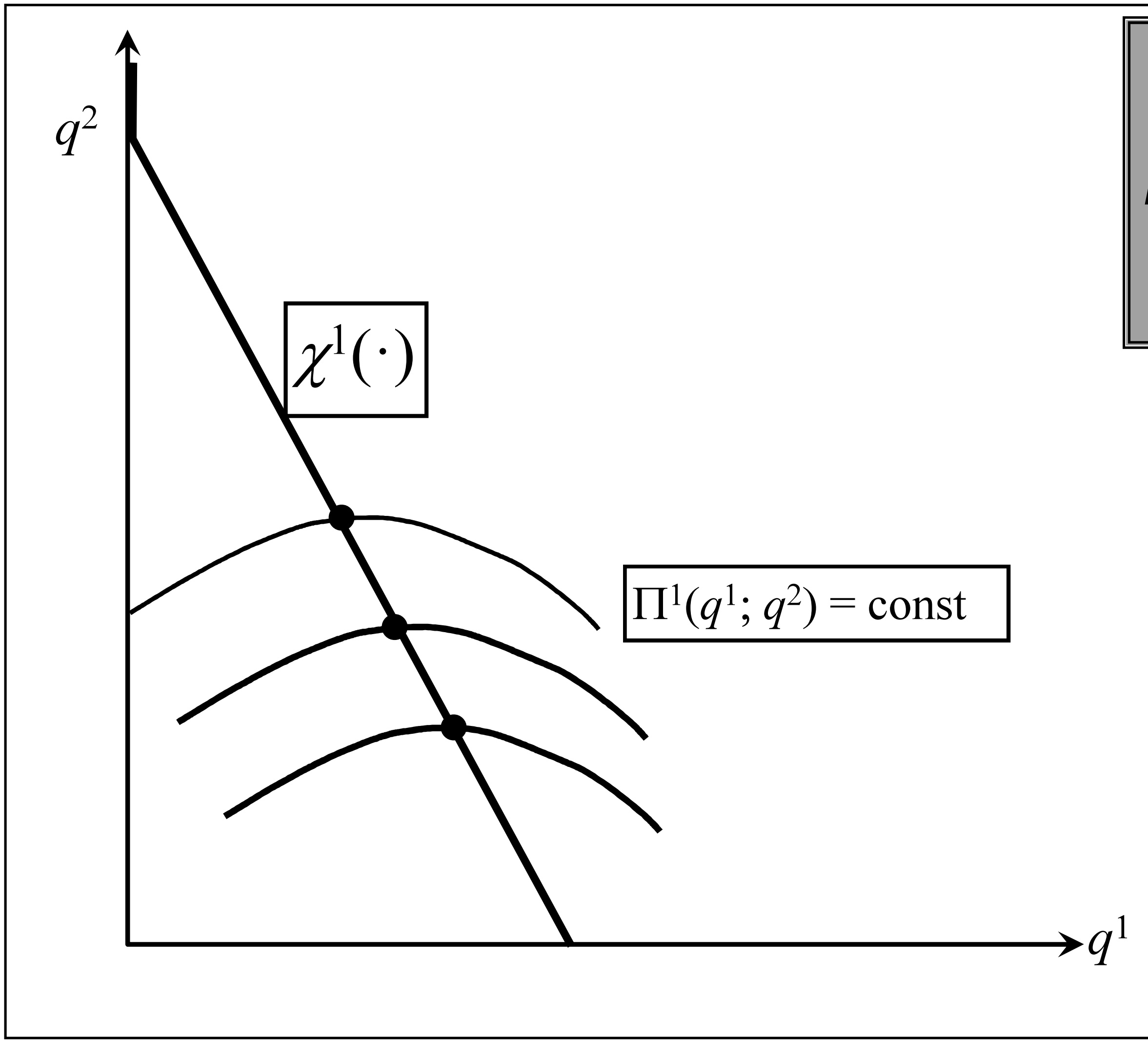
- Suppose firm 1's profits are Π.
- Then, rearranging, the iso-profit curve for firm 1 is:

$$q^2 = \frac{\beta_0 - c^1}{\beta} - q^1 - \frac{C_0^1 + \Pi}{\beta\, q^1}$$

Cournot – solving the linear example

- Firm 1's profits are given by
  - $\Pi^1(q^1; q^2) = [\beta_0 - \beta q]\, q^1 - [C_0{}^1 + c^1 q^1]$
- So, choose q^1 so as to maximise this.
- Differentiating we get:
  - $$\frac{\partial \Pi^1(q^1; q^2)}{\partial\, q^1} = -2\beta q^1 + \beta_0 - \beta q^2 - c^1$$
- FOC for an interior solution ($q^1 > 0$) sets this equal to zero.
- Doing this and rearranging, we get the reaction function:
  - $$q^1 = \max\left\{ \frac{\beta_0 - c^1}{2\beta} - \tfrac{1}{2} q^2 \;,\; 0 \right\}$$

The reaction function again



- *Firm 1's Iso-profit curves*
- *Firm 1 maximises profit, given q^2.*
- *The reaction function*

Finding Cournot-Nash equilibrium

- Assume output of both firm 1 and firm 2 is positive.
- Reaction functions of the firms, $\chi^1(\cdot)$, $\chi^2(\cdot)$ are given by:

$$q^1 = \frac{a - c^1}{2b} - \tfrac{1}{2}q^2 \ ; \qquad q^2 = \frac{a - c^2}{2b} - \tfrac{1}{2}q^1 \ .$$

- Substitute from χ^2 into χ^1:

$$q_C^1 = \frac{a - c^1}{2b} - \tfrac{1}{2}\left[\frac{a - c^2}{2b} - \tfrac{1}{2}q_C^1\right] .$$

- Solving this we get the Cournot-Nash output for firm 1:

$$q_C^1 = \frac{a + c^2 - 2c^1}{3b} \ .$$

- By symmetry get the Cournot-Nash output for firm 2:

$$q_C^2 = \frac{a + c^1 - 2c^2}{3b} \ .$$

Cournot – identical firms

Reminder

- Take the case where the firms are *identical*.
  - This is useful but very special.
- Use the previous formula for the Cournot-Nash outputs.

$$q_C^1 = \frac{a + c^2 - 2c^1}{3b} \quad ; q_C^2 = \frac{a + c^1 - 2c^2}{3b} \quad .$$

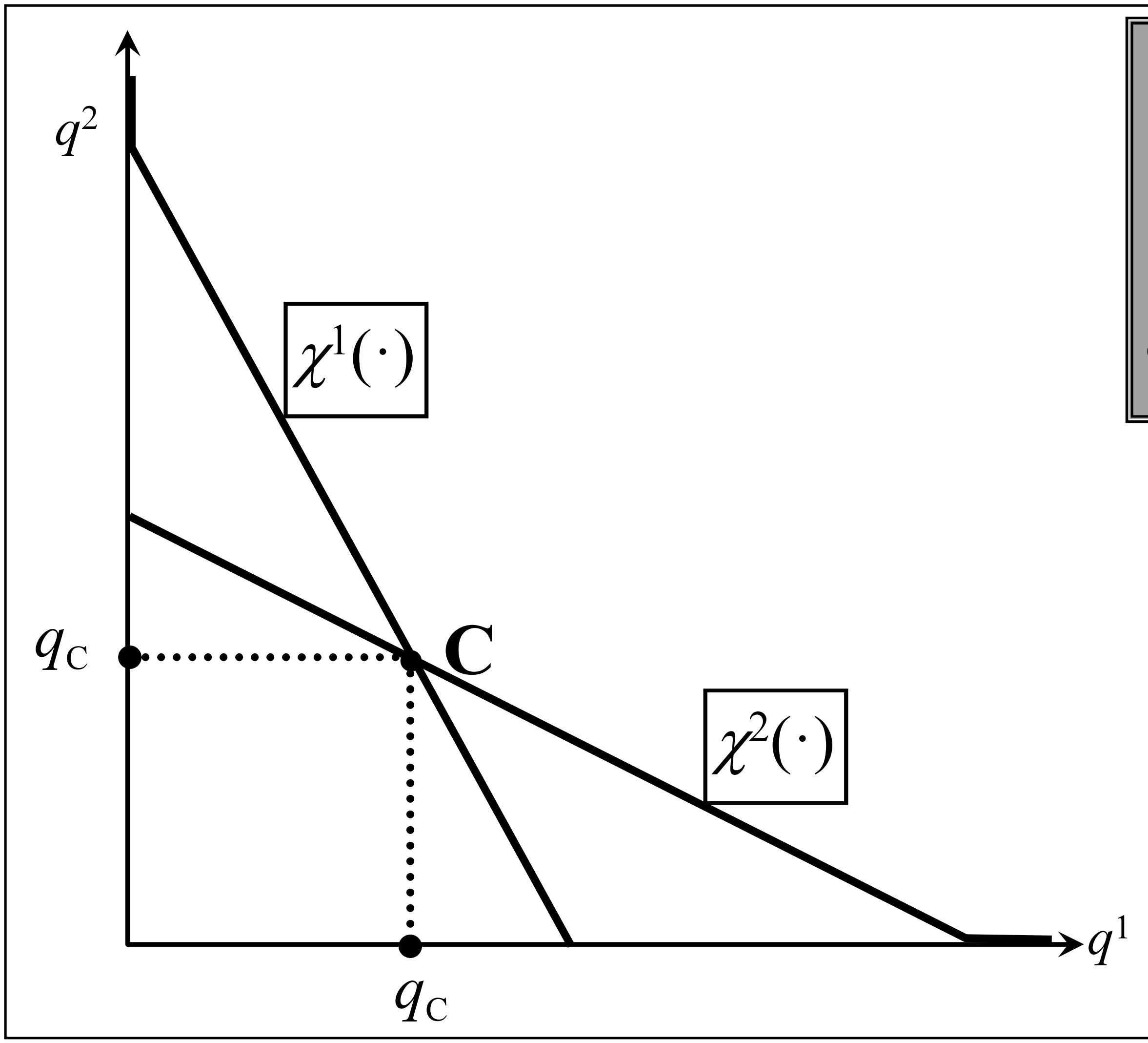
- Put $c^1 = c^2 = c$. Then we find $q_C^1 = q_C^2 = q_C$ where

$$q_C = \frac{a - c}{3b} \quad .$$

- From the demand curve the price in this case is $\frac{1}{3}[a+2c]$
- Profits are

$$\Pi_C = \frac{[a - c]^2}{9b} - C_0 \, .$$

Symmetric Cournot



- *A case with identical firms*
- *Firm 1's reaction to firm 2*
- *Firm 2's reaction to firm 1*
- *The Cournot-Nash equilibrium*

Cournot – assessment

- Cournot-Nash outcome straightforward.
  - Usually have continuous reaction functions.
- Apparently “suboptimal” from the selfish point of view of the firms.
  - Could get higher profits for all firms by collusion.
- Unsatisfactory aspect is that price emerges as a “by-product.”
  - Contrast with Bertrand model.
- Absence of time in the model may be unsatisfactory.

Overview...

Sequential "competition" in quantities

- Duopoly
  - Background
  - Price competition
  - **Quantity competition**
    - Collusion
    - The Cournot model
    - **Leader-Follower**
  - Assessment

Leader-Follower – basic set-up

- Two firms choose the quantity of output.
  - Single homogeneous output.
- Both firms know the market demand curve.
- But firm 1 is able to choose first.
  - It announces an output level.
- Firm 2 then moves, knowing the announced output of firm 1.
- Firm 1 knows the reaction function of firm 2.
- So it can use firm 2's reaction as a "menu" for choosing its own output…

Leader-follower – model

- Firm 1 (the leader) knows firm 2's reaction.
  - If firm 1 produces q^1 then firm 2 produces $\chi^2(q^1)$.
- Firm 1 uses χ^2 as a feasibility constraint for its own action.
- Building in this constraint, firm 1's profits are given by

  $$p(q^1 + \chi^2(q^1))\, q^1 - C^1(q^1)$$

- In the "linear" case firm 2's reaction function is

  $$q^2 = \frac{a - c^2}{2b} - ½q^1 .$$

Reminder

- So firm 1's profits are

  $$\left[a - b\left[q^1 + [a - c^2]/2b - ½q^1\right]\right]q^1 - \left[C_0^1 + c^1 q^1\right]$$

Solving the leader-follower model

- Simplifying the expression for firm 1's profits we have:

$$\tfrac{1}{2}\,[a + c^2 - bq^1]\,q^1 \;-\,[C_0{}^1 + c^1 q^1]$$

- The FOC for maximising this is:

$$\tfrac{1}{2}\,[a + c^2] \;- bq^1 \;-\; c^1 = 0$$

- Solving for q^1 we get:

$$q_S^1 = \frac{a + c^2 - 2c^1}{2b} \quad .$$

- Using 2's reaction function to find q^2 we get:

$$q_S^2 = \frac{a + 2c^1 - 3c^2}{4b} \quad .$$

Leader-follower – identical firms

Of course they still differ in terms of their strategic position – firm 1 moves first.

- Again assume that the firms have the same cost function.
- Take the previous expressions for the Leader-Follower outputs:

Reminder

$$q_S^1 = \frac{a + c^2 - 2c^1}{2b} \quad ; \quad q_S^2 = \frac{a + 2c^1 - 3c^2}{4b} \quad .$$

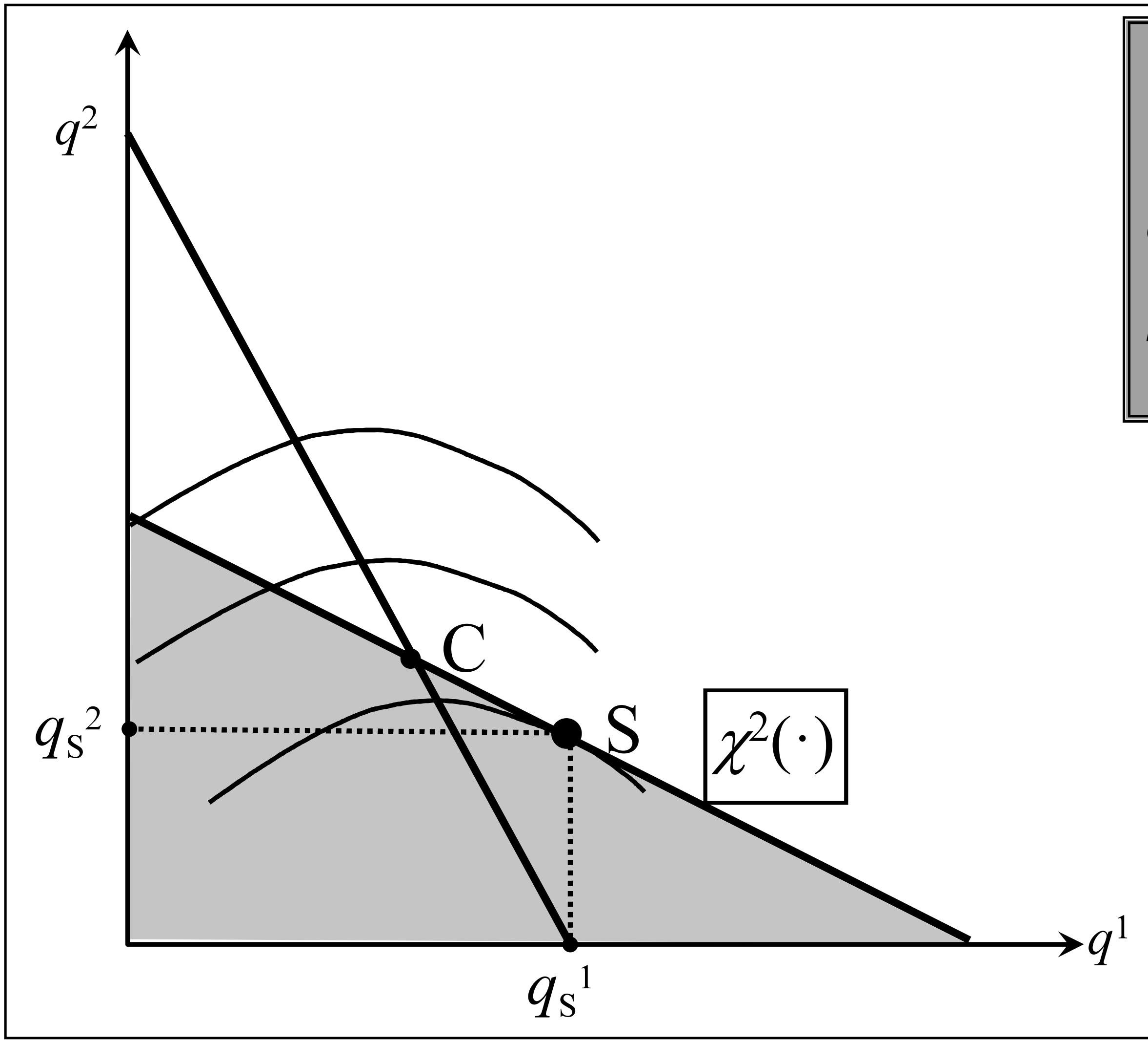
- Put $c^1 = c^2 = c$; then we get the following outputs:

$$q_S^1 = \frac{a - c}{2b} \quad ; \quad q_S^2 = \frac{a - c}{4b} \quad .$$

- Using the demand curve, market price is ¼ $[a + 3c]$.
- So profits are:

$$\Pi_S^1 = \frac{[a-c]^2}{8b} - C_0 \, ; \quad \Pi_S^2 = \frac{[a-c]^2}{16b} - C_0 \quad .$$

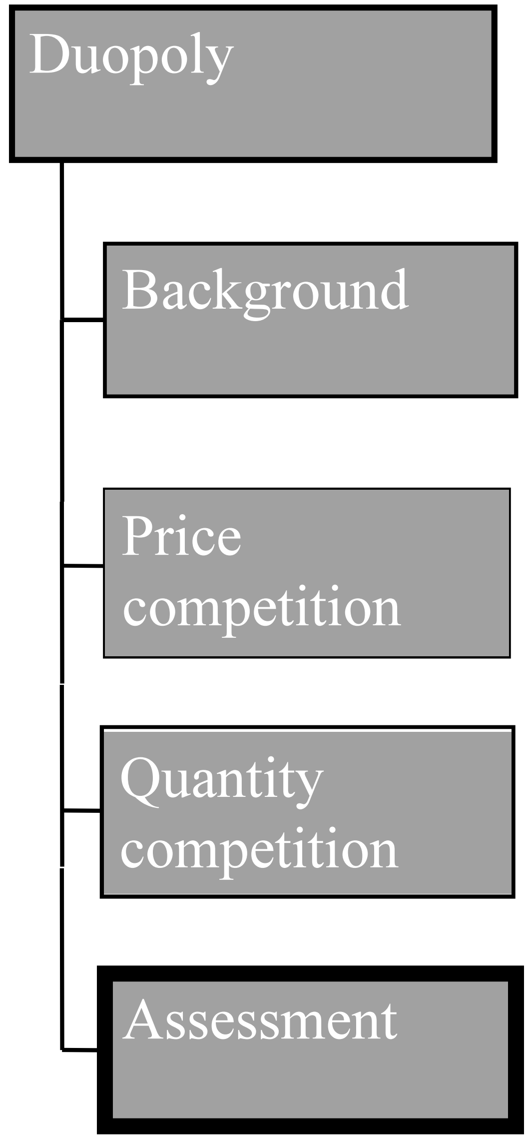
Leader-Follower



- ***Firm 1's Iso-profit curves***
- ***Firm 2's reaction to firm 1***
- ***Firm 1 takes this as an opportunity set…***
- ***…and maximises profit here***
- ***Firm 2 follows suit***

- *Leader has higher output (and follower less) than in Cournot-Nash*
- *"S" stands for <u>von Stackelberg</u>*

Overview...

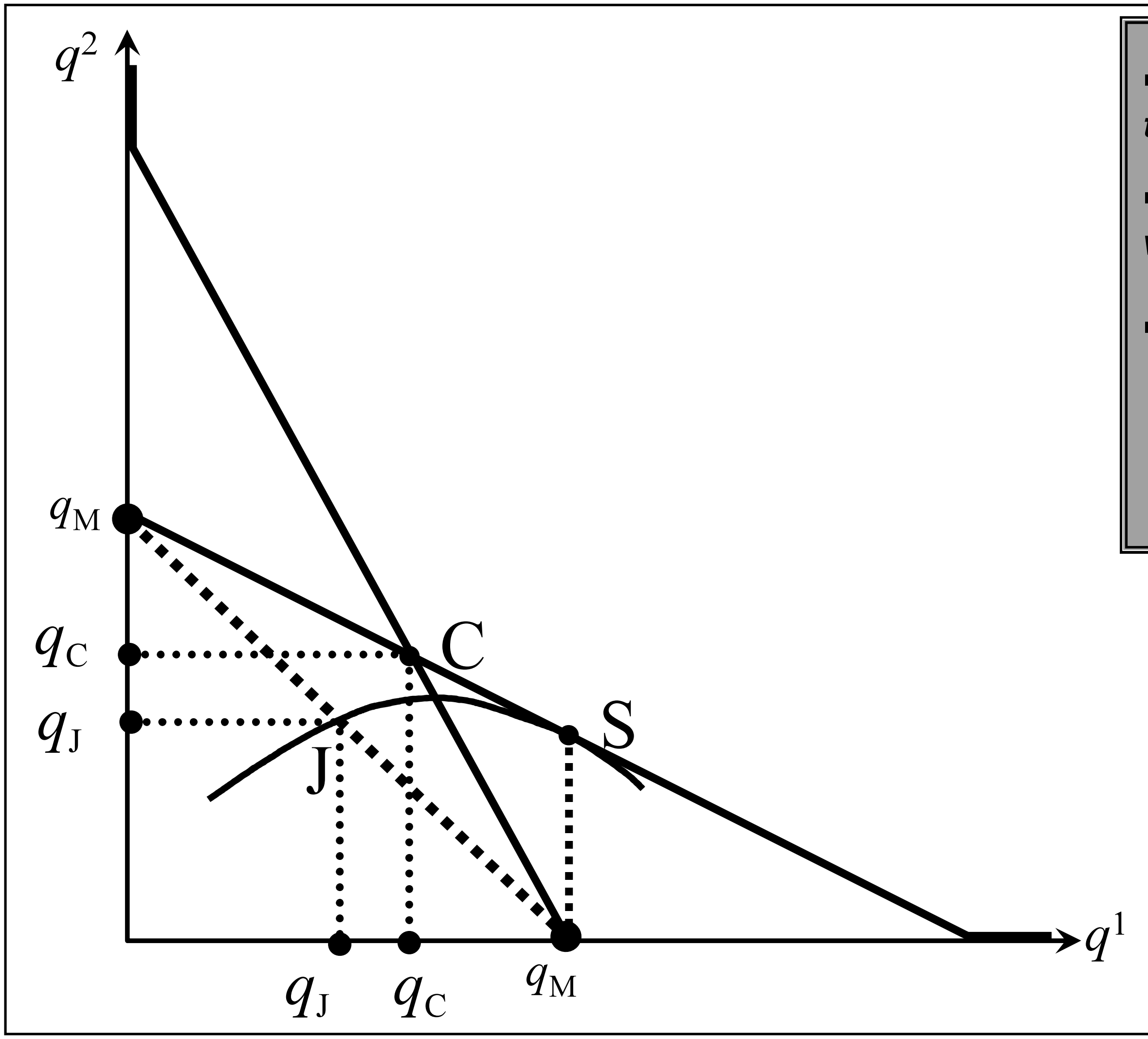


How the simple price- and quantity-models compare.

Comparing the models

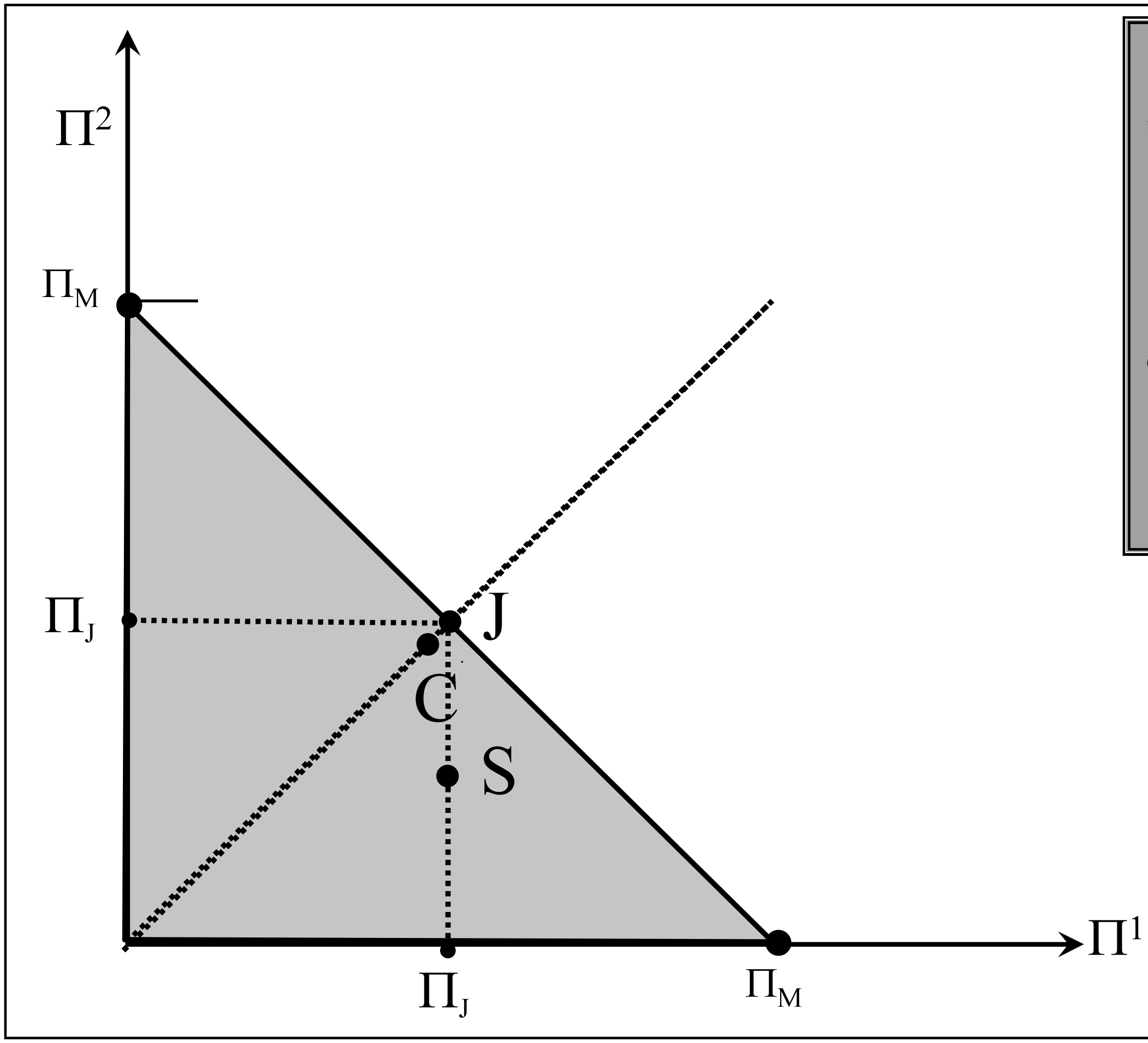
- The price-competition model may seem more "natural"
- But the outcome (p = MC) is surely at variance with everyday experience.
- To evaluate the quantity-based models we need to:
  - Compare the quantity outcomes of the three versions
  - Compare the profits attained in each case.

Output under different regimes



- *Reaction curves for the two firms.*
- *Joint-profit maximisation with equal outputs*
- *Cournot-Nash equilibrium*
- *Leader-follower (Stackelberg) equilibrium*

Profits under different regimes



- ***Attainable set with transferable profits***
- ***Joint-profit maximisation with equal shares***
- ***Profits at Cournot-Nash equilibrium***
- ***Profits in leader-follower (Stackelberg) equilibrium***

- *Cournot and leader-follower models yield profit levels <u>inside</u> the frontier.*

What next?

- Introduce the possibility of entry.
- General models of oligopoly.
- Dynamic versions of Cournot competition