CASE A: TWO QL U FNS (NORMAL GOODS) } COMP E & WELFARE CH 10
TWO PDN FNS / COST FN

CASE B: ADD TO ABOVE EXTERNALITY IN CN (ONE GOOD, ONE BAD)
SHOW WHY WE CAN FOCUS ON EXTERNALITY – INDIRECT UTILITY

CASE C: PUBLIC GOODS
MANY QL UTILITY FNS – REP. COST FN

CASE D: MULTILATERAL DEPLETABLE EXTERNALITY
SINGLE PRODUCER/CONSUMER SPECIAL CASE

FOCUS ON INDIRECT U/PROFIT FN EXTERNALITY ONLY

$$u_1(m_1, x_1, h) = m_1 + 5\ln(1+x_1) + 10h - \frac{h^2}{2}$$

$$u_2(m_2, x_2, h) = m_2 + 3\ln(1+x_2) - h^2$$

$$v_1(p, w_i, h) = \max_{m_1, x_1} u_1(m_1, x_1, h)$$

[note that in M.W.G x_i represents vector of L goods $x_1, x_2, \ldots x_L$ where x_1 is numeraire w_1]

IN THIS CASE $\max m_1 + 5\ln(1+x_1) + 10h - \frac{h^2}{2}$

s.t. $m_1 + p x_1 \leq w_1$

$$L = m_1 + 5\ln(1+x_1) + 10h - \frac{h^2}{2} + \lambda(w_1 - m_1 - p x_1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial m_1} &= 1 - \lambda = 0 \\ \frac{\partial L}{\partial x_1} &= \frac{5}{1+x_1} - \lambda p = 0 \end{aligned}\right\} \quad \frac{1+x_1}{5} = \frac{1}{p} \Rightarrow x_1 = \frac{5}{p} - 1$$

$$\frac{\partial L}{\partial \lambda} = w_1 - m_1 - p x_1 = 0 \Rightarrow w_1 - m_1 - p\left(\frac{5}{p} - 1\right) = 0 \Rightarrow m_1 = w_1 - 5 + p$$

$$\Rightarrow v_1(p, w_1, h) = w_1 - 5 + p + 5\ln\left(\frac{5}{p}\right) + 10h - \frac{h^2}{2}$$

note by similarity

$$v_2(p_2, w_2, h) = w_2 - 3 + p + 3\ln\left(\frac{3}{p}\right) - h^2$$

THIS CAN BE REWRITTEN AS

$$v_1(p_1, w_1, h) = \phi_1(p, h) + w_1 = \underbrace{p + 5\ln\left(\frac{5}{p}\right) - 5 + 10h - \frac{h^2}{2}}_{\phi_1(p,h)} + w_1$$

SINCE p ASSUMED UNAFFECTED BY ANY CHANGES BEING CONSIDERED AND 5 AND w_1 WILL NOT CHANGE WE CAN FOCUS ON THE SINGLE MKT (EXTERNALITY)

$$\phi_1(h) = 10h - \frac{h^2}{2}$$

$$\phi_2(h) = -h^2$$

$\phi_1'(l^*) = 0 \Rightarrow 10 - l = 0 \Rightarrow l^* = 10$

BUT FOR P.O.

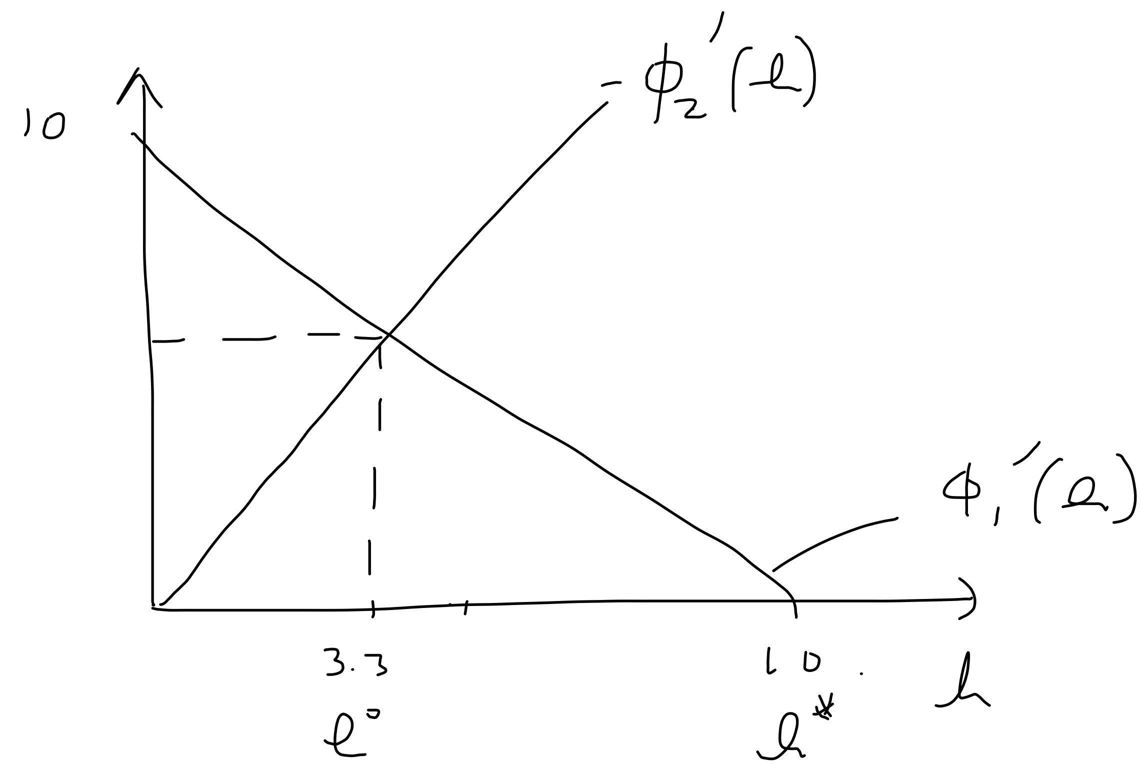
$$\max_{l \geq 0} \quad \phi_1(l) + \phi_2(l)$$

$$\max_{l} \quad 10l - \frac{l^2}{2} - l^2$$

FOC $\quad \phi_1'(l^\circ) = -\phi_2'(l^\circ)$

$$10 - l = 2l$$

$$\Rightarrow 10 = 3l \Rightarrow l^\circ = \frac{10}{3}$$



SHOW WITH QUASI-LINEAR MODEL HOW UPC IS LINEAR (2 goods, 2 firms)

$u_1(m_1, x_2, h) = m_1 + 5\ln(1+x_1)$

$u_2(m_2, x_2, h) = m_2 + 3\ln(1+x_2)$

$C_1(q_1) = 0.3 q_1^2$ where q is output of x $(q=x)$ produced from m

$C_2(q_2) = 0.5 q_2^2$

suppose good l fixed at some level $(\bar{x}_1, \bar{x}_2, \bar{q}_1, \bar{q}_2)$

with these c^n and p^{dn} levels fixed total amount of numeraire available for distribution among consumers is

$$w_m - C_1(q_1) - C_2(q_2)$$

note $w_m = w_{m1} + w_{m2}$

$$u_1 + u_2 \le \phi_1(\bar{x}_1) + \phi_2(\bar{x}_2) + w_m - C_1(\bar{q}_1) - C_2(\bar{q}_2)$$

i.e. specific total number and because w enters directly u. fns can transfer 1:1

quasi-linear UPFrontier — allows unit-for-unit transfer of numeraire

⇒ every P.O. MUST INVOLVE QUANTITIES $x_1^*, x_2^*, q_1^*, q_2^*$ THAT EXTEND BOUNDARY AS FAR OUT AS POSSIBLE

⇒ OPTIMAL CONSUMPTION & PRODUCTION LEVEL OF GOOD l CAN BE OBTAINED AS THE SOLUTION TO

$$\text{Max } \phi_1(x_1) + \phi_2(x_2) - C_1(q_1) - C_2(q_2) + w_m$$

$$\text{s.t. } x_1 + x_2 - q_1 - q_2 = 0$$

WE KNOW FROM CH. 10 (SHOW) THAT PERFECTLY COMP ECONOMY ACHIEVES P.O. OUTCOME (DISTRIBUTION DEPENDS ON ENDOWMENTS)

NOW IF WE INCLUDE EXTERNALITY TO THIS MODEL THE P.O. OUTCOME WILL INVOLVE IN ADDITION MAX OF $\phi_1(h) + \phi_2(h)$ WHICH ESSENTIALLY PUSHES UPC FURTHER OUT FROM COMP. EQ. h^* (VERSUS h°)

ASSUME CONSUMER 2 HAS RIGHT TO ~~EXTERNALITY~~ FREE ENV

CONSUMER 1 SOLVES

$$\max_{l_1} \quad \phi_1(l_1) - p_l \cdot l_1 \;=\; 10l - \frac{l^2}{2} - p_l \cdot l$$

CONSUMER 2 SOLVES $\qquad -l^2 + p_l \cdot l \qquad ($

$FOC_1: \quad 10 - l = p_l$

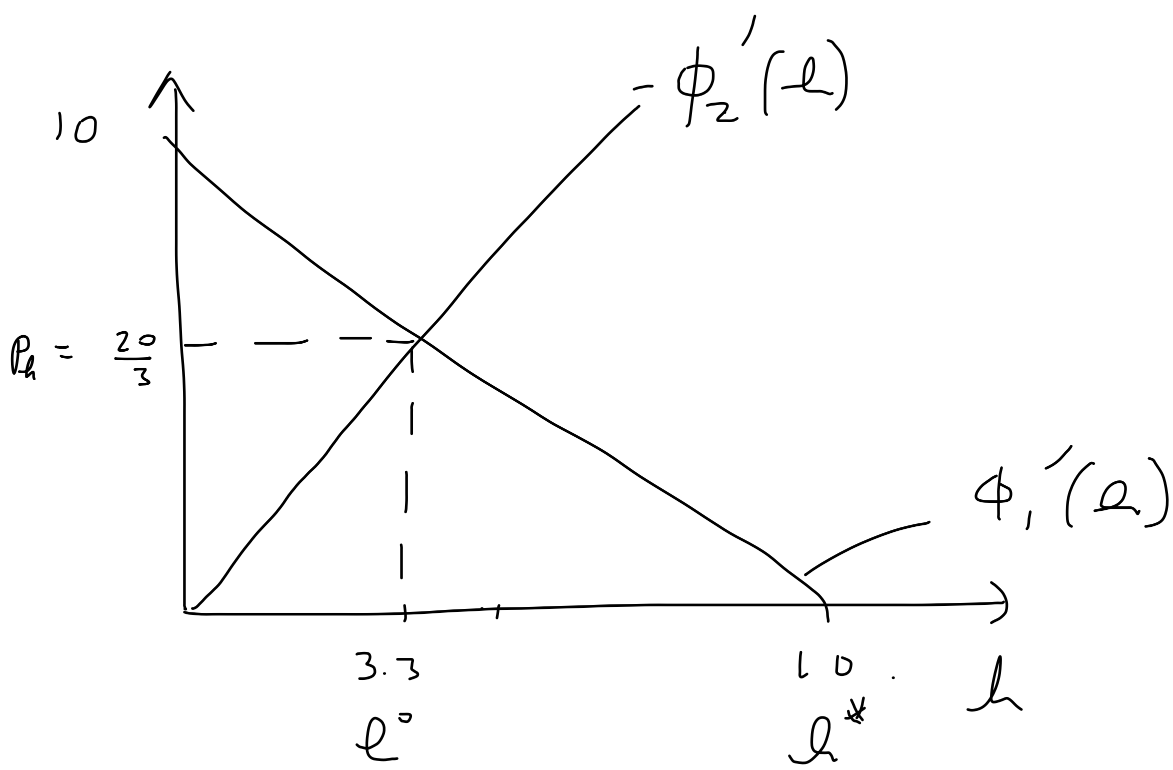
$FOC_2: \; -2l = -p_l \qquad (2l = p_l)$

COMP. EQM MKT MUST CLEAR (AND $l_1 = l_2$)

$$\Rightarrow \phi_1'(l^{**}) = -\phi_2'(l^{**})$$

$$\Rightarrow \quad 10 - l = 2l = p_l \quad \Rightarrow \; l^{**} = \frac{10}{3} \Rightarrow p_l = \frac{20}{3}$$

So markets ensure optimal outcome



EQUILIBRIUM UTILITY ARE

$$\phi_1(l^o) - p_l^* \cdot l^o = 10\left(\frac{10}{3}\right) - \frac{\left(\frac{10}{3}\right)^2}{2} - \frac{20}{3} \cdot \frac{10}{3}$$

$$\phi_2(l^o) + p_l^* l^o = \left(\frac{10}{3}\right)^2 + \frac{20}{3} \cdot \frac{10}{3}$$

SUPPOSE x IS PUBLIC GOOD

$u_1(m_1, x) = m_1 + 5\ln(1+x)$ no subscript for x

$u_2(m_2, x) = m_2 + 3\ln(1+x)$

$C(q) = 0.5q^2$

$\phi_1(q) = 5\ln(1+q)$

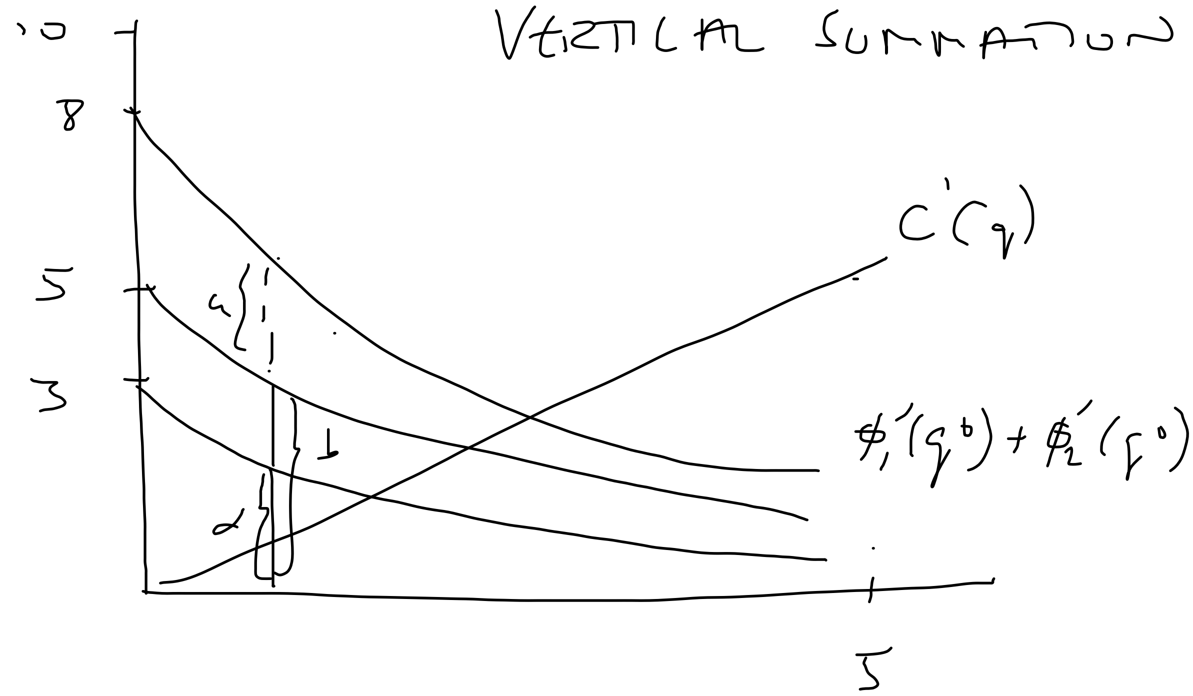
$\phi_2(q) = 3\ln(1+q)$

$\max_{q \geq 0} \sum_{i=1}^{2} \phi_i(q) - C(q) = 5\ln(1+q) + 3\ln(1+q) - 0.5q^2$

$\Rightarrow \sum \phi_i'(q^0) \leq c'(q^0)$

FOC $\frac{5}{1+q^0} + \frac{3}{1+q^0} = q^0$

VERTICAL SUMMATION



$$\max_{x_i \geq 0} \ \phi_i\Big(x_i + \sum_{k \neq i} x_k\Big) - p^* x_i$$

FOC $\quad \phi_i'\Big(x_i^* + \sum_{k \neq i} x_k\Big) \leq p^*$ $\qquad$ NASH: CONSUMERS TAKE GIVEN AMOUNT PURCHASED BY OTHERS

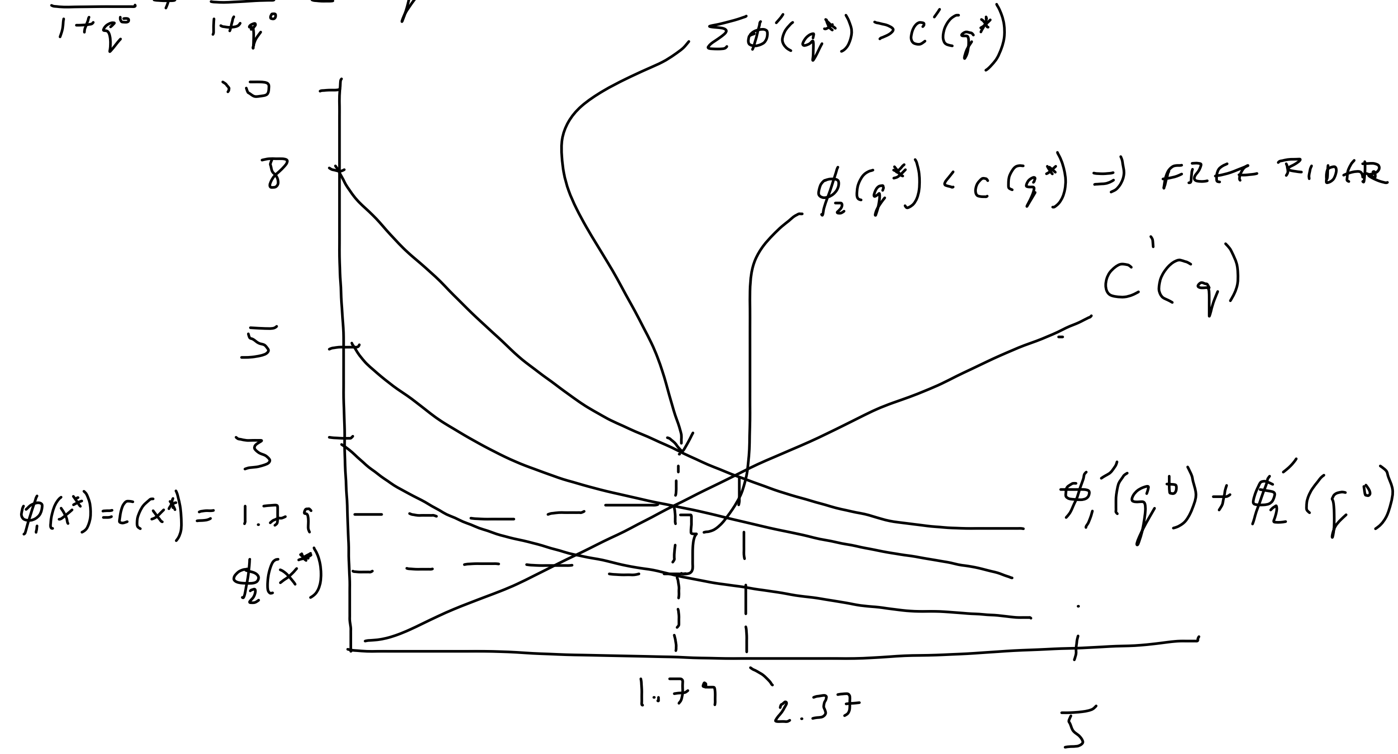
LETTING $x^* = \sum x_i^*$ EQM LEVEL OF PUBLIC GOOD FOR EACH CONSUMER i

$$\sum_i \delta_i \left[\phi_i'(q^*) - c'(q^*)\right] = 0 \Rightarrow \delta_1 \left[\frac{5}{1+q^*} - q^*\right] + \delta_2 \left[\frac{3}{1+q^*} - q^*\right]$$

WHERE $\delta_1 = 1$ AND $\delta_2 = 0$ SINCE $\dfrac{3}{1+q^*} < q^*$

RECALL PUBLIC GOOD OPTIMAL

FOC $\quad \dfrac{5}{1+q^o} + \dfrac{3}{1+q^o} = q^o$



$$\frac{5}{1+q^*} = q^* \Rightarrow q^* + q^{*2} - 5 = 0$$

$$\Rightarrow q^* = 1.79$$

IMAGINE MANY CONSUMERS $\Rightarrow$ GAP BETWEEN OPTIMAL & PRIVATE C.E. LARGE

DEPLETABLE EXTERNALITIES

USE INITIAL UTILITY FN WITH CONSUMER 2 WHO DISLIKES (NEGATIVE) ~~EXTERNALITY~~

$$u_1 = m_1 + 3\ln(1+x_1) - h^2 = m_1 + \phi_1(p, h)$$

$$u_2 = m_2 + 4\ln(1+x_2) - \frac{h^2}{2} = m_2 + \phi_2(p, h)$$

WITH PARTIAL EQUILIBRIUM ANALYSIS WE CAN ASSUME OTHER MARKETS ARE EFFICIENT AND WE FOCUS ON EXTERNALITIES (DERIVED UT. FN) SO WE ONLY NEED TO MAX $\phi_1(\tilde{h}) + \phi_2(\tilde{h})$

FIRMS NOW REDUCE COST BY POLLUTING

EACH FIRM SETS LEVEL OF THE (NEGATIVE) EXTERNALITY SATISFYING

$$\Pi_j'(h_j^*) \leq 0, \text{ WITH EQUALITY IF } h_j^* > 0.$$

ASSUME

$$C_1(q_1, h) = q_1^2 + (10-h)^2 \qquad \pi_1(q_1, h) = p \cdot q_1 - q_1^2 - (10-h)^2$$

$$C_2(q_2, h) = q_2^2 + (5-h)^2 \qquad \pi_2(q_2, h) = p \cdot q_2 - q_2^2 - (5-h)^2$$

FIRM'S DERIVED PROFIT FN (PLAYS SAME ROLE AS UTILITY FN) CAN BE WRITTEN AS

$$\pi_1(h) = -(10-h)^2$$

$$\pi_2(h) = -(5-h)^2$$

FOR TWO CONSUMERS (UTILITY) - TWO FIRMS - HOMOGENEOUS EXTERNALITY (DEPLETABLE) MUST SOLVE

$$\max_{\substack{h_1, h_2 \geq 0 \\ \tilde{h}_1, \tilde{h}_2 \geq 0}} \phi_1(\tilde{h}_1) + \phi_2(\tilde{h}_2) + \pi_1(h_1) + \pi_2(h_2) \quad \text{s.t.} \quad h_1 + h_2 = \tilde{h}_1 + \tilde{h}_2$$

$$L = \phi_1(\tilde{h}_1) + \phi_2(\tilde{h}_2) + \pi_1(h_1) + \pi_2(h_2) + \mu(\tilde{h}_1 + \tilde{h}_2 - h_1 - h_2)$$

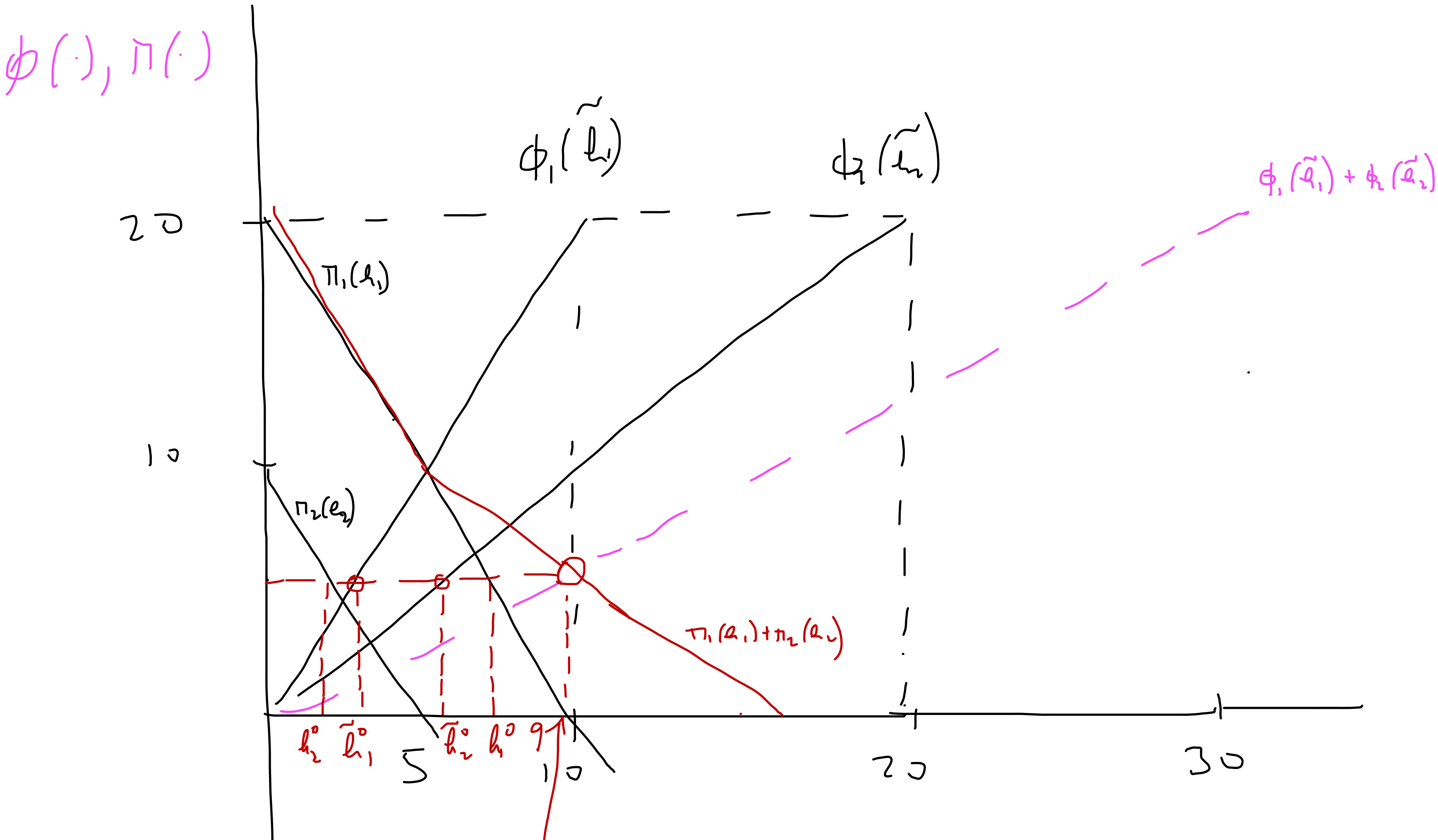
$$= -\tilde{h}_1^2 - \frac{\tilde{h}_2^2}{2} - (10-h_1)^2 - (5-h_2)^2 + \mu(\tilde{h}_1 + \tilde{h}_2 - h_1 - h_2)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial \tilde{h}_1} &= -2\tilde{h}_1 + \mu = 0 \\ \frac{\partial L}{\partial \tilde{h}_2} &= -\tilde{h}_2 + \mu = 0 \end{aligned}\right\} \Rightarrow \tilde{h}_1 + \tilde{h}_2 = -\frac{3}{2}\mu$$

$$\left.\begin{aligned} \frac{\partial L}{\partial h_1} &= -2(10-h_1) - \mu = 0 \\ \frac{\partial L}{\partial h_2} &= -2(5-h_1) - \mu = 0 \end{aligned}\right\} \Rightarrow h_1 + h_2 = \mu + 15$$

$$\Rightarrow \mu = -\frac{30}{5} = -6 \Rightarrow \begin{aligned} \tilde{h}_1 &= 3 \\ \tilde{h}_2 &= 6 \\ h_1 &= 7 \\ h_2 &= 2 \end{aligned}$$

$$\frac{\partial L}{\partial \mu} = \tilde{h}_1 + \tilde{h}_2 - h_1 - h_2 = 0$$



$$\tilde{l}_1^\circ + \tilde{l}_2^\circ = l_1^\circ + l_2^\circ = 9$$

$$3 + 6 = 7 + 2 = 9$$

your cost is less than c with probability $p(\bar{c}) = \frac{\bar{c}}{\alpha}$

$$\max_{p} \; p(p)(b-p) = \frac{p(b-p)}{\alpha} \Rightarrow FOC_r \left(\frac{pb - r^2}{\alpha} \right) = \frac{b-2p}{\alpha} = 0 \Rightarrow p = \frac{b}{2}$$

rp

$\frac{b}{2} < c < b$

$$\max_{T} \; \underbrace{(1 - G(T))}_{\text{prob that } b \geq T} \underbrace{(T - c)}$$

$$\left(1 - \frac{T}{\alpha}\right)(T - c) = T - c - \frac{T^2}{\alpha} - \frac{T}{\alpha}c$$

$$1 - \frac{2T}{\alpha} - \frac{c}{\alpha} = 0 \Rightarrow \frac{2T}{\alpha} = 1 - \frac{c}{\alpha}$$

$$T = \frac{\alpha}{2} - \frac{c}{2}$$

$c < b < T_c^*$ or $c < b < \frac{\alpha}{2} - \frac{c}{2}$ <u>REJECT</u>

CONSUMER'S COST / FIRM'S BARGAIN / CONSUMER OFFER

SIMPLY MAKE UP NO.s FOR c, b, α