EXTERNALITIES AND PUBLIC GOODS

MWG Ch. 11
Introduction

A simple bilateral Externality

Public Goods

Multilateral externalities

Private information and second-best solutions
“Surprisingly….a fully satisfying definition of an externality has proved elusive” (MWG)

Definition 11.B.1: An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

“Directly”: excludes effects mediated by prices
INTRODUCTION

• Examples: Fishery affected by emissions from a nearby oil refinery
• Note: impact of oil price is a “pecuniary externality”
• Presence of externality is a function of the set of markets in existence
Externality and Public Goods

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A SIMPLE BILATERAL EXTERNALITY

- Consider a simple two agent partial equilibrium model
- Assume actions of consumers do not affect prices of L traded goods
- Consumer i’s utility function takes the form $u_i(x_{1i}, \ldots, x_{Li}, h)$ where choice of $h$ by consumer 1 affects consumer 2, i.e.,
  $$\frac{\partial u_2(x_{12}, \ldots, x_{L2}, h)}{\partial h} \neq 0,$$  e.g., music, polluting river
A SIMPLE BILATERAL EXTERNALITY

• Convenient to define for each consumer i a derived utility function over level of $h$ assuming optimal commodity purchases by consumer i of traded goods at $p \in \mathbb{R}^L$ and wealth $w_i$:

$$v_i(p, w_i, h) = \max_{x_i \geq 0} u_i(x_i, h)$$

Such that $px_i \leq w_i$
A SIMPLE BILATERAL EXTERNALITY

• Assume quasilinear form (zero wealth effects)
• We can write derived utility function $v_i(\cdot)$ as

$$v_i(p, w_i, h) = \phi_i(p, h) + w_i$$

• Since we assume prices of L traded goods are unaffected (by changes) we shall suppress price vector and just write $\phi_i(h)$
• Assume $\phi_i(h)$ is twice differentiable and $\phi_i''(\cdot) < 0$ (not innocent)
Everything applies to the derived profit function so that $\pi_j(h)$ has the same role in the analysis as $\phi_i(h)$. 
• Assume the competitive equilibrium with commodity prices $p$, i.e., both consumers maximize their utility limited only by wealth & prices of traded goods

• It must be the case that consumer $i$ (who controls $h$ outside the market) will choose her level of $h \geq 0$ to maximize $\phi_1(h)$
Equilibrium $h^*$ satisfies first order conditions

$$\phi'_1(h^*) \leq 0 \quad \text{with equality if} \quad h > 0$$

interior solution: $$\phi'_1(h^*) = 0$$
NONOPTIMALITY OF THE COMPETITIVE OUTCOME

• In contrast, in any Pareto optimal allocation, the optimal level of \( h \), \( h^o \) must maximize the joint surplus of the two consumers

\[
\max_{h \geq 0} \phi_1(h) + \phi_2(h)
\]

\text{FOC: } \phi_1'(h^o) \leq -\phi_2'(h^o) \text{ with equality if } h^o > 0

Equilibrium level not optimal unless \( h^* = h^o = 0 \)
NONOPTIMALITY OF THE COMPETITIVE OUTCOME

• For Pareto Optimal allocation in which \( h^o = h \) and \( w_i \) is consumer \( i \)'s wealth level for \( i = 1, 2 \), it must be impossible to change \( h \) to make a Pareto improvement

\[(h^o, 0) \text{ must solve }\]

\[
\max_{h, T} \phi_1(h) + w_1 - T
\]

\[s.t.: \quad \phi_2(h) + w_2 + T \geq \bar{u}_2\]

Constraint holds with equality so substituting into objective function \( h^o \) must max \( \phi_1(h) + \phi_2(h) \)
NONOPTIMALITY OF THE COMPETITIVE OUTCOME

Shows the case of negative externality

\[ \phi_1'(h^o) = -\phi_2'(h^o) > 0 \]

Because \( \phi_1'(\cdot) < 0 \)
and \( \phi_1'(h^*) = 0 \)
\[ \Rightarrow h^* > h^o \]

Note: optimality does not usually entail the complete elimination of a negative externality
TRADITIONAL SOLUTIONS TO THE EXTERNALITY PROBLEM

1. Direct approach: command and control or government mandate
2. Tax (subsidy) on externality-generating activity
3. Fostering bargaining over bargaining: enforceable property rights
• Tax on externality generating activity (Pigouvian tax)

\[ t_h = -\phi_2'(h^o) > 0 \]

• Consumer 1 will choose the level of \( h \) that solves

\[ \max_{h \geq 0} \phi_1(h) - t_h h \]

\[ \text{FOC} \quad \phi'_1(h) \leq t_h \quad \text{With equality if } h > 0 \]
Consumer 1 effectively *internalizes* the externality.
**SUBSIDY FOR POSITIVE EXTERNALITY**

- Principle is same with a *positive* externality only now we set

  \[ t_h = -\phi'(h^o) < 0 \]

- Where \( t_h \) becomes a per-unit subsidy
TAXING OR SUBSIDIZING A NEGATIVE EXTERNALITY

• Optimality can be achieved by taxing a negative externality or subsidizing its reduction

\[ s_h = -\phi'_2(h^o) > 0 \]

• For every unit that consumer 1’s choice is below \( h^* \)

\[ \text{Max } \phi_1(h) + s_h(h^* - h) = \phi_1(h) - t_h h + t_h h^* \]

• Equivalent to a tax \( t_h \) per unit \( h \) combined with a lump sum transfer of \( t_h h^* \), i.e., like tax
• In general it is essential to tax the externality-generating activity directly
• E.g., tax music equipment vs. loudness, or output vs. pollution, will be inefficient except in special case of 1:1 relationship
• Tax and quota can both achieve optimality but informational burden for government can be high
BARGAINING

• Suppose we establish property rights and assign an “externality free” environment to consumer
• Suppose a take-it-or-leave-it offer, e.g., 2 asks for T from 1 to allow him to generate externality
• 1 agrees iff $\phi_1(h) - T \geq \phi_1(0)$
• 2 chooses offer $(h, T)$ to solve
BARGAINING

\[
\max_{h \geq 0, T} \frac{\phi_2(h)}{T} + T
\]

subject to \( \phi_1(h) - T \geq \phi_1(0) \)

- Constraint is binding in any solution so \( T = \phi_1(h) - \phi_1(0) \)
- Consumer 2’s optimal offer involves level of \( h \) that solves

\[
\max_{h \geq 0} \phi_2(h) + \phi_1(h) - \phi_1(0)
\]

- But this is precisely \( h^o \) (socially optimal level)
BARGAINING

• Note: precise allocation of rights between consumers is inessential to achieve this result
• E.g., consumer 1 has right to generate
• Consumer 2 must offer a $T < 0$ (pay) to have $h < h^*$
• Consumer 1 will agree to level of $h$ iff $\phi_1(h) - T \geq \phi_1(h^*)$, where $T = \phi_1(h) - \phi_1(h^*)$
• Consumer will offer to set $h$ at level that solves

$$\max_{h \geq 0} \phi_2(h) + \phi_1(h) - \phi_1(h^*)$$

• Optimal $h^0$ results

• $h^0$ invariant to property rights (zero wealth effects) though outcomes differ in final wealth of two consumers
BARGAINING

• 1st case consumer 1 pays $\phi_1(h^o) - \phi_1(0)$ to be allowed

• 2nd case consumer 2 pays $\phi_1(h^o) - \phi_1(h^*)$ to set $h^o < h^*$

• Instance of Coase Theorem
BARGAINING OUTCOMES

a zero externality without transfers
b $h^*$ externality transfers
c power with 1 by consumer 1
d t.i.o.l.i. by consumer bargaining

e t.i.o.l.i. by consumer 2
f After bargaining under

g Other possible outcomes of alternative bargaining mechanism, e.g., comp market
• Note: property rights must be well defined & enforced for optimality proponents of this “approach” focus on absence of legal institutions as central to sub-optimality

• Advantage over taxes & quotas: consumers must know each other’s preferences but government need not

• Note: firms, mergers one possibility

• Externality measurable: may be costly
EXTERNALITIES AND MISSING MARKETS

• Market system as a particular type of trading procedure
• Suppose property rights well defined & enforced and a comp mkt for rights to engage in externality-generating activity exists
• Assume consumer 2 has right to externality-free environment
EXTERNALITIES AND MISSING MARKETS

• Consumer 1 solves

\[ \frac{\text{Max}}{h_1} > \phi_1(h_1) - p_h h_1 \]

\[ FOC: \phi_1'(h_1) \leq p_h \text{ with equality if } h_1 > 0 \]

• Consumer 2 (decides how many rights to sell)

\[ \frac{\text{Max}}{h_2 \geq 0} > \phi_2(h_2) + p_h h_2 \]

\[ FOC: \phi_2'(h_2) \leq -p_h \]
EXTERNALITIES AND MISSING MARKETS

- In competitive equilibrium, the market must clear: $h_1 = h_2$

$$\phi'_1(h^{**}) = -\phi'_2(h^{**})$$

Equilibrium price: $p^*_h = \phi'_1(h^o) = -\phi'_2(h^o)$

- Consumer 1 and 2's equilibrium utilities are then:

$$\phi_1(h^o) - p^*_h h^o \quad \phi_2(h^o) + p^*_h h^o$$

- The market works like a particular bargaining procedure for splitting gains from trade, e.g., point g in Fig.
MISSING MARKETS

- Externalities can be seen as inherently tied to absence of markets of certain comp mkts
- Missing market: point noted by Meade (1952) and extended by Arrow (1969)
- Comp mkt in this example (two consumers/producers) unrealistic (price taking)
- Most externalities involve many agents (multilateral externalities)
- Extension of missing market approach depends on public or private nature of goods
Definition 11.C.1: A public good is a commodity for which one unit of the good by one agent does not preclude its use by other agents.

Possess features:
- non depletable
- No-excludability
CONDITIONS FOR PARETO OPTIMALITY IN PUBLIC GOOD

- I consumers and one public good
- Partial equilibrium perspective by assuming quantity of public good has no effect on prices of L traded goods
- Each consumer’s utility function is quasi-linear wrt same numeraire (traded commodity)
• Consumer i’s utility from public good is $\phi_i(x)$, $x$ has no subscript to indicate public nature, $\phi_i''(x) < 0$ at all $x > 0$

• In this quasi-linear model any P.O. allocation must max the algebraic surplus

• Public good level solves:
CONDITIONS FOR PARETO OPTIMALITY IN PUBLIC GOOD

\[
\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q)
\]

\[
\sum_{i=1}^{I} \phi'_i(q^0) \leq c'(q^0) \quad \text{equality if } q^0 > 0
\]

- Classic optimality condition derived by Samuelson (1954, 1955)

Interior solution

\[
\sum_{i=1}^{I} \phi'_i(q^0) = c'(q^0)
\]
CONDITIONS FOR PARETO OPTIMALITY IN PUBLIC GOOD
INEFFICIENCY OF PRIVATE PROVISION OF PUBLIC GOOD

- Consider case where public good provided by means of private purchases by consumers
- Each consumer chooses how much public good to buy at $x_i \geq 0$ at mkt price $p$, with total purchases $x = \sum_i x_i$
- Supply side single profit max firms with cost function $c(\cdot)$ taking mkt price as given
INEFFICIENCY OF PRIVATE PROVISION OF PUBLIC GOOD

- At competitive eq $p^*$ each consumer $i$’s purchases max utility to solve

$$\max_{x_i \geq 0} \phi_i(x_i + \sum_{k \neq i} x_i) - p^* x_i$$

- Consumers take as given amount of private good being purchased by others (Nash eq concept)

FOC

$$\phi_i'(x_i^* + \sum_{k \neq i} x_k^*) \leq p^* \text{ equality if } x_i^* > 0$$
Letting $x^* = \sum_i x_i^*$ eq level of public good for each consumer, we must have

$$\phi'_i(x^*) \leq p^* \quad \text{equality if } x_i^* > 0$$

The firm’s supply $q^*$ must solve

$$\max_{q \geq 0} (pq^* - c(q)) \Rightarrow p \leq c'(q^*)$$

at comp eq if $q^* = x^*$
Letting \( \delta_i = 1 \) if \( x_i^* > 0 \) and \( \delta_i = 0 \) if \( x_i^* = 0 \), we have

\[
\sum_i \delta_i \left[ \phi'_i \left( q^* \right) - c' \left( q^* \right) \right] = 0
\]

Recalling that \( \phi'_i (\cdot) > 0 \) and \( c'(q^*) > 0 \) this implies that whenever \( I > 1 \) and \( q^* > 0 \) (so \( \delta_i = 1 \) for some \( i \)) we have

\[
\sum_i \phi'_i > c'(q^*)
\]
INEFFICIENCY OF PRIVATE PROVISION OF PUBLIC GOOD
• Public good provision too low
• Externality interpretation: everybody’s purchase provides benefit to others (free rider problem)
• Stark form of free rider here
• Suppose we can order consumers according to their marginal benefits $\phi'_1(x) < \cdots < \phi'_i(x)$ at all $x$, then condition $\phi'_i(x^*) \leq p^*$ can hold with equality for only a single consumer and equilibrium is $\phi'_i(q^*) = c'(q^*)$
STARK FORM OF FREE RIDER
PUBLIC GOOD REMEDIES

• Government can remedy inefficiency of private provision: just as with externalities can happen through quantity-based intervention (provision) or “price-based” (taxes, subsidies)

• Essentially externality of consumer’s purchases are subsidized according to marginal benefit provided to others (see MWG)
PUBLIC GOOD REMEDIES

• E.g., two consumers with benefit functions

\[ \phi_1 (x_1 + x_2) \text{ and } \phi_2 (x_1 + x_2) \]

• \( x_i \) amount of good purchased by \( i \)

• A subsidy to each consumer \( i \) per unit purchased of \( s_i = \phi'_i(q^0) \) with \( q^0 > 0 \), or tax if purchase below some level

• \((\tilde{x}_1, \tilde{x}_2)\) competitive eq levels of public good purchased by two consumers given these subsidies, \( \tilde{p} \) eq price
PUBLIC GOOD REMEDIES

• E.g., two consumers with benefit functions

\[
\max_{x_i \geq 0} \phi_i \left( x_i + \tilde{x}_j \right) + s_i x_i - \tilde{p} x_i
\]

FOC

\[
\phi_i' \left( \tilde{x}_1 + \tilde{x}_2 \right) + s_i \leq \tilde{p}
\]

• Substitute for \( s_i \) and using c.e. conditions and market eq clearing \( \tilde{x}_1 + \tilde{x}_2 = \tilde{q} \) is total pub good in c.e. iff

• \( \phi_i' (\tilde{q}) + \phi_{-i}' (q^o) \leq c' (\tilde{q}) \) from Samuelson public good optimality we see that \( \tilde{q} = q^o \)
PUBLIC GOOD REMEDIES

\[ \phi_1 (q_0) + s_1 = \tilde{p} = \phi_2 (q_0) + s_2 \]

[Graphical representation]
Markets for public goods “as experienced by consumer i”

Think of each consumer’s consumption of the public good as a distinct commodity (market)

In theory this ‘privatization’ of public good (personalization) eliminates the externality each consumer faces a ‘private’ price
Realism questionable:
- Ability to exclude consumers from use essential for consumers to believe that without purchase they will get nothing
- Price-taking behavior not realistic with single agent on demand side
- $p_i$ price of personalized good
- Suppose given $p_i^{**}$ eq price each consumer $i$ sees herself as deciding on the total amount of the public good she will consume
LINDAHL EQUILIBRIUM

\[
\max_{x_i \geq 0} \phi_i (x_i) - p_i^{**} x_i
\]

FOC \quad \phi_i' (x_i^{**}) \leq p_i^{**} \quad (11.C.6)

- Firm viewed as producing a bundle of \( l \) goods with fixed proportions technology

\[
\max_{q \geq 0} \left( \sum_{i=1}^{l} p_i^{**} q \right) - c(q)
\]

Suff. FOC \quad \sum p_i^{**} \leq c'(q^{**}) \quad (11.C.7)
LINDAHL EQUILIBRIUM

• (11.C.6) and (11.C.7) and market clearing conditions \( x_i^{**} = q^{**} \) for \( i \)

\[
\Rightarrow \sum \phi_i'(q^{**}) \leq c'(q^{**}) \ \text{w.e.} \ if \ q^{**} > 0
\]

• Can see that \( q^{**} = q^o \)

• Externalities eliminated and we have private prices for public goods
Problems:
Ability to exclude
Price taking
Externality and Public Goods

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- Multilateral externalities
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MULTILATERAL EXTERNALITIES

- Most externalities are felt & generated by numerous parties, e.g., pollution
- Important distinction: depletable (private or rivalrous) and nondepletable (public or non-rivalrous)
- E.g., depletable: garbage dumped on piece of property depletes potential for further dumping
- Nondepletable: air pollution, amount of air pollution experienced by one agent not affected by fact that others are experiencing it (note: can be rivalrous in alternative uses and production)
MULTILATERAL EXTERNALITIES

• Depletable externalities amenable to market solutions
• Assume agents generating externalities distinct from those experiencing (not essential)
• Firms generate while consumers experience
• P.E. approach: J firms with $\pi(h)$ function, I consumers with quasi-linear derived utility functions $\phi_i(\tilde{h}_i)$, focus on negative externalities $\phi_i'(\cdot) < 0$
In comp eq each firm $\pi_j(h_j^*) \leq 0$ with equality if $h_j^* > 0$ and this would not be P.O.

For P.O. we would need $(\tilde{h}_1^o, ..., \tilde{h}_l^o, h_1^o, ..., h_j^o)$ that solve

$$\begin{align*}
\max & \quad \sum_{i=1}^J \phi_i(\tilde{h}_i) + \sum_{j=1}^J \pi_j(h_j) \\
\text{s.t.} & \quad \sum_{j=1}^J h_j = \sum_{i=1}^I \tilde{h}_i
\end{align*}$$
DEPLETABLE MULTILATERAL EXTERNALITIES

- Constraints reflect the depletability: if $\tilde{h}_i$ is increased by one unit there is one unit less of the externality that needs to be experienced by others

$$\phi_i'\left(\tilde{h}_i^0\right) \leq \mu_i \text{ with eq if } \tilde{h}_i^0 > 0, i = 1, \ldots, I$$

- Characterise optimal levels of externality generation and consumption and parallel efficiency conditions for a private good where $-\pi_j'(h_j^0)$ is interpreted as firm J’s marginal cost of producing more externality
• Assume the level of externality experienced by each consumer is $\sum_j h_j$, the total level of externality produced by firms.

• In unfettered c.e. each firm’s externality generation would continue till it’s marginal benefit is zero $h_j^*$ but P.O. allocation involves $(h_1^0, \ldots, h_j^0)$ that solve

$$\max_{(h_1, \ldots, h_j) \geq 0} \left( \sum_{i=1}^J \phi_i \left( \sum_{j=1}^J h_j \right) \right) + \sum_{j=1}^J \pi_j \left( h_j \right)$$
The F.O.C. for each firm j’s optimal level of externality generation $h_j^o$ are

$$\sum_{i=1}^l \phi_i' \left( \sum_j h_j^o \right) \leq -\pi_j' \left( h_j^o \right)$$

This condition is exactly analogous to the optimality condition for a public good where $-\pi_j' (\cdot)$ is firm j’s marginal cost of externality production.
The introduction of a ‘standard’ market for externality will not lead to an optimal outcome.

\[ \sum_{i=1}^{l} \phi_i'(\sum_{j} h_j^0) \leq -\pi_j'(h_j^0) \]
• Free rider problem reappears so that the equilibrium level of (negative) externality will exceed the optimal level

• A market-based solution would require personalized markets (Lindahl with its problems)

• Given adequate information (strong assumption!) the government could set quotas or taxes to achieve optimality, e.g., in quota case set upper bound to optimal level for each firm $j h_j^o$
Optimality-restoring taxes face each firm with the marginal social cost of their externality.

Optimal tax is identical for each firm and equal to

\[ t_h = - \sum_i \phi'_i(\sum_j h_j^o) \]

per unit of the externality generated.

Given this tax, each firm j solves
NONDEPLETABLE EXTERNALITY

\[ \max_{h_j \geq 0} \pi_j \left( h_j \right) - t_h h_j \]

\[ \pi_j' \left( h_j \right) \leq t_h \]

Given
\[ t_h = - \sum_i \phi_i' \left( \sum_j h_j^o \right) \]

Optimal choice for firm j is \[ h_j = h_j^o \]
PARTIAL MARKET-BASED APPROACH

• Specification of a quota on the total level of the externality and distribution of that number of tradeable externality permits.

• Suppose that $h^o = \sum_j h_j^o$ permits are given to the firms, with firm $j$ receiving $\bar{h}_j$ of them.

• Let $p^*_h$ denote the equilibrium price of these permits.
PARTIAL MARKET-BASED APPROACH

- Each firm j’s demand for permits solves
  \[ \max_{h_j \geq 0} (\pi_j(h_j) + p_h^*(\bar{h}_j - h_j)) \] and FOC \( \pi_j'(h_j) \leq p_h^* \).

- In addition market clearing in permits requires that
  \( p_h^* = -\sum_i \phi_i'(\sum_j h_j) \) and each firm j using \( h_j \) permits and so yields optimal allocation.

- Informational advantage when government has limited information about \( \pi_j(h_j) \) though it has enough information to compute the optimal aggregate level \( h^0 \).
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PRIVATE INFORMATION AND SECOND-BEST SOLUTIONS

• Presence of privately held (or asymmetrically held) information can confound centralized & decentralized attempts to achieve optimality

• Suppose we can write consumer’s derived utility function from externality level as $\phi(h, \eta)$ where $\eta$ is consumer’s type affecting costs from externality. Also $\pi(h, \theta)$

• Actual values of $\eta, \theta$ are privately observed, but ex ante likelihoods (prob distributions) of various values of $\eta, \theta$ are publically known
PRIVATE INFORMATION AND SECOND-BEST SOLUTIONS

• Assume for convenience that $\eta, \theta$ are independently distributed
• Assume consumer has right to externality-free environment
• Assume simple take-it-or-leave-it offer to firm
• Assume only two possible levels of externality $(0, \bar{h})$ and assume negative externality
PRIVATE INFORMATION AND SECOND-BEST SOLUTIONS

- Define $b(\theta) = \pi(\bar{h}, \theta) - \pi(0, \theta) > 0$ firm’s benefit when type $\theta$
- Define $c(\eta) = \phi(0, \eta) - \phi(\bar{h}, \eta) > 0$ consumer’s cost from $h$ when type $\eta$
- In this setting only aspect of types that matter are values of $b$ and $c$
- $G(b), F(c)$ are distribution functions of these variables (induced by $\theta, \eta$)
- Since consumer has right to ext-free env will always insist on $h = 0$
PRIVATE INFORMATION AND SECOND-BEST SOLUTIONS

• However P.O. of values b and c, firms should be allowed to set $h = \bar{h}$ whenever $b > c$

• Consumer can demand $T$ to allow $h$, firm will pay $T$ only if $b \geq T$

• Probability that firm will accept an offer = prob that $b \geq T = 1 - G(T)$

• Given that cost $c > 0$ (assume risk neutrality) consumer optimally chooses the value $T$ to solve
PRIVATE INFORMATION AND SECOND-BEST SOLUTIONS

• To solve

$$\max_T (1 - G(T))(T - c)$$

- Probability firm accepts
- Net gain to consumer

- Strictly positive for all $T > c$, zero when $T = c$

• Solution $T^*_c > c$ implies that bargaining process must result in strictly positive probability of an inefficient outcome since whenever $c < b < T^*$ firm will reject consumer’s offer ($h = 0$) even though optimality requires $h = \bar{h}$
• Just as decentralized bargaining will involve inefficiencies in the presence of privately held information, so too will the use of quotas and taxes

• Weitzman (1974): in the presence of asymmetrically held info two policy instruments no longer perfect substitutes

• Given $\theta$ and $\eta$ aggregate surplus resulting from externality level $h$ is $\phi(h, \eta) + \pi(h, \theta)$, so externality level that max agg surplus depends on realized values of $(\theta, \eta)$
SURPLUS MAXIMIZING LEVELS OF EXTERNALITY AND TYPES

Different optimal levels depending on types
• Suppose quota level $\hat{h}$ is fixed
• Firm will choose level of externality to solve

$$\max_{h \geq 0} \pi \left( h, \theta \right) \text{ s.t. } h \leq \hat{h}$$

• Denote its optimal choice $h^q(\hat{h}, \theta)$
• Typical effect of quota is to make externality much less sensitive to the values of $\theta$ and $\eta$ than is required by optimality
• If quota $\hat{h}$ is such that $\frac{\partial \pi(\hat{h}, \theta)}{\partial h} > 0 \ \forall \theta \Rightarrow h^q(\hat{h}, \theta) = \hat{h} \ \forall \theta$
\[ \phi(h^q(\hat{h}, \theta), \eta) + \pi(h^q(\hat{h}, \theta), \theta) - \phi(h^o(\theta, \eta), \eta) - \pi(h^o(\theta, \eta), \theta) \]

Net b's at \( \hat{h} \)  
- \( h \) determined by firm given firm type and quota

Net b's at \( h^o \)  
- Optimal \( h \) determined by both types

\[
= \int_{h(\theta, \eta)}^{h^q(\hat{h}, \theta)} \left( \frac{\partial \pi(h, \theta)}{\partial h} + \frac{\partial \phi(h, \theta)}{\partial h} \right) dh.
\]

Integral of net loss of \( h^q(\hat{h}, \theta) \) vs \( h^o(\theta, \eta) \)
QUOTA LOSS

Quota determined by average of types of firms and consumers.
TAX LOSS
QUOTA LOSS AND TAX LOSS

Quota determined by average of types of firms and consumers
• Now assuming tax is set at \( t = -\frac{\partial \phi(h^o(\bar{\theta}, \bar{\eta}), \bar{\eta})}{\partial h} \)

• Note that under tax and quota the level of externality is responsive to changes in marginal benefits of the firm but not to changes in the marginal cost of the consumer.
QUOTA OR TAX BETTER?

• It depends
• Suppose $\eta$ constant then for $\theta$ such that benefits to the firm are high a quota will typically miss optimal externality level by not allowing the externality to increase above the quota
• On the other hand a fixed tax by not reflecting increasing cost to consumer may lead to excessive externality
• Intuitively, when optimal externality varies little with $\theta$, we expect a quota to be better
Quota at $h^*$ may aggregate surplus at any value of $(\theta, \eta)$

Note: here we really need $h^*$ slightly less than $h^*$ infinite costs
In this case tax best to account for the firms’ changing benefits, therefore optimal that $h$ changes
QUOTA OR TAX

If expected value of agg surplus as welfare measure than either policy instrument may be preferable

Note also that either tax or quota will work better than bargaining procedure discussed earlier
MORE GENERAL POLICY MECHANISM

• Tax and quota schemes considered completely unresponsive to change in the marginal costs to consumers (in this case)
• Could there be schemes that do better?
• Problem: b’s and c’s unobservable and parties involved may not have incentive to reveal truthfully if asked
• E.g., suppose government simply asks and sets instrument based on answer: firms and consumers exaggerate
Return to the possible levels of externality 0 and $\bar{h}$

Can we design a scheme that achieves optimal externality for every realization of $b$ and $c$?

Answer: Yes
REVELATION MECHANISM

• Firm and consumer each asked to report their value of $b$ and $c$
• Let $\hat{b}$ and $\hat{c}$ denote announcements
• For each pair $(\hat{b}, \hat{c})$ government sets an allowed level of externality as well as tax or subsidy payment for each of the two agents
• E.g., government declares it will max agg surplus given the announcements that is $h = \bar{h}$ iff $\hat{b} > \hat{c}$
In addition if externality generation allowed the government will tax firm an amount equal to $\hat{c}$ and subsidize consumer with payment of $\hat{b}$.

If producer pays tax by amount of damage announced by consumer and consumer is paid subsidy by amount of benefit announced by firm then both are incentivized to reveal truthfully.
REVELATION MECHANISM

• To see this consider consumer’s optimal announcement when her cost level is $c$
• if firm announces some $\hat{b} > c$ then the consumer prefers to have externality activity allowed (she gains $\hat{b} - c$ relative to prevented)
• Therefore optimal announcement satisfies $\hat{c} < \hat{b}$
• Also since any such announcement will give her that benefit she might as well be truthful
REVELATION MECHANISM

• If firm announced $\hat{b} \leq \hat{c}$ consumer prefers to have zero externality hence announce $\hat{c} \geq \hat{b}$ (might as well say $\hat{c} = c$)

• So whatever firm announces consumer’s dominant strategy is to be truthful (weakly dominant strategy)

• Same for firm

• Scheme is an example of Clarke-Grove mechanism

• Problem of balanced budget too
Figure 8.3 Uncertainty about abatement costs – costs overestimated
Figure 8.4 Uncertainty about abatement costs – costs underestimated
Figure 8.5 Uncertainty about abatement costs – costs overestimated
Figure 8.6 Uncertainty about abatement costs – costs underestimated
Figure 8.7 Uncertainty about damage costs – damages underestimated