Practice Exercises

2. Consider the agency problem with the risk-neutral principal and the risk-averse agent examined in Case 1. Assume that the principal's and the agent's objective functions are as above, $\pi_L = 9$, $\psi(e_0) = 1$, $\psi(e_1) = 2$, and the rest of the values are as above.

   a) Set up formally the principal's programme and find the optimal contract in the case of symmetric information (effort observable and verifiable).

   b) Set up formally the principal's programme, and write the first-order conditions for the case of asymmetric information (unobservable/ non-verifiable effort).

   c) Find explicitly the optimal contract under asymmetric information. Is it different from the first-best contract?

   d) Is a first best attained under asymmetric information? Explain intuitively why or why not.

a) *[continued from class]*

- As we discussed in our last tutorial, in the case of symmetric information the principal and the agent can write an enforceable contract stipulating that the latter should exert a specific level of effort.
- Since the principal is assumed to have all the market power, she needs to offer to the agent just the minimum salary required in order for him to exert that given level of effort (i.e. participation constraint satisfied with equality).
- Then the principal will choose the level of effort that maximizes her expected benefit (profit in this case), specify that in the contract, and offer to the agent the minimum salary satisfying the corresponding participation constraint.
- In particular, we saw that these salaries were $w_0^{SI} = 4$ and $w_1^{SI} = 9$, with the corresponding expected benefits for the principal being

$$\mathbf{E}\left[B\left(\pi - w_0^{SI}\right)|e = e_0\right] = \frac{1}{4} \cdot 20 - \frac{3}{4} \cdot 9 - 4 = 7.75$$
$$\mathbf{E}\left[B\left(\pi - w_1^{SI}\right)|e = e_1\right] = \frac{3}{4} \cdot 20 - \frac{1}{4} \cdot 9 - 9 = 8.25.$$

- The principal will therefore choose a contract that stipulates high effort, and pay the corresponding salary.

b)

- The agent's salary can no longer be contingent on his effort level, but only on the realized gross profit, so that $w \in \{\ w(\pi_i)\ \}_{i\in\{H,L\}}$.
- The corresponding participation constraint of course still needs to be satisfied at the effort level e_j , $j \in \{0,1\}$ chosen by the principal:

$$p_j\sqrt{w(\pi_H)} + \left(1 - p_j\right)\sqrt{w(\pi_L)} - \psi(e_j) \geq \bar{u}.$$

- If furthermore the principal would like the agent to exert high effort, the incentive compatibility constraint (IC) must also be satisfied:

$$\mathbf{E}[U(w(\pi), e)|e = e_1] \geq \mathbf{E}[U(w(\pi), e)|e = e_0]$$

$$p_1 u\big(w(\pi_H)\big) + (1 - p_1)u\big(w(\pi_L)\big) - \psi(1) \geq\ p_0 u\big(w(\pi_H)\big) + (1 - p_0)u\big(w(\pi_L)\big) - \psi(0)$$

$$\frac{3}{4}\sqrt{w(\pi_H)} + \frac{1}{4}\sqrt{w(\pi_L)} - 2 \geq \frac{1}{4}\sqrt{w(\pi_H)} + \frac{3}{4}\sqrt{w(\pi_L)} - 1$$

$$\frac{1}{2}\sqrt{w(\pi_H)} - \frac{1}{2}\sqrt{w(\pi_L)} \geq 1$$

The principal's problem will be

$$\max_{\{e,w(\pi)\}} \mathbf{E}[B(\pi - w(\pi))] = p_1(\pi_H - w(\pi_H)) + (1 - p_1)(\pi_L - w(\pi_{\mathrm{L}}))$$

$$\text{s.t.} \quad \mathbf{E}[U(w(\pi), e) | e = 1] \geq \bar{u}. \qquad \text{(PC)}$$

$$\mathbf{E}[U(w(\pi), e) | e = 1] \geq \mathbf{E}[U(w(\pi), e) | e = 0] \qquad \text{(IC)}$$

or

$$\max_{\{e,w(\pi)\}} \tfrac{3}{4}(20 - w(\pi_H)) + \tfrac{1}{4}(9 - w(\pi_{\mathrm{L}}))$$

$$\text{s.t.} \quad \frac{3}{4}\sqrt{w(\pi_H)} + \frac{1}{4}\sqrt{w(\pi_L)} \geq 3 \qquad \text{(PC)}$$

$$\frac{1}{2}\sqrt{w(\pi_H)} - \frac{1}{2}\sqrt{w(\pi_L)} \geq 1. \qquad \text{(IC)}$$

The Lagragian function will be

$$\begin{aligned}\mathcal{L}(w(\pi_H), w(\pi_L), \lambda, \mu) = \quad & \tfrac{3}{4}(20 - w(\pi_H)) + \tfrac{1}{4}(9 - w(\pi_\mathrm{L})) \\ & +\lambda\left[\tfrac{3}{4}\sqrt{w(\pi_H)} + \tfrac{1}{4}\sqrt{w(\pi_L)} - 3\right] \\ & +\mu\left[\tfrac{1}{2}\sqrt{w(\pi_H)} - \tfrac{1}{2}\sqrt{w(\pi_L)} - 1\right]\end{aligned}$$

and the corresponding FOC

$$w(\pi_H): \quad -\tfrac{3}{4} + \tfrac{3}{4}\lambda\frac{1}{2\sqrt{w_{H,1}}} + \tfrac{1}{2}\mu\frac{1}{2\sqrt{w_{H,1}}} = 0 \quad (1)$$

$$w(\pi_L): \quad -\tfrac{1}{4} + \tfrac{1}{4}\lambda\frac{1}{2\sqrt{w_{L,1}}} - \tfrac{1}{2}\mu\frac{1}{2\sqrt{w_{L,1}}} = 0 \quad (2)$$

$$\lambda: \quad \tfrac{3}{4}\sqrt{w_{H,1}} + \tfrac{1}{4}\sqrt{w_{L,1}} - 3 \geq 0 \quad (3)$$

$$\mu: \quad \tfrac{1}{2}\sqrt{w_{H,1}} - \tfrac{1}{2}\sqrt{w_{L,1}} - 1 \geq 0 \quad (4)$$

c)

- We can formally solve the above maximization problem following the Kuhn–Tucker method. Yet, by noticing that the principal's objective function is decreasing in $w(\pi_{H,1})$ and $w(\pi_{L,1})$ we can infer that that both constraints must bind at the optimum. [Otherwise, since u is a continuous function, the would be able to increase her expected benefit by decreasing either $w(\pi_{H,1})$ or $w(\pi_{L,1})$, or both.] This implies that $\lambda > 0$ and $\mu > 0$.

- Hence from expressions (3) and (4) we get that under the optimal contract w_H and w_L must satisfy:

$$\tfrac{3}{4}\sqrt{w_{H,1}} + \tfrac{1}{4}\sqrt{w_{L,1}} - 3 = 0$$
$$\tfrac{1}{2}\sqrt{w(\pi_{H,1})} - \tfrac{1}{2}\sqrt{w(\pi_{L,1})} - 1 = 0$$

- Solving this system we find the profit-contingent salaries that maximize the principal's expected benefit if the agent is to exert high effort:

$$w_{H,1} = 12.25$$
$$w_{L,1} = 2.25.$$

- The principal's expected benefit will be

$$E[B(\pi - w(\pi))|e = e_1] = \tfrac{3}{4}(20 - 12.25) + \tfrac{1}{4}(8 - 2.25) = 7.5$$

- But we are not done yet. We also need to examine the case where the agent exerts low effort.
- It is easy to notice that, unless provided with relevant incentives, (cf. incentive compatibility constraint), the optimal choice for the agent would be to exert low effort. In order therefore for the principal to induce low effort (e_0) from the agent, she only needs to see that his participation constraint is satisfied.
- The principal's problem becomes thus identical to the one under symmetric information, where it was shown that

$$E[B(\pi - w(\pi))|e = e_0] = 7.75.$$

- It is hence optimal for the principal to induce low effort from the agent, but offering him a contract with practically fixed salary $w_{L,0} = w_{H,0} = 4$.
- This contract is different from the one under symmetric information.
- Agent's expected utility remains at its reservation level, yet the principal's expected benefit is now lower.
- Due to the presence of moral hazard, social welfare is now lower, and only a *second best* may be achieved.

d)

- A first-best is not attained. Agent’s expected utility remains at its reservation level, yet the principal’s expected benefit is now lower.
- In order to compensate for that and induce the agent to accept the contract, the principal needs to increase his salary in the “good” state of the world, and more than proportionally so, since the agent is risk-averse.
- In order to compensate for the relatively lower salary in the case of low profits, the principal needs to provide a significantly higher salary in the case of high profits.
- The difference is not worth the higher expected profit generated by a higher effort level on behalf of the agent.
- Due to the presence of moral hazard, social welfare is lower, and only a *second best* may be achieved.