Solutions for problem Set 2

1)

The Nash solution comes from the following problem:

$$\max_{x_i} \sum_{i=1}^{k} n_i log x_i$$
$$s.t. \sum_{i=1}^{k} x_i = 1$$

We form the Lagrangean:

$$\mathcal{L} = \sum_{i=1}^{k} n_i log x_i - \lambda\left(\sum_{i=1}^{k} x_i - 1\right)$$

From the FOCs we get:

$$n_i \frac{1}{x_i} - \lambda = 0 \Longrightarrow n_i = \lambda x_i$$

And

$$\sum_{i=1}^{k} x_i = 1$$

Now, if we sum the FOCs for all $i = 1, \dots, k$ we get:

$$\sum_{i=1}^{k} n_i = \sum_{i=1}^{k} \lambda x_i \Longrightarrow \sum_{i=1}^{k} n_i = \lambda \sum_{i=1}^{k} x_i \Longrightarrow n = \lambda$$

If we substitute in the FOC, we get that $x_i = \frac{n_i}{n}$.

2) I) Plurality: A

Borda: A

Condorcet: A

ii) If C was not available B would win. IIA is violated

3)

4) a) It satisfies IIA

b) It satisfies Absences of Dictatorship, as there is no dictator

c) It does not satisfy the Pareto Principle since, even if all the agents agreed on a particular ordering, this SWF would still give us the same ordering.

5) Breaking the cycle at c>a we get c>b>d>a

6)

i) Bob does not believe his mother's promise because she cannot be consistent with Arrow's criteria and still resolve a future "pizza versus burger" conflict differently. If she decides to let Ann get her way this Saturday and eat pizza, she must let Ann get her way every time the same conflict appears, or else she would violate IIA! This means that Ann would become the dictator over Saturday's dinner!

ii) The SWF proposed satisfies the Pareto Principle since it always gives as output one of the children's preferences.
It also satisfies Absence of Dictatorship because neither Ann's nor Bob's preferences always coincide with the social ordering.
However it does not satisfy IIA. Assume that we have the following preferences for Ann and Bob:

$b \succ_B s \succ_B p$ and $b \succ_A p \succ_A s$
Then the social ordering would coincide with Bob's ordering since $b \succ_B s$, that is:

$$b \succ_S s \succ_S p$$

We observe that there was a conflict between p and s and it was resolved in favor of s.
Now, we have the new preference profiles:
$s \succ_B p \succ_B b$ and $p \succ_A s \succ_A b$
Since we have that $b \prec_B s$ the new social ordering will be Ann's preferences:

$$p \succ_S s \succ_S b$$

We now observe that the conflict between p and s appeared again and now the social ordering is in favor of p, thus violating IIA.

7) For the first profile:
z>y from Pareto.
For the second:
Y>z>x from Pareto.
For the third:
x>y from Pareto.
z>x from IIA. So: z>x>y from transitivity

For the fourth there was a mistake, replace the table with:

3	y		
2			z
1		x	
	1	2	3

So, x>z from Pareto, y>x from IIA and y>x>z from transitivity.