Voting and Social Choice

Chapter 4 Moulin

Ordinal welfarism

- Ordinal welfarism pursues the welfarist program in those situations where cardinal measurement of individual welfare is either unfeasible, unreliable or ethically untenable
- In most real life elections voters are not asked to express more than an "ordinal" opinion of the names on the ballot
- If the outcome depends on intensity of voters' feelings, a minority of fanatics will influence the outcome more than a quiet majority

Ordinal welfarism

- The identification of welfare with preferences, and of preferences with choice, is an intellectual construction at the center of modern economic thinking
- Social choice theory adapts the welfarist program to the ordinalist approach
- Individual welfare can no longer be separated from the set A of outcomes to which it applies

Ordinal welfarism

- In the ordinal world collective decision making can only be defined if we specify the set A of feasible outcomes (states of the world), and for each agent i a preference relation Ri on A.
- The focus is on the distribution of decision power
- Two central models of social choice theory: a *voting* problem and a *preference aggregation* problem
- These are the most general microeconomic models of cdm because they make no restrictive assumptions neither on the set A of outcomes or on the admissible preference profile of the agents.

Condorcet versus Borda

- Plurality voting is the most widely used voting method
- Each voter chooses one of the competing candidates and the candidate with the largest support wins
- Condorcet and Borda argued that plurality voting is seriously flawed because it reflects only the distribution of the "top" candidates and fails to take into account entire relation of voters

Where Condorcet and Borda agree

- 21 voters and three candidates a,b,c
- Plurarily elects a yet b is more convincing compromise (a more often below b)
- Borda tally: Score a=16,b=27,c=20
- Condorcet winner b: bPc, bPa,cPa

No. voters	6	7	8
Top	b	c	a
	c	b	b
Bot	a	a	c

Where Borda and Condorcet Disagree

- The profile of 26 voters and three candidates
- Plurality winner "a" (also Condorcet winner)
- Borda winner is "b" – eleven "minority" voters dislike "a" more than fifteen "majority" dislike "b"

No of voters	15	11
	a	b
	b	c
	c	a

Where Borda and Condorcet Disagree

- Borda's argument relies on scoring convention
- General family of scoring include Borda's and plurality as special cases:
- Plurality: s1=1, sk=0 for all k
- Borda sk=p-k for k=1,…,p
- In this example depending on scores either a or b selected but never c (this flexibility contrasts Condorcet)

No of voters	15	11
	a	b
	b	c
	c	a

Condorcet against Scoring Method

- 81 voters, 3 candidates
- “b” is plurality and Borda winner
- Condorcet winner "a" aPb by 42/29 and aPc by 58/23

30	3	25	14	9
a	a	b	b	c
b	c	a	c	a
c	b	c	a	b

Condorcet against Scoring Method

- b wins for any choice of scores, s between [0,1] with s=0 plurality, s=1/2 Borda
- c fares badly in both scoring and Borda (c much more often between b and a when b is first choice than between a and b when a is first choice
- a Condorcet winer and is unaffected by the position of a sure loser c

30	3	25	14	9
a	a	b	b	c
b	c	a	c	a
c	b	c	a	b

score (b) = 39+30s >score (a)=33+34s > score (c) = 9+17s

Top score = 1, bottom 0 and s middle

Condorcet cycle

- Majority relation may cycle
- n1+n2>n3=>aPb
- n1+n3>n2=>bPc
- n2+n3>n1=>cPa
- No Condorcet winner
- Proposed to break cycle at weakest link

n1	n2	n3
a	c	b
b	a	c
c	b	a

The Reunion Paradox

Two disjoint groups (34 and 35 members each) who vote for same candidates

Candidate “a” is majority winner among bottom group (right-handed)

Among top group (left-handed) we have a cycle and removing weakest link leads to “a”

10	6	6	12
a	b	b	c
b	a	c	a
c	c	a	b

18	17
a	c
c	a
b	b

Voting over Resource Allocation

- For political elections with a few candidates arbitrary preferences are a reasonable assumption
- When the issue concerns allocation of resources some important restrictions come into play

Voting over resource allocation

- Majority voting works well in a number of allocation problems but produces systematic cycling in others
- Scoring methods are hopelessly impractical when the set of A outcomes is large (and typically modelled as an infinite set), also because of IIA property

Voting over Time shares ex. 4.5

- Can choose any mixture (x1,…x5) where xi represents time share and sum to one
- Set N agents partitioned into five disjoint groups of one-minded fans
- If one group has a majority (>n/2) then that station is a Condorcet winner and it is played all the time
- If no group has an absolute majority then the majority relation is strongly cyclic.
- Destructive competition: failure of the logic of private contracting (negative externalities)=> instability and unpredictability

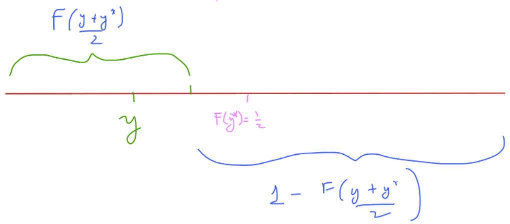
Single-Peaked Preferences

- Example 2.6: Location of a Facility

$$u_i(y) = -|y - x_i|$$

$$F(z)$$

$$y < y^* \Rightarrow F(\frac{y+y^*}{2}) < F(y^*) = \frac{1}{2} \Rightarrow 1 - F(\frac{y+y^*}{2}) > \frac{1}{2}$$



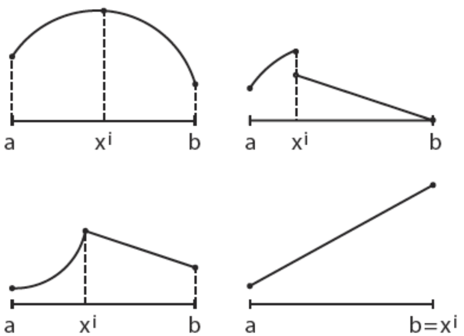
Single-Peaked Preferences

- The coincidence of Condorcet and Utilitarian optimum depends on particular assumption of common utility = distance
- However, *median* of distribution is a Condorcet winner (if not util optimum) for a much larger *domain* of individual preferences called *single-peaked preferences*

Single-Peaked Preferences

- Given an ordering of the set A, we write x<y when x on left of y
- *we say that z is "between" x and y if either x ≤ z ≤ y or y ≤ z ≤ x*
- The preference relation Ri is *single-peaked* with *peak* xi if xi is the top outcome of Ri and for all other outcomes x prefers any outcome in between.

Single-Peaked preferences



Single-Peaked preferences and IIA

- Definition of feasible set far away from A does not matter, e.g., [0,100] median 35

$$x^1 = 35 \quad x^2 = 10 \quad x^3 = 22 \quad x^4 = 78 \quad x^5 = 92 \quad x^6 = 18 \quad x^7 = 50$$

$$B = [20, 75]: \quad \text{peaks } x^1 = 35,\ \tilde{x}^2 = \tilde{x}^6 = 20,\ x^3 = 22,\ \tilde{x}^4 = \tilde{x}^5 = 75,\ x^7 = 50$$

$$C = [20, 40]: \quad \text{peaks } x^1 = 35,\ \tilde{x}^2 = \tilde{x}^6 = 20,\ x^3 = 22,\ \tilde{x}^4 = \tilde{x}^5 = \tilde{x}^7 = 40$$

Condorcet method is *strategy-proof*

- A voter has no incentive to lie strategically when reporting a peak of her preferences
- Even if a group of voters join forces to jointly misrepresent their peaks, they cannot find a move from which they all benefit

Strategy proofness example

N_- the set of agents whose peak is (strictly) to the left of y^* on A ($x^i < y^*$)

N_+ the set of those whose peak is to its right ($y^* < x^i$)

N_0 those with $x^i = y^*$

Suppose that the coalition T of voters agree to alter their reported peaks

from the true peak x^i to a fake $\tilde{x}^i$

while the rest of the agents report their peak truthfully as before.

z^* the new median of the reported peaks: we show that either $z^* = y^*$

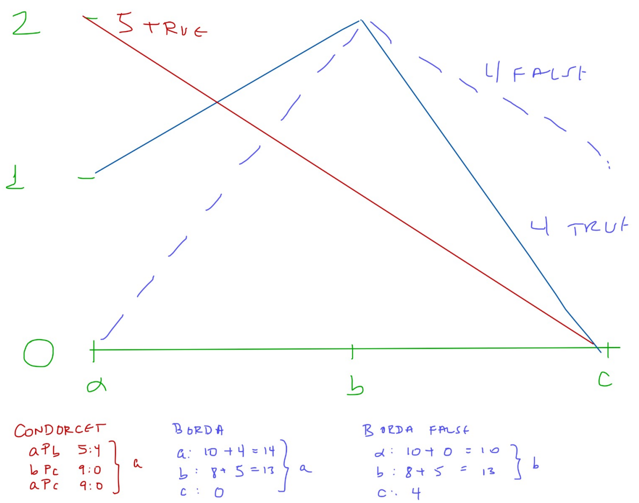
or at least one agent in T strictly prefers y^* to z^*

Proof:

The proof is by contradiction. Suppose that $z^* \neq y^*$ and that no i in T strictly prefers y^* to z^*. Say that z^* is to the right of y^* in A ($y^* < z^*$). Because preferences are single peaked, everyone in N_- and in N_0 strictly prefers y^* to z^*; therefore T is contained in N_+. By definition of the median, $N_- \cup N_0$ forms a strict majority, and we just proved that they all still report their true peak; therefore a majority prefers y^* to z^*, and z^* cannot be chosen when T misreports, contradiction. A symmetrical argument applies when z^* is to the left of y^*.

Strategy Proofness

- Ultimate test of incentive-compatibility in mechanism design
- Simple truth is always best move (whether or not I have information about other agents messages)
- Two important examples of strategy-proof mechanisms: majority voting over single-peaked preferences and atomistic competitive equilibrium



Gibbard-Satterthwaite theorem

- Any voting method defined for *all* rational preferences over a set A of three or more outcomes must *fail* the strategy proofness property: at some preference profile some agent will be able to “rig” the election to her advantage by reporting untruthfully
- Technically equivalent to Arrow’s IT