Simple Partial equilibrium example with linear functions

Linear PE example

- Two individuals with the following utility functions:
- $u_1(m_1, x_1) = m_1 + 5x_1 - 0.5x_1^2$
- $u_2(m_2, x_2) = m_2 + 3x_2 - 0.5x_2^2$
- $\phi_1 = 5x_1 - 0.5x_1^2$ and $\phi_2 = 3x_2 - 0.5x_2^2$

Linear PE example

- Two firms with the following cost functions:
- $C_1(q_1) = 0.3q_1^2$
- $C_2(q_2) = 0.6q_2^2$

Linear PE example

- Competitive equilibrium requires all agents maximize subject to their constraints:
- $Max\ u_1(m_1, x_1)$ s.t. $m_1 + px_1 \leq \omega_{m_1} + \theta_{11}\big(pq_1 - c_1(q_1)\big) + \theta_{12}(pq_2 - c_2(q_2))$
- Substitute m_1 in utility function to get
- $\underset{x_1}{Max}\ 5x_1 - 0.5x_1^2 - px_1 + [\omega_{m_1} + \theta_{11}\big(pq_1 - c_1(q_1)\big) + \theta_{12}\big(pq_2 - c_2(q_2)\big)]$

Linear PE example

- $\underset{x_1}{Max}\ 5x_1 - 0.5x_1^2 - px_1 + [\omega_{m_1} + \theta_{11}\big(pq_1 - c_1(q_1)\big) + \theta_{12}\big(pq_2 - c_2(q_2)\big)]$
- FOC gives us consumer 1's demand function
- $5 - x_1 - p = 0 \Rightarrow x_1 = 5 - p$
- Can easily see that consumer 2 will have:
- $3 - x_2 - p = 0 \Rightarrow x_2 = 3 - p$

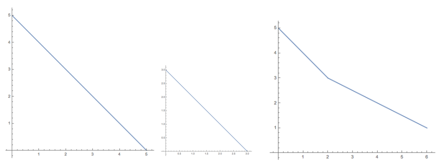
Profit maximization

- $\underset{q_1}{Max}\ pq_1 - C_1(q_1) or\ pq_1 - 0.3q^2$
- FOC $p - 0.6q_1 = 0 \Rightarrow q_1 = \frac{p}{0.6}$
- Likewise for firm 2 $\Rightarrow q_2 = \frac{p}{1.2}$

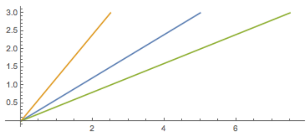
Competitive Equilibrium

- For a C.E. we must have markets clearing
- $x_1 + x_2 = q_1 + q_2$
- So
- $5 - p + 3 - p = \frac{p}{0.6} + \frac{p}{1.2}$
- Solving for p we find $p = 1.77$
- Competitive equilibrium allocation is
- $(x_1^*, x_2^*, q_1^*, q_2^*; p^*) = (3.22, 1.22, 2.96, 1.48)$
- You may note that in the market clearing there is no wealth (this has dropped out due to demand for good being independent of wealth)

Aggregate demand

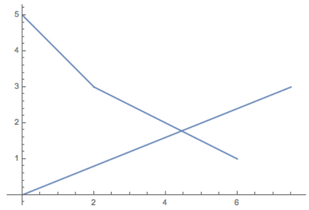


Aggregate Supply



$q_1 + q_2 = 2.5\,p \Rightarrow Q = 2.5p \text{ or } p = \frac{Q}{2.5}$

Competitive equilibrium



Pareto optimal allocations

- Suppose consumption and production levels of the good in question is fixed at $(\overline{x_1}, \overline{x_2}, \overline{q_1}, \overline{q_2})$
- With these production levels total amount of numeraire available for distribution is $\omega_m - \sum_j C_j(\overline{q_j}) = \omega_m - C_1(\overline{q_1}) - C_2(\overline{q_2})$
- The set of attainable utilities by our two consumers are:
- $\{(u_1, u_2): u_1 + u_2 \leq \phi_1(\overline{x_1}) + \phi_2(\overline{x_2}) + \omega_m - C_1(\overline{q_1}) - C_2(\overline{q_2})\}$

Pareto optimal allocations

- It follows that the optimal consumption and production levels of the good in question can be obtained as the solution to
- $\underset{\substack{x_1, x_2 > 0 \\ q_1, q_2 > 0}}{Max}\ \phi_1(x_1) + \phi_2(x_2) + \omega_m - C_1(q_1) - C_2(q_2)$
- Subject to $x_1 + x_2 = q_1 + q_2$

Pareto optimal allocations

- With μ as multiplier in constraint the optimal values $(x_1^*, x_2^*, q_1^*, q_2^*)$ satisfy the following 2+2+1 conditions:

$$\mu \leq C_1'(q_1^*)$$
$$\mu \leq C_2'(q_2^*)$$
$$\phi_1'(x_1') \leq \mu$$
$$\phi_2'(x_2') \leq \mu$$
$$x_1 + x_2 = q_1 + q_2$$

Linear utility possibility frontier