9 Externalities and Public Goods

Another form of market failure concerns externalities and public goods. In particular, an externality emerges when the well-being of an agent (or the production possibilities of a firm) is *directly* affected by the actions of another agent in the economy. A typical example of negative externality is the case of a refinery producing a good which, as a by-product, pollutes a lake where several fishermen operate. In this context their production possibilities (fish captures) are directly affected by the pollutants that the refinery dumps into the lake. Importantly, externalities describe effects from one agent to another that are *not* captured by the price system. In the example above, if the refinery voluntarily increases production and, as a consequence, the price of oil decreases, thus reducing the costs of fishermen, we do not claim that fishermen benefit from a positive externality. (Instead, effects transmitted through the price system are referred to as "pecuniary externalities.") In section 9.1 we analyze negative (positive) externalities and how markets produce too much (too little) of them when left unregulated, relative to what a social planner would produce.

Section 9.2 studies a solution to the externality problem originally proposed by Coase (1960) that does not involve direct government intervention. Instead, it requires that the agents producing and affected by the externality bargain among themselves. If agents are perfectly informed about each other's benefits and costs, and if negotiation costs are sufficiently low, the so-called Coase theorem suggests that agents can reach socially optimal outcomes without the need of intervention. We then examine other approaches seeking to correct the excessive (or insufficient) externality production using a more direct policy intervention: (1) quotas that determine a maximum amount of pollution, (2) emission fees on every unit of pollution that firms emit (i.e., Pigouvian taxation), and (3) tradable emission permits distributed among firms. Sections 9.3 and 9.4 extend our results to industries in which a single firm operates (monopoly) or a few firms do (oligopoly). In both of these settings the regulator faces two market failures: the externality that pollution generates (which he seeks to reduce) and the insufficient production of monopolies or oligopolies (which he would like to increase). We explore the regulator's trade-off in this context, and how optimal regulation varies as more firms operate in the industry.

Under complete information, the regulator can induce socially optimal outcomes by using alternative policy tools; that is, he can use either quotas or emission fees and yield the same social welfare. However, under incomplete information about the firm's profits (which can occur if, for instance, the regulator does not perfectly observe the firm's production costs) or about the damage to affected individuals, each policy tool can induce different externality levels, and as a consequence the social welfare that emerges when using quotas or emission fees can radically differ. Sections 9.5 through 9.7 examine the regulator's decision on how to set quotas or emission fees under such an incomplete information setting; section 9.8 then compares the welfare arising in each case in order to determine which policy instrument is socially preferred.

Still in the context of externalities, section 9.9 describes the famous "tragedy of the commons," first showing that exploitation of common pool resources (e.g., fishing grounds and aquifers) is socially excessive, and second analyzing how such result is affected by the number of firms exploiting the common pool resource. Section 9.10 analyzes settings where firms simultaneously determine how many units of output to produce and how much abatement effort to exert in order to reduce their pollutant emissions. In this context, regulation sets an emission fee that seeks to induce firms to produce at the socially optimal level. For generality, we consider two forms of pollution: uniform and nonuniform, where the latter allows for pollution from one firm j to be registered in the monitoring station the regulator has next to another firm k.

In section 9.11 we switch our focus to public goods. Unlike private goods, the cost of excluding additional users of a public good is extremely high (i.e., the good is nonexcludable) and the utility that existing users derive from the good is unaffected if more individuals enjoy the good (i.e., the good is nonrival). In sections 9.12 and 9.13 we demonstrate that, if the provision of public goods depends on private donations, the total amount provided lies below the social optimum (the amount of public good that a benevolent social planner would select). Given this result, a common question is whether a central planner could set an income tax on each individual in order to fund the production of the public good. Unfortunately, the "crowding-out" effect shows that such a tax scheme would decrease private donations to the public good by exactly the amount of the tax (i.e., public contributions crowd out private donations), thus leaving the total production of public good unaffected. We also review the main results in the experimental literature that tested whether subjects in controlled settings behave as prescribed by the theoretical results, and in which contexts theory and data differ.

Section 9.14 then explores solutions to the underprovision of public goods. In section 9.15 we discuss an alternative solution to the public good problem, where the social planner sets "personalized prices" on each individual enjoying the public good, a set of prices commonly known as Lindahl prices. Section 9.16 extends our analysis to a setting where public goods experience congestion, and thus the nonrival property is imperfect, and how equilibrium results differ from those in which congestion is absent. Finally, section 9.17 considers the introduction of behavioral motives such as warm-glow, status acquisition, and social preferences, evaluating how equilibrium donations to a public good are affected.

9.1 Externalities

In this section we focus on a setting with a polluting firm (agent 1) and an individual affected by such pollution (agent 2). (We briefly mention positive externalities, but most of our analysis focuses on negative externalities, such as CO_2 pollution.) In particular, $\pi(p,x)$ represents the firm's profit function as a function of the price vector p and the amount of externality generated x. For simplicity, we consider here that prices are given, but subsequent sections examine how our results are affected when firms sustain some market power, when they are monopolies or oligopolies in the product market. Since price vector p acts as a parameter, we state the profit function more compactly as $\pi(x)$. In the present setting, assume that $\pi'(x)>0$ and $\pi''(x)<0$, thus indicating that the marginal benefit from the externality-generating activity is positive for the firm, but that such marginal benefit is decreasing in the level of the externality-generating activity x.[1] Hence the firm obtains a positive and significant benefit from the first unit of the externality-generating activity, but the *additional* benefit from further units is decreasing. This could be the case, for instance, of a firm generating pollution as a side effect of its production process.

Finally, agent 2's utility is given by $u(q,x)$, where $q\in\mathbb{R}^L$ denotes a vector of L-tradable goods, and $x\in\mathbb{R}_+$ represents the negative externality, where $\partial u/\partial u<0$ and $\partial u/\partial q_k\geq 0$ in every good k. Let $q^*(p,w,x)$ denote agent 2's Walrasian demand, which depends on the price vector p and the agent's wealth level w but, for generality, can also be a function of the externality level x. Let $v(x)=u(q^*(p,w,x),x)$ be the indirect utility that results from his utility maximization problem (UMP) where $v'(x)<0$ for all $x>0$. For instance, the firm's profits can be given by $\pi=py-cy^2$ where $y\in\mathbb{R}^L_+$ denotes

1. For an example on externalities in consumption (production), see exercise 10 (9, respectively) at the end of the chapter.

output, and $p>c>0$. Every unit of output y can generate more than one unit of pollution, namely $y=\alpha x$, where $\alpha>1$, or less than one unit of pollution, $\alpha\in(0,1)$. Therefore we can rearrange $y=\alpha x$ as $y/\alpha=x$, which yields a profit function

$$\pi(x)=p\frac{x}{\alpha}-c\left(\frac{x}{\alpha}\right)^2.$$

In this context

$$\pi'(x^*)=\frac{p}{\alpha}-2c\frac{x^*}{\alpha}\frac{1}{\alpha},$$

which, as required, decreases in x since $\pi''(x)=-2c/\alpha^2<0$ for all x. Setting $\pi'(x^*)=0$ and solving for x^*, we obtain a competitive equilibrium level of pollution $x^*=\alpha(p/2c)$.

9.1.1 Competitive Equilibrium

In the competitive equilibrium the firm independently chooses the level of the externality-generating activity, x, that solves its profit-maximization problem (PMP)

$$\max_x \pi(x).$$

Taking first-order conditions with respect to x, we obtain
$\pi'(x^*)\leq 0$, with equality if $x>0$ (interior solution).

This result is graphically represented (for an interior solution where $\pi'(x^*)=0$) in figure 9.1, where the firm increases the externality-generating activity until the point where the marginal benefit from an additional unit, $\pi'(x)$, are exactly zero. This occurs at x^*.

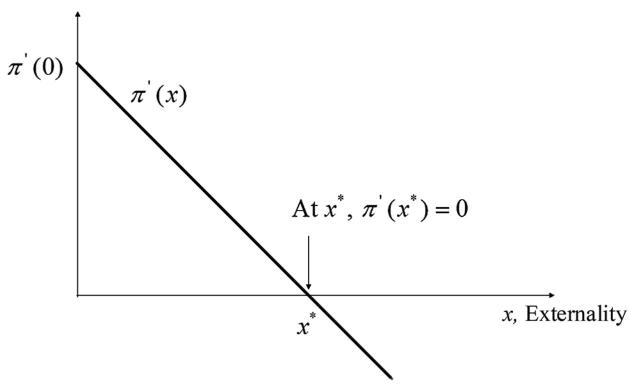

Figure 9.1
Equilibrium externality level

At the competitive equilibrium allocation, all agents independently and simultaneously solve their UMP (for consumers) or their PMP (for firms). The UMP of the individual affected by pollution (agent 2) is not that interesting because he cannot affect the level of the externality-generating activity x as he maximizes $u(q,x)$ subject to $pq \leq w$, where $p \in \mathbb{R}^L_+$ is the given price vector and $q \in \mathbb{R}^L$ does not include pollution as one of the L-tradable goods (but we relax this assumption below). For this reason our discussions of competitive equilibrium allocations will be silent about agent 2's UMP.

9.1.2 Pareto Optimality

In contrast, the social planner selects the level of x that maximizes social welfare, that is

$$\max_{x \geq 0} \ \pi(x) + v(x).$$

The first-order condition for a maximum is

$$\pi'(x) + v'(x) \leq 0, \quad \text{with equality if } x^0 > 0,$$

where x^0 is the Pareto optimal amount of the externality. That is,
$\pi'(x^0) \leq v'(x^0)$, or $\pi'(x^0) = -v'(x^0)$ in the case of interior solutions.
Intuitively, this condition states that, at a Pareto optimal (and interior) solution the marginal profit that the firm obtains from an additional unit of the externality-generating activity, $\pi'(x)$, must be equal to the marginal cost that the affected individual suffers, $-v'(x)$, as figure 9.2 depicts.[2]

Figure 9.2 represents the case of a *negative* externality, entails $v'(x) < 0$, whereby x reduces the utility of the affected individual (e.g., loud music). In addition, since $v'(x) < 0$ for all x, then $-v'(x) > 0$ for all x, lying in the positive quadrant of the figure. Intuitively, $v'(x)$ can then be interpreted as the (negative) marginal benefit from the externality while $-v'(x)$ denotes the marginal cost that the affected individual suffers from the externality. In this case we have that $x^* > x^0$, where too much externality x is produced in the competitive equilibrium relative to the Pareto optimum.[3]

2. For an example of an industry in which firms simultaneously generate positive and negative externalities on each other's costs, see exercise 3 at the end of the chapter.

3. For an example of an externality that arises only *in expectation*, how equilibrium and socially optimal amounts of the externality-generating activity differ, and how to restore optimality, see exercise 1 at the end of the chapter.

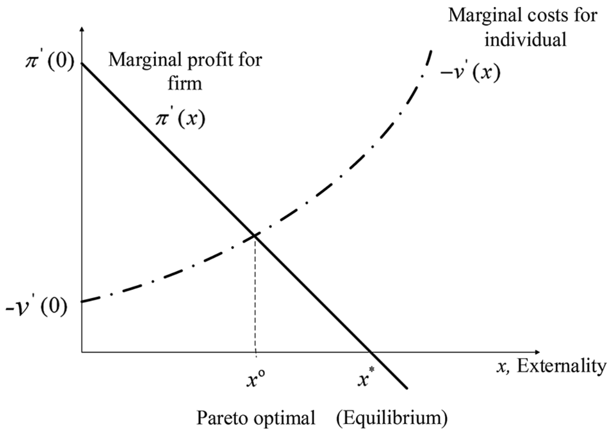

Figure 9.2
Pareto optimal and equilibrium externality level (negative externality)

Example 9.1: ***Equilibrium and socially optimal level of externality*** Consider a firm with profit function as that defined above, $\pi = py - cy^2$, where $y = \alpha x$ denotes that every unit of output generates $\alpha > 0$ units of pollution x. Rearranging $y = \alpha x$ as $y/\alpha = x$ yields a profit function

$$\pi(x) = p\frac{x}{\alpha} - c\left(\frac{x}{\alpha}\right)^2.$$

Differentiating with respect to x, we obtain the marginal profit of increasing pollution

$$\pi'(x) = \frac{p}{\alpha} - \frac{2c}{\alpha^2}x.$$

For compactness, let $a \equiv p/\alpha$ and $a \equiv 2c/\alpha^2$, which helps us express the marginal profit function as

$$\pi'(x) = a - bx \quad \text{where } a, b > 0,$$

which is decreasing in x. However, a consumer is damaged by the negative externality, according to the marginal damage function

$$v'(x) = c + dx \quad \text{where } c, d > 0,$$

which is increasing in x. In this context, if the firm is left unregulated, the competitive equilibrium amount of the externality, x^*, satisfies $\pi'(x^*) = 0$, that is, $a - bx^* = 0$, or

$x^*=a/b$. If, however, the social planner chooses the social optimal level of the externality-generating activity, x^0, that solves $\pi'(x^0)=-v'(x^0)$, we obtain

$$a-bx^0=c+dx^0, \quad \text{or } x^0=\frac{a-c}{b+d},$$

which is positive as long as the standard assumption $\pi'(0)>-v'(0)$ holds, where x is evaluated at the origin $x=0$, given $a>c$. ■

If, in contrast, the activities of the firm produce a *positive* externality in the individual's wellbeing, then $v'(x)>0$ and $-v'(x)<0$, as indicated in figure 9.3, where $-v'(x)<0$ lies in the negative quadrant, while the marginal profit function $\pi'(x)$ is unaffected by the type of externality. In this setting the equilibrium level of externality, x^*, satisfies $x^*<x^0$, and there is an underproduction of the externality-generating activity relative to the Pareto optimal amount, as figure 9.4 illustrates.[4]

Negative externalities are not necessarily eliminated at the Pareto optimal solution. Indeed this would only occur at the extreme case in which the externality-generating activity produces a sufficiently high damage for the affected individual such that the first unit of externality satisfies $-v'(0)>\pi'(0)$ at the vertical intercept of figure 9.3. In this setting curves $\pi'(x)$ and $-v'(x)$ do not cross, and the Pareto optimal solution only occurs at the corner where $x^0=0$. Intuitively, the marginal damage of the first unit of pollution is so high for the individual that they do offset the marginal profit that the firm would obtain from producing such a unit.

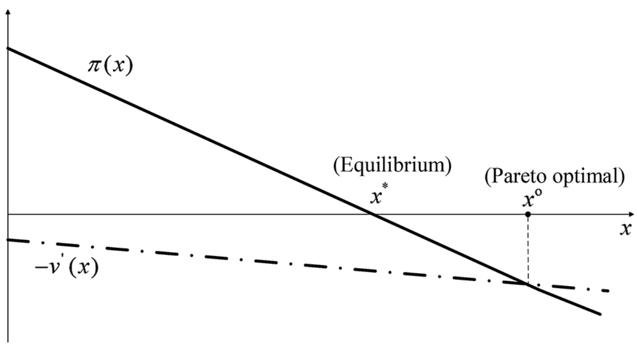

Figure 9.3
Pareto optimal and equilibrium externality level (positive externality)

4. Note that $-v'(x)$ could decrease faster than $-\pi'(x)$, implying that they do not cross, and a corner solution for the social planner's problem arises. In particular, $x^0=+\infty$, since the marginal benefits that the individual obtains from the positive externality are larger than those the firm captures from larger amounts of x. This case emerges, for instance, when $|-v''(x)|>|\pi''(x)|$.

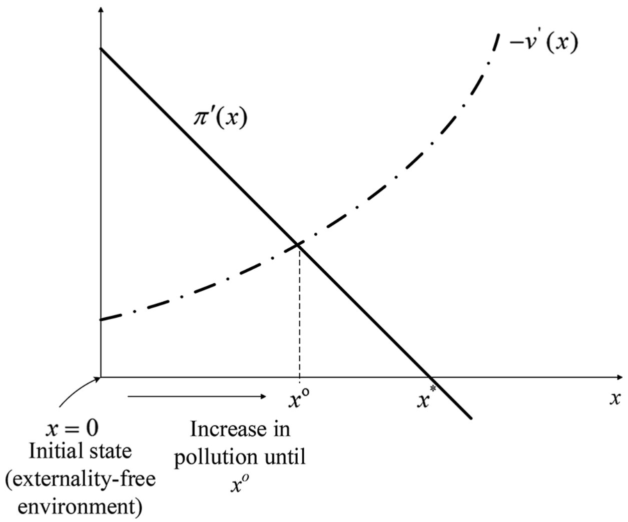

Figure 9.4
Coase Theorem with property rights assigned to the affected individual.

Example 9.2: *Positive externalities* Consider two neighboring countries, $i,j=\{1,2\}$ where $j\neq i$, simultaneously choosing how many resources (in hours) to spend in recycling activities, r_i. The net benefit from spending r_i dollars on recycling is

$$\pi_i(r_i,r_j)=\left(a-r_i+\frac{r_j}{2}\right)r_i-br_i,$$

where $a,b>0$, and b denotes the marginal cost of recycling (in terms of the opportunity cost if resources dedicated to recycling were directed to other public projects). Country i's average benefit, $(a-r_i+(r_j/2))$ is increasing in the resources that neighbor country j spends on recycling, r_j, because a clean environment produces positive external effects on the other country.

Let us first find the competitive equilibrium allocation. Taking the first-order conditions with respect to r_i and solving for r_i yields country i's best-response function (BRF_i):

$$r_i(r_j)=\frac{r_j}{4}+\frac{a-b}{2}.$$

Symmetrically, country j's best-response function is

$$r_j(r_i)=\frac{r_i}{4}+\frac{a-b}{2}.$$

Importantly, the fact that both countries' best-response functions are positively sloped indicates that countries' recycling activities are strategic complements, since an increase in r_j induces country i to strategically increase its own level of recycling, r_i, by ¼. Solving simultaneously for both best-response functions yields

$$r_1 = \frac{(r_1/4)+((a-b)/2)}{4} + \frac{a-b}{2},$$

and after rearranging, we obtain an equilibrium level of recycling of $r_1^* = \tfrac{2}{3}(a-b)$ for country 1, and similarly for country 2. If, instead, a social planner simultaneously selects r_i and r_j in order to maximize social welfare, he would solve

$$\max_{r_i, r_j} \left(a - r_i + \frac{r_j}{2}\right) r_i - br_i + \left(a - r_j + \frac{r_i}{2}\right) r_j - br_j.$$

Taking first-order conditions with respect to r_i yields

$$a - 2r_i + \frac{r_j}{2} - b + \frac{r_j}{2} = 0,$$

and with respect to r_j yields

$$a - 2r_j + \frac{r_i}{2} - b + \frac{r_i}{2} = 0.$$

Simultaneously solving for r_i and r_j in the first-order conditions above, we obtain socially optimal levels of recycling, $r_i^0 = a - b$ for every country i, that are larger than the competitive equilibrium level of recycling, r_i^*, since $r_i^0 = a - b > \tfrac{2}{3}(a-b) = r_1^*$. ■

9.2 Common Pool Resources

In the middle ages the commons was a meadow that belonged to all farmers of a community (commoners) and every farmer could decide how many cows to graze on the commons. Since every farmer only considered his private benefits and costs when bringing cows to the commons, but ignored the external effect that his cows imposed on other farmers (less graze available), the commons were overexploited. This problem has been recurrent in other common pool resources (CPRs), such as fishing grounds, forests, and aquifers simultaneously exploited by several firms. In this section we analyze equilibrium behavior in CPRs and compare it with the social optimum level of exploitation.

9.2.1 Tragedy of the Commons

Consider a setting in which each firm exploiting the CPR independently chooses its individual appropriation level $x_i \in \mathbb{R}_+$ (e.g., its exploitation of the resource). In particular, every firm i's profit function is

$$\pi_i(x_i, X_{-i}) = p(X)x_i - c(x_i, X_{-i}),$$

where $p(X)$ denotes the inverse demand function, where $X \equiv \sum_{i=1}^{N} x_i$, $p'(X) < 0$ and $p''(X) \geq 0$, and where $c(x_i, X_{-i})$ represents firm i's costs of appropriating x_i units of the resource when its rivals extract $X_{-i} \equiv \sum_{j \neq i} x_j$. Costs are increasing and convex in firm i's own appropriation, $\partial c(\cdot)/\partial x_i > 0$ and $\partial^2 c(\cdot)/\partial x_i^2 \geq 0$, and marginal costs increase in the appropriation from any other firm $j \neq i$, $\partial^2 c(\cdot)/\partial x_i \partial x_y > 0$, implying that the CPR becomes more scarce and more difficult to exploit by firm i. For instance, fisherman i will need to spend more time looking for fish when the appropriation by other fishermen, X_{-i}, grows.

Equilibrium Every firm i simultaneously and independently chooses its appropriation level x_i to maximize its profit $\pi_i(x_i, X_{-i})$. Taking FOC with respect to x_i, and assuming an interior solution, we obtain

$$p(X^*) + p'(X^*)x_i^* = \frac{\partial c(x_i^*, X_{-i}^*)}{\partial x_i}, \tag{1}$$

where $X^* = x_i^* + X_{-i}^*$. Intuitively, the marginal revenues and marginal costs from a larger individual approximation x_i must offset each other in equilibrium. Solving for x_i in the FOC above, we find firm i's best-response function $x_i^*(X_{-i})$, describing firm i's appropriation as a function of its rivals' X_{-i}.

Example 9.3: *Equilibrium behavior in the commons* Consider a commons with linear inverse demand $p(X) = a - bX$, where $a, b > 0$, and cost function $c(x_i, X_{-i}) = cx_i(1 + \alpha X_{-i})$, where $c > 0$, and where $\alpha \geq 0$ represents how other firms' appropriation, X_{-i}, affects firm i's costs. That is, when $\alpha = 0$ costs reduce to cx_i, but otherwise they increase in α. In this setting the FOC for equilibrium appropriation becomes

$$a - b(x_i + X_{-i}) + (-b)x_i = c(1 + \alpha X_{-i}).$$

Solving for firm i's appropriation x_i yields a best-response function of

$$x_i^*(X_{-i}) = \frac{a-c}{2b} - \frac{b + c\alpha}{2b} X_{-i},$$

which is decreasing in other firms' appropriation X_{-i}, thus reflecting that appropriation of firms i and j are strategic substitutes, that is, as firm j increases its appropriation firm i chooses to decrease his own. Note that when $\alpha=0$, the best-response function collapses to $(a-c)/2b-X_{-i}/2$, thus coinciding with the standard best response in the Cournot model when firms face linear inverse demand and constant marginal costs.

In a symmetric equilibrium where $x_i^*=x_j^*$ for all firm $i\neq j$, the preceding best-response function becomes

$$x_i^*=\frac{a-c}{2b}-\frac{b+c\alpha}{2b}(N-1)x_i^*,$$

since $X_{-i}=(N-1)x_i^*$. Solving for x_i^*, we obtain an equilibrium approximation of

$$x_i^*=\frac{a-c}{(N+1)b+c\alpha(N-1)}.$$

which is decreasing in the number of firms N, and in the external effect α. For instance, when $N=2$ this equilibrium appropriation becomes $x_i^*=(a-c)/(3b+c\alpha)$. Finally, note that our results embody the special case in which firms are price takers. This can happen, for instance, if firms sell their product (e.g., fish) at the international market, and thus their sales do not affect market prices. In that context, $p(x)=a$, entailing that $b=0$ in our model above, yielding an equilibrium appropriation level of $x_i^*=(a-c)/(c\alpha(N-1))$. ■

In order to more generally identify conditions under which the best-response function of firm i, $x_i^*(X_{-i})$ decreases in X_{-i}, we next differentiate FOC (1) with respect to x_j,

$$\underbrace{\frac{\partial p(X^*)}{\partial x_j}}_{-}+\underbrace{\frac{\partial p(X^*)}{\partial x_i \partial x_j}\cdot x_i}_{+}-\underbrace{\frac{\partial c(x_i^*,X_{-i}^*)^2}{\partial x_i \partial x_j}}_{+},$$

which is negative if

$$\frac{\partial p(X^*)}{\partial x_i \partial x_j}x_i<-\frac{\partial p\left(X^*\right)}{\partial x_j}+\frac{\partial c(x_i^*,X_{-i}^*)^2}{\partial x_i \partial x_j}.$$

In the special case in which demand is linear, $p(x)=a-bX$, then $\partial p(X)/\partial x_i=-b$, which implies a cross-partial derivative of zero, namely $\partial p(X^*)/\partial x_i \partial x_j=0$; thus the inequality above unambiguously holds. In addition, if prices are given at the international market, meaning $p(x)=a$, then $\partial p(X^*)/\partial x_i=\partial p(X^*)/\partial x_j=\partial p(X^*)/\partial x_i \partial x_j=0$, and the inequality above collapses to a condition on costs,

$$0 < -\frac{\partial c(x_i, X_{-i})^2}{\partial x_i \partial x_j},$$

which holds by definition.

Social Optimum Let us next show that equilibrium appropriation levels (those solving FOC for every firm i) are socially excessive. In particular, if all firms maximize their joint profits, their maximization problem becomes

$$\max_{x_1, x_2, \ldots, x_N} \; p(X) \cdot X - \sum_{i=1}^{N} c(x_i, X_{-i}),$$

since $\sum_{i=1}^{N} p(X) x_i = p(X) \cdot X$. Taking FOCs with respect to x_i for every firm i, and assuming interior solutions, yields

$$p(X^{SO}) + p'(X^{SO}) \cdot x_i^{SO} + p'(X^{SO}) \cdot \sum_{j \neq i} x_j^{SO} = \frac{\partial c(x_i^{SO}, X_{-i}^{SO})}{\partial x_i} + \sum_{j \neq i} \frac{\partial c(x_j^{SO}, X_{-j}^{SO})}{\partial x_i} \qquad (2)$$

for every firm i and $j \neq i$. This FOC differs from that of equilibrium behavior in two ways. First, it considers aggregate (rather than individual) marginal revenue on the left-hand side because firms now internalize the effect that selling more units has on the revenues of all other firms (see third term on the left-hand side) rather than on their own revenues alone. Second, it includes the increase in marginal costs that other firms experience as a result of a larger approximation by firm i (i.e., a negative externality in costs). Writing down the difference between the FOC (1) at equilibrium appropriation levels and the FOC (2) at the social optimum, we find that for all x_i,

$$\left[p(X) + p'(X) x_i - \frac{\partial c(x_i, X_{-i})}{\partial x_i} \right]$$
$$- \left[p(X) + p'(X) x_i + p'(X) \sum_{j \neq i} x_j - \frac{\partial c(x_i, X_{-i})}{\partial x_i} - \sum_{j \neq i} \frac{\partial c(x_j, X_{-j})}{\partial x_i} \right],$$

which simplifies to

$$-p'(X) \sum_{j \neq i} x_j + \sum_{j \neq i} \frac{\partial c(x_j, X_{-j})}{\partial x_i}.$$

Since $p'(X) \leq 0$ by definition (i.e., by the law of demand), both terms in this expression are positive, entailing that equilibrium appropriation is socially excessive, that is, $x_i^* > x_i^{SO}$ for every firm i. Such as result is often referred to as the "tragedy of the commons," and indicates that individual incentives lead each firm (e.g., fisherman) to exploit the CPR above its socially optimal level.

Importantly, the expression above is positive, and thus $x_i^* > x_i^{SO}$, in two extreme cases that help us illustrate the two types of external effects present in this model:

1. The ranking $x_i^* > x_i^{SO}$ holds when firm i's appropriation does not increase its rivals' costs, which entails $\partial c(x_j, X_{-j}) / \partial x_i = 0$, but demand is negatively sloped $p'(X) < 0$. In this case, the joint profit-maximization problem presented above simplifies to the profit-maximization problem of a standard cartel (as described in chapter 8), whereby firms collude to reduce their production (appropriation in this context) and thus increase profits. (In other words, in the equilibrium appropriation x_i^* every firm i does not internalize the negative effect that its additional appropriation imposes on firm j's profits via a lower market price; thus creating a pecuniary externality. Such overexploitation is corrected at the social optimum.)
2. The ranking $x_i^* > x_i^{SO}$ also holds when firms take prices as given, $p'(X) = 0$, but every firm i's appropriation increases its rivals' costs, that is, $\partial c(x_j, X_{-j}) / \partial x_i > 0$. This may occur if, for instance, firms sell their appropriation (e.g., fish) at the international market where many other firms sell the same product.[5] In this case the inverse demand function collapses to a (exogenous) price p, and the result $x_i^{SO} < x_i^*$ now indicates that the social optimum internalizes the external effect that each firm's appropriation generates on its rival's costs.

Finally, both external effects are present if $p'(X) < 0$ and $\partial c(x_j, X_{-j}) / \partial x_i > 0$. The following example illustrates these effects, and compares equilibrium and optimal appropriation.

Example 9.4: *Social optimum in the commons* Let us now return to our example with linear demand $p(X) < a - bX$ and cost function $c(x_i, X_{-i}) = cx_i(1 + \alpha X_{-i})$, in order to find the social optimal appropriation, and how it differs from the equilibrium appropriation identified in example 9.3. For simplicity, consider $N = 2$ firms. In this context, the joint profit-maximization problem becomes

$$\max_{x_i, x_j} \; \left[a - b\left(x_i + x_j\right)\right] \cdot \left(x_i + x_j\right) - cx_i\left(1 + \alpha x_j\right) - cx_j\left(1 + \alpha x_i\right).$$

Taking FOC with respect to every x_i yields

5. In the case of fisheries, the two firms that exploit a specific commons sell their fish at the international market where firms exploiting other commons offer the same type of fish.

$$a-2b(x_i+x_j)-c(1+2\alpha x_j)\le 0 \quad \text{for all } i=\{1,2\} \text{ and } j\neq i\,.$$

Assuming an interior solution and solving for x_i, we obtain

$$x_i=\frac{a-c}{2b}-\frac{2(b+c\alpha)}{2b}x_j.$$

Since we find a symmetric expression for x_j, we can simultaneously solve for x_i and x_j to obtain the socially optimal appropriation levels

$$x_i^{SO}=x_j^{SO}=\frac{a-c}{2(2b+c\alpha)},$$

which are lower than equilibrium appropriation x_i^*, since $(a-c)/2(2b+c\alpha)<(a-c)/(3b+c\alpha)$ simplifies to $-c\alpha<b$, which holds given that $c,b>0$ and $\alpha\ge 0$ by definition. For a numerical example, consider parameter values $a=b=1$, $c=1/2$, and an external effect on costs of $\alpha=1/4$. In this setting, equilibrium appropriation becomes $x_i^*=0.16$, while optimal appropriation is only $x_i^{SO}=2/17\cong 0.11$.

Interestingly, $x_i^{SO}<x_i^*$ holds in the two extreme cases described above: (1) when external effects are absent from the cost function, $\alpha=0$, but the inverse demand is negatively sloped, $b>0$, since $(a-c)/4b<(a-c)/3b$; and (2) when $\alpha>0$ but firms take market prices as given, in that $b=0$, which yields a (exogenous) price of $p=a$, and appropriation levels of $x_i^{SO}=(a-c)/2c\alpha$ and $x_i^*=(a-c)/c\alpha$, where $x_i^{SO}<x_i^*$. ■

9.2.2 Tragedy of the Commons and the Market Structure

Our presentation of the "tragedy of the commons" analyzes equilibrium exploitation levels and shows that they are socially excessive given the externality that each firm imposes on its rivals. While in subsequent sections we explore policy tools that help firms internalize such external effects for a given number of firms, we now examine an alternative policy tool: limiting the number of firms to those that yield the highest social welfare. We will then first analyze equilibrium behavior

For simplicity, assume that firms face a market demand function $p(X)=1-X$, where as usual X denotes aggregate output. Each firm i has a convex cost function, $c(x_i)=\theta\cdot x_i^2$, where $\theta\ge 1$.

Equilibrium Every firm i chooses its output level x_i to solve the problem

$$\max\ (1-x_i-X_{-i})x_i-\theta x_i^2,$$

where $X_{-i} = \sum_{j \neq i} x_j$ represents the aggregate output of all firms but i. Taking first-order conditions with respect to x_i yields

$$1 - 2(1+\theta)x_i - X_{-i} = 0,$$

entailing a best-response function

$$x_i(X_{-i}) = \frac{1}{2(1+\theta)} - \frac{1}{2(1+\theta)} X_{-i}.$$

Intuitively, if firm i is the only firm exploiting the commons, it produces $x_i(0) = 1/2(1+\theta)$, but its output decreases as the aggregate output of other firms, X_{-i}, increases. In a symmetric equilibrium $x_i^* = x_j^* = x^*$, yielding

$$x^* = \frac{1}{2(1+\theta)} - \frac{N-1}{2(1+\theta)} x^*,$$

which entails an equilibrium output of

$$x^* = \frac{1}{N+1+2\theta}$$

and equilibrium profits of

$$\pi^* = \left(1 - x^* - (N-1)x^*\right)x^* - \theta(x^*)^2$$
$$= \frac{(1+\theta)}{(N+1+2\theta)^2},$$

which decrease in N but only approach zero when $N \to \infty$. Hence, if entry is endogenous, every firm i enters until the point in which its profits are zero, $\pi^* = 0$, leading to an infinitely large number of firms, each earning zero economic profits.

Joint Profits This outcome is, however, not the number of firms that maximizes joint profits. In particular, aggregate profits are

$$\Pi^* = N\pi^* = \frac{N(1+\theta)}{(N+1+2\theta)^2},$$

which reach their maximum when

$$\frac{\partial \Pi^*}{\partial N} = \frac{(1+\theta)(1-N+2\theta)}{(1+N+2\theta)^3} = 0.$$

Solving for N yields $N^* = 1 + 2\theta$. That is, if firms could limit the number of entrants, they would choose a CPR with $N^* = 1 + 2\theta$ firms.

Social Optimum Social welfare is given by

$$SW = CS + \Pi - ED,$$

where CS denotes consumer surplus, Π represents aggregate profits, and ED indicates environmental damage generated from exploiting the CPR (e.g., exploitation of the commons reduces biodiversity). Specifically, since the inverse demand function is linear, consumer surplus becomes$CS^* = (X^*)^2/2$. Given that $X^* = Nx^* = N/(N+1+2\theta)$, we obtain a consumer surplus of $CS^* = N^2 = 2(N+1+2\theta)^2$. As found above, aggregate profits are $\Pi^* = N(1+\theta)/(N+1+2\theta)^2$. Finally, we consider a convex environmental damage function $ED = dX^2$, which reflects that the marginal damage of extraction, $2dX$, increases as the resources becomes more depleted. As a consequence environmental damage evaluated at the equilibrium aggregate output X^* is

$$ED^* = d\left(X^*\right)^2 = d\frac{N^2}{(N+1+2\theta)^2},$$

which results in a social welfare of

$$SW^* = CS^* + \Pi^* - ED^* = \frac{N^2}{2(N+1+2\theta)^2} + \frac{N(1+\theta)}{(N+1+2\theta)^2} - d\frac{N^2}{(N+1+2\theta)^2}.$$

Note that consumer surplus increases in the number of firms N,

$$\frac{\partial CS^*}{\partial N} = \frac{2\theta N + N}{(N+1+2\theta)^3} > 0,$$

while the difference $\Pi^* - ED^*$ decreases in N,

$$\frac{\partial[\Pi^* - ED^*]}{\partial N} = -\frac{2(2d\theta N + \theta + dN + 1)}{(N+1+2\theta)^3} < 0.$$

Intuitively, increasing the number of firms, N, increases consumer surplus (as a larger output entails lower prices) but decreases industry profits and generates more environmental damages. Overall, the first (positive) effect coincides with the second and third (negative) effects when $\partial SW/\partial N = 0$, which occurs when

$$\frac{2\theta N + N}{(N+1+2\theta)^3} = \frac{2(2d\theta N + \theta + dN + 1)}{(N+1+2\theta)^3},$$

or, solving for N, we find $\bar{N} = 2(1+\theta)/(1-2d+2\theta-4d\theta)$. Comparing our cutoffs, we obtain that $\bar{N} < N^*$ holds for all $d > (4\theta^2+2\theta-1)/2(1+2\theta)^2 \equiv \bar{d}$. In words, when environmental damage is relatively strong, the number of firms that a social planner would choose to maximize social welfare is lower than the number of firms the industry would choose to maximize aggregate profits (which, in turn, are both lower than the number of firms entering in equilibrium). For instance, at $\theta=1$ cutoff $\bar{d}$ becomes $\bar{d} = 5/18$, and increases in θ approaching $1/2$ when $\theta \to +\infty$.

Further reading For more references on the extensive literature analyzing the tragedy of the commons, both from a theoretical and empirical approach, see the textbook by Ostrom (1990) and the survey article by Faysee (2005).

9.3 Solutions to the Externality Problem

Let us examine some of the approaches to solve the externality problem, starting from the least invasive approach (i.e., allowing the agents producing and affected by the externality to voluntarily bargain), and continuing with more direct policy interventions (quotas, taxes, and tradable permits).

9.3.1 Bargaining over the Externality and the Coase Theorem

The success of this system depends on the assignment of property rights. The result is that, as long as property rights are clearly assigned, the two parties will negotiate in such a way that the optimal level of the externality-producing activity is implemented (known as the Coase theorem, after Coase 1960). Unlike the previous solutions such as quotas, taxes or subsidies, bargaining does not imply government intervention.

Property Rights Assigned to Agent 2 Let us first assume that we assign property rights to the individual suffering the negative externality so that at the initial state no externality is generated, $x = 0$. We refer to this state as the “externality-free” environment. In this context, the firm must pay the affected individual if it seeks to increase the externality over zero. In particular, consider that the affected individual makes a take-it-or-leave-it offer where the firm pays T dollars in exchange of x units of pollutionin order to be allowed by the affected individual to produce pollution. Specifically, the firm agrees to pay \$$T$ to the affected individual (in order to pollute x units) if and only if

$$\pi(x) + w_1 - T \geq \underbrace{\pi(0)}_{\text{Current state}} + w_1 \Leftrightarrow \pi(x) - T \geq \pi(0),$$

where w_1 denotes agent 1's wealth. Given this constraint on the set of acceptable offers, the affected individual will choose the pair (x, T) in order to solve the problem

$$\max_{x\geq 0, T} \; v(x)+w_2+T$$

subject to $\pi(x)-T\geq \pi(0)$.

Note that the constraint of the UMP is binding (holding with equality), since the affected individual will raise the fixed fee $\$T$ that he charges to the firm until the point where the firm is made indifferent between accepting and rejecting such offer. That is, $\pi(x)-T=\pi(0)$ or $\pi(x)-\pi(0)=T$. Plugging this result into the affected individual's UMP, we obtain

$$\max_{x\geq 0} \; v(x)+w_2+\pi(x)-\pi(0).$$

Taking first-order conditions with respect to x (the only choice variable now) yields

$$v'(x)+\pi'(x)\leq 0 \in \;\Leftrightarrow\; \pi'(x)\leq -v'(x).$$

Importantly, this first-order condition coincides with that solving the social planner's problem that we described in previous sections of this chapter. Therefore bargaining allows for the level of the externality x to reach the optimal level $x = x^0$. Figure 9.4 illustrates this result. In particular, starting from an initial state where $x = 0$ (externality free environment at the origin of the figure), the result above shows that the polluter is willing to pay $\$T$ to the affected individual in order to increase pollution from $x = 0$ to $x = x^0$ (rightward movement in the figure).[6]

Property Rights Assigned to the Firm What happens if, instead, the property rights are assigned to the polluter? First note that, if there is no bargaining between the firm and the affected individual, the firm would pollute until it exhausts the marginal benefits from the externality, $\pi'(x^*)=0$ at $x = x^*$. However, the affected individual can pay $\$T$ to the firm in exchange of a lower level of pollution, x, where x is reduced from x^*. Note that the polluter is willing to accept this offer if and only if

6. Note that the polluter does not have incentives to raise pollution from x^0 meaning to $x^0+\varepsilon$, where $\varepsilon>0$, since the payment he would make to the affected individual (in order to compensate him for his marginal costs), $-v'(x^0+\varepsilon)$, lies above the marginal benefit the polluter obtains from additional units of the externality, $\pi'(x^0+\varepsilon)$.

$$\pi(x) + w_1 + T \geq \underbrace{\pi(x^*)}_{\text{Current state}} + w_1,$$

$$\text{or } \pi(x) + T \geq \pi(x^*).$$

Plugging this constraint into the affected individual's UMP, his maximization problem becomes

$$\max_{x \geq 0, T} \; v(x) + w_2 - T$$

$$\text{subject to } \pi(x) + T \geq \pi(x^*).$$

(Note that fee $\$T$ now enters negatively into the affected individual's utility, since he pays it, but positively into the firm's, since it receives this fee; as opposed to the case in which property rights were assigned to the affected individual.) As in our previous discussion, the affected individual reduces the fee T that he provides to the firm in order to reduce pollution until the point where the firm is indifferent between accepting and rejecting the offer T. That is, the constraint in the above UMP holds with equality, $\pi(x) + T = \pi(x^*)$ or $T = \pi(x^*) - \pi(x)$. Inserting this result into the affected individual's UMP, we obtain

$$\max_{x \geq 0} \; v(x) + w_2 - \pi(x^*) + \pi(x),$$

and after taking first-order conditions with respect to x, we have

$$v'(x) + \pi'(x) \leq 0 \iff \pi'(x) \leq -v'(x),$$

which, again, coincides with the first-order conditions at the optimal level of the externality (social planner's problem) where $x = x^0$. Figure 9.5 depicts the voluntary reduction of the externality associated to the bargaining process. Specifically, starting from an initial situation where $x = x^*$, the affected individual pays $\$T$ to the firm in order to reduce pollution from $x = x^*$ to $x = x^0$ (leftward move in the figure).[7]

Hence, regardless of the initial assignment of property rights over the externality-generating activity (independently on whether the firm or the affected individual have the property rights over the resource), agents can negotiate to increase or decrease

7. Note that the affected individual is not willing to reduce pollution below x^0, meaning to $x^0 - \varepsilon$, since he would have to compensate the firm for his relatively high marginal benefits, i.e., large $\pi'(x^0 - \varepsilon)$. Because the affected individual's marginal cost of additional units of pollution lies below the firm's marginal benefits from such pollution, $v'(x^0 - \varepsilon) < \pi'(x^0 - \varepsilon)$, the affected individual is thus not willing to further reduce pollution below x^0.

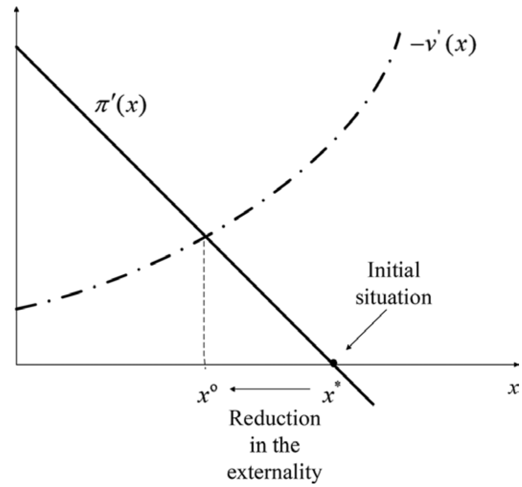

Figure 9.5
Coase theorem with property right assigned to the firm

the externality level x, until reaching the Pareto optimal level x^0. This result is usually referred to as the Coase theorem, and we present it below.

Coase theorem If bargaining between the agents generating and affected by the externality is possible, and if transaction costs are zero, the initial allocation of property rights does not affect the level of the externality. In particular, bargaining helps set the level of the externality at the optimal level $x = x^0$.

While the result is efficient regardless of the initial assignment of property rights, the final wealth of each agent depends on such assignment:

1. If property rights are assigned to the individual affected by the externality, the firm must pay $T = \pi(x^0) - \pi(0)$ to the affected individual, in order to increase the externality from $x = 0$ to $x = x^0$. Hence, if property rights are allocated to the affected individual, his utility level is

$$v(x^0) + T, \quad \text{or} \quad v(x^0) + \pi(x^0) - \pi(0),$$

while that of the firm is

$$\pi(x^0) - T, \quad \text{or} \quad \pi(x^0) - (\pi(x^0) - \pi(0)).$$

Hence the utility of the agent who has the property rights (the affected individual) is larger than that of the agent who does not hold these property rights (the firm) if

$$v\left(x^{0}\right)+\pi\left(x^{0}\right)-\pi(0)>\pi(0) \Leftrightarrow \pi\left(x^{0}\right)+v\left(x^{0}\right)>2\pi\left(x^{0}\right).$$

2. If, instead, property rights are assigned to the firm (the individual producing the externality-generating activity, i.e., the polluter), the affected individual must pay $T=\pi(x^*)-\pi(x^0)$ to the firm in order to reduce his externality, from $x=x^*$ to $x=x^0$. Therefore, if property rights are allocated to the firm, its utility level is

$$\pi\left(x^{0}\right)+T, \quad \text{or } \pi\left(x^{0}\right)+\pi\left(x^{*}\right)-\pi\left(x^{0}\right)=\pi\left(x^{*}\right),$$

while that of the affected individual is

$$v\left(x^{0}\right)-T, \quad \text{or } v\left(x^{0}\right)-\left(\pi\left(x^{*}\right)-\pi(x^{0})\right).$$

Hence the utility of the individual who owns the property right (the firm) is larger than that of the individual who does not (the affected individual) if

$$\pi\left(x^{*}\right)>v\left(x^{0}\right)-\pi\left(x^{*}\right)+\pi\left(x^{0}\right), \quad \text{or } \; 2\pi\left(x^{*}\right)>\pi\left(x^{0}\right)+v\left(x^{0}\right).$$

Therefore, combining the inequalities found in the two scenarios above, we obtain that the agent with the bargaining power has a total utility higher than the agent without the bargaining power if

$$2\pi\left(x^{*}\right)>\pi\left(x^{0}\right)+v\left(x^{0}\right)>2\pi(0),$$

where $\pi(x^0)+v(x^0)$ can be intuitively understood as the aggregate welfare that this society enjoys when consuming the Pareto optimal level of externality x^0.

Let us finally emphasize some of the advantages and disadvantages of bargaining between the affected parties as a solution to the externality problem. The main disadvantage of the Coase theorem is its assumption about transaction costs to be zero between the negotiating parties. Second, it requires perfectly defined property rights. In several settings, agents might not know who holds the property rights of a polluted resource; thus they cannot easily identify who they should bargain with. In these cases the externality problem might never be solved with the voluntary bargaining mechanism proposed by the Coase theorem. Third, property rights must be perfectly enforced, in that the level of x must be perfectly observable and measurable by both parties so that agents can check if the other party is complying with the terms of their agreement. This might be technologically feasible for some types of externalities but not others, especially when several polluters might be responsible for the externality. Nonetheless, in settings where property rights are well defined and enforceable (and negotiation costs are sufficiently low), the Coase theorem presents an important advantage over other solutions to the externality problem, such as taxes, subsidies, and quotas. In particular, only the parties involved must know the marginal benefits and costs associated to the externality, which is to say, the regulator does not need to know anything! However, note that this assumption is also relatively strong, since, under this setting, the polluter must know the cost that the externality imposes on the affected consumers, and similarly consumers must know by how much the profits of the firm increase as a result of higher emissions, namely the polluter's marginal profit function.[8]

Further reading For a more detailed discussion on the criticisms of the Coase theorem from the law and economics perspective, see Medena and Zerbe (1999), and for its connections with cooperative game theory, see Starrett (2003). Finally, note that the Coase theorem has be recurrently tested in controlled laboratory experiments, but their findings do not generally coincide with the equilibrium predictions and report the presence of endowment effects; see Hoffman and Spitzer (1982, 1985), Harrison and McKee (1985), and Kahneman et al. (1990).

9.3.2 More Intrusive Approaches to Correct Externalities

Quotas If the social planner is perfectly informed about the benefits and damages of the externality for all consumers, he can choose to set a quota (emission standard) banning production levels higher than the Pareto optimal level x^0.

Pigouvian Taxation[9] This policy sets a tax t_x per unit of the externality-generating activity x. What is the level of tax t_x that restores efficiency? In order to answer this question, let us start by re-writing the firm's PMP including the tax, as follows:

$$\max_{x \geq 0} \ \pi(x) - t_x \cdot x.$$

8. Note that, if the two parties are firms (e.g., a fishery and a refinery), a form of bargaining could be the sale of one firm to the other, creating a merger. This would imply the selection of a Pareto efficient level of the externality, since the now merged firm would internalize the effects that pollution imposes on the production process of the fishery. A seminal article showing this result is Davis and Whinston (1962), which argues that the merger between two firms that produce an externality to one another (what they refer as a "technological externality") can be welfare superior to other forms of government intervention.

9. For additional references about this policy tool, see chapter 12 in Koldstad's (2011) textbook on environmental economics.

Taking first-order conditions with respect to x, we obtain

$$\pi'(x) - t_x \leq 0 \Rightarrow \pi'(x) \leq t_x,$$

which, in the case of interior solutions, implies $\pi'(x) = t_x$. Intuitively, this result entails that the agent producing the externality-generating activity increases x until the point where the marginal benefits from an additional unit of x coincide with the associated cost he must bear, namely the per-unit tax t_x. Since we know that at the optimal level, x^0, we must have $\pi'(x^0) = -v'(x^0)$, the tax t_x needs to be set at $t_x = -v'(x^0)$, which is positive since $v'(x) < 0$ by definition. Intuitively, tax $t_x = -v'(x^0)$ forces the firm to internalize the negative externality that his production generates on consumer's well-being at x^0. This tax leads the firm to choose a level of x equal to x^0, implementing the social optimum, as figure 9.6 illustrates.

Graphically, the tax produces a downward shift in the curve representing the firm's net marginal benefit from additional units of pollution, $\pi'(x)$, as figure 9.7 depicts. This allows for the new curve of marginal benefits to exactly cross the horizontal axis at x^0, indicating that, after the tax is imposed, the firm voluntarily chooses a level of x that coincides with the Pareto optimal level x^0.

Intuitively, note that the optimality-restoring tax t_x is equal to the marginal externality that the firm causes on the consumer's well-being at the optimal level x^0. That is, it is equal to the amount of money that the consumer would be willing to pay to the firm in order to reduce x slightly from its optimal level x^0. As suggested above, the tax t_x induces the firm to internalize the externality that it causes on the consumer. These types of optimality-restoring taxes are often referred to as Pigouvian taxes, after Pigou

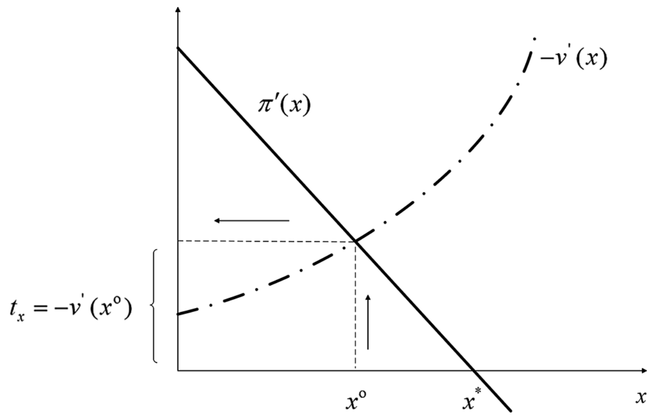

Figure 9.6
Tax on the externality generating individual

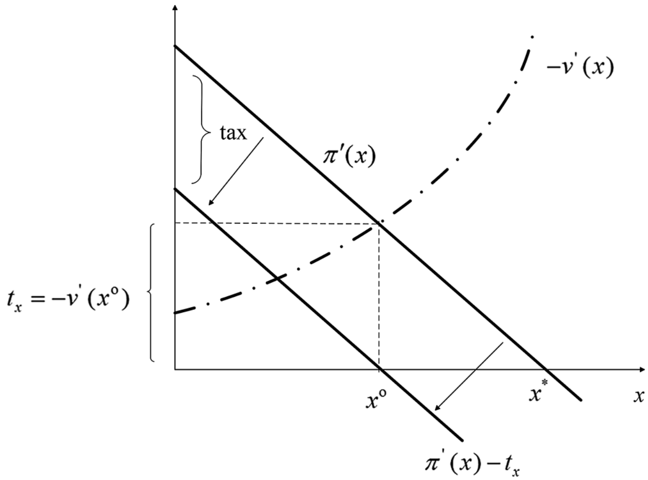

Figure 9.7
Effect of a Pigouvian tax.

(1920). Finally, in the event that such negative externality is very substantial (and the socially optimal externality level would recommend $x^0 = 0$), we would need to impose a tax $t_x = \pi'(0)$, as illustrated in figure 9.8.[10]

All our previous discussion can also be extended to positive externalities by similarly setting a tax $t_x = -v'(x^0)$. However, since now $v'(x^0) > 0$ (given that x increases consumer's welfare), the optimality-correcting tax $t_x = -v'(x^0) < 0$ becomes in fact a subsidy to the firm for every unit of the positive externality x that it generates. Graphically, this per-unit subsidy produces an upward shift in the curve representing the marginal benefits that the firm obtains from the externality-generating activity, as figure 9.9 represents. This implies that the firm has incentives to increase x beyond the competitive equilibrium level x^* until reaching the Pareto optimal level x^0.

Let us highlight a few important points about Pigouvian taxation:

1. A tax t_x on the negative externality is equivalent to a subsidy inducing agents to reduce the externality until the Pareto optimal level x^0. In particular, consider that the social planner sets a subsidy $s_x = -v'(x^0) > 0$ for every unit that the firm's pollution, x, is below the equilibrium level of x^*. Hence, the firm's PMP becomes

10. Note that a tax slightly lower than $t_x = -v'(0)$would also induce the firm to reduce the externality-generating activity to zero, whereby $x^* = 0$. In particular, a tax $t_x = -\pi'(0) < -v'(0)$would cause the firm to voluntarily choose $x^* = 0$.

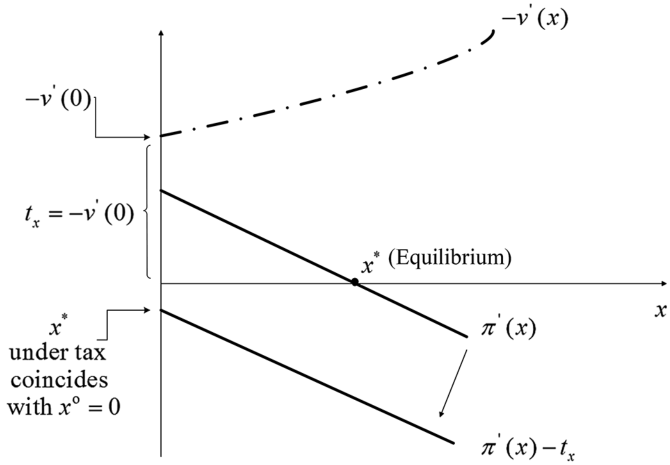

Figure 9.8
Effect of a Pigouvian tax that seeks to induce $x^0 = 0$

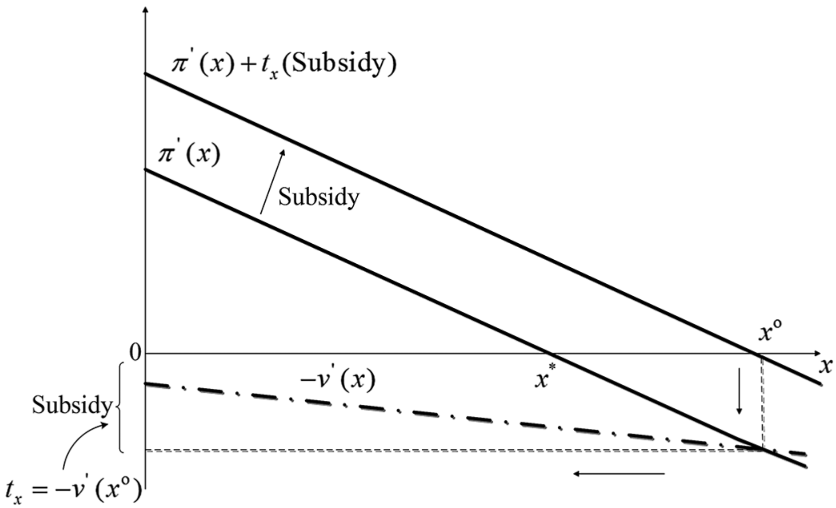

Figure 9.9
Effect of a Pigouvian subsidy on positive externality

$\max_{x \geq 0} \pi(x) + s_x(x^* - x)$. Taking first-order conditions with respect to x, we obtain $\pi'(x^0) - s_x \leq 0$, meaning $\pi'(x^0) \leq s_x$. Importantly, this first-order condition coincides with that under the Pigouvian taxation described above (taxing the negative externality at a rate t_x), plus a lump-sum tax of $t_x x^*$. Hence a subsidy for the reduction of the externality (combined with a lump-sum tax $t_x x^*$) can exactly replicate the outcome of the Pigouvian tax. This implies that, if all tax collection from the Pigouvian tax is afterward distributed among the affected agents as a lump-sum transfer, they will still have incentives to behave as prescribed by the Pigouvian tax. Intuitively, the social planner only needs to alter the relative cost of the externality for the agents, even if their total wealth is unaffected.

2. The Pigouvian tax levies a tax on the externality-generating activity (e.g., pollution) but not on the output that generated such pollution. In this sense, the externality-generating activity is *directly* taxed. If instead,output was taxed, the firm would reduce output which does not necessarily guarantees a reduction in pollutant emissions. This result was initially shown by Weitzman (1974), and we will explore it at the end of this chapter.[11]
3. The quota and the Pigouvian tax are equally effective under complete information, whereby the social planner has accurate information about all agents' benefit and cost functions. This might not be the case if governments lack relevant information about the benefits and costs of the externality for consumers and firms. We explore regulation under incomplete information at the end of the chapter.

Tradable Externality Permits Regulators might instead use permits to solve the externality problem. Every permit grants the right to generate one unit of the externality. Suppose that the regulator chooses a number of total permits equal to the socially optimal aggregate externality, x^0, that is, $x^0 = \sum_j x_j^0$. In particular, every firm receives $\overline{x}_j$ permits.[12] Additionally assume that there is a sufficiently large number of firms that

11. See Weitzman (1974). There is, however, one exemption: if emissions maintain a fixed monotonic relationship with the level of output, then every unit of output generates a constant proportion (e.g., α units) of emissions. In such a case, emissions could be measured by simply observing output, and a Pigouvian tax on output would induce the firm to reduce output (and emissions) to its optimal level. Therefore imposing a direct tax on emissions or an indirect tax on output would yield the same results in terms of total pollution.

12. The particular procedure by which externality permits are assigned to firms is not explicitly described here, but it could be done, for instance, according to every firm's history of emissions, or using an auction. Note that assigning pollution permits according to every firm's emission history

they can regard the price of externality permits as given (i.e., they are price takers in their purchases of permits). Specifically, let p_x* denote the equilibrium price of these permits. Therefore every firm j's PMP now becomes

$$\max_{x \geq 0} \ \pi_j(x_j) + p_x^*(\bar{x}_j - x_j),$$

where firm j must pay a price p_x* for every permit it needs to buy in excess of its initial endowment $\bar{x}_j$.[13] Taking first-order conditions with respect to x_j, we obtain[14]

$$\pi'_j(x_j) - p_x^* \leq 0,$$

or $\pi'_j(x_j) = p_x^*$ in the case of interior solutions, $x_j > 0$. If all J firms carry out this PMP, and the market clearing condition $x^0 = \sum_j x_j$ holds, we can restore efficiency by setting a price per permit of $p_x^* = -\sum_{i=1}^{I} v_i'(x^0)$. Indeed, setting this price, we modify firm j's first-order condition as

$$\pi'_j(h_j^0) + \sum_{i=1}^{I} v_i'(x^0) \leq 0, \quad \text{with equality if } x_j > 0,$$

where $x^0 = \sum_j x_j^0$ or $\pi'_j(x_j^0) = -\sum_{i=1}^{I} v_i'(x^0)$ in the case of interior solutions, $x_j^0 > 0$. Importantly, this condition exactly coincides with the first-order condition that solves the social planner's problem. Therefore every firm j is induced to voluntarily choose an optimal externality level, $x_j = x_j^0$.

Interestingly, the advantage of tradable externality permits, relative to other policy instruments such as quotas or taxes, is that government officials do not need access to so much information. In particular, they only need data about the optimal level of pollution, x^0. This implies having information about aggregate firms' profits (industry profits) and on consumers' damage from the externality in aggregate terms, but not necessarily about profit functions of every individual firm or the damage function of

might induce firms to strategically increase pollution before the pollution permit program is implemented. An assignment using auctions partially solves this problem; see Montero (2008). For a discussion of different methods to assign emission permits, see Kolstad (2011).

13. Note that if the firm *sells* permits (because the firm does not need to use its initial endowment of $\bar{x}_j$ permits, i.e., $\bar{x}_j > x_j$), its profits increase, while if the firm needs to *buy* permits (beyond its initial endowment of $\bar{x}_j$, i.e., $\bar{x}_j < x_j$), its profits decrease.

14. Note that second-order conditions are also satisfied since the profit function is concave in x_j, $\pi''_j(x_j) < 0$, by definition.

every individual consumer. Once the aggregate amount of externality permits x^0 is determined by the regulator, the market for pollution permits assigns them efficiently. That is, the amount of permits finally owned by firm j (after purchasing or selling permits) leads every firm j to generate an amount of externality, x_j, that coincides with the Pareto optimal level x_j^0.[15]

9.4 Regulating a Polluting Monopolist

Let us now extend our previous analysis of emission fees to a setting in which a monopolist is the only firm emitting pollution. Consider a monopolist facing a linear inverse demand curve $p(x)=1-x$ and constant marginal production costs $c<1$. Assume that the monopolist's production generates an environmental damage measured by $ED=d(x)^2$, where $d>0$. Intuitively, a marginal increase in output entails a positive and increasing environmental damage, meaning pollution is convex in output. Suppose that the social planner seeks to induce a socially optimal output level, x_{SO}, that maximizes the welfare function

$$W(x)=CS(x)+PS(x)+T-d(x)^2,$$

where $CS(x)\equiv \frac{1}{2}(x)^2$ and $PS(x)\equiv(1-x)x-(c+t)x$ denote consumer and producer surplus, respectively, and $T\equiv tx$ represents tax revenue from the emission fee. Intuitively, this implies that the fee is revenue neutral. We next examine the monopolist's pollution decision in the absence of regulation, then find the socially optimal amount of pollution that a regulator would set, and finally identify the emission fee that induces the monopolist modify its emissions to coincide with the social optimum (i.e., the fee that restores optimality).

9.4.1 Unregulated Equilibrium

Let us first find the monopolist's profit-maximizing production level for a given emission fee t. Hence, for a given emission fee t, the monopolist chooses x to solve

$$\max_x \ (1-x)x-(c+t)x.$$

Taking first-order conditions with respect to x yields

15. This is an active area of research, with models analyzing, for instance, how to design the initial distribution of permits, and the consequences of having a dominant firm in the industry that holds monopsonistic power in their purchases of externality permits, and so on.

$$1-2x-(c+t)=0.$$

Then solving for x, we obtain an output function of $x(t)=(1-(c+t))/2$, which is, of course, decreasing in the emission fee t.

Social Optimum The social planner chooses an output level x that solves

$$\max_x \; CS(x)+PS(x)+T-d(x)^2.$$

Taking first-order conditions with respect to x, we obtain

$$2\frac{1}{2}x+[1-2x-(c+t)]+t-2dx=0,$$

or after rearranging,

$$1-c=x(1+2d),$$

which, after solving for x, yields a socially optimal output $x_{SO}=(1-c)/(1+2d)$.

9.4.2 Restoring the Social Optimum

Once we found the socially optimal output that the regulator seeks to induce, we can find the emission fee t that induces the monopolist to produce x_{SO} by solving

$$\frac{1-(c+t)}{2}=\frac{1-c}{1+2d},$$

that is,

$$t=(2d-1)\frac{1-c}{1+2d},$$

or more compactly, $t=(2d-1)x_{SO}$. Note that the emission fee is positive as long as $d>1/2$. Otherwise, it would be negative (thus becoming a subsidy to the monopolist). Intuitively, this reflects that, when the market failure arising from the environmental externality is sufficiently large, $d>1/2$, it dominates the market failure emerging from the low production of the monopolist; and ultimately the regulator is led to reduce the production of the polluting monopolist. If, in contrast, $d<1/2$, the market failure from the externality is less damaging (in terms of social welfare) than the underproduction of the monopolist, leading the regulator to subsidize the monopolist's production in order to induce it to increase its output level.

9.5 Regulating a Polluting Oligopoly

Let us now extend the previous model to a polluting industry with two firms competing in quantities.[16] Consider a Cournot oligopoly facing a linear inverse demand curve $p(X)=1-X$, where X denotes aggregate output. For generality, let us allow for firms to differ in their constant marginal production costs. Specifically, consider that an incumbent firm can enjoy a cost advantage relative to the firm that recently entered the industry (entrant), that is, $c_{inc}<c_{ent}<1$, or face the same production costs, $c_{inc}=c_{ent}<1$. As in the previous analysis, assume that the oligopoly generates an environmental damage measured by $ED=d(X)^2$, where $d>0$, and so pollution is convex in output. Finally, assume that the social planner seeks to induce a socially optimal output level, x_{SO} that maximizes the welfare function

$$W(X)=CS(X)+PS(X)+T-d(X)^2,$$

where $CS(X)$and $PS(x)$ denote consumer and producer surplus, respectively, and $T\equiv tX$- represents tax revenue from the emission fee. (For simplicity, we assume that $d>1/2$.) As in the previous section, we first analyze equilibrium production when regulation is absent, then we find the social optimum, and finally we evaluate the emission fee that induces firms to voluntarily produce the social optimum.

9.5.1 Unregulated Equilibrium

Let us find the oligopoly's profit-maximizing production level for a given emission fee t. In the case of entry, the incumbent solves

$$\max_{x_{inc}} \ (1-x_{inc}-x_{ent})x_{inc}-(c_{inc}+t_2)x_{inc},$$

while the entrant solves

$$\max_{x_{ent}} \ (1-x_{ent}-x_{inc})x_{ent}-(c_{ent}+t_2)x_{ent}.$$

Taking first-order conditions with respect to x_i, where $i=\{inc, ent\}$ yields

$$1-2x_i-x_j-(c_i+t)=0,$$

16. This section is based on Espinola-Arredondo and Munoz-Garcia (2013a). While this section only considers a complete information setting, the article allows for incomplete information between firms, and subsequently analyzes under which conditions the presence of environmental regulation can help the incumbent firm conceal information about its relative efficiency from potential entrants.

where $j \neq i$. Solving for x_i, we obtain firm $i's$ best-response function

$$x_i(x_j) = \frac{1-(c_i+t)}{2} - \frac{1}{2}x_j,$$

which decreases in firm i's own costs, c_i, in the emission fee, t, and in the production level of its rival, x_j. Plugging firm j's best-response function into firm i's, we find

$$x_i = \frac{1-(c_i+t)}{2} - \frac{1}{2}\left(\frac{1-(c_j+t)}{2} - \frac{1}{2}x_i\right),$$

and solving for x_i yields an equilibrium output function

$$x_i(t) = \frac{1-2c_i+c_j-t}{3}$$

for every firm i, which diminishes in its own cost, c_i, and in the emission fee, t, but increases in its rival's cost, c_j.

Social Optimum The aggregate output level that maximizes social welfare solves

$$\max_X \quad CS(X) + PS(X) + T - d \times X^2,$$

where $X \equiv x_{inc} + x_{ent}$, $CS(X) \equiv \frac{1}{2}(X)^2$, $PS(X) \equiv (1-X)X - (c_{inc}+t_2)X$, and $T \equiv tX$.[17] Taking first-order conditions, we obtain the aggregate socially optimal output

$$X_{SO} = \frac{1-c_{inc}}{1+2d},$$

which coincides with X_{SO} in the previous section where we analyze the case of only one polluting firm.

9.5.2 Restoring the Social Optimum

Let us now find the emission fee t that induces the industry to produce an aggregate output that coincides with the socially optimal output level X_{SO}. In order to find the

17. Note that producer surplus $PS(X)$ considers the incumbent's marginal costs. This is due to the fact that, in order to allocate the production decision of the socially optimal output, a benevolent social planner would produce using the most efficient firm. Specifically, when the incumbent's costs are lower than those of the entrant, all socially optimal output would be produced by this firm; however, when firms' costs coincide, incumbent and entrant are equally efficient, $c_{inc}=c_{ent}$, and hence the socially optimal output can be equally split among the two firms.

emission fee t and the individual output levels that the incumbent and entrant will be induced to produce, $x_{inc,SO}$ and $x_{ent,SO}$ the social planner must simultaneously solve

$$x_{inc,SO} + x_{ent,SO} = \frac{1 - c_{inc}}{1 + 2d} \tag{1}$$

(the sum of incumbent's and entrant's output coincides with the socially optimal output X_{SO}) and

$$x_{inc}(t) = \frac{1 - c_{inc} + c_{ent} - t_2}{3}, \tag{2}$$

$$x_{ent}(t) = \frac{1 - 2c_{ent} + c_{inc} - t_2}{3}. \tag{3}$$

For simplicity, let us next find the solution of this system of equations by first focusing on the case of cost symmetry, namely $c_{\text{inc}} = c_{\text{ent}}$, and then on the case in which the incumbent firm enjoys a cost advantage, $c_{\text{inc}} < c_{\text{ent.}}$

Case 1: *Cost symmetry*, $c_{inc} = c_{ent}$ Simultaneously solving equations (1) to (3) yields the emission fee

$$t = \frac{(4d - 1)}{2} \frac{1 - c_{inc}}{1 + 2d},$$

or

$$t = (4d - 1)\frac{X_{SO}}{2},$$

which is strictly positive if $d > 1/4$. That is, the negative externality from pollution dominates the market failure that arises from the underproduction in oligopoly when $d > 1/4$. Intuitively, this occurs for a larger set of environmental damages than under monopoly (see the previous exercise where emissions were positive if $d > 1/2$) because the market failure that emerges from the underproduction in oligopoly is actually smaller than that under monopoly. As a consequence the regulator imposes emission fees on the oligopoly even in settings in which he would not impose a fee to a monopoly, whereby $d \in [\frac{1}{4}, \frac{1}{2}]$. Substituting this fee t into the output function of every firm i, $x_{\text{i}}(t)$ yields

$$x_{inc}(t) = x_{ent}(t) = \frac{1}{2}\left[\frac{1 - c_{inc}}{1 + 2d}\right] = \frac{X_{SO}}{2}.$$

Case 2: *Cost asymmetry*, $c_{inc} < c_{ent}$ Simultaneously solving equations (1) to (3) when the incumbent's costs are lower than the entrant's yields an emission fee

$$t = \frac{A(1-c_{ent}) - B(1-c_{inc})}{2A},$$

where $A \equiv 1 + 2d$ and $B \equiv 2 - 2d$. Hence the equilibrium output levels evaluated at fee t are

$$x_{inc}(t) = \frac{1 + Ac_{ent} - (2+2d)c_{inc})}{2A} \quad \text{and} \quad x_{ent}(t) = \frac{1 - Ac_{ent} + Bc_{inc}}{2A},$$

which are positive if, $c_{ent} > ((2+2d)c_{inc} - 1)/A$ and $c_{ent} < (1+2dc_{inc})/A$, respectively. Moreover the emission fee $t = (A(1-c_{ent}) - B(1-c_{inc}))/2A$ is positive if $c_{ent} < (4d - 1 + Bc_{inc})/A$.

We now compare the above three conditions for c_{ent}, separately checking which of them are binding. First, note that the condition that guarantees a positive output from the incumbent, $c_{ent} = ((2+2d)c_{inc} - 1)/A$, holds for all $c_{ent} > c_{inc}$, since cutoff $((2+2d)c_{inc} - 1)/A$ originates at the negative quadrant (when $c_{inc} = 0$, the cutoff originates at $-(1/A)$ and reaches $c_{ent} = 1$ when $c_{inc} = 1$). Therefore cutoff $((2+2d)c_{inc} - 1)/A$ lies below the 45-degree line for all values of c_{inc}, as depicted in the lowest line of figure 9.10. Since we only focus on cost pairs above the 45-degree line, which is, $c_{ent} > c_{inc}$, condition $c_{ent} > ((2+2d)c_{inc} - 1)/A$ is hence not binding.

Second, the condition that guarantees that the entrant produces a positive output, $c_{ent} < (1+2dc_{inc})/A$, is more restrictive than the condition implying a positive emission fee, $c_{ent} = (4d - 1 + Bc_{inc})/A$. Indeed both cutoffs reach $c_{ent} = 1$ when $c_{inc} = 1$, but $(1+2dc_{inc})/A$ originates at $1/A$ while $(4d - 1 + Bc_{inc})/A$ originates at a higher vertical intercept $(4d - 1)/A$ since $4d - 1 > 1$ given that $d > 1/2$ by definition (for a graphical illustration of these two cutoffs and their vertical intercepts, see figure 9.11). As a consequence condition $c_{ent} < (4d - 1 + Bc_{inc})/A$ is not binding either.

In summary, only condition $c_{ent} < (1+2dc_{inc})/A$ is binding, and in order to have a positive emission fee that induces positive output levels from both firms, we only need firms' costs to be relatively symmetric, that is,

$$c_{inc} < c_{ent} < \frac{1 + 2dc_{inc}}{A},$$

as indicated in figure 9.10 by the region above the 45-degree line and below the dashed line originating at $1/A$.

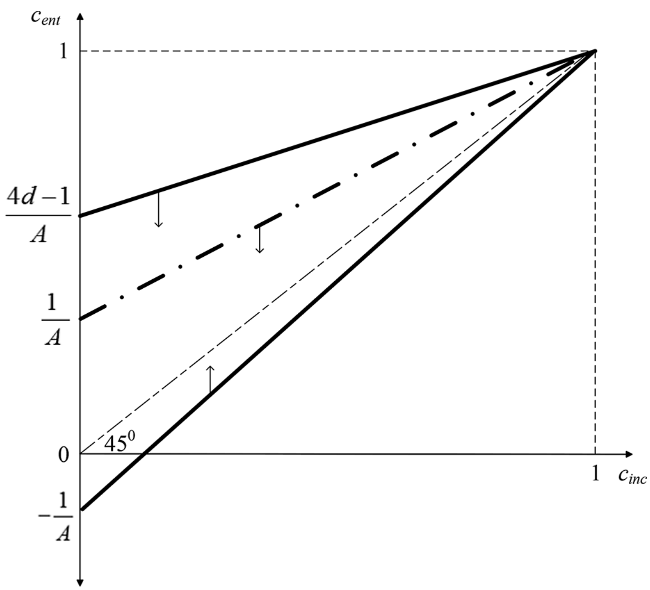

Figure 9.10
Costs that guarantee positive production and fees

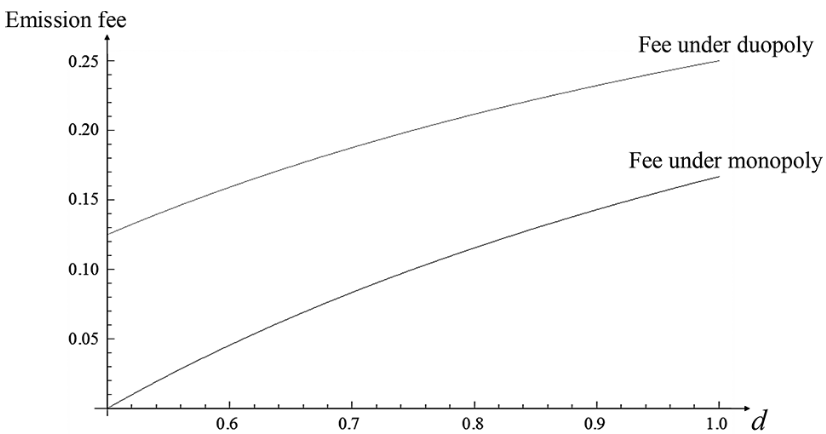

Figure 9.11
Emission fees under different market structures

9.6 Fee Comparison

Relative to the emission fees under monopoly in the previous section, the regulator sets more stringent fees to the duopolists than to the monopolist, as in Buchanan (1969). Indeed, for the case of cost symmetry, $c_{inc} = c_{ent}$,

$$(4d-1)\frac{X_{SO}}{2} > (2d-1)X_{SO} \Leftrightarrow 4d-1 > 4d-2.$$

Figure 9.11 depicts the emission fee to a duopoly market, $(4d-1)(X_{SO}/2)$, and that to a monopoly, $(2d-1)\,X_{SO}$, where $X_{SO}=(1-c_{inc})/(1+2d)$. (For simplicity, figure 9.11 assumes a marginal cost of $c_{inc}=1/2$, but similar results arise for a different marginal cost.) Intuitively, the amount of pollution generated by the unregulated duopoly is larger than that created by the unregulated monopoly, inducing the regulator to set a more stringent fee on the latter. This argument extends to the regulation of oligopolies with more than two polluting firms, whereby environmental policy needs to be even more stringent than in the duopoly we just analyzed in order to curb additional pollution of several firms.

9.6.1 Presence of Asymmetric Information in Externality Problems

In several settings, agents might privately observe their own marginal profits and damage function, while the regulator does not observe this information. This section analyzes such an information context, following the seminal work of Weitzman (1974, 1978). In particular, consider a setting in which firms generate an externality whereas consumers suffer from such an externality. Let $v(x, \eta)$ represent the derived utility that a consumer of type $\eta \in \mathbb{R}$ experiences amount x of the externality, and let $\pi(x, \theta)$ be the derived profit function of a firm of type $\theta \in \mathbb{R}$ that generates an amount x of externality. Consider additionally that parameters η and θ are privately observed by the consumer and firm, respectively. Agents do not observe each other's types but know the ex ante likelihoods (the probability distribution) of η and θ. For simplicity, we consider that parameters η and θ are independently distributed.[18] Finally, as in previous sections, functions $v(x, \eta)$ and $\pi(x, \theta)$ are strictly concave in the externality x for any value of η and θ.

18. Intuitively, under correlated benefits and costs, the firm, after observing whether its marginal benefits from pollution are high or low, can more accurately infer the marginal cost that pollution generates on consumers. In contrast, under independently distributed marginal benefits and costs, agents could not use their private information to infer the other agent's marginal costs or benefits. Stavins (1996) extends this model to a setting in which the types of firms and consumers are correlated.

9.6.2 Bargaining

Let us first consider the decentralized bargaining procedure we examined in previous sections. Specifically, we will show that bargaining in the presence of asymmetric information does not necessarily lead to an efficient level of the externality x^0. (Note that this result differs with that in the context of symmetric information, where x^0 units of the externality were generated, regardless of the initial allocation of property rights.) Suppose that the consumer has the right to an externality-free environment, and he makes a take-it-or-leave-it offer to the firm. For simplicity, assume that there are two levels of the externality: either $x = 0$ or $x = \bar{x}$. Since we are dealing with negative externalities, the consumer prefers $x = 0$ to $x = \bar{x}$, whereas the firm prefers $x = \bar{x}$ to $x = 0$. For compactness, let's measure the benefits that a firm with type θ obtains from having an externality level $x = \bar{x}$ as

$$b(\theta) = \pi(\bar{x}, \theta) - \pi(0, \theta) > 0$$

and similarly let's measure consumer η's costs from having an externality level $x = \bar{x}$, rather than the externality-free environment $x = 0$, as

$$c(\eta) = v(0, \eta) - v(\bar{x}, \eta) > 0.$$

In this setting, the only elements that matter in the negotiation between the consumer and the firm are the precise values of the benefit function $b(\theta)$ and the cost function $c(\eta)$, which we can denote more compactly as b and c, respectively. Before analyzing equilibrium behavior in the bargaining game, let us examine how $b(\theta)$ and $c(\eta)$ are distributed. In particular, they behave according to cumulative distribution functions $G(b)$ and $F(c)$ for b and c, respectively (where these cumulative distribution functions represent the probability distribution of parameters η and θ), with associated density functions $g(b) > 0$ for all $b > 0$ and $f(c) > 0$ for all $c > 0$.

Finally, in the absence of an agreement, the level of the externality remains at $x = 0$, since this is the initial state of the externality given that we assumed the consumer has the property rights over the resource. Moreover note that in any arrangement that guarantees Pareto optimal outcomes, the firm should be allowed to set a level of the externality $x = \bar{x}$ whenever $b > c$. Intuitively, in this setting the firm would be willing to pay the consumer more than the damage that the consumer suffers from the externality and, hence, a positive level of the externality $x = \bar{x}$ would be agreed by a firm and consumer if they were perfectly informed about each other's marginal benefits and costs (i.e., complete information setting).

Let us now start analyzing equilibrium strategies in this context. First, let us analyze what amount should the consumer demand from the firm (a take-it-or-leave-it-offer) when his cost of the externality is exactly $c(\eta) = c$. We know that a θ-type firm will agree to pay \$$T$ if and only if its benefits, $b(\theta) = b$, satisfy $b \geq T$. Hence the consumer knows that if she demands a payment of \$$T$, the probability of the firm accepting is equal to the probability that $b \geq T$, that is, $1 - G(T)$. Figure 9.13 depicts this probability. In particular, the figure represents the range of all possible benefits that the firm enjoys from increasing the externality level from zero to $\bar{x}$ in the horizontal axis, and its cumulative probability on the vertical axis. Therefore, for a given offer \$$T$, the region of benefits lying to the right-hand side of \$$T$ represents that the offer will be accepted, namely $b \geq T$, which occurs with an associated probability $1 - G(T)$. In contrast, the region of benefits to the left-hand side of \$$T$ implies that the offer will be rejected, namely $b < T$, which occurs with an associated probability $G(T)$.

Therefore the consumer chooses the value of the offer \$$T$ that maximizes his expected utility

$$\max_{T \geq 0} [1 - G(T)](T - c),$$

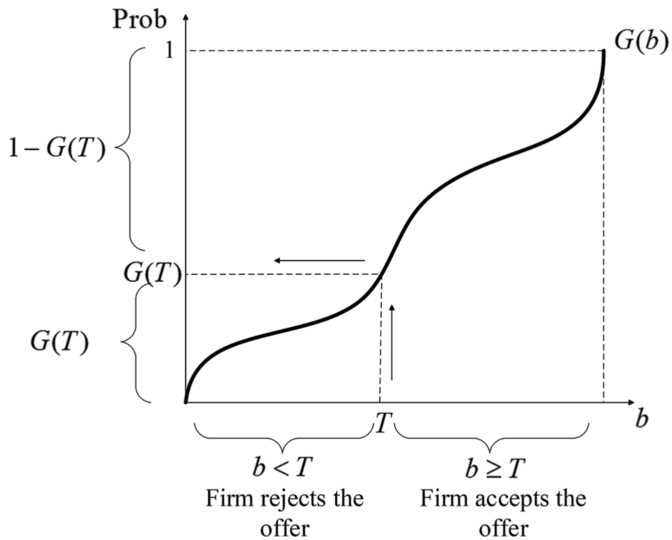

Figure 9.12
Probability distribution of firm benefits, b

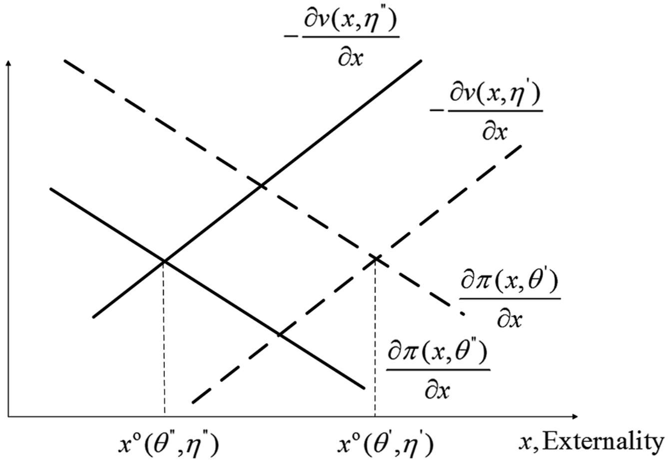

Figure 9.13
Marginal benefits and costs under incomplete information

where $1-G(T)$ represents the probability that an offer of $\$T$ is accepted by the firm, whereas $T-c$ denotes the net gain that a consumer (with cost c) obtains if the offer is accepted.[19] Taking first-order conditions with respect to T, yields

$$\left[1-G\left(T_c^*\right)\right]-g\left(T_c^*\right)\left(T_c^*-\mathrm{c}\right)\leq 0,$$

and in interior solutions, $1-G\left(T_c^*\right)=g\left(T_c^*\right)\left(T_c^*-\mathrm{c}\right)$, or after rearranging,

$$\frac{1-G\left(T_c^*\right)}{g\left(T_c^*\right)}+c=T_c^*.$$

Finally, since the ratio $\left[1-G\left(T_c^*\right)\right]/g\left(T_c^*\right)\neq 0$, we have that $T_c^*>c$. Intuitively, this implies that the firm rejects the consumer's offer since there exists a range of benefits for the firm, b, satisfying $T_c^*>b>c$. This is, however, an inefficient outcome. Indeed, if $b>c$, Pareto optimality requires that the externality is increased until $x=\bar{x}$, but in this setting the consumer's offer is rejected with positive probability for all benefits b between T_c^* and c, which is to say, $T_c^*>b>c$. Consequently, while the firm and consumers would be willing to bargain and have the externality produced (implying a

19. Note that if the offer is rejected, which occurs with probability $G(T)$, the consumer obtains a zero payoff, since his payoff when both parties do not reach an agreement is zero. If the disagreement payoff is different from zero, such as $\$D$, then the expected utility maximization problem becomes $[1-G(T)](T-c)+G(T)D$.

welfare improvement for both parties) when they are perfectly informed about their benefits and costs, the lack of information hinders the success of this mutually beneficial agreement.

9.7 Setting Quotas under Incomplete Information

The previous result shows that decentralized bargaining does not necessarily yield efficient outcomes if agents are not perfectly informed about each other's types. Natural candidates to restore efficiency are quotas or taxes, which we demonstrated to induce Pareto optimal allocations under complete information contexts. In settings where agents are asymmetrically informed, however, we will show that these policy tools do not necessarily achieve efficient outcomes. Moreover, unlike complete information settings, quotas or taxes are not perfectly substitutable between one another when agents are asymmetrically informed.

First, note that for given parameters θ and η, the aggregate surplus resulting from externality level x is

$$v(x,\eta)+\pi(x,\theta).$$

That is, the Pareto optimal level of the externality, denoted as $x(\eta,\theta)$, must be a function of the realizations of parameter θ and η, and solves

$$\max_{x\geq 0}\ v(x,\eta)+\pi(x,\theta).$$

Taking first-order conditions with respect to x, we obtain $\partial v(x,\eta)/\partial x+\partial\pi(x,\theta)/\partial x\leq 0$ or, at an interior optimum,

$$\frac{\partial v(x,\eta)}{\partial x}=-\frac{\partial \pi(x,\theta)}{\partial x}.$$

Figure 9.13 graphically represents these first-order conditions, where the firm's marginal profit function and the consumer's marginal damage function are evaluated at two different pairs of parameters θ and η: (θ', η'), as depicted with dashed lines, and (θ'', η''), represented in solid lines. In particular, when parameters take the realization (θ', η'), the Pareto optimal level of the externality is $x^0(\theta',\eta')$, whereas when their realization is (θ'', η'') efficiency is attained when the externality reaches a level $x^0(\theta'',\eta'')$, where $x^0(\theta'',\eta'')<x^0(\theta',\eta')$.

Suppose now that a quota is fixed at the level of the externality $x=\hat{x}$. Then the firm's PMP becomes

$$\max_{x \geq 0} \ \pi(x, \theta)$$

$$\text{subject to} \quad x \leq \hat{x}.$$

Now let $x^q(\hat{x}, \theta)$ denote the externality level that solves this PMP. Since the firm's PMP does not depend on η, the firm's choice of the externality level x is completely insensitive to η. This result implies that the level of the externality cannot be efficient, since, for efficiency to emerge, we need the externality level $x^0(\theta, \eta)$, which is a function of both parameters θ and η. Moreover, if the level of the quota $\hat{x}$ is such that $\partial\pi(\hat{x}, \theta)/\partial x$ for all $\theta > 0$, then the profit-maximizing level of the externality is $x^q(\hat{x}, \theta) = \hat{x}$.[20] That is, the firm would like to increase the externality x beyond $\hat{x}$, but it cannot because it has already reached the quota. We can formally measure the welfare loss caused by the imposition of a quota that, rather than using the actual realization of parameters θ and η, and thus inducing the socially optimal level of externality $x^0(\theta, \eta)$, uses the average of these parameters $\bar{\theta}$ and $\bar{\eta}$, as an approximation of their true value.

$$\underbrace{\left[v\left(x^q(\hat{x}, \theta), \eta\right) + \pi\left(x^q(\hat{x}, \theta), \theta\right)\right]}_{\text{Aggregate surplus with the quota } \hat{x}} - \underbrace{\left[v\left(x^0(\theta, \eta), \eta\right) + \pi\left(x^0(\theta, \eta), \theta\right)\right]}_{\text{Aggregate surplus at the PO}},$$

where note that, with the quota, the firm's profit function and the consumer's damage function are both evaluated at the profit-maximizing level of the externality $x^q(\hat{x}, \theta)$ that solves the firm's PMP when its type is θ and the firm is subject to a quota $\hat{x}$. We can express the difference as

$$= \int_{x^o(\theta,\eta)}^{x^q(\hat{x},\theta)} \left(\frac{\partial \pi(x, \theta)}{\partial x} + \frac{\partial v(x, \eta)}{\partial x} \right) dx.$$

Graphically, this integral represents the area below the marginal profit function $\partial\pi(x, \theta)/\partial x$and above the marginal damage function $-\partial v(x, \theta)/\partial x$, between the quota $\hat{x}$ (set by an uninformed regulator, who considers the average value of parameters θ and η, $\bar{\theta}$, and $\bar{\eta}$, respectively) and the optimal value of the externality $x^o(\theta, \eta)$ (which only a perfectly informed regulator would be able to identify).

Figure 9.14 measures the welfare loss that the uninformed regulator generates when imposing a quota $\hat{x}$ based on the averages $\bar{\theta}$ and $\bar{\eta}$. In particular, $\partial\pi(x, \theta)/\partial x$ represents

20. Note that condition $\partial\pi(\hat{x}, \theta)/\partial x > 0$ graphically implies that the firm's marginal profit function, despite increasing in the externality, has not yet crossed the horizontal axis at the quota $\hat{x}$. Hence the quota is smaller than the level of externality that the firm would select in an unregulated context.

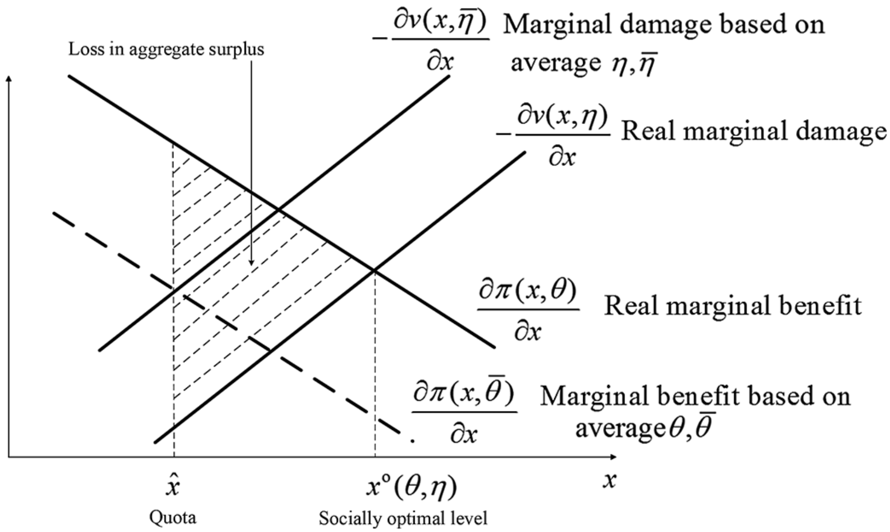

Figure 9.14
Welfare loss of a quota set under incomplete information

the firm's marginal profit function given the true realization of parameter θ, whereas $\partial\pi(x,\overline{\theta})/\partial x$ reflects the marginal profit function based on the average value $\overline{\theta}$. Similarly $\partial v(x,\eta)/\partial x$ denotes the marginal damage function for a consumer whose type is η, while $\partial v(x,\overline{\eta})/\partial x$ represents the marginal damage function based on the average value of η, $\overline{\eta}$. Given the average value of parameters θ and η, the regulator uses the crossing point between $\partial\pi(x,\overline{\theta})/\partial x$ and $\partial v(x,\overline{\eta})/\partial x$ in order to determine the level of the quota $\hat{x}$. Such level of the externality, however, cannot be optimal, since at $\hat{x}$ the firm's real marginal profit from additional units of pollution is larger than the consumer's real marginal damage function, thus allowing both parties to negotiate a further increase in the level of the externality (i.e., the firm is willing to compensate the consumer) and producing a welfare improvement. The extent of the inefficiency arising from the imposition of a quota $\hat{x}$ that differs from the socially optimal level, $x^{o}(\theta,\eta)$, is captured by the shaded area in figure 9.14.

9.8 Setting Emission Fees under Incomplete Information

Let us now consider the possibility that the regulator imposes a tax t per unit of the externality. Then the firm's PMP becomes

$$\max_{x\geq 0} \ \pi(x,\theta) - tx,$$

and taking first-order conditions with respect to x, we obtain $\partial\pi(x,\theta)/\partial x \leq t$. In the case of interior solutions, let $x^t(t,\theta)$ denote the amount of the externality that solves the first-order condition $\partial\pi(x,\theta)/\partial x = t$.[21] We can now measure the welfare loss caused by the imposition of a tax that, rather than considering the actual realization of parameters θ and η, uses the average of these parameters as an approximation of their true value. In particular, the loss in aggregate surplus arising from the tax is given by

$$\underbrace{\left[v\left(x^t(t,\theta),\eta\right)+\pi\left(x^t(t,\theta),\theta\right)\right]}_{\text{Aggregate surplus with tax } t}-\underbrace{\left[v\left(x^0(\theta,\eta),\eta\right)+\pi\left(x^0(\theta,\eta),\theta\right)\right]}_{\text{Aggregate surplus at the PO}},$$

or, more compactly,

$$=\int_{x^o(\theta,\eta)}^{x^t(t,\theta)}\left(\frac{\partial\pi(x,\theta)}{\partial x}+\frac{\partial v(x,\eta)}{\partial x}\right)dx.$$

As we did for the case of the quota, we can graphically represent this welfare loss using figure 9.15. In particular, note that now the tax must be set at the point that maximizes aggregate surplus, evaluated at the average value of θ and η, $(\bar{\theta},\bar{\eta})$, that is,

$$t=-\frac{\partial v\left(x^o(\bar{\theta},\bar{\eta}),\bar{\eta}\right)}{\partial x}.$$

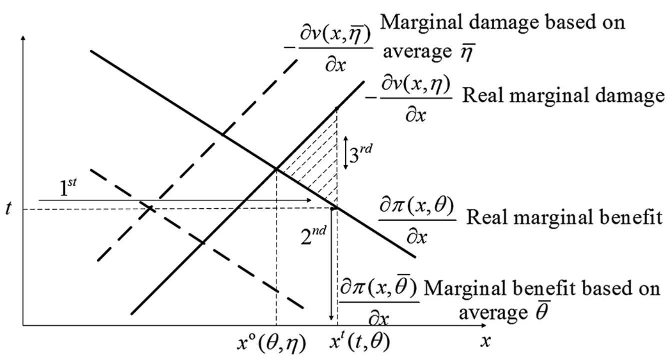

Figure 9.15
Welfare loss from setting an emission fee under incomplete information

21. As in the case of a quota, the profit-maximizing level of the externality under an emission fee, $x^t(t,\theta)$depends on the specific value of the emission fee t and the realization of parameter θ, but it is independent on the realization of parameter measuring consumer's sensitivity to the externality, η.

In figure 9.15, the socially optimal level that an uninformed regulator would seek to promote, $x^0(\bar{\theta},\bar{\eta})$, guarantees that the average marginal profit function crosses the average marginal damage function. Hence this externality level $x^0(\bar{\theta},\bar{\eta})$ can be plugged into the average marginal damage function, $-\partial v(x^0(\bar{\theta},\bar{\eta}),\bar{\eta})/\partial x$, in order to obtain the size of the tax t that induces the firm to produce exactly $x^0(\bar{\theta},\bar{\eta})$. Such a tax is graphically represented as the height of the average marginal damage function when evaluated at $x^0(\bar{\theta},\bar{\eta})$. This emission fee, however, induces the firm (whose actual marginal profit function is $\partial\pi(x,\theta)/\partial x$)) to respond, choosing a level of externality given by $x^t(t,\theta)$, since the firm increases x until the point where $\partial\pi(x,\theta)/\partial x = t$. (This is graphically represented by steps 1 and 2 in figure 9.15.) This level of the externality is, however, not optimal, since at $x^t(t,\theta)$ the marginal damage of pollution exceeds its marginal profit. So there is still room for the two parties generating and being affected by the externality to negotiate and reduce the amount of the externality from $x^t(t,\theta)$ to $x^0(\theta,\eta)$. Graphically, the welfare loss from the imposition of fee t is represented by the shaded area.

9.9 Comparing Policy Instruments under Incomplete Information

Given that both quotas and emission fees create inefficiencies in settings of incomplete information, a natural question is which instrument, despite being imperfect, performs better. We hence search for a "second-best" policy, since the uninformed regulator cannot achieve a policy that exactly induces socially optimal amounts of the externality. As we show next, the answer depends on the elasticity of the marginal damage and marginal profit functions.

Let us first consider a setting where the realization of parameter θ is $\theta = \theta_1$. For simplicity, let us assume that the regulator has relatively precise information about the marginal damage function, but he is uncertain about the firm's marginal profit function, that is, he could not observe the precise realization of parameter θ. Figure 9.16 illustrates this case, where the regulator uses the average marginal profit function $\partial\pi(x,\bar{\theta})/\partial x$. As described in previous sections, if the regulator sets a quota $\hat{x}$ at which the (observed) marginal damage function crosses the (average) marginal profit function, this policy yields a deadweight loss, graphically represented by the area below the marginal damage function and above the true marginal profit function (for a range of x between the Pareto optimal level of the externality $x^o(\theta_1,\eta)$ and the quota $\hat{x}$). If instead the regulator sets an emission fee t (at the height at which the true marginal damage function crosses the average marginal profit function), the firm responds by selecting an externality level of $x^t(t,\theta_1)$, below the social optimum, entailing an inefficiency

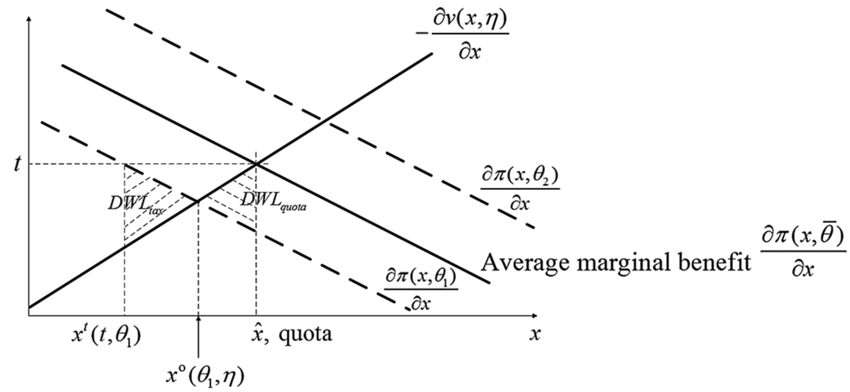

Figure 9.16
Welfare comparison between quota and tax—I

measured by the shaded deadweight loss in the left-hand side of figure 9.16. In this case the deadweight loss generated from the imposition of a tax is larger than that associated with a quota, implying that the quota (despite creating inefficiencies) performs better that the tax.

In the case we just described, the marginal damage function was relatively sensitive to the level of the externality. Intuitively, pollution was very damaging for consumers. As a consequence the marginal damage function was rapidly increasing in x. Let us next analyze what happens if instead the marginal damage function is not very sensitive to x. Figure 9.17 illustrates this case, where the true realization of parameter θ is still $\theta = \theta_1$. Both policy instruments still generate inefficiencies, measured by the shaded deadweight losses, but the deadweight loss associated with the quota is now significantly larger than that of the tax. Hence, in this setting, the tax (despite creating inefficiencies) performs better than the quota.

Summarizing, for a given elasticity of the marginal profit function at the socially optimal level of the externality, the quota (emission fee) performs better when the marginal damage function is relatively inelastic (elastic, respectively).

9.10 Pollution Abatement

Let us now analyze emission fees that induce firms to reduce their emissions in the least costly method (i.e., using end-of-pipe technologies, redesigning their production process, or just reducing their output). In particular, we first examine optional tax setting in contexts where the environmental damage of every unit of pollution is

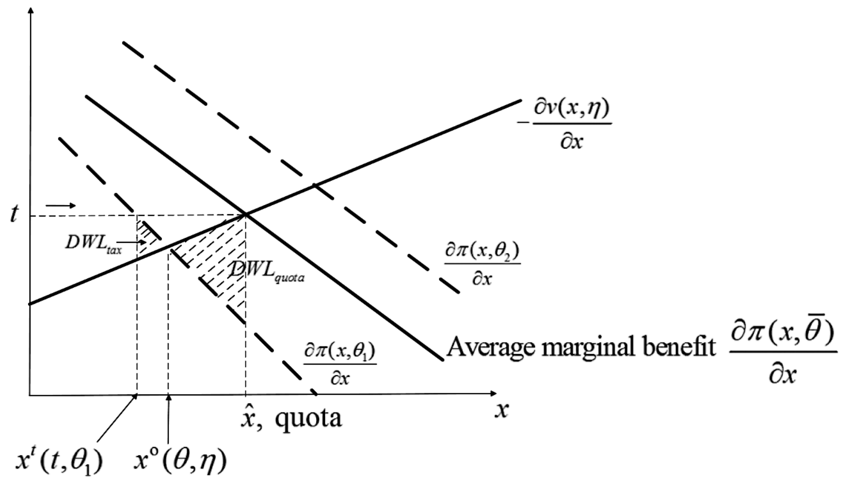

Figure 9.17
Welfare comparison between quota and tax—II

uniformly distributed (i.e., similar across firms), and subsequently explore how our results are affected when such environmental damage is not uniformly distributed.

9.10.1 Uniform Pollutants

Consider an environmental regulator seeking to limit total pollution to a maximum level x^0, so $x^0 \geq \sum_j x_j$. Firms' production functions are given by $y_j(p,w)$ where, as usual, p indicates the price of output while w represents input prices (potentially a vector). Firm j's abatement efforts in order to limit their emissions to a given level x_j are captured by the abatement function $a_j(y_j, V_j)=x_j$, where V_j denotes the use of abatement inputs, each with a price p_V. Firm j can reach a target emission level x_j producing few units of output and using few abatement inputs (i.e., low y_j and V_j) or producing a large amount of output but using large amounts of abatement inputs (i.e., high y_j and V). That is, for a given x_j, depicting the level curve $a_j(y_j, V_j)=x_j$ in the (y_j, x_j) quadrant yields a positively sloped level curve.

If a social planner had the ability to choose the use production and abatement inputs across firms, that is, $(z_1, \dots z_y)$ and $(V_1, \dots, V)$, he would solve

$$\min \sum_j (w * z_j + p_V * V_j)$$

subject to $\quad y_j(p,w) = \overline{y}_j,$

$$\sum_j a_j(\bar{y}_j, V_j) \leq x^0 \text{, and}$$

$x_j \geq 0$ for every firm j.

Intuitively, the social planner seeks to minimize the total cost of inputs (used in either production or abatement) for all J firms in the economy, subject to (1) a given output $\bar{y}_j$ being reached per firm, (2) each firm using its abatement function $a_j(\bar{y}_j, V)$ in order to reach $\bar{y}_j$ units of output, and (3) total pollution not exceeding x^0. (Note that the problem could alternatively be approached as maximizing aggregate profits $\sum_j (p\bar{y}_j - wz_j - p_v V_j)$ as the price vector is given.)

The Lagrangian of this constrained maximization problem is

$$\mathcal{L} = \sum_j (wz_j - p_V V_j) + \sum_j \lambda_j [\bar{y}_j - y_j(p, w)] + \mu \left[\sum_j a_j(\bar{y}_j, V_j) - x^0 \right].$$

Taking first-order conditions with respect to z_j yields

$$w = \lambda_j \frac{\partial y_j(p, w)}{\partial w}$$

for every firm j. Taking first-order conditions with respect to V_j, we obtain

$$p_V = -\mu \frac{\partial a_j(\bar{y}_j, V_j)}{\partial w}$$

for every firm j. That is, production input z_j should be increased until the cost of an additional unit, w, coincides with its marginal benefit in terms of additional production. Similarly abatement input V_j should be increased until its costs, p_V, coincide with its marginal benefit (as a facilitator of abatement efforts). Importantly, while the first condition also converged in settings without externalities (i.e., standard production theory), the second condition arises now because the planner seeks firms to abate pollution (hiring V_j inputs) until the second condition is satisfied. In particular, for a profit maximizing firm to voluntarily select V_j at the socially optimal level, we need to set an emission fee on pollution, t_j^*, that coincides with μ. In order to explicitly show this result, consider the cost-minimizing problem of firm j

$$\min_{z_j, v_j} \; wz_j + p_V V_j + t_j x_j$$

subject to $y_j(p, w) = \bar{y}_j$, and $a_j(\bar{y}_j, V_j) = x_j$, which allows us to rewrite the program as

$$\min_{z_j, v_j} \; wz_j + p_V V_j + t_j a_j(\bar{y}_j, V_j)$$

subject to $y_j(p,w) = \bar{y}_j$.

Hence the Lagrangian for j is

$$\mathcal{L} = wz_j - p_V V_j + t_j a_j(\bar{y}_j, V_j) + \theta_j[\bar{y}_j - y_j(p,w)].$$

Taking first-order conditions with respect to z_j yields a similar condition as the one we found for the social planner,

$$w = \theta_j \frac{\partial y_j(p,w)}{\partial w}$$

while, taking the first-order conditions with respect to V_j, we obtain

$$p_V = -t_j \frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j},$$

which coincides with the first-order condition we found for the social planner as long as $t_j = \mu$, and thus the emission fee on firm j must coincide with the marginal benefit that firm j obtains from dedicating one more input into abatement. In addition, solving for t_j, we find that

$$t_j = -\frac{p_V}{\partial a_j(\bar{y}_j, V_j)/\partial V_j}.$$

Given that, for optimality, we need $t_j = \mu$ to hold for every firm j, we thus require that, for every two firms j and k, $k \neq j$,

$$-\frac{p_V}{\partial a_j(\bar{y}_j, V_j)/\partial V_j} = -\frac{p_V}{\partial a_k(\bar{y}_k, V_k)/\partial V_k},$$

or after rearranging $\partial a_k(\bar{y}_k, V_k)/\partial V_k = \partial a_j(\bar{y}_j, V_j)/\partial V_j$, which are their marginal benefits from dedicating more inputs to abatement coincide at the social optimum.

9.10.2 Nonuniform Pollutants

Consider now nonuniform pollution sources, such as rivers, whereby the amount of pollution measured at a particular monitoring station, m_j, not only depends on total pollution, x, but also on how pollution from a different firm k transfers to the station located nearby firm j, as captured by d_{kj}. That is, the measurement at station m_k

can be expressed as $m_k = \sum_j d_{kj} x_j$. If a regulator seeks to limit the measurement in each station k so it does not exceed a cutoff $\overline{m}_k$, namely $m_k = \sum_j d_{kj} x_j \leq \overline{m}_k$, the social planner problem now becomes

$$\min_j \sum_j \left(wz_j + p_v V_j\right)$$

subject to $y_j(p, w) = \overline{y}_j$ and

$$\sum_j d_{kj}\left[a_j\left(\overline{y}_j, V_j\right)\right] \leq \overline{m}_k,$$

since $a_j\left(\overline{y}_j, V_j\right) = x_j$ by definition. Therefore the Lagrangian to this program is

$$\mathcal{L} = \sum_j \left(wz_j - p_V V_j\right) + \sum_j \left[\overline{y}_j - y_j(p, w)\right] + \sum_k \mu_k \left(\sum_j d_{kj} a_j\left(\overline{y}_j, V_j\right) - \overline{m}_k\right).$$

Taking first-order conditions with respect to z_j yields $w = \sum_j \partial y_j(p, w)/\partial w$ for every firm j, as under uniform pollutants. However, taking first-order conditions with respect to the abatement input V_j, we obtain

$$p_V = -\sum_k \mu_k d_{kj} \frac{\partial a_j\left(\overline{y}_j, V_j\right)}{\partial V_j}.$$

For instance, in the case of two firms,

$$p_V = -\mu_1 d_{12} \frac{\partial a_1\left(\overline{y}_1, V_1\right)}{\partial V_1} - \mu_2 d_{22} \frac{\partial a_2\left(\overline{y}_2, V_2\right)}{\partial V_2},$$

which differs from the solution we found under uniform pollution. In order to set an emission fee to firm 1 in this example, t_1, we thus need

$$-t_1 \frac{\partial a_1\left(\overline{y}_1, V_1\right)}{\partial V_1} = -\mu_1 d_{12} \frac{\partial a_1\left(\overline{y}_1, V_1\right)}{\partial V_1} - \mu_2 d_{22} \frac{\partial a_2\left(\overline{y}_2, V_2\right)}{\partial V_2},$$

or, solving for t_1, we have

$$t_1 = -\mu_1 d_{12} + \mu_2 d_{22} \left(\frac{\partial a_2/\partial V_2}{\partial a_1/\partial V_1}\right).$$

One suggestion by Tietenberg (1973) is to set taxes based on the pollution recorded at each monitoring point, m_j, rather than on emission fees. This would imply that every firm j pays taxes at each monitoring point depending on whether its pollution affects points $k \neq j$, as determined by coefficients d_{jk}. That is, at every measurement point k, firm j pays a tax $d_{jk}\mu_j$ per unit of emissions. This solution, however, entails high administrative costs for the regulator.

9.11 Public Goods

A good is a pure public good if, once produced, the cost of excluding users from enjoying it is extremely high (i.e., the good is nonexcludable) and if the utility that existing consumers derive from the good is unaffected if an additional consumer enjoys it (i.e., the good is nonrival). Table 9.1 presents a taxonomy of four different types of goods depending on whether they satisfy either rivalry or excludability.

1. *Private goods* (e.g., an apple) These goods are rival in consumption, since the consumption of the good by one individual reduces the amount available to other individuals, and they are excludable in consumption, since it is easy to exclude an individual who did not pay for the good.
2. *Club goods* (e.g., golf course) These goods are nonrival in consumption, since the consumption of the good by one individual does not reduce the amount available to other individuals,[22] but they are excludable in consumption, since it is easy to exclude an individual who did not pay for the good (e.g., asking for a membership fee).

Table 9.1
Taxonomy of goods

		Player 2	
		C	*NC*
Player 1	*C*	*a, a*	*c, b*
	NC	*b, c*	*d, d*

22. Club goods, however, assume that the number of users is sufficiently low so that no congestion effects emerge, reducing the utility of previous users. Otherwise, they would become rival in consumption, just as regular private goods. This would be applicable, for instance, to a very small gym or golf club that could become easily congested.

3. *Common property resources* (e.g., fishing grounds) These goods are rival in consumption, since the consumption of the good by one individual (e.g., appropriation of fish by a fisherman) reduces the amount of the good available to other individuals (to other fishermen in the same fishing ground), but they are nonexcludable, since the costs of excluding additional vessels from a large area of the sea would be extremely high.
4. *Public goods* (e.g., national defense) These goods are nonrival, since my enjoyment (e.g., of national defense) does not reduce your enjoyment, and they are nonexcludable, since the costs of excluding noncontributing individuals from enjoying the good would be extremely costly.[23]

Consider I consumers, one public good x, and L traded private goods. Every consumer i's marginal utility from the consumption of x units of a public good is $v_i'(x)$, where note that x does not have a subscript because of nonrivalry, meaning the total amount of public good in the economy, x, is enjoyed not only by individual i but also by all other individuals. We consider the case of a public good, where $v_i'(x) > 0$ for every individual i, but a "public bad" would simply imply $v_i'(x) < 0$ for every individual i. In addition, assume that $v_i''(x) < 0$, which represents a positive but decreasing marginal utility from additional units of the public good. Figure 9.18 illustrates the marginal benefit from the public good for individual i.

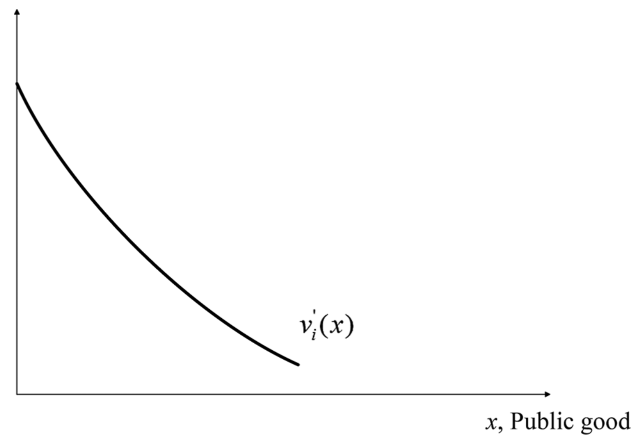

Figure 9.18
Marginal benefit from the public good

23. How would you exclude an individual who did not pay his/her taxes from benefiting of national defense? Exile him out of the country?

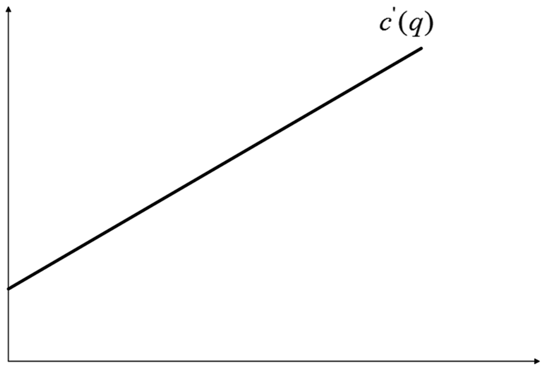

Figure 9.19
Marginal costs from providing the public good

Moreover we assume that the marginal utility from the public good, $v_i'(x)$, is independent on the private good. However, the cost of supplying x units of the public good is $c(x)$, where $c'(x)>0$ and $c''(x)>0$ for all x, which tells us that the costs of providing the public good are increasing and convex in q. Figure 9.20 depicts the marginal cost function.[24]

Let us first find the Pareto optimal allocation. In particular the social planner maximizes aggregate surplus, as follows:

$$\max_{x\geq 0} \sum_{i=1}^{I} v_i(x) - c(x).$$

Taking first-order conditions with respect to x, yields

$$\sum_{i=1}^{I} v_i'(x^o) - c'(x^o) \leq 0, \quad \text{with equality if } x^o > 0,$$

and the second-order conditions are also satisfied, since $\sum_{i=1}^{I} v_i''(x^o) - c''(x^o) \leq 0$. Hence, in the case of an interior solution, the first-order conditions above establish that the optimal level of public good is achieved for the level of x^o that solves

24. Note that if we were describing a public bad, such as pollution, we would need $c'(x)<0$, since reducing x is costly but increasing x is not costly.

$$\sum_{i=1}^{I} v_i'(x^o) = c'(x^o).$$

Intuitively, this condition implies that the social planner should increase the public good until the point in which the sum of the consumers' marginal benefit from an additional unit of the public good (also referred to as the marginal social benefit) is equal to its marginal cost. This condition is commonly known as the Samuelson rule, after Samuelson (1954). Importantly, the Pareto optimal level of public goods does not coincide with that of private goods where, for interior solutions, a benevolent social planner would only increase the amount of the private good until the point in which every individual i's private marginal benefit is equal to the marginal cost producing that good, that is, $v_i'(x_i^*) = c_j'(x_j)$.

Example 9.5: *Discrete public good* Consider a public good with $x = \{0, 1\}$, meaning it is either produced or not. Every individual i has a valuation $v_i(x) = \alpha_i x$ for the (discrete) public good where $\alpha_i \geq 0$ is individual i's value for this good. If the total cost of producing the public good is $C \cdot X$, where $C > 0$, then the Pareto optimal condition found in the previous section requires

$$\sum_{i=1}^{I} v'_i(x) = c,$$

which in this discrete context implies that the public good is produced if $\sum_{i=1}^{I} v'_i(x) > c$, that is, if the aggregate marginal valuation is weakly higher than its marginal cost. (Note that in this case, since the public good is discrete, the interpretation above is equivalent to "if the aggregate valuation is weakly larger than its cost.") ■

9.12 Inefficiency of the Private Provision of Public Goods

Let us next see how the creation of a market in which every individual purchases amounts of the public good does not eliminate the divergence between the Pareto optimal and the equilibrium amount of the public good. In particular, let us consider the case in which a market exists for the public good and that each consumer i chooses how much of the public good to buy, denoted as $x_i \geq 0$ units, taking as given a market price of p. The total amount of the public good purchased by all I individuals is hence[25] $x=\sum_{i=1}^{I} x_i$. Consider a single producer of the public good (e.g., federal government) with a cost function $c(x)$.[26] Formally, for a given competitive equilibrium price p^*, each consumer i's purchase of the public good, x_i^*, must solve his own utility maximization problem

$$\max_{x_i \geq 0} v_i\left(x_i + \sum_{k \neq i} x_k^*\right) + w_i - p^* x_i,$$

where w_i denotes consumer i's wealth. The first term reflects that individual i benefits from both the x_i units of the public good he purchases and the $\sum_{k \neq 1} x_k^*$ units of the public good that all other individuals acquire, since the public good is nonrival by assumption. Hence, when determining his purchases of the public good, individual i takes the purchases of all the other individuals as given, $\sum_{k \neq 1} x_k^*$. In this regard other individuals' purchases are a form of positive externality to individual i. Finally, note that consumer i pays $p^* x_i$ when acquiring x_i units of the public good. Taking first-order conditions with respect to x_i, we obtain

$$v_i'\left(x_i^* + \sum_{k \neq 1} x_k^*\right) - p^* \leq 0, \quad \text{with equality if } x_i^* > 0.$$

For compactness, let x^* denote the total purchases of public goods by all individuals, whereby $x^* = x_i^* + \sum_{k \neq i} x_k^*$. Hence we can rewrite the first-order condition above as

$$v_i'(x^*) - p^* \leq 0, \quad \text{with equality if } x_i^* > 0.$$

Intuitively, individual i increases his purchases of the public good x_i until the point in which the marginal benefit he obtains from the aggregate amount of the public good (including not only his own purchases but also those of all other individuals), $v_i'(x^*)$,

25. At this point you might start thinking intuitively about the incentives of every consumer in this model: if the amounts of public goods purchased by all other individuals in a society are nonrival (i.e., you can benefit from them even if you did not contribute to their provision), you would probably not have the incentive to buy several units of the public good.

26. We could extend this setting in order to consider J firms producing the public good, and then aggregate the cost function for the entire industry so that it exactly coincides with $c(x)$. (Note that we can do this because of the price-taking assumption, as we did in perfectly competitive markets (Chapter 6), where we horizontally summed the marginal cost functions of all J firms in the industry.)

coincides with the market price of acquiring further units of the good, p^*. Yet, the firm producing the public good must solve the PMP,

$$\max_{q \geq 0} \; pq - c(q).$$

Then taking first-order conditions with respect to x, yields

$$p^* - c'(x^*) \leq 0, \quad \text{with equality if } x^* > 0,$$

which captures the usual intuition of increasing output until marginal costs of providing additional units coincide with their corresponding marginal revenues (which are equal to prices in perfectly competitive markets such as the one we consider here). Finally, the market-clearing condition implies that the total amount of the public goods produced coincides with the amount consumed by all individuals. Combining the first-order conditions for consumers and the firm, we obtain

$$v_i'(x^*) = c'(x^*) \text{ if } x^* > 0 \quad \text{and} \quad v_i'(x^*) < c'(x^*) \text{ if } x^* = 0.$$

Figure 9.20 illustrates this expression for the case of interior solutions. Intuitively, individual i increases his consumption of the public good until the point in which his marginal benefit from the public good equals the marginal cost.

If, in contrast, a corner solution exists, the marginal cost of providing the first unit of the public good is higher than the marginal benefit that individual i would obtain from such a unit, that is, $v_i'(0) < c'(0)$, as figure 9.21 depicts in the vertical axis.

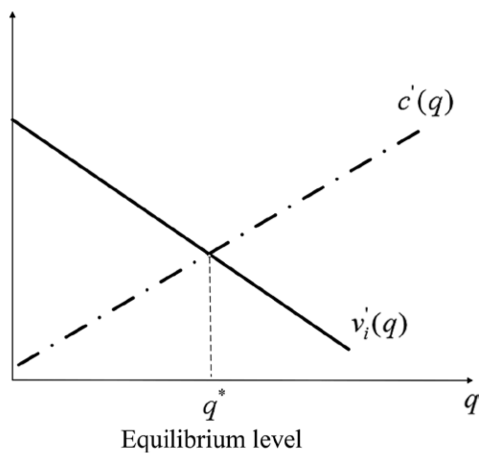

Figure 9.20
Equilibrium level of public good (interior solution).

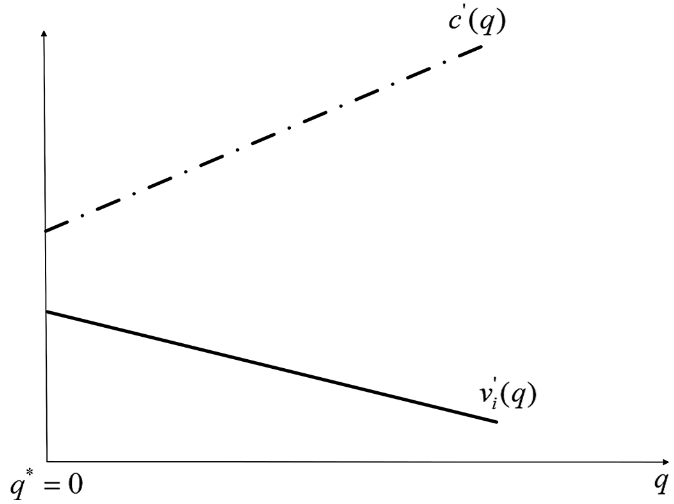

Figure 9.21
Equilibrium level of public good (corner solution)

9.12.1 Social Optimum

Recall that at the Pareto optimality, we must have $\sum_{i=1}^{I} v_i'(x^o) = c'(x^o)$. Graphically, this implies a *vertical* summation of the marginal benefit that all individuals obtain from the public good.[27] This result is graphically represented in figure 9.22, which shows that there is an underprovision of the public good in the competitive equilibrium relative to the optimal allocation as long as the marginal cost curve is not vertical, namely $c''(x) \neq +\infty$.

Intuitively, individual i's purchases of the public good benefit not only him but also all other individuals in the economy. In other words, every individual does not internalize the positive externalities that his individual purchases of the public good generate on other individuals. Hence every consumer i does not have enough incentives to purchase sufficient amounts of the public good, leading to the standard free-rider problem, whereby the public good is underprovided.[28]

27. This result differs from that in private goods where, in order to obtain aggregate demand, we *horizontally* summed individual demands. In that case we found, for a given price p, how many units were demanded by all consumers in the economy. In the case of public goods, however, for a given amount of the public good, x, we find the marginal social benefit that all individuals in the economy obtain from a particular level of public good being provided, which is enjoyed by all individuals given its nonrival nature.

28. This underprovision result is originally due to Bergstrom et al. (1986) who show it for a general class of utility functions. The exercises at the end of the chapter study different private contributions to a public good using a parametric example, and then compare the donations against the social optimum in order to measure individuals' free-riding incentives.

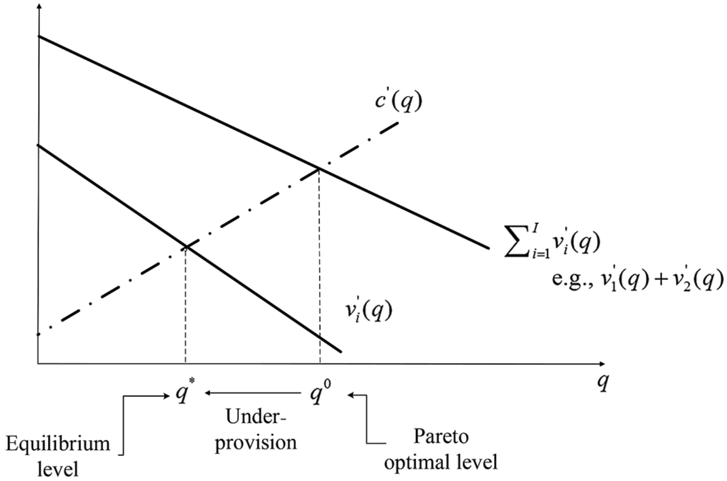

Figure 9.22
Pareto optimal level and equilibrium level of public good

Example 9.6: ***Private contributions to a public good*** Consider an economy with two individuals $i=\{1,2\}$ with quasi-linear utility function$u_i(x,y_i)=y_i+\alpha_i\log(x)$, where $\alpha_i>0$ denotes the value that individual i assigns to total contributions to the public good, $x=x_i+x_j$, and y_i represents a composite private good commodity . Assume that $\alpha_1\geq\alpha_2$. For simplicity, we consider that the price of both private and public good is one, thus entailing a budget constraint $x_i+y_i=w$ for every individual$i$. Using the constraints, we can transform the maximization problem into an unconstrained program as follows:

$$\max_{x_i\geq 0}\; w-x_i+\alpha_i\log(x_i+x_j),$$

where $w-x_i$ represents the utility that individual i obtains from buying units of the private good with the money that has not been contributed to the public good, x_i. Taking first-order conditions with respect to x_i we obtain

$$-1+\frac{\alpha_i}{x_i+x_j}=0.$$

Solving for x_i produces a best-response function $x_i(x_j)$ given by

$$x_i(x_j)=\begin{cases}\alpha_i-x_j & \text{if } x_j<\alpha_i,\\ 0 & \text{otherwise.}\end{cases}$$

Figure 9.23 depicts $x_i(x_j)$, which originates at a donation of $x_i = \alpha_i$ when individual j does not contribute to the public good, $x_j = 0$, but decreases as individual j raises his contribution, ultimately becoming zero when individual j's donation is sufficiently high, at $x_j \geq \alpha_i$

Individual j's best-response function, $x_j(x_i)$, is analogous. We can hence find the equilibrium level of (x_i^*, x_j^*) by simultaneously solving for x_i and x_j in both individuals' best-response functions $x_i(x_j)$ and $x_j(x_i)$ to obtain $x_1^* = \alpha_1 > 0$ and $x_2^* = 0$, since $\alpha_1 \geq \alpha_2$ by definition. This equilibrium is depicted in figure 9.24, where we superimposed both individuals' best-response functions.

In contrast, a social planner would select a higher contribution to the public good in the social optimum. In particular, he would solve

$$\max_{x_i, x_j} w - x_i + \alpha_i \log(x_i + x_j) + w - x_j + \alpha_j \log(x_i + x_j).$$

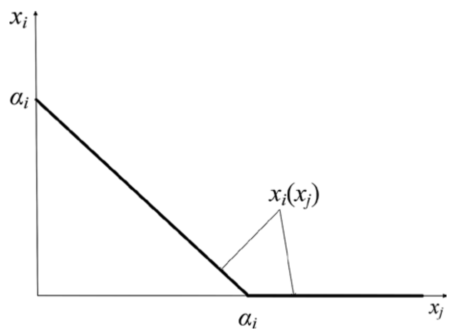

Figure 9.23
Individual i's best-response function, $x_i(x_j)$

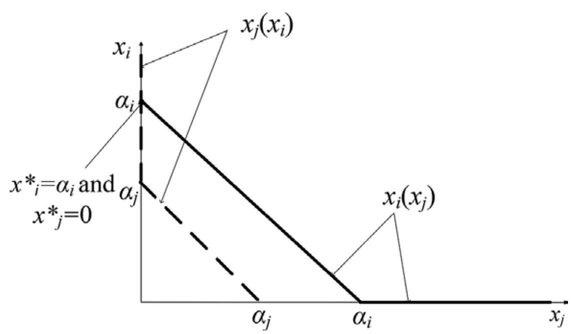

Figure 9.24
Competitive equilibrium in a public good game

Taking first-order conditions with respect to x_i yields $-1+((\alpha_i+\alpha_j)/(x_i+x_j))=0$ (and similarly for the first-order conditions with respect to x_j). Solving for x_i, we hence obtain a continuum of Pareto optimal allocations $x_i^{so}=\alpha_i+\alpha_j-x_j^{so}$, as depicted in figure 9.25. ■

9.12.2 Experiments in Public Goods

A large body of experimental evidence has studied public good games in controlled experiments over the last decades. Their general findings could be summarized as follows: (1) contributions in the first rounds of interaction are generally high (subjects donate about half of the monetary amounts available to them, i.e., w); (2) contributions decrease as individuals interact with one another during more rounds of cooperation (rapidly in some experiments); (3) average contributions increase in the benefit individuals obtain from the public good, m; (4) pre-play communication helps subjects to significantly increase their contributions; and (5) allowing subjects to punish other individuals after each round of interaction increases contributions. For more details, see excellent surveys on this literature by Ledyard (1995) and Vesterlund (2014), and the article by Chaudhuri (2011).

9.13 Neutrality and the Crowding-out Effect

In this section we consider revenue neutral policies (i.e., an income tax to one individual that is entirely allocated as a transfer to another individual), and their effect on private donations to the public good. In particular, consider a Cobb–Douglas utility function $u_i(x_i, G)=x_1^{\alpha}G^{1-\alpha}$, where x_i denotes the private good and G represents total

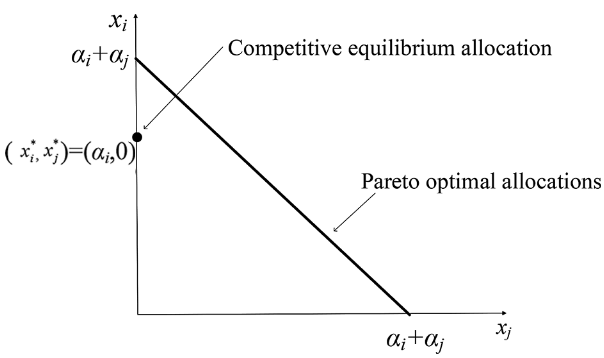

Figure 9.25
Competitive equilibrium versus Pareto optimum

contributions to the public good. Assume, for simplicity, that the price of the private and public good is 1, and that individual i's income is w_i. In this context, every individual $i=\{1,2\}$ solves the utility maximization problem

$$\max_{x_i, g_i} \; x_i^{\alpha}\left(g_i+g_j\right)^{1-\alpha}$$

$$\text{subject to} \quad x_i+g_i=w_i\,,$$

since $G=g_i+g_j$. Using the constraint $x_i=w_i-g_i$ the maximization problem above can be reduced to an unconstrained program with a unique choice variable, g_i, as follows:

$$\max_{g_i} \; \left(w_i-g_i\right)^{\alpha}\left(g_i+g_j\right)^{1-\alpha}.$$

Taking first-order conditions with respect to g_i yields

$$-\alpha\left(w_i-g_i\right)^{\alpha-1}\left(g_i+g_j\right)^{1-\alpha}+(1-\alpha)\left(w_i-g_i\right)^{\alpha}\left(g_i+g_j\right)^{-\alpha}\leq 0.$$

In the case of interior solutions, we can solve for g_i to find donor i's best-response function

$$g_i\left(g_j\right)=\begin{cases}(1-\alpha)w_i-\alpha g_j \;\text{ if }\; g_j<\dfrac{(1-\alpha)w_i}{\alpha}, \\ 0 \;\text{ otherwise.}\end{cases}$$

Simultaneously solving for g_i and g_j, for instance, plugging $g_j(g_i)$ into $g_i(g_j)$, yields the equilibrium donation

$$g_i^* = \frac{w_i-\alpha w_j}{1+\alpha}$$

and an aggregate contribution of

$$G^* = g_i^* + g_j^* = \frac{(1-\alpha)(w_1+w_2)}{1+\alpha}.$$

Note that the aggregate donation in equilibrium lies below the socially optimal donation that a benevolent planner would select, that is, $G^* < G^{SO}$. As a practice, show that in this context $G^{SO}=(1-\alpha)(w_1+w_2)$.

Now consider a tax $dw_2<0$ to individual 2 that is entirely given to individual 1 as a transfer $dw_1>0$, so that $dw_1+dw_2=0$, or more simply $dw_1=-dw_2$. Individual i's equilibrium contribution g_i^* is affected as follows:

$$dg_i^* = \frac{dw_i - \alpha dw_j}{1+\alpha}$$

Since $dw_i = -dw_j$, we can rewrite the expression above as

$$dg_i^* = \frac{dw_i + \alpha dw_i}{1+\alpha} = \frac{(1+\alpha)dw_i}{1+\alpha} = dw_i.$$

Hence individual i's contribution is exactly increased by the amount of the transfer he receives (if $dw_i > 0$) or reduced by the tax he bears (if $dw_i < 0$). However, his contribution change dg_i^* is unaffected by the initial income distribution (i.e., the initial values of w_i and w_j). As a consequence aggregate donations are unaffected by income redistributions, since

$$dG^* = dg_i^* + dg_2^* = dw_1 + dw_2,$$

which is identically zero by definition (i.e., $dw_1 = -dw_2$). The crowding-out effect of levying taxes to fund the production of the public good is obvious in this setting. That is, a \$1 tax, which is expressed as $dw_i = -1 < 0$, reduces every individual i's private contributions to the public good (e.g., donations to charities and NGOs) by exactly \$1, since $dg_i^* = dw_i = -1$.

Example 9.7: ***Increasing the number of contributors*** Let us extend the previous setting of two individuals with Cobb–Douglas preferences to a context with N individuals. We assume that all donors have the same income, $w_1 = w_2 = \ldots = w_n = w$. In this setting, individual i's best-response function becomes

$$g_i(g_{-i}) = (1-\alpha)w - \alpha\sum_{j\neq i} g_j.$$

Invoking symmetry in equilibrium contributions, $g_1 = g_2 = \ldots = g_n = g$, yields

$$g = (1-\alpha)w - \alpha(N-1)g.$$

Solving for g, we obtain an equilibrium donation of

$$g^* = \frac{(1-\alpha)w}{1+\alpha\ (N-1)},$$

and an aggregate contribution of

$$G^* = Ng^* = \frac{N(1-\alpha)w}{1+\alpha(N-1)}.$$

We can now examine the effect of increasing the number of contributors in our equilibrium results. In particular,

$$\frac{\partial g^*}{\partial N} = \frac{-\alpha(1-\alpha)w}{[1+\alpha(N-1)]^2} < 0$$

and

$$\frac{\partial G^*}{\partial N} = (1-\alpha)w\frac{1+\alpha(N-2)}{[1+\alpha(N-1)]^2} > 0,$$

thus indicating that, while individual contributions decrease as a result of more donors potentially contributing to the public good, the overall effect of adding more donors is still positive. Figure 9.26 illustrates this comparative statics results where, for simplicity, we assume that $\alpha = \frac{1}{2}$ and $w=10$, and thus the expressions above become $g^*=10/(1+N)$ and $G^*=N(10/(1+N))$. ■

9.14 Remedies to the Underprovision of Public Goods

Let us now analyze some remedies to the above free-riding problem. The first remedy is a quantity-based intervention (i.e., a basic governmental provision of the public good). The second remedy is a price-based intervention via taxes or subsidies. In order to understand the effects of these subsidies, assume two consumers with benefit functions $v_1(x_1+x_2)$ and $v_2(x_1+x_2)$, respectively, where x_i denotes the amount of the public good purchased by consumer i. Similarly to our analysis of externalities, we can provide a subsidy s_i per unit of the public good purchased by every consumer i that induces him to take into account the positive external effect that his purchases of public goods produce on other individuals' welfare. Hence the subsidy must be $s_i = v'_{-i}(x^o)$, where $v'_{-i}(x^o)$ reflects the marginal benefit that all other individuals obtain from enjoying x^0 units of the public good.[29] In order to show this result, note that the consumer i's UMP becomes that of selecting the equilibrium level of his contribution to the public good, $\tilde{x}_i$, for a given level of the other individual's contribution, $\tilde{x}_j$:

$$\max_{x_i \geq 0} \; v_i(x_i + \tilde{x}_j) + s_i x_i - \tilde{p} x_i.$$

Taking first-order conditions with respect to x_i obtains

29. Note that this analysis is equivalent to that of imposing a tax $t_i = -v'_{-i}(x^o)$ per unit of the public good when the overall amount of the public good falls below x^o.

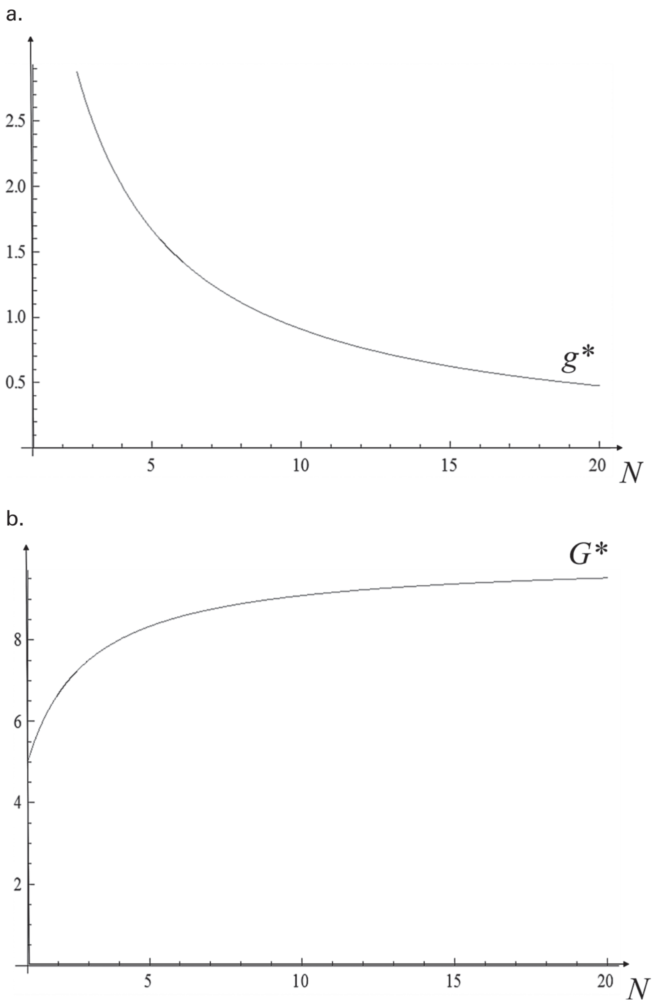

Figure 9.26
Individual and aggregate contributions as a function of N

$$v_i'(\tilde{x}_i + \tilde{x}_j) + s_i - \tilde{p} \leq 0, \quad \text{with equality if } \tilde{x}_i > 0.$$

Using the market-clearing condition $\tilde{x} = \tilde{x}_i + \tilde{x}_j$, and the fact that in a competitive equilibrium the PMP implies $\tilde{p} = \tilde{c}'(\tilde{x})$, we can rewrite the previous first-order condition as

$$v_i'(\tilde{x}) + s_i \leq c'(\tilde{x}).$$

Finally, recall that for a subsidy s_i to be optimal, we need $s_i = v_{-i}'(x^o) = v_j'(x^o)$, which allows us to rewrite the first-order condition above as

$$v_i'(x^o) + v_j'(x^o) \leq c'(x^o).$$

Hence we just need a subsidy $s_i = v_j'(x^o)$ to consumer i in the case of only two consumers, and $s_i = v_j'(x^o) + v_k'(x^o) + \cdots = \sum_{j \neq i} v_j'(x^o)$ in the case of N consumers. The introduction of a subsidy may seem at first like an effective and easy solution to the underprovision problem in public goods (the free-riding problem). However, we cannot ignore that we assumed that the regulator has access to information about the marginal benefits of the public good for every consumer. This assumption might be difficult to support in certain cases.

9.15 Lindahl Equilibria

We know that the private provision of a public good results in inefficiencies, whereby $x^* < x^0$, a problem that can be solved by the use of quantity-based regulation or by the use of subsidies, as we described above. There is, however, a market solution that *in principle* could achieve optimality. Let us consider such a solution for a market where every individual's consumption of the public good is a distinct commodity with its own market. We can denote the price of this personalized good by p_i, which is allowed to differ across consumers. If consumer i faces a price p_i^{**}, his UMP is

$$\max_{x_i \geq 0} v_i(x_i) + w_i - p_i^{**} x_i.$$

Taking first-order conditions with respect to x_i yields

$$v_i'(x_i^{**}) - p_i^{**} \leq 0, \quad \text{with equality if } x_i^{**} > 0,$$

implying that, at the aggregate level, $\sum_{i=1}^{I} v_i'(x_i^{**}) \leq \sum_{i=1}^{I} p_i^{**}$. Let us now analyze the firm producing the public good. Because the public good is individual specific, the firm produces a bundle of I goods (one for each consumer), and its PMP becomes

$$\max_{q\geq 0} \sum_{i=1}^{I}\left(p_i^{**}x\right)-c(x),$$

where the first term represents the total revenue for the firm, from selling x goods to I consumers, each of them paying a personalized price p_i^{**}. Taking first-order conditions with respect to x yields

$$\sum_{i=1}^{I} p_i^{**}-c\left(x^{**}\right)\leq 0, \quad \text{with equality if } q^{**}>0,$$

or $\sum_{i=1}^{I} p_i^{**} \leq c'(x^{**})$. Combining this condition with that which we found for consumers yields

$$\sum_{i=1}^{I} v_i'\left(x_i^{**}\right)\leq \sum_{i=1}^{I} p_i^{**} \leq c'(x^{**}) \Rightarrow \sum_{i=1}^{I} v_i'\left(x_i^{**}\right)\leq c'(x^{**}).$$

Finally, using the market-clearing condition $x^{**}=q^{**}$, we obtain $\sum_{i=1}^{I} v_i'(x_i^{**}) \leq c'(q^{**})$. This inequality coincides with the first-order condition for the social planner. Thus the equilibrium level of the public good that every individual i purchases is exactly the efficient level, that is, $q^{**} = q^{o}$. The equilibrium in personalized markets for the public good is usually known as the *Lindahl equilibrium*, after Lindahl (1919).

Intuitively, these markets attain efficiency because, first, we define personalized markets for the public good, and second, each consumer, taking the price of her personalized good as given, fully determines her own level of consumption of the public good. Intuitively, the positive externalities arising in the private provision of public goods are absent in these personalized markets. Given the efficient equilibrium predictions of Lindahl markets, a natural question is whether these personalized markets of the public good are realistic. Unfortunately, for a personalized market to exist, we need excludability between the different personalized public goods each consumer enjoys, which might only be applicable to very specific cases, such as some forms of health care or college education. Furthermore, even if excludability was possible, these personalized markets would be monopsonistic (since there is only one buyer on the demand side, i.e., each consumer i) entailing that the price-taking assumption is probably difficult to support.

Example 9.8: *Calculating a Lindahl equilibrium* Three first-year graduate students, Eric (E), Chris (C), and Matt (M), have decided to purchase a microwave for their office. There is argument, however, over how much each person should contribute to the purchase of this microwave: how good of a microwave should be bought, and how much should each student contribute to its purchase? We can express the utility function of each student as

$$u_i(x_i, y) = \ln x_i + \alpha_i \ln y,$$

where x_i represents the utility gained by student $i=\{E,C,M\}$ from private purchases (i.e., all other goods), y represents the utility gained by the total amount spent on a new microwave by the three students, and α_i denotes the benefit student i obtains from the microwave.

For simplicity, assume that both the price of the private good, x_i, and the wealth of each student is 1. Set up the utility-maximization problem for student i as

$$\max_{x_i} \ \ln x_i + \alpha_i \ln y$$

$$\text{subject to} \quad x_i + p_i y \leq 1,$$

where p_i represents that Lindahl price (effectively a tax) each student pays, and $p_E+p_c+p_M=1$ Since the budget constraint holds with equality, we can rewrite the program as

$$\max_{y} \ \ln(1-p_i y)+\alpha_i \ln y.$$

Now, taking first-order conditions with respect to y yields

$$\frac{p_i}{1-p_i y}=\frac{\alpha_i}{y}.$$

Rearranging these terms, we see that each individual i's purchase of the public good must satisfy

$$p_i y=\frac{\alpha_i}{1+\alpha_i},$$

which is increasing and thus is each individual's benefit from the public good, α_i. Summing across all three students yields

$$\sum_{i=\{E,C,M\}} p_i y=(p_E+p_C+p_M)y=y=\frac{\alpha_E}{1+\alpha_E}+\frac{\alpha_C}{1+\alpha_C}+\frac{\alpha_M}{1+\alpha_M},$$

since $p_E+p_c+p_M=1$ by definition. We can substitute this value for y in equation $p_i y=\alpha_i/(1+\alpha_i)$ to solve for the Lindahl prices

$$p_i^* = \frac{\dfrac{\alpha_i}{1+\alpha_i}}{\dfrac{\alpha_E}{1+\alpha_E}+\dfrac{\alpha_C}{1+\alpha_C}+\dfrac{\alpha_M}{1+\alpha_M}}.$$

For instance, $\alpha_E=1$ and $\alpha_c=\alpha_M=0.6$ would imply that Eric heats his lunch every day, but Chris and Matt like cold sandwiches a few times a week. With these parameters we would obtain $y=1.25$ spent on the new microwave and Lindahl prices of $p_E=0.4$ and $p_c=p_M=0.3$. ■

9.16 Public Goods That Experience Congestion

Let us now consider that the number of individuals consuming the public good *reduces* the benefit that each user i enjoys from the good. As a consequence each user's utility function now becomes $v_i(x_i,x_{-i})+w_i$, which depends on the same arguments in previous sections (the contribution of individual i, x_i, and those of all other individuals, x_{-i}), but x_{-i} enters now negatively in $v_i(\cdot)$, in order to capture congestion effects. In particular, utility increases in x_i but *decreases* in the amount of the public good consumed by other individuals x_i as in the case of negative externalities.

Let us next examine how the Samuelson rule for optimal provision of public goods is affected by the presence of congestion. In particular, the social planner's maximization problem is

$$\max_{x_1,\ldots,x_n} \sum_{i=1}^{N}[v(x_i,\, x_{-i})+w_i]-C(x).$$

Taking first-order conditions with respect to x_i yields

$$\frac{\partial v_i(x_i,\, x_{-i})}{\partial x_i}+\sum_{j\neq i}\frac{\partial v_j(x_i,\, x_{-i})}{\partial x_j}-\frac{\partial C(x)}{\partial x_i}\leq 0 \quad \text{for all } i,$$

which in the case of interior solutions becomes

$$\frac{\partial v_i(x_i,\, x_{-i})}{\partial x_i}+\sum_{j\neq i}\frac{\partial v_j(x_i,\, x_{-i})}{\partial x_j}=\frac{\partial C(x)}{\partial x_i}.$$

Summing over all N individuals, we obtain

$$\sum_{j=1}^{N}\frac{\partial v_i(x_i,\, x_{-i})}{\partial x_i}+\sum_{i=1}^{N}\sum_{j\neq i}^{N}\frac{\partial v_j(x_i,\, x_{-i})}{\partial x_j}=\sum_{i=1}^{N}\frac{\partial C(x)}{\partial x_i},$$

which coincides with the standard Samuelson rule for the optimal provision of public goods, except for the second term. Intuitively, this term reflects the negative externality that individual i suffers from a larger consumption of the public good by all other individuals. (More precisely, $\sum_{j\neq i}\partial v_j(x_i,x_{-i})/\partial x_i$ indicates how all individuals $j\neq i$ are affected by a marginal increase in x_i, whereas the aggregation $\sum_{i=1}^{N}\sum_{j\neq i}^{N}\partial v_j(x_i,x_{-i})/\partial x_i$ measures this effect when the consumption of *all* individuals, and not only that of individual i, is increased.) As a consequence the socially optimal amount of public good will tend to be smaller than in the absence of congestion effects.

9.17 Behavioral Motives in Public Good Games

9.17.1 Warm-Glow Benefits in Private Contributions to the Public Good

Andreoni (1990) developed one of the most cited behavioral motives in the literature on public good games, by assigning a "warm-glow" benefit to individuals who make donations to the public good. Specifically, he assumes that individual i's utility function, $u_i(x_i, G, g_i)$ increases in his consumption of the private good x_i, the total contributions to the public good G, and in the warm glow of donating g_i dollars to the public good. As we next demonstrate, the presence of warm-glow in the donors' utility function prevents the "crowding-out" result studied in section 9.14 from arising under relatively large conditions. In particular, let us first specify donor i's UMP in this setting

$$\max_{x_i,g_i,G} \; u_i(x_i, G, g_i)$$

$$\text{subject to} \quad x_i + g_i = w_i \;\; \text{and}$$

$$g_i + G_{-i} = G,$$

where $G_{-i} = \sum_{j\neq i} g_j$ denotes total donations from all other individuals. Since $g_i = G - G_{-i}$ and, as a consequence, $x_i = w_i - g_i = w_i - (G - G_{-i}) = w_i - G + G_{-i}$, we can simplify the program above to

$$\max_{G} \quad u_i(w_i - G + G_{-i}, G, G - G_{-i}).$$

Differentiating with respect to G yields a total donations function of

$$G = f_i(w_i + G_{-i}, G_{-i}),$$

which only depends on the elements of $u_i(\cdot)$ that are different to G. Therefore the individual donation function of player i, $g_i = G - G_{-i}$, is

$$g_i = f_i\left(w_i + G_{-i}, G_{-i}\right) - G_{-i}.$$

The first term in $f_i(\cdot)$ is common to other public good games in which donors do not benefit from warm glow. In contrast, the second term of $f_i(\cdot)$ is novel to this setting, and arises because of the warm-glow benefits that donors obtain. Intuitively, the first term captures what Andreoni refers to as altruistic motivations in public goods, whereas the second component measures egoistic motivations (because the warm-glow benefit is private).

Let $f_{ia}(f_{ie})$denote the first-order derivative of $f_i(\cdot)$ with respect to the first argument (altruism) and the second argument (egoism) where $f_{ia} \in [0,1]$, $f_{ie} \geq 0$, and $0 < f_{ia} + f_{ie} \leq 1$ (see Andreoni 1990 for more details on these bounds). This notation helps us obtain the following "altruism coefficient":

$$\alpha_i = \frac{\partial f_i / \partial w_i}{\partial f_i / \partial G_{-i}} = \frac{f_{ia}}{f_{ia} + f_{ie}},$$

which becomes $\alpha_i = 1$ for pure altruists, where $f_{ie} = 0$, and $\alpha_i = f_{ia}$ for pure egoists since $f_{ia} + f_{ie} = 1$. Let us now consider a transfer from individual 1 to 2, namely $d_{w_1} > 0$ and $d_{w_2} < 0$, where $d_{w_1} = -d_{w_2}$, and let us examine how total donations G are affected. With this goal, let us first analyze how individual contributions are affected by the transfer. We can totally differentiate g_i to obtain

$$dg_i = f_{ia}\left(dw_i + dG_{-i}\right) + f_{ie} dG_{-i} - dG_{-i}.$$

Then, after factoring out dG_{-i}, we have

$$dg_i = f_{ia}\, dw_i + \left(f_{ia} + f_{ie} - 1\right) dG_{-i}.$$

Now, since $G_{-i} = G - g_i$ by definition, we can substitute $dG_{-i} = dG - dg_i$ in the expression above to obtain

$$dg_i = f_{ia} dw_i + \left(f_{ia} + f_{ie} - 1\right) dG - \left(f_{ia} + f_{ie} - 1\right) dg_i.$$

Rearranging yields

$$\left(f_{ia} + f_{ie}\right) dg_i = \left(f_{ia} + f_{ie} - 1\right) dG + f_{ia} dw_i.$$

We then divide both sides by $(f_{ia} + f_{ie})$ and use the definition of α_i to get

$$dg_i = \frac{f_{ia} + f_{ie} - 1}{f_{ia} + f_{ie}} dG + \alpha_i dw_i,$$

which captures how the donation of individual i is affected by a change in his wealth-dw_i. We can now aggregate across donors to obtain the change in aggregate contributions, dG:

$$\sum_{i=1}^{I} dg_i = \sum_{i=1}^{I} \frac{f_{ia} + f_{ie} - 1}{f_{ia} + f_{ie}} dG + \sum_{i=1}^{I} \alpha_i dw_i$$

$$\text{and since } \sum_{i=1}^{I} dg_i = dG,$$

$$dG = \sum_{i=1}^{I} \frac{f_{ia} + f_{ie} - 1}{f_{ia} + f_{ie}} dG + \sum_{i=1}^{I} \alpha_i dw_i.$$

We can now rearrange and solve for dG to obtain

$$dG = \frac{1}{1 - \sum_{i=1}^{I} [(f_{ia} + f_{ie} - 1)/(f_{ia} + f_{ie})]} \sum_{i=1}^{I} \alpha_i dw_i.$$

For compactness, we let

$$c \equiv \frac{1}{1 - \sum_{i=1}^{I} [(f_{ia} + f_{ie} - 1)/(f_{ia} + f_{ie})]},$$

which helps us express the previous result as

$$dG = c \sum_{i=1}^{I} \alpha_i dw_i.$$

Last, since $dw_1 = -dw_2$ and $dw_j = 0$ for all other individuals $j \neq 1 \neq 2$, the expression above becomes

$$da = c[\alpha_1 dw_i + \alpha_2 (-dw_2)] = c(\alpha_1 dw_1 - \alpha_2 dw_2),$$

thus implying that $dG \geq 0$ only holds if $\alpha_1 \geq \alpha_2$, but dG is negative otherwise. In words, the transfer from individual 1 to 2 is not necessarily neutral, as it can increase total donations to the public good if and only if individual 1 is more altruistic than individual 2. Andreoni (1990) then elaborates on other related results, such as that subsidies from less to more altruistic individuals can increase the total supply of the public good; and that the crowding-out effect between private and public provision of the public good is incomplete. We next provide a numerical example of this result.

Example 9.9: ***Warm glow in public goods*** Consider a public good game with two individuals $i = \{1,2\}$, each with Cobb–Douglas utility function

$$u_i(x_i, G, g_i) = a \log x_i + b \log G + c_i \log g_i,$$

where $c_1 > c_2$ represent the warm-glow benefit that individuals obtain from their contributions to the public good. Since $x_i = w_i - g_i$ and $G = g_i + g_j$, we can rewrite the expression above as

$$u_i(w_i - g_i, g_i + g_j, g_i) = a \log(w_i - g_i) + b \log(g_i + g_j) + c_i \log g_i.$$

Then, taking first-order conditions with respect to g_i, we obtain

$$-\frac{a}{w_i - g_i} + \frac{b}{g_i + g_j} + \frac{c_i}{g_i} = 0.$$

For simplicity, consider $w_i = 10$ for both individuals, $a = 1$, $b = \frac{1}{3}$, and warm-glow parameters $c_1 = \frac{1}{4}$ and $c_2 = \frac{1}{5}$. In this setting, we can solve for g_1 to obtain player 1's best response function, $g_1(g_2)$. Operating similarly for player 2, we find his best-response function $g_2(g_1)$. Simultaneously solving for g_1 and g_2 yields equilibrium contribution levels of $g_1^* = 2.995$ and $g_2^* = 2.623$, thus entailing aggregate donations of $G^* = 5.61$. In this setting, we can implement a \$2 transfer from the individual with high warm-glow parameter (and thus low altruistic concerns, i.e., individual 1) to that with low warm-glow parameter (and high altruistic concerns, individual 2). In particular, after implementing the transfer from individual 1 to 2, the wealth levels become $w_1 = \$8$ and $w_2 = \$12$, which modifies individual donations to $g_1^* = 2.2$ and $g_2^* = 3.45$, thus implying a larger amount of total contributions, $G^* = 5.65$, as predicted by the general result we described above. ■

9.17.2 Social Preferences

Consider the public good game shown in table 9.2, where players are asked to simultaneously and independently choose between contributing (C) or not contributing (NC) to the public good. Here both players' payoffs satisfy $b > a > d > c$, indicating that both players have incentives to free ride. Indeed, every player's best response is to not contribute, both when his opponent contributes (given that $b > a$) and when he does not (since $d > c$). In fact both players find C to be strictly dominated by NC, implying that the strategy profile (NC, NC) is ultimately the unique equilibrium of the unrepeated game.

Table 9.2
Payoff matrix under no social preferences

		Player 2	
		C	*NC*
Player 1	*C*	a, a	$c-\alpha_1(b-c), b-\beta_2(b-c)$
	NC	$b-\beta_1(b-c), c-\alpha_2(b-c)$	d, d

Table 9.3
Payoff matrix under no social preferences

	Rival	*Nonrival*
Excludable	Private good	Club good
Nonexcludable	Common pool resource	Public good

Consider, instead that players exhibit Fehr and Schmidt (1999) type social preferences, as we described in chapter 1. In particular, for the case of two players, Fehr and Schmidt's (1999) utility function reduces to

$$U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\},$$

where x_i is player i's payoff, and x_j is the payoff of his opponent (player j). Recall that parameter α_i represents player i's disutility from envy, which occurs when $x_i < x_j$ and thus the second term satisfies $\max\{x_j - x_i, 0\} = x_j - x_i > 0$, whereas the third term is null as $\max\{x_i - x_j, 0\} = 0$, yielding a utility of $x_i - \alpha_i(x_j - x_i)$. In contrast, parameter β_i captures player i's disutility from guilt when he earns a higher payoff than player j. Indeed, when $x_i > x_j$ the second term cancels while the third term is positive, entailing a utility level $x_i - \beta_i(x_i - x_j)$. Additionally Fehr and Schmidt (1999) assume that the players' envy is always stronger than their guilt, which is captured by assuming that $\alpha_i \geq \beta_i$ and $1 > \beta_i \geq 0$. Now, by taking social preferences into account, we can reformulate the payoff matrix as shown in table 9.3.

In the table, every player i's utility level decreases when he is either: the player with the highest payoff in the group (due to guilt, e.g., player 1 under outcome (NC, C)), or when he is the player with the lowest payoff in the group (due to envy, e.g., player 1 under outcome (C, NC)). In this setting let us first analyze player i's best-response function. When player j contributes, player i prefers to defect if $a \leq b - \beta_i(b-c)$, or after solving for β_i, if $\beta_i \leq (b-a)/(b-c)$. When instead player j does not contribute, player i prefers to not contribute if $b - \alpha_i(b-c) < d$, which holds given that

$(c-d)/(b-c)<0\leq\alpha_i$ by definition. Hence, if parameter β_i satisfies $\beta_i\leq(b-a)/(b-c)$ NC becomes a strictly dominant strategy for player i. If instead $\beta_i>(b-a)/(b-c)$, then player i's best response to C is C since $a>b-\beta_i(b-c)$ for all $\beta_i>(b-a)/(b-c)$, but his best response to NC is NC given that $c-\alpha_i(b-c)<d$ for all $(c-d)/(b-c)<0\leq\alpha_i$. That is, when $\beta_i>(b-a)/(b-c)$, player i's best response is to select the same action as his opponent.

Given players' best responses, if either $\beta_i\leq(b-a)/(b-c)$ or $\beta_j\leq(b-a)/(b-c)$, then both players NC, and the unique Nash equilibrium of the game is (NC, NC). Otherwise (if both $\beta_i>(b-a)/(b-c)$ and $\beta_j>(b-a)/(b-c)$), then both players' best response to C is C, and both players' best response to NC is NC. Hence, when $\beta_i,\beta_j>(b-a)/(b-c)$, (C, C) and (NC, NC) are Nash equilibria of the game in pure strategies.[30] Summarizing, if at least one player has relatively low concerns about guilt, the unique Nash equilibrium of the game, (NC, NC), coincides with that where players have no concerns about the fairness of the payoff distribution (standard preferences); see unshaded areas of figure 9.27. However, when *both* individuals are sufficiently concerned about fairness—the shaded area of figure 9.27— we can identify

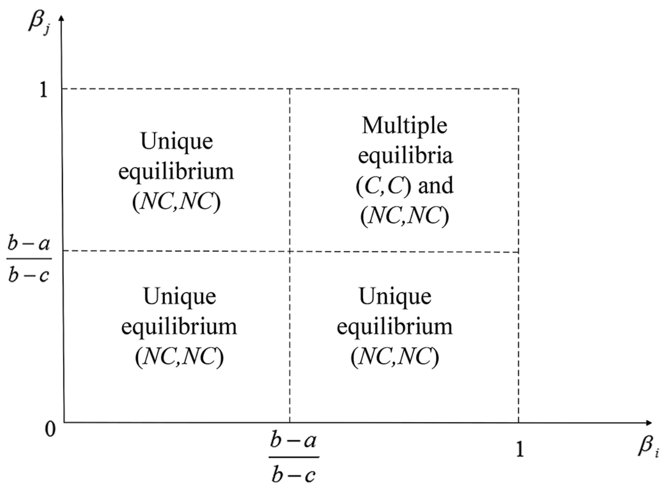

Figure 9.27
Equilibrium behavior as a function of fairness concerns

30. For simplicity, we do not consider here the mixed strategy equilibrium of this game (which arises when both β_i and β_j satisfy $\beta_i>(b-a)/(b-c)$ and $\beta_j>(b-a)/(b-c)$), where both players randomize between C and NC. For more details about this strategy, and for an extension of this model to infinitely repeated games, see Duffy and Munoz-Garcia (2012).

two different Nash equilibria: one in which both players cooperate, and one in which no player cooperates. The introduction of sufficient concerns about fairness by both players thus transforms the payoff structure of the game from a Prisoner's Dilemma to a Pareto-rankable coordination game, where every player's best response is to select the same action as his opponent, but both players strictly prefer outcome (C, C) to (NC, NC).[31]

9.17.3 Competition for Status Acquisition[32]

Let us consider a public good game with two agents privately contributing to its provision. In addition to the return $m \in [0, \infty)$ that every player obtains from total contributions to the public good, $G = g_i + g_j$, each benefits from status acquisition if his donation is larger than his rival's. In particular, let g_i denote subject i's voluntary contributions to the public good, and let $x_i \geq 0$ represent his consumption of private goods. Additionally assume that every player is endowed with w monetary units that she can distribute between private and public good consumption, and that the marginal utility individual i derives from his consumption of the private good is one.[33] Specifically, the representative contributor's utility function is

$$u_i(x_i, G) = x_i + \ln\left[mG + \alpha_i\left(g_i - g_j\right)\right].$$

In this setting, consider that the status subject i acquires by contributing g_i is given by the difference between his contribution and that of the other player, $g_i - g_j$. That is, subject i enhances his relative status if his contribution is greater than individual j's; otherwise, subject i perceives himself as an individual with lower status than subject j. This difference is scaled by α_i, indicating the importance of relative status for subject i, where $\alpha_i \in [0, +\infty)$. Additionally all the elements of the game, including the particular values of α_i, are assumed to be common knowledge among the players. Every player i's UMP is hence

31. Note that this best-response function is similar to what Cooper et al. (1996) call " best response altruists," namely players for whom cooperate (defect) is their best response to cooperation (defection, respectively), as opposed to what Cooper et al. (1996) refer to as "dominant strategy altruists" for whom cooperation is always a best response, regardless of other players' strategies. Our results are also connected with those in Bolton and Ockenfels (2000), who allow for every individual's payoff thresholds to be private information.

32. This subsection is based on Munoz-Garcia (2011), in which, besides simultaneous contributions to the public good, I analyze the sequential contribution mechanism, and ultimately provide a ranking of individual and aggregate donations across contribution mechanisms.

33. Allowing for asymmetric monetary endowments, $w_i \neq w_j$, would not change our results, since players' utility function is quasi-linear in w. Moreover we assume that w is sufficiently large.

$$\max_{x_i, G} \; x_i + \ln\left[mG + \alpha_i\left(g_i - g_j\right)\right]$$

$$\text{subject to } \; x_i + g_i = w,$$

$$g_i + g_j = G, \text{ and}$$

$$g_i, g_j \geqslant 0.$$

Using $x_i = w - g_i \geq 0$ and $g_i + g_j = G$, we can simplify the preceding program to

$$\max_{g_i \geq 0} \; w - g_i + \ln\left[m(g_i + g_j) + \alpha_i\left(g_i - g_j\right)\right].$$

In particular, the first term, $w - g_i$, represents the utility from consuming the remaining units of money that have not been contributed to the public good. The second term denotes, on the one hand, the utility that individual i gets from the consumption of the total contributions to the public good $g_i + g_j$, and on the other hand, the utility derived from relative status acquisition.

Intuitively, note that in this setting an increase in player j's contribution, g_j, imposes both a positive and a negative externality on player i's utility level. The *positive* externality from g_j on player i's utility is just the usual one arising from the public good nature of player j's contributions. Player j's donations, however, impose also a *negative* externality on player i since this donation reduces the status perception of player i, which is to say, higher g_j decreases $\alpha_i(g_i - g_j)$ for a given g_i. Let us next analyze player i's best-response function, $g_i(g_j)$,

$$g_i(g_j) = \begin{cases} 1 + \dfrac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left[0, \dfrac{m + \alpha_i}{m - \alpha_i}\right], \\ 0 & \text{if } g_j > \dfrac{m + \alpha_i}{m - \alpha_i}, \end{cases}$$

if $\alpha_i < m$, and thus player i's best-response function is decreasing in g_j. In contrast, when $\alpha_i > m$, his best-response function is positively sloped, which we write as $g_i(g_j) = 1 + ((\alpha_i - m)/(\alpha_i + m))g_j$ for all g_j as depicted in figure 9.28a and b.

In particular, when $\alpha_i < m$, the positive externality that player j's donations impose on player i's utility dominates the negative one, and player i considers player j's contributions as strategic *substitutes* of his own (i.e., he is a net free-rider), as in the usual PGG models without status. In contrast, when $\alpha_i > m$, the negative externality resulting from player j's contributions is higher than the positive externality originated from the public good nature of his contributions. In this case, player i considers player j's a.

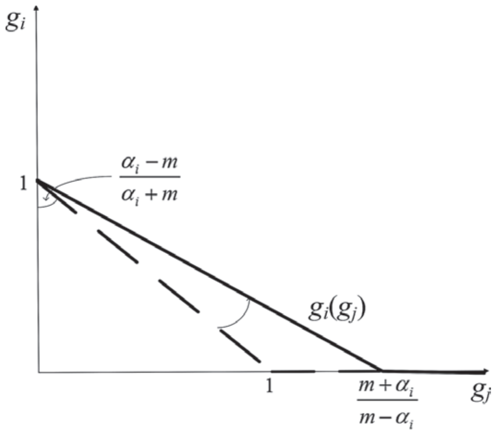

Figure 9.28a
Best response function when $\alpha_i < m$

b.

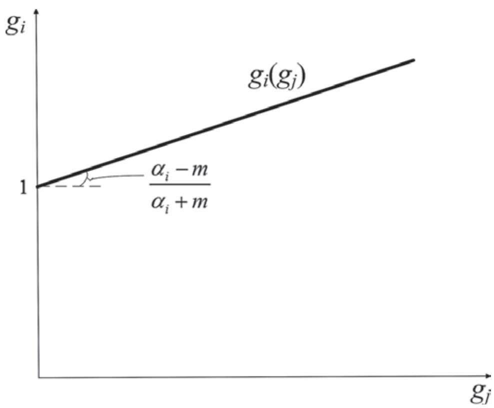

Figure 9.28b
Best response function when $\alpha_i > m$

donations as strategic *complements* to his own (i.e., he is a net status-seeker, which leads to the positively sloped best-response function depicted in figure 9.28b). The slope of player i's best-response function also increases in his value to status, α_i. As α_i increases, $g_i(g_j)$ pivots upward, from its center at $g_i=1$: from a negative slope when $\alpha_i<m$ to a positive slope when $\alpha_i>m$.

Given these best-response functions, we can now characterize equilibrium donations in this simultaneous PGG, g_i^{Sm}. For presentation purposes we next describe the three conditions under which player i's and j's best-response functions can cross each other.

1. From player i's best-response function, $g_i(g_j)$, we have $g_i^{Sm}=1$ only when: (1) the slope of player j's best-response function, $g_j(g_i)$, is smaller than -1 and (2) the horizontal intercept of player i's best-response function, $g_i(g_j)$, is higher than 1. Otherwise, both players' best-response functions would cross each other at an interior point. That is, $g_i^{Sm}=1$ if and only if $((\alpha_j-m)/(\alpha_j+m))\leq-1$, which simplifies to $\alpha_j\leq 0$. And $((m+\alpha_i)(m-\alpha_i))\geq 1$ if and only if $\alpha_i>0$. Since $\alpha_i, \alpha_j\geq 0$ by definition, these conditions on player i and j's concerns about status are $\alpha_i\geq 0$ and $\alpha_j=0$. Hence $g_i^{Sm}=1$ if and only if $\alpha_i\geq 0$ and $\alpha_j=0$.
2. We have that $g_i^{Sm}=0$ only when the opposite conditions of case 1 happen, which is when $\alpha_i=0$ and $\alpha_j\geq 0$.
3. When none of the conditions above are satisfied, which is when $\alpha_i>0$ and $\alpha_j>0$, we have an interior solution. Solving for g_i and g_j in a system of two equations, we obtain $g_i^{Sm}=\left(\alpha_i\left(\alpha_j+m\right)/\left(\alpha_i+\alpha_j\right)m\right)$, as the interior Nash equilibrium contribution level.

We can hence summarize these three cases in terms of the equilibrium contribution of player i, g_i^{Sm}, as follows:

$$g_i^{Sm}=\begin{cases}1 & \text{if } \alpha_i>0 \text{ and } \alpha_j=0,\\ \dfrac{\alpha_i\left(\alpha_j+m\right)}{\left(\alpha_i+\alpha_j\right)m} & \text{if } \alpha_i>0 \text{ and } \alpha_j>0,\\ 0 & \text{if } \alpha_i=0 \text{ and } \alpha_j>0,\end{cases}$$

and $g_i^{Sm}+g_j^{Sm}=1$ if $\alpha_i=\alpha_j=0$. Figure 9.29 illustrates the set of parameter values that support these different contribution levels, as a function of player i's and j's concerns for status acquisition. In particular, $g_i^{Sm}=1$ on the vertical axis of the figure where $\alpha_j=0$; $g_i^{Sm}=0$ on the horizontal axis, where $\alpha_i=0$; and $g_i^{Sm}=\left(\alpha_i\left(\alpha_j+m\right)/\left(\alpha_i+\alpha_j\right)m\right)$ at strictly interior points when α_i, $\alpha_j>0$. Intuitively, player i submits $g_i^{Sm}=1$ when he assigns a value to status and player j does not; submits a zero contribution when he

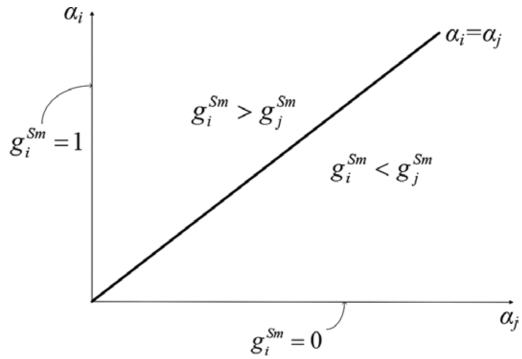

Figure 9.29
Individual contributions in the (α_i, α_j)-quadrant

does not assign any value to status, $\alpha_i = 0$, and player *j* does, $\alpha_j > 0$; and finally he submits $g_i^{Sm} = (\alpha_i(\alpha_j + m)/(\alpha_i + \alpha_j)m)$ when both players assign a value to status.
As a consequence total contributions to the public good are

$$G^{Sm} = \begin{cases} 1 & \text{if } \alpha_j = 0 \text{ and } \alpha_i > 0, \\ 1 + \dfrac{2\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} & \text{if } \alpha_i > 0 \text{ and } \alpha_j > 0, \\ 1 & \text{if } \alpha_i = 0 \text{ and } \alpha_j \geq 0, \end{cases}$$

where G^{sm} is weakly increasing in both α_i and α_j, and maximized for (α_i, α_j) pairs such that $\alpha_i = \alpha_j = \alpha$. Hence total contributions when *either* player values status coincides with total contributions when *neither* does, $G^{sm} = 1$ (i.e., total donations in a standard public good game without status concerns). Alternatively, an increase in the status concerns of only one individual does no raise total contributions. Finally, G^{sm} is higher when players' value of status acquisition are relatively homogeneous (e.g., $\alpha_i = \alpha_j = \alpha$) than when they are heterogeneous ($\alpha_i \neq \alpha_j$).

Appendix: More General Policy Mechanisms

From the chapter's discussion it is clear that in the presence of incomplete information about the firms' marginal profit function and the consumers' marginal damage function, standard policy tools (quotas and emission fees) entail welfare losses. Let us now examine more general policy mechanisms that attempt to maximize social surplus in the context of incomplete information. In particular, we consider mechanisms in which we ask agents to self-report their types, That is, we ask the firm: What is your benefit from increasing the externality level from $x = 0$ to $x = \bar{x}$, whereby $b = b(\theta)$, given your private observation of θ? And we ask the consumer: What is your damage from the externality $c = c(\eta)$ given your private observation of η? At first glance, this type of mechanisms may seem to induce firms to underreport their benefits, stating a benefit $\hat{b}$ below their true benefit b, $\hat{b} < b$, in order to reduce the compensation that they have to provide to those consumers affected by the externality. Similarly one might anticipate these mechanisms to provide incentives to consumers to overestimate their damages, stating a cost $\hat{c} > c$, in order to guarantee that the externality is not allowed by the regulator or, if so, they are substantially compensated for the cost they suffer (beyond the true damage they experience from the externality).

The mechanisms we are interested in, instead, focus on providing incentives to all parties to guarantee that a truth-telling equilibrium emerges, that is, on achieving that the firm reports its true benefit from increasing the externality from $x = 0$ to $x = \bar{x}$, whereby $\hat{b} = b$, and the consumer reports the costs he experiences from the externality, $\hat{c} = c$. One such mechanism is the so-called Groves–Clark–Vickrey mechanism (GCV),[34] in which the regulator declares that it will set the level of the externality at $x = \bar{x}$ if the reports received from the firm and the consumers satisfy $\hat{b} > \hat{c}$. (Otherwise, the regulator keeps the level of the externality at $x = 0$.) If this is the case, the government pays $\hat{b}$ to the consumer and charges $\hat{c}$ to the firm. Let us next demonstrate that this simple mechanism induces truth-telling from both parties, by separately analyzing the incentives of every agent.[35]

Consumer

Consider a consumer with real cost $c = c(\eta)$. Let us first examine her optimal announcement, $\hat{c}$, given a firm's announcement of a benefit $\hat{b} > c$; as depicted in figure A9.1.

Note first that this consumer does not have incentives to overreport her cost, $\hat{c} > c$ (as indicated by the arrow at the right-hand side of figure A9.1 illustrating a deviation

34. See the seminal articles of Groves (1973) and Clarke (1971). Moulin (1988) and Green and Laffont (1979) provide a detailed presentation of this mechanism.

35. Though the GCV mechanism induces truth-telling as a weakly dominant strategy, it does not necessarily entail a balanced budget for the authority implementing the public project. In particular, if the profile of marginal benefits and costs satisfies $b > c$, then the GCV mechanism predicts that both agents will truthfully reveal their privately observed information, inducing the regulator to implement the public project, and require the firm to compensate with c dollars the consumer, but paying b dollars to the consumer and ultimately leading to a $b - c$ budget deficit. Some extensions of the GCV mechanism avoid this problem, but only under certain conditions. For more details about this type of mechanisms, see chapter 23 in Mas-Collel et al. (1995).

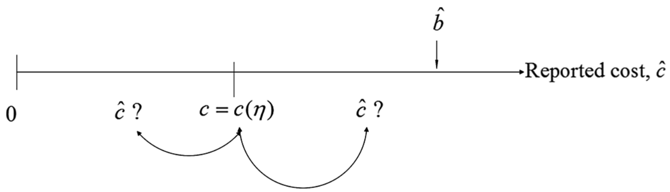

Figure A9.1
GCV mechanism—Consumer's reports when $\hat{b} > c$

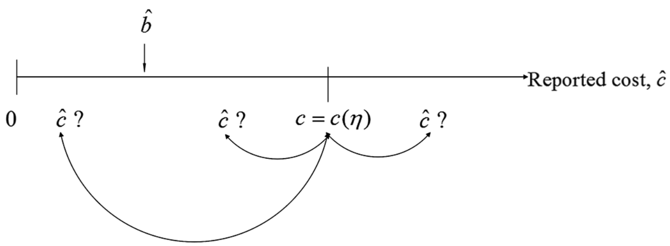

Figure A9.2
GCV mechanism—Consumer's reports when $\hat{b} < c$

from reporting $c = c(\eta)$ to $\hat{c} > c$), or underreport it, $\hat{c} < c$ (left-hand arrow), since in both cases the compensation she receives is $\hat{b}$. Intuitively, the compensation that the consumer receives is unaffected by her report, inducing the consumer to truthfully reveal her cost $c = c(\eta)$. If, instead, the consumer overreports her costs, reporting a level of $\hat{c}$ beyond $\hat{b}$, and the regulator observes $\hat{c} > \hat{b}$, he would determine that the damages of increasing the externality from $x = 0$ to $x = \overline{x}$ are too large, thus deciding to not allow the externality. This is, however, an outcome that yields a lower payoff for the consumer than the outcomes above, whereby a report $\hat{c} = c$ yields a compensation of $\hat{b}$ from the firm.

A similar argument is applicable to the case where $\hat{b} < c$, as depicted in figure A9.2. In particular, overreporting the cost (right-hand side arrow) implies that the government observes reports satisfying $\hat{c} > \hat{b}$, leading the regulator to not allow the increase in the externality level.[36] If, instead, the consumer slightly underreports her cost $\hat{c}$ to points between $\hat{b}$ and $\hat{c}$ (see figure A9.2), the externality is still not allowed by the regulator, given that reports satisfy $\hat{c} > \hat{b}$. Finally, an extreme underreport of her costs

36. The government official would indeed consider that the damage that the externality imposes on the consumer exceeds the profit that the externality provides to the firm.

to values below $\hat{b}$ is not sensible either: while the externality is now allowed (given that reports satisfy $\hat{b} > \hat{c}$), the consumer receives a subsidy $\hat{b}$ below her true cost c, and thus a negative utility level.

In summary, the consumer has incentives to truthfully reveal the damage she suffers from the externality, $\hat{c} = c(\eta)$, regardless of the precise report $\hat{b}$ that the firm makes, such that truthfully reporting her cost is a weakly dominant strategy for the consumer because it yields a weakly larger payoff than reporting a different cost, $\hat{c} \neq c$, regardless of the firm's report $\hat{b}$.

Firm Let us now analyze the firm with a benefit from the externality given by $b = b(\theta)$. Following a similar approach to that for the consumer, let us first consider the case in which the consumer's report, $\hat{c}$, lies below the firm's true benefit from the externality, b, as figure A9.3 illustrates.

The firm has no incentives to overreport its true benefit b, thus making a report $\hat{b}$ such that $\hat{b} > b$, as indicated by the right-hand arrow of figure A9.3. In particular, the firm would have to pay the same compensation to the consumer, $\hat{c}$, and the externality would still be allowed since reports satisfy $\hat{b} > \hat{c}$. Likewise the firm has no incentives to underreport its true benefit by declaring a value of $\hat{b}$ in the interval between $\hat{c}$ and b (see figure A9.3). Indeed the compensation that the firm has to pay is still $\hat{c}$and the externality is allowed, since reports still satisfy $\hat{b} > \hat{c}$. Finally, the firm has no incentives to extremely underreport its true benefit to values below $\hat{c}$, since in this case the externality would not be allowed by the government given that reports satisfy $\hat{b} < \hat{c}$. Let us now consider the case in which, in contrast, the consumer's report $\hat{c}$ lies above the firm's true benefit b, as depicted in figure A9.4.

First, note that the firm would not like the externality to be allowed, since the true benefit that the firm obtains from the externality lies below the cost $\hat{c}$ that the consumer declared to experience (which, in this mechanism, also represents the

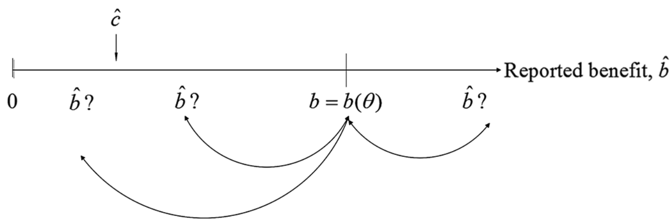

Figure A9.3
GCV mechanism—Firm's reports when $\hat{c}<b$

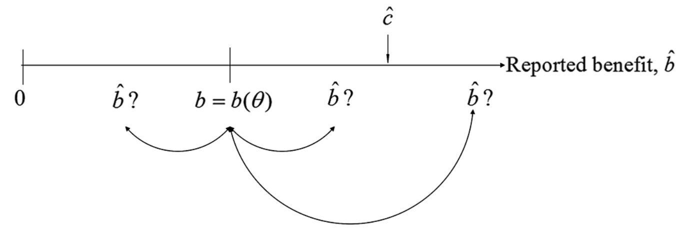

Figure A9.4
GCV mechanism—Firm's reports when $\hat{c}>b$

compensation that the firm must pay consumers). If, rather than truthfully revealing its benefit b from the externality, the firm overreports to values above $\hat{c}$, the externality would be allowed, since reports would satisfy $\hat{b} > \hat{c}$, and the government would ask the firm to pay a compensation $\hat{c}$ to the consumer, which is higher than the real benefit the firm obtains from the externality, $\hat{c} > b = b(\theta)$. Similarly overreporting to values of $\hat{b}$ between b and $\hat{c}$ is insensible as well, given that the externality would be allowed (since reports satisfy $\hat{b} > \hat{c}$), and the firm would still have to pay a compensation to the consumer, $\hat{c}$, above its true benefit, $\hat{c} > b = b(\theta)$. Finally, underreporting (any report $\hat{b}$ such that $\hat{b} < b$) is not strictly profitable for the firm either, since in this case the externality will not be allowed (given that reports would now satisfy $\hat{b} < \hat{c}$), yielding the same payoff for the firm that obtains from truthfully revealing its benefits, $\hat{b} = b = b(\theta)$. As a consequence the firm has no incentives to deviate from a truthful revelation of the benefits it obtains from the externality, so $\hat{b} = b(\theta)$ regardless of the precise report from the consumer, $\hat{c}$. Hence truthfully reporting its benefit from the externality is a weakly dominant strategy for the firm.

Further reading For more references about the use of mechanism design in environmental economics, see the excellent survey by Baliga and Maskin (2003) in the *Handbook of Environmental Economics* and references therein.

Exercises

1. **Externalities and car accidents** Consider an economy with two individuals $i = \{1, 2\}$ with the following quasi-linear utility function:

$$u_i(s^i, q^i) = v^i(s^i) + \alpha w^i,$$

where s^i denotes the speed at which individual i drives his car, w^i is his wealth, and $\alpha > 0$. The utility that individual i obtains from driving fast is $v^i(s^i)$, which is increasing but concave in speed, whereby $\partial v^i(s^i)/\partial s^i > 0$ and $\partial^2 v^i(s^i)/(\partial s^i)^2 < 0$. Driving fast, however, increases the probability of suffering a car accident, represented by $\gamma(s^i, s^j)$. This probability is increasing both in the speed at which individual i drives, s^i, and the speed at which other individuals drive, s^j, where $j \neq i$. Hence the speed of other individuals imposes a negative externality on driver i, since it increases his risk of suffering a car accident. If individual i suffers an accident, he bears a cost of $c^i > 0$, which intuitively embodies the cost of fixing his car, health care expenses, and so on.

a. *Unregulated equilibrium* Set up individual i's expected utility maximization problem. Take first-order conditions with respect to s^i, and denote the (implicit) solution to this first-order condition as $\hat{s}^i$.

b. *Social optimum* Set up the social planner's expected welfare maximization problem. Take first-order conditions with respect to s^1 and s^2. Denote the (implicit) solution to this first-order condition as $\overline{s}^i$.

c. *Comparison* Show that drivers have individual incentives to drive too fast, relative to the socially optimal speed, that is, show that $\hat{s}^i > \overline{s}^i$.

d. *Restoring the social optimum* Let us now evaluate the effect of speeding tickets (fines) to individuals driving too fast, namely to those drivers with a speed $\hat{s}^i$ satisfying, $\hat{s}^i > \overline{s}^i$. What is the dollar amount of the fine m^i that induces every individual i to fully internalize the externality he imposes onto others?

e. Let us now consider that individuals obtain a utility from driving fast, $v^i(s^i)$, only in the case where no accident occurs. Repeat steps a through c , finding the optimal fine m^i that induces individuals to fully internalize the externality.

2. **Pollution and income level** According to the so-called Kuznet's curve, when a country becomes rich, pollution tends to decrease. Although there is mixed empirical evidence supporting and rejecting this hypothesis, let us next study the conditions under which the predictions of this curve can be sustained. Consider an economy with two goods, good 2 being produced from good 1 with production function $y_2 = f(z_1)$, where z_1 is the quantity of good 1 used as input, y_2 is the quantity of good 2 produced and $f(\cdot)$ is a differentiable, increasing, and concave production function. The production of good 2 creates harmful emissions, e, that can be reduced by using additional good 1 in a pollution abatement technology. Thus the level of emissions is described by

$$e = g(y_2, z_e),$$

where z_e is the quantity of good 1 used to reduce pollution and $g(\cdot)$ is a convex and differentiable function, being increasing in output, y_2, and decreasing in abatement efforts, z_e. Assume that consumers have a utility function $u(x_1, x_2, e)$, which is differentiable and concave, increasing in the first two components (consumption of goods 1 and 2) but decreasing in the third component (emissions). The consumer is endowed with w units of good 1, where w represents his income level.

a. Find the socially optimal allocation.

b. Interpret the first-order conditions in terms of marginal costs and benefits.

c. Let us develop a parametric example of the exercise. First, assume that the utility function is

$$u(x_1, x_2, e) = \ln(x_1) + \ln(x_2) - \gamma \ln(e), \text{ where } \gamma < 1 \text{ and } w > 2$$

while the production and emission functions are given by

$$f(z_1) = z_1, \text{ and } g(y_2, z_e) = \frac{ky_2}{1 + z_e}, \text{ where } k > 0$$

Find the Pareto optimal allocation of this economy.

d. Discuss how this Pareto optimal allocation depends on the income level, w.

3. **Positive and negative externalities** Consider an economy with two firms that produce an homogeneous good. Firm 1 produces q_1 units of the good, and its cost function is $c_1(q_1, q_2) = 2q_1^2 + 5q_1 + q_2$, while firm 2 produces q_2 units of the same good and its cost function is $c_2(q_2, q_1) = q_2^2 + 3q_2 - 4q_1$. Note that every firm i's costs depends on its rival's output, q_j, where $j \neq i$. Finally, inverse market demand is given by $p(Q) = 34 - Q$, where $Q = q_1 + q_2$ denotes aggregate output.

a. *Unregulated equilibrium* Considering that every firm independently and simultaneously selects its production level, determine equilibrium output q_1 and q_2. Which are the associated profits for each firm? Measure consumer surplus, profits, and social welfare.

b. *Merger* Assume that the government is aware of these mutual externalities between firm 1 and 2, but does not want to directly regulate their production by the imposition of quotas or fees. Instead, the regulator allows both firms to merge. Determine the equilibrium level of q_1 and q_2 that the newly merged firm will choose, and check if firm 1 and 2 have incentives to merge.

c. *Comparisons* Compare consumer surplus, profits and welfare after the merger (as you found in part b) and before the merger (as found in part a). Does the merger ameliorate the negative externality that the production of firm 2 generates? Does social welfare increase as a result of the merger?

4. **Flexible and inflexible environmental policy** Consider an industry with an incumbent monopolist in period $t=1$ and a duopoly (i.e., the incumbent and an entrant) in period $t=2$. For simplicity, assume that both firms face the same constant marginal cost $c>0$, and a linear inverse demand curve $p(Q)=1-Q$, where Q represent aggregate output. Their output generates an environmental externality given by the convex damage function $ED(Q)=d\cdot Q^2$, where $d>0$. Assume that the social welfare function that the Environmental Protection Agency (EPA) considers is

$$SW = CS + PS + T - ED,$$

where CS (PS) denotes consumer (producer) surplus, respectively, and $T \equiv t\cdot Q$ represents total revenue from emission fees.[37]

a. *Flexible policy* Assume that the EPA can easily adjust emission fees between the first and second period. Find the emission fee it sets to the monopolist in period 1, t_1, and to the duopolists in period 2, t_2.

b. *Inflexible policy* Assume now that the EPA cannot adjust environmental regulation after industry conditions change. Such inflexible policy may be due to the institutional setting requiring that changes in environmental regulation be approved by the Congress. Find the unique emission fee t that the EPA sets across both time periods. [*Hint*: The EPA anticipates that such a policy will generate inefficiencies in one (or both) periods, but seeks to minimize the sum of such inefficiencies. For simplicity, assume no time discounting.]

c. *Comparison.* Compare the flexible emission fees you found in part a with the inflexible fee found in part b. Interpret.

5. **Regulating externalities under incomplete information** Consider a polluting firm with profit function $\pi(q)=10q-q^2$, where q denotes units of the externality-generating activity (e.g., q can represent units of output if each unit generates one unit of pollution). Pollution damage to consumers is given by the convex damage function $d(q)=3q^2$. Let us analyze a context in which the regulator does not observes the firm's profit function but observes the damage that such additional pollution causes on consumers. In particular, the regulator estimates that marginal profits are

$$\frac{\partial\pi(q,a)}{\partial q}=10-2aq,$$

37. This exercise is based on Espinola-Arredondo et al. (2014), who extend the model to a setting of incomplete information between firms in order to evaluate whether environmental regulation can entail more entry-deterring effects when such policy is flexible or inflexible.

where the random parameter a takes two equally likely values, $a=1$ or $a=\frac{1}{2}$. (Note that in our description we assume that the firm privately observes that the realization of parameter a is $a=1$, thus yielding a marginal profit function of $10-2q$.) We will first determine which are the best quota and emission fee that the regulator can design, given that he operates under incomplete information. Afterward we will evaluate the welfare that arises under each of these policy instruments, to determine which is better from a social point of view.

a. *Unregulated equilibrium* Find the equilibrium amount of pollution, q^E, if the firm is unregulated and no bargaining occurs between the affected consumers and the firm.

b. *Setting a quota* In this incomplete information setting, determine which is the best quota x_q that a social planner can select in order to maximize the expected value of aggregate surplus.

c. *Setting an emission fee* Find the best tax t^* that this social planner can set under the context of incomplete information described above.

d. *Policy comparison* Compare the emission fee and the quota in terms of their associated deadweight loss. Under which conditions an uninformed regulator prefers to choose the emission fee?

6. **The problem of the commons** Let us assume that Lake Washington can be freely accessed by fishermen. The cost of sending a boat out on the lake is $r>0$. When b boats are sent out onto the lake, $f(b)$ fish are caught in total (for simplicity, assume that each boat catches an equal share of $f(b)/b$ fish), where total production $f(b)$ is increasing and concave in the number of boats, meaning $f'(b)>0$ and $f''(b)<0$ for all $b\geq 0$. The international price of fish is $p\geq 0$, which is unaffected by the level of catch in Lake Washington, since this catch represents a negligible share of world catches of this specific fish.

a. *Unregulated equilibrium* Characterize the equilibrium number of boats that are sent out on the lake.

b. *Social optimum* Characterize the optimal number of boats that should be sent out on the lake.

c. Compare your answers in parts a and b. Explain.

d. Suppose instead that the lake is owned by a single individual who can choose how many boats to send out. What number of boats would this owner choose?

7. **Entry in the commons** Consider a common pool resource initially operated by a single firm during two periods, appropriating x_i units in the first period and q_i

units in the second period. In particular, assume that its first-period cost function is x_i^2/θ, where $\theta > 0$, while second-period cost function is

$$\frac{q_i^2}{\theta-(1-\beta)x_i}.$$

Intuitively, parameter θ reflects the initial abundance of stock; that is, a large θ decreases the firms' first and second-period costs, while β denotes the regeneration rate of the resource. Hence, if regeneration is complete, $\beta=1$, first- and second-period costs coincide, but if regeneration is null, $\beta=0$, second period costs become $q_i^2/(\theta-x_i)$ and thus every unit of first-period appropriation x_i increases the firm's second-period costs. For simplicity, assume that every unit of output is sold at a price of $1 at the international market.[38]

a. Assuming no entry during both periods (i.e., the incumbent operates alone in both periods), find the profit-maximizing second-period appropriation, q_i^{NE}, and its first-period appropriation, x_i^{NE}, where superscript *NE* denotes no entry. [*Hint*: Use backward induction.]

b. Assume that entry occurs in the second period, and that the second-period cost function for both incumbent and entrant becomes $(q_i+q_j)q_i/(\theta-(1-\beta)x_i)$. Find the profit-maximizing second-period appropriation, q_i^E and q_j^E, and first-period appropriation, x_i^E, where superscript *E* denotes entry.

8. **Private contributions to a public good** Consider an economy with two consumers, Alessandro and Beatrice, $i=\{A,B\}$, one private good x, and one public good G. Let each consumer have an income M. For simplicity, let the prices of both the public and private good to be 1. Additionally the utility functions of consumer A and B are

$$U^A=\log(x^A)+\log(G) \quad \text{for individual } A,$$

$$U^B=\log(x^B)+\log(G) \quad \text{for individual } B.$$

Assume that the public good G is only provided by the contributions of these two individuals, that is, $G=g^A+g^B$.

a. Find Alessandro's best-response function. Depict it in a figure with his contribution, g^A, on the vertical axis and Beatrice's contribution, g^B, on the horizontal axis.

38. This exercise is based on Espinola-Arredondo and Munoz-Garcia (2013b). The exercise, however, focuses on a complete information setting, whereas the article examines how the presence of incomplete information affects equilibrium appropriation, and ultimately welfare levels.

b. Identify Beatrice's best-response function. Depict it in a figure with her contribution, g^B, on the horizontal axis and Alessandro's contribution, g^A, on the vertical axis.

c. *Unregulated equilibrium* Find the equilibrium contributions to the public good by Alessandro and Beatrice, that is, the Nash equilibrium of this public good game.

d. *Social optimum* Find the efficient (socially optimal) contribution to the public good by Alessandro and Beatrice.

e. Use a figure to contrast the Pareto efficient level of private provision and the Nash equilibrium level of provision.

9. Voluntary contributions to a public good with Cobb–Douglas preferences Consider a setting with N individuals, each of them simultaneously and independently deciding how many dollars to contribute to a public good. Assume that each individual has a Cobb–Douglas utility function $u(x_i, G) = x_i^{1-\alpha} G^{\alpha}$ where $G = \sum_{j=1}^{n} g_j$ denotes aggregate contributions and $\alpha \in (0, 1)$ for all $i = 1, \dots, N$. For simplicity, normalize the price of the public good.

a. Set up the utility maximization problem of agent i. Find the demand functions denoted $(x_i(\cdot), G(\cdot))$, for the private and public good.

b. Suppose that individuals are ranked according to wealth, whereby $\omega_1 \geq \omega_2 \geq \ldots \geq \omega_n$. Find conditions on ω_i and α for an equilibrium in which $g_2^* = \ldots = g_n^* = 0$ and agent 1 is the only contributor (only the richest individual contributes).

c. Let G_k denote aggregate donations in equilibrium when the total wealth W is divided equally among k individuals.

i. Suppose first that we divide the wealth W among 2 individuals. Find aggregate donations in this case, G_2, and show that they are lower than aggregate donations when a single individual holds all the wealth, whereby $G_2 < G_1$

ii. More generally, suppose that the wealth is divided into k equal shares W/k among k consumers. Compute the equilibrium value of G_k and show that $G_k \to 0$ when $k \to +\infty$. (The smallest amount of public production is supplied when everyone is a contributor).

10. Production and externalities According to some residents, a firm's production of paper at Lewiston, Idaho, generates a smelly gas as an unpleasant side product. Let $c(y, m; \mathbf{w})$ denote the (minimum) input cost of producing y tons of paper and m cubic meters of gas, where input prices are given by the vector $\mathbf{w} \gg \mathbf{0}$. Let $p > 0$

denote the market price of paper. Assume that the cost function satisfies $\partial c/\partial y>0$ and $\partial c/\partial m<0$, and that $c(y,m;\mathbf{w})$ is strictly convex in y and m. Let stars * denote solutions and assume throughout that the firm produces positive amounts of paper $y^*>0$.

a. Show that the cost function $c(y,m;\mathbf{w})$ is concave in input prices, $\mathbf{w}$.

b. *Setting a quota* Suppose that the government imposes a ceiling on gas emissions such that $m\leq\bar{m}$, (a quota). Assuming that this constraint binds, write down the firm's profit maximization problem with respect to y, and find necessary and sufficient conditions for the firm's cost-minimizing production, y^*.

c. *Comparative statics* Under which condition on the cost function $c(y,m;\mathbf{w})$ can we guarantee that an increase in the ceiling on gas emissions, $\bar{m}$, produces a raise in the firm's cost-minimizing production, y^*, whereby $\partial y^*/\partial\bar{m}>0$?

d. *Emission fee* Suppose now that the government abandons its emissions ceiling and replaces it with a tax $t>0$ on gas emissions. Thus the new cost of producing (y,m) is given by $c(y,m;\ \mathbf{w})+tm$. Show that maximized profits are convex in t, and that the firm's choice of pollution decreases in the pollution tax, that is, $\partial m^*/\partial t\leq 0$.

11. Externalities in consumption Consider two consumers with utility functions over two goods, x_1 and x_2, given by

$$u_A=\log(x_1^A)+x_2^A-\frac{1}{2}\log(x_1^B)\quad\text{for consumer } A,$$

$$u_B=\log(x_1^B)+x_2^B-\frac{1}{2}\log(x_1^A)\quad\text{for consumer } B,$$

where the consumption of good 1 by individual $i=\{A,B\}$ creates a negative externality on individual $j\neq i$ (see the third term, which enters negatively on each individual's utility function). For simplicity, consider that both individuals have the same wealth, m, and that the price for both goods is 1.

a. *Unregulated equilibrium* Set up consumer A's utility maximization problem, and determine his demand for goods 1 and 2, as x_1^A and x_2^A. Then operate similarly to find consumer B's demand for good 1 and 2, as x_1^B and x_2^B.

b. *Social optimum* Calculate the socially optimal amounts of x_1^A, x_2^A, x_1^B, and x_2^B, considering that the social planner maximizes a utilitarian social welfare function, namely $W=U_A+U_B$.

c. *Restoring efficiency* Show that the social optimum you found in part b can be induced by a tax on good 1 (so the after-tax price becomes $1+t$) with the revenue returned equally to both consumers in a lump-sum transfer.[39]

12. **The Porter hypothesis (based on Porter and van der Linde 1995)** Consider two symmetric firms competing à la Cournot with inverse demand $p(Q)=1-Q$ and constant marginal costs $c<1$. Before their production decision each firm can simultaneously and independently invest in a green plant at a cost of $k>0$.

    a. If firms are unregulated (no taxes or subsidies), show that investing is a strictly dominated strategy for both firms.

    b. Assume that the government imposes a fine of $\$T$ on those firms that did not invest in the green plant. Under which conditions of T both firms invest in clean technologies?

    c. Let us now alter the setting in part b by assuming that, if a firm does not invest in green technologies its marginal production cost is zero, while if it invests in green technologies its marginal cost is $c<1$.

13. **Reference points in public good games** Consider a sequential public good game where a first-mover (player 1) is asked to submit a donation, $g_1 \in [0,1]$, for the provision of a public good, and observing her donation, a follower (player 2) responds selecting his own contribution, $g_2 \in [0,1]$. In particular, leader and follower's utility functions are

$$u_1(g_1, g_2) = w - g_1 + [m(g_1 + g_2)]^{0.5},$$

$$u_2(g_1, g_2) = w - g_2 + \left[m(g_1 + g_2)\left(1 + \alpha\left(g_1 - g_1^R\right)\right)\right]^{0.5}.$$

Both of these functions are linear in money, w. The nonlinear part of their utility function takes into account the utility derived from the total public good provision $G=g_i+g_j$ (relevant for both players) but for the follower also considers the distance $\alpha\left(g_1 - g_1^R\right)$, which compares the first-mover's actual donation against a reference point, g_1^R. For simplicity, assume that the follower uses the same reference contribution g_1^R to evaluate all donations of the leader. Finally, $m \geq 0$ denotes the return every player obtains from total contributions to the public good. In particular, note that when $\alpha=0$, the follower only cares about private and public good

39. As in sections 9.4 and 9.5 on a polluting monopoly or oligopoly subject to emission fees, we assume that tax revenue is entirely returned to the agents being taxed as a lump-sum transfer. This assumption guarantees that the tax is revenue neutral, but it helps modify agents' incentives, ultimately correcting the externality and inducing the social optimum.

consumption. However, when $\alpha > 0$, he experiences a higher utility from contributing to the public good when the leader's donation is higher than the reference point, $g_1 > g_1^R$, but a lower utility otherwise, $g_1 < g_1^R$.

a. Find the follower's best response function, $g_2(g_1, g_1^R)$, and explain how it is affected by changes in his reference point g_1^R.

b. Find the leader's equilibrium donation in this sequential public good game. Under which conditions such donation is strictly positive? Interpret.

14. Social planner preferring Cournot or Bertrand competition? Consider an industry with n symmetric firms, each facing a constant marginal cost $c > 0$ and inverse demand function $p(Q) = 1 - Q$, where $1 > c$. Additionally firms' production generates a linear environmental externality (damage) measured by $ED(Q) = d \times Q$.

a. Assuming that firms compete à la Cournot, find their equilibrium individual and aggregate output, the equilibrium profits, the associated consumer surplus, and overall social welfare.

b. Assuming that firms compete à la Bertrand, find their equilibrium individual and aggregate output, the equilibrium profits, the associated consumer surplus, and overall social welfare.

c. Compare the social welfare arising when firms compete à la Cournot (found in part a) and à la Bertrand (found in part b). Under which conditions does the social planner prefer that firms compete à la Cournot? Interpret.

References

Andreoni, J. 1990. Impure altruism and donations to public goods: A theory of warm glow giving. *Economic Journal* 100: 464–77.

Baliga, S., and E. Maskin. 2003. Mechanism design for the environment. In *Handbook of Environmental Economics*. vol. 1, ed. K. G. Mäler and J. R. Vincent. Amsterdam: Elsevier.

Bergstrom, T., L. Blume, and H. Varian. 1986. On the private provision of public goods. *Journal of Public Economics* 29: 25–49.

Bolton, G. E., and A. Ockenfels. 2000. ERC: A theory of equity, reciprocity, and competition. *American Economic Review* 90: 166–93.

Buchanan, J. 1969. External diseconomies, corrective taxes and market structure. *American Economic Review* 59: 174–77.

Chaudhuri, A. 2011. Sustaining cooperation in laboratory public goods experiments: A selective survey of the literature. *Experimental Economics* 14 (1): 47–83.

Clarke, E. H. 1971. Multipart pricing of public goods. *Public Choice* 11: 17–33.

Coase, R. 1960. The problem of social cost. *Journal of Law and Economics* III: 1–44.

Cooper, R., D. V. DeJong, R. Forsythe, and T. W. Ross. 1996. Cooperation without reputation: Experimental evidence from prisoner's dilemma games. *Games and Economic Behavior* 12: 187–218.

Davis, O. A., and A. Whinston. 1962. Externalities, welfare and the theory of games. *Journal of Political Economy* 70 (3): 241–62.

Duffy, J., and F. Munoz-Garcia. 2015. Cooperation and signaling with uncertain social preferences. *Theory and Decision* 78 (1): 45–75.

Espinola-Arredondo, A., and F. Munoz-Garcia. 2013a. When does environmental regulation facilitate entry-deterring practices? *Journal of Environmental Economics and Management* 65 (1): 133–52.

Espinola-Arredondo, A., and F. Munoz-Garcia. 2013b. Asymmetric information may protect the commons: The welfare benefits of uninformed regulators. *Economics Letters* 121: 463–66.

Espinola-Arredondo, A., F. Munoz-Garcia, and J. Bayham. 2014. The entry-deterring effects of inflexible regulation. *Canadian Journal of Economics* 47 (1): 298–324.

Faysse, N. 2005. Coping with the tragedy of the commons: Game structure and design of rules. *Journal of Economic Surveys* 19 (2): 239–61.

Fehr, E., and K. Schmidt. 1999. A theory of fairness, competition and cooperation. *Quarterly Journal of Economics* 114: 817–68.

Green, J., and J.-J. Laffont. 1979. *Incentives in Public Decision Making*. Amsterdam: North-Holland.

Groves, T. 1973. Incentives in teams. *Econometrica* 41: 617–31.

Harrison, G. W., and M. McKee. 1985. Experimental evaluation of the Coase theorem. *Journal of Law and Economics* 28 (3): 653–70.

Hoffman, E., and M. Spitzer. 1982. The Coase theorem: Some experimental tests. *Journal of Law and Economics* 25 (1): 73–98.

Hoffman, E., and M. Spitzer. 1985. Entitlements, rights, and fairness: An experimental examination of subjects' concepts of distributive justice. *Journal of Legal Studies* 15: 254–97.

Kahneman, D., J. L. Knetsch, and R. H. Thaler. 1990. Experimental tests of the endowment effect and the Coase theorem. *Journal of Political Economy* 98 (6): 1325–48.

Koldstad, C. D. 2011. *Environmental Economics*, 2nd ed. New York: Oxford University Press.

Lindahl, E. 1919. Just taxation—A positive solution. In *Classics in the Theory of Public Finance (1958)*, ed. R. A. Musgrave and A. T. Peacock. London: Macmillan.

Ledyard, J. O. 1995. Public goods: A survey of experimental research. In *The Handbook of Experimental Economics*, ed. John H. Kagel and Alvin E. Roth. Princeton: Princeton University Press.

Mas-Colell, A., M. Whinston, and J. Green. 1995. *Microeconomic Theory*. New York: Oxford University Press.

Medema, S. G., and R. O. Zerbe, Jr. 1999. The Coase theorem. In *Encyclopedia of Law and Economics*, ed. B. Bouckaert and G. De Geest. Cheltenham, UK: Edward Elgar.

Montero, J.-P. 2008. A simple auction mechanism for the optimal allocation of the commons. *American Economic Review* 98 (1): 496–518.

Moulin, H. 1988. *Axioms of Cooperative Decision Making*. Cambridge, UK: Cambridge University Press.

Munoz-Garcia, F. 2011. Competition for status acquisition in public good games. *Oxford Economic Papers* 63:549–67.

Ostrom, E. 1990. *Governing the Commons*. Cambridge, UK: Cambridge University Press.

Pigou, A. C. 1920. *The Economics of Welfare*. London: Macmillan.

Porter, M. E., and C. van der Linde. 1995. Toward a new conception of the environment-competitiveness relationship. *Journal of Economic Perspectives* 9: 97–118.

Samuelson, P. A. 1954. The pure theory of public expenditure. *Review of Economics and Statistics* 36 (4): 386–89.

Starrett, D. A. 2003. Property rights, public goods, and the environment. *Handbook of Environmental Economics* (K.-G. Mäler and J. R. Vincent, eds.). North Holland, Amsterdam, 97–125.

Stavins, R. 1996. Correlated uncertainty and policy instrument choice. *Journal of Environmental Economics and Management* 30 (2): 218–32.

Tietenberg, T. H. 1973. Controlling pollution by price and standard systems: A general equilibrium analysis. *Swedish Journal of Economics* 75: 193–203.

Vesterlund, L. 2014. Charitable giving: A review of experiments on voluntary giving to public goods. In *Handbook of Experimental Economics*. vol. 2, ed. C.R. Plott and V. L. Smith. Princeton: Princeton University Press.

Weitzman, M. L. 1974. Prices vs. quantities. *Review of Economic Studies* 41 (4): 477–91.

Weitzman, M. L. 1978. Optimal rewards for economic regulation. *American Economic Review* 68 (4): 683–91.