General Equilibrium

General Equilibrium

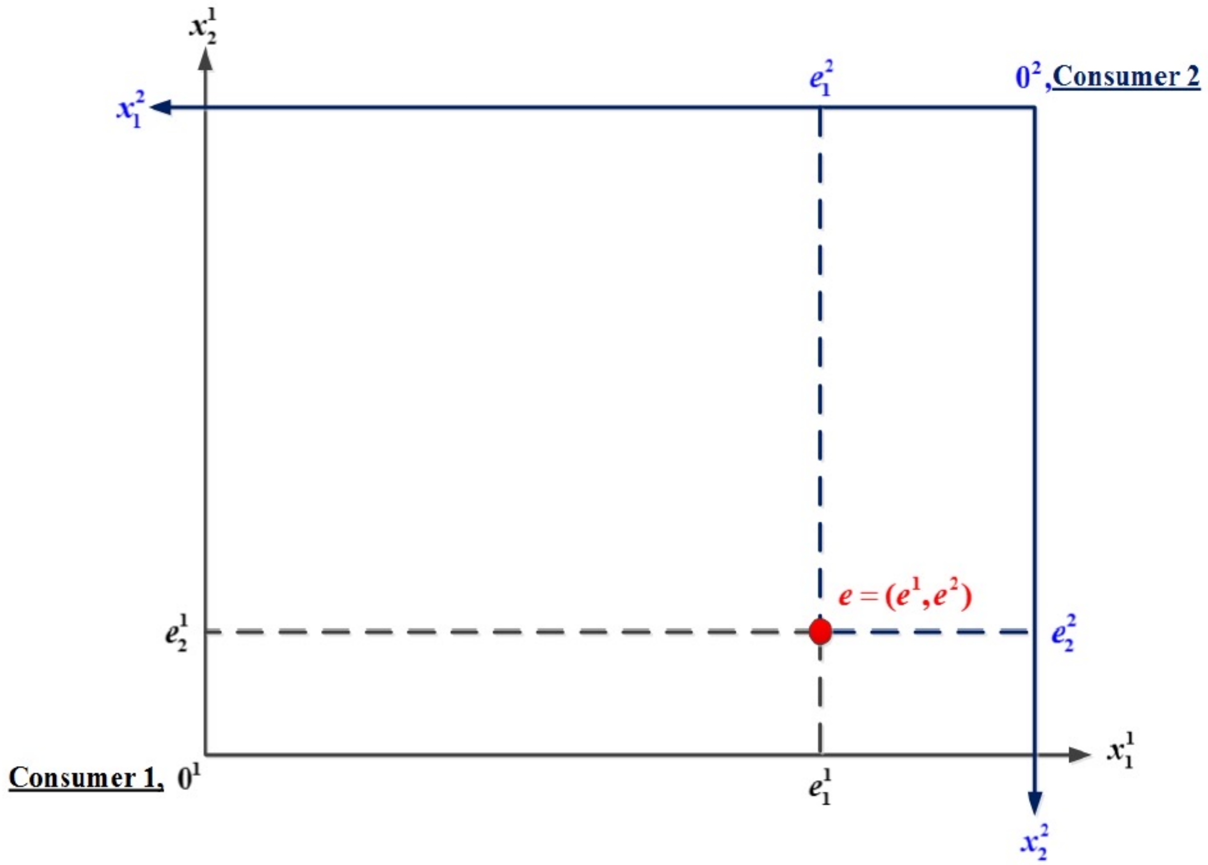
- So far, we explored equilibrium conditions in a single market with a single type of consumer.
- Now we examine settings with markets for different goods and multiple consumers.

General Equilibrium: No Production

- Consider an economy with two goods and two consumers, $i = \{1,2\}$.
- Each consumer is initially endowed with $\mathbf{e}^i \equiv (e_1^i, e_2^i)$ units of good 1 and 2.
- Any other allocations are denoted by $\mathbf{x}^i \equiv (x_1^i, x_2^i)$.

General Equilibrium: No Production

- ***Edgeworth box***:

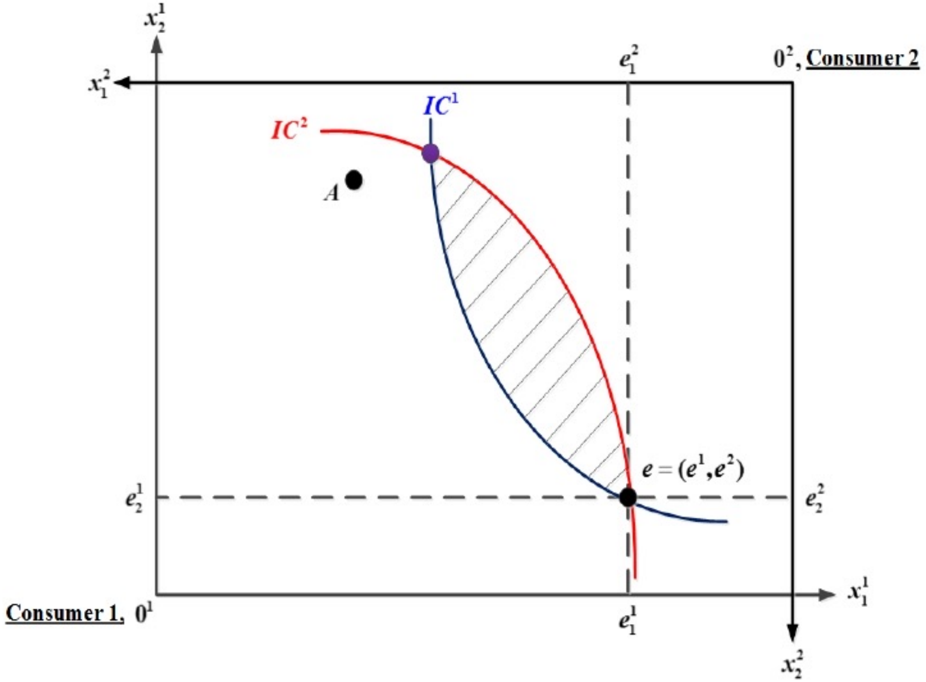


General Equilibrium: No Production

- IC^i is the indifference curve of consumer i, which passes through his endowment point $\mathbf{e}^i$.
- The shaded area represents the set of bundles (x_1^i, x_2^i) for consumer i satisfying

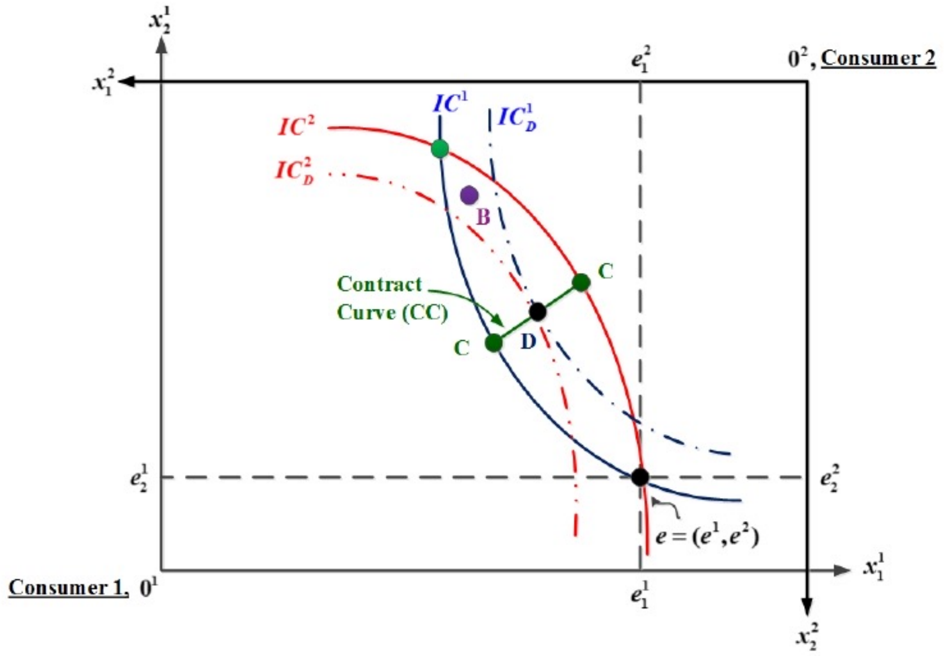
$$u^1(x_1^1, x_2^1) \geq u^1(e_1^1, e_2^1)$$
$$u^2(x_1^2, x_2^2) \geq u^2(e_1^2, e_2^2)$$

- Bundle A cannot be a barter equilibrium:
  - Consumer 1 does not exchange $\mathbf{e}$ for A.



General Equilibrium: No Production

- Not all points in the lens-shaped area is a barter equilibrium!
- Bundle B lies inside the lens-shaped area
  - Thus, it yields a higher utility level than the initial endowment **e** for both consumers.
- Bundle D, however, makes both consumers better off than bundle B.
  - It lies on "***contract curve***," in which the indifference curves are tangent to one another.
  - It is an equilibrium, since Pareto improvements are no longer possible



General Equilibrium: No Production

- ***Feasible allocation***:
  - An allocation $\mathbf{x} \equiv (\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^I)$ is *feasible* if it satisfies

$$\sum_{i=1}^{I} \mathbf{x}^i \leq \sum_{i=1}^{I} \mathbf{e}^i$$

  - That is, the aggregate amount of goods in allocation $\mathbf{x}$ does not exceed the aggregate initial endowment $\mathbf{e} \equiv \sum_{i=1}^{I} \mathbf{e}^i$.

General Equilibrium: No Production

- ***Pareto-efficient allocations***:
  - A feasible allocation $\mathbf{x}$ is *Pareto efficient* if there is no other feasible allocation $\mathbf{y}$ which is weakly preferred by all consumers, i.e.,$\mathbf{y}^i \succsim \mathbf{x}^i$ for all $i \in I$, and at least strictly preferred by one consumer, $\mathbf{y}^i \succ \mathbf{x}^i$.
  - That is, allocation $\mathbf{x}$ is Pareto efficient if there is no other feasible allocation $\mathbf{y}$ making all individuals at least as well off as under $\mathbf{x}$ and making one individual strictly better off.

General Equilibrium: No Production

- ***Pareto-efficient allocations***:
  - The set of Pareto efficient allocations $(\mathbf{x}^1, \ldots, \mathbf{x}^I)$ solves

$$\max_{\mathbf{x}^1, \ldots, \mathbf{x}^I \geq 0} u^1(\mathbf{x}^1)$$
$$\text{s.t.} \quad u^j(\mathbf{x}^j) \geq u^{-j} \text{ for } j \neq 1, \text{ and}$$
$$\sum_{i=1}^{I} \mathbf{x}^i \leq \sum_{i=1}^{I} \mathbf{e}^i \quad \text{(feasibility)}$$

  where $\mathbf{x}^i = (x_1^i, x_2^i)$.

  - That is, allocations $(\mathbf{x}^1, \ldots, \mathbf{x}^I)$ are Pareto efficient if they maximizes individual 1's utility without reducing the utility of all other individuals below a given level u^{-j}, and satisfying feasibility.

General Equilibrium: No Production

– The Lagrangian is

$$L(\mathbf{x}^1, \ldots, \mathbf{x}^I; \lambda^2, \ldots, \lambda^I, \mu) =$$
$$u^1(\mathbf{x}^1) + \lambda^2[u^2(\mathbf{x}^2) - u^{-2}] + \cdots$$
$$+\lambda^I[u^I(\mathbf{x}^I) - u^{-I}] + \mu\left[\sum_{i=1}^I \mathbf{e}^i - \sum_{i=1}^I \mathbf{x}^i\right]$$

– FOC wrt $\mathbf{x}^1 = (x_1^1, x_2^1)$ yields

$$\frac{\partial L}{\partial x_k^1} = \frac{\partial u^1(\mathbf{x}^1)}{\partial x_k^1} - \mu \leq 0$$

for every good k of consumer 1.

– For any individual $j \neq 1$, the FOCs become

$$\frac{\partial L}{\partial x_k^j} = \lambda^j \frac{\partial u^j(\mathbf{x}^j)}{\partial x_k^1} - \mu \leq 0$$

General Equilibrium: No Production

– FOCs wrt λ^j and μ yield $u^j(\mathbf{x}^j) \geq u^{-j}$ and $\sum_{i=1}^{I} \mathbf{x}^i \leq \sum_{i=1}^{I} \mathbf{e}^i$, respectively.

– In the case of interior solutions, a compact condition for Pareto efficiency is

$$\frac{\frac{\partial u^1(\mathbf{x}^1)}{\partial x_k^1}}{\frac{\partial u^1(\mathbf{x}^1)}{\partial x_2^1}} = \frac{\frac{\partial u^j(\mathbf{x}^j)}{\partial x_k^1}}{\frac{\partial u^j(\mathbf{x}^j)}{\partial x_2^j}} \text{ or } MRS_{1,2}^1 = MRS_{1,2}^j$$

for every consumer $j \neq 1$.

– Graphically, consumers' indifference curves become tangent to one another at the Pareto efficient allocations.

General Equilibrium: No Production

- ***Example 6.5*** (Pareto efficiency):
  - Consider a barter economy with two goods, 1 and 2, and two consumers, A and B, each with the initial endowments of $\mathbf{e}^A = (100{,}350)$ and $\mathbf{e}^B = (100{,}50)$, respectively.
  - Both consumers' utility function is a Cobb-Douglas type given by $u^i(x_1^i, x_2^i) = x_1^i x_2^i$ for all individual $i = \{A, B\}$.
  - Let us find the set of Pareto efficient allocations.

General Equilibrium: No Production

- ***Example*** (continued):
  - Pareto efficient allocations are reached at points where the $MRS^A = MRS^B$. Hence,

$$MRS^A = MRS^B \Longrightarrow \frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} \text{ or } x_2^A x_1^B = x_2^B x_1^A$$

  - Using the feasibility constraints for good 1 and 2, i.e.,

$$e_1^A + e_1^B = x_1^A + x_1^B$$
$$e_2^A + e_2^B = x_2^A + x_2^B$$

  we obtain

$$x_1^B = e_1^A + e_1^B - x_1^A$$
$$x_2^B = e_2^A + e_2^B - x_2^A$$

General Equilibrium: No Production

- ***Example*** (continued):
  - Combining the tangency condition and feasibility constraints yields

$$x_2^A \underbrace{(e_1^A + e_1^B - x_1^A)}_{x_1^B} = \underbrace{(e_2^A + e_2^B - x_2^A)}_{x_2^B} x_1^A$$

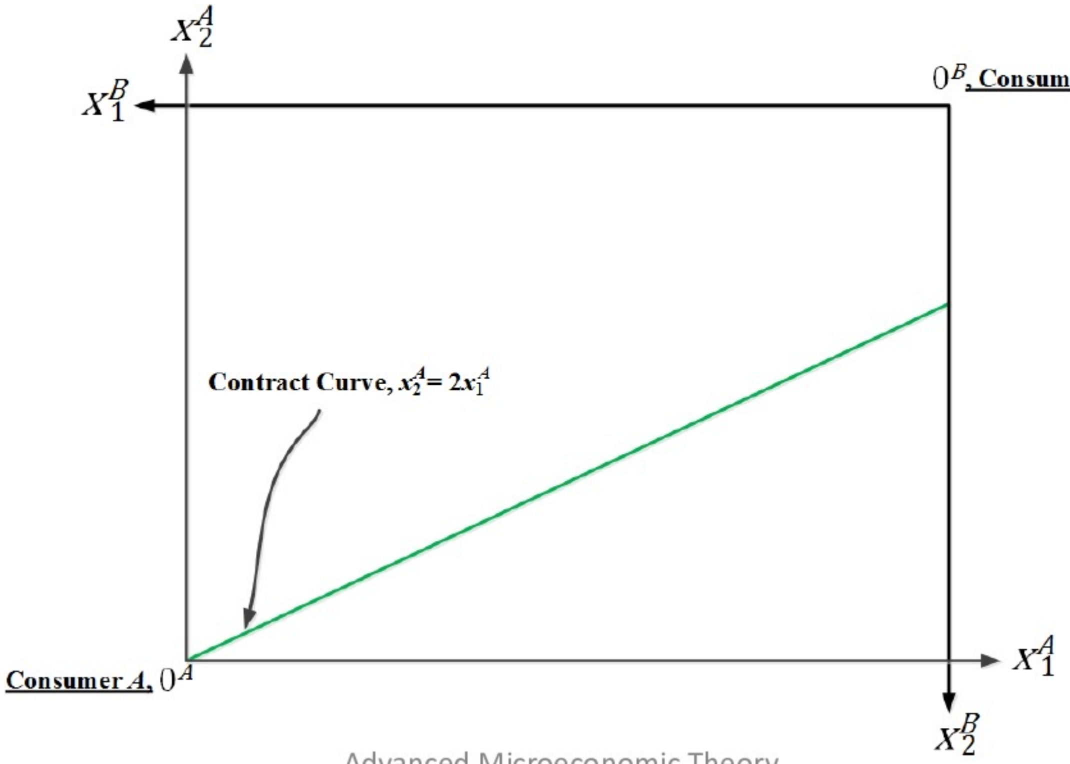
which can be re-written as

$$x_2^A = \frac{e_2^A + e_2^B}{e_1^A + e_1^B} x_1^A = \frac{350+50}{100+100} x_1^A = 2x_1^A$$

for all $x_1^A \in [0,200]$.

General Equilibrium: No Production

- ***Example*** (continued):
  - The line representing the set of Pareto efficient allocations



General Equilibrium: No Production

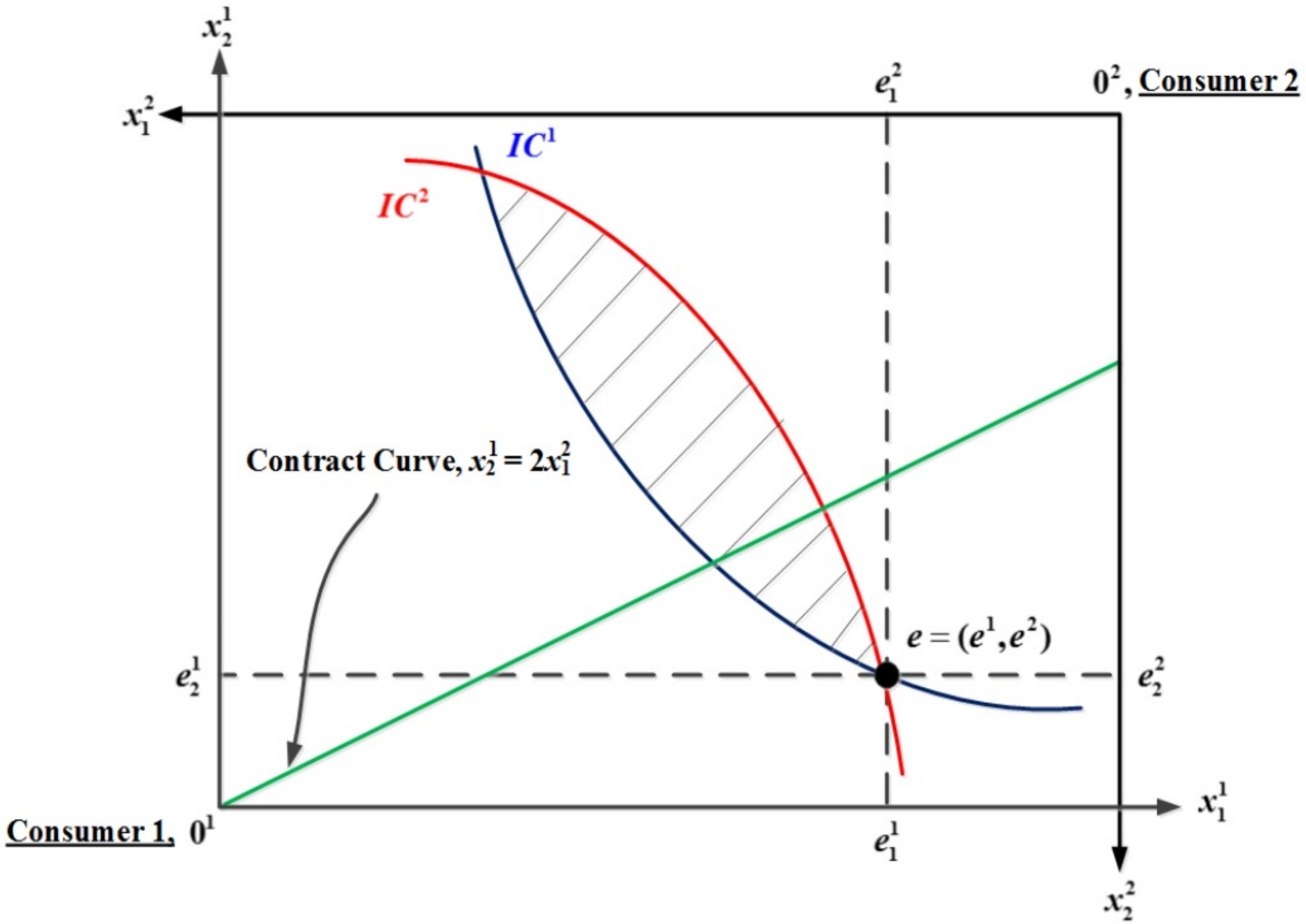
- ***Blocking coalitions***: Let $S \subset I$ denote a coalition of consumers. We say that S *blocks* the feasible allocation $\mathbf{x}$ if there is an allocation $\mathbf{y}$ such that:

  1) *Allocation is feasible for S.* The aggregate amount of goods that individuals in S enjoy in allocation $\mathbf{y}$ coincides with their aggregate initial endowment, i.e., $\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i$; and
  2) *Preferable*. Allocation $\mathbf{y}$ makes all individuals in the coalition weakly better off than under $\mathbf{x}$, i.e., $\mathbf{y}^i \gtrsim \mathbf{x}^i$ where $i \in S$, but makes at least one individual strictly better off, i.e., $\mathbf{y}^i \succ \mathbf{x}^i$.

General Equilibrium: No Production

- ***Equilibrium in a barter economy***: A feasible allocation $\mathbf{x}$ is an equilibrium in the exchange economy with initial endowment $\mathbf{e}$ if $\mathbf{x}$ is *not blocked* by any coalition of consumers.
- ***Core***: The core of an exchange economy with endowment $\mathbf{e}$, denoted $C(\mathbf{e})$, is the set of all *unblocked* feasible allocations $\mathbf{x}$.
  - Such allocations:
    a) mutually beneficial for all individuals (i.e., they lie in the lens-shaped area)
    b) do not allow for further Pareto improvements (i.e., they lie in the contract curve)

General Equilibrium: No Production



General Equilibrium: Competitive Markets

- Barter economy did not require prices for an equilibrium to arise.
- Now we explore the equilibrium in economies where we allow prices to emerge.
- Order of analysis:
  - consumers' preferences
  - the excess demand function
  - the equilibrium allocations in competitive markets (i.e., *Walrasian equilibrium allocations*)

General Equilibrium: Competitive Markets

- ***Consumers***:
  - Consider consumers' utility function to be continuous, strictly increasing, and strictly quasiconcave in $\mathbb{R}^n_+$.
  - Hence the UMP of every consumer i, when facing a budget constraint

    $$\mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i \text{ for all price vector } \mathbf{p} \gg \mathbf{0}$$

    yields a unique solution, which is the Walrasian demand $\mathbf{x}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$.
  - $\mathbf{x}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$ is continuous in the price vector $\mathbf{p}$.

General Equilibrium: Competitive Markets

– Intuitively, individual i's income comes from selling his endowment $\mathbf{e}^i$ at market prices $\mathbf{p}$, producing $\mathbf{p} \cdot \mathbf{e}^i = p_1 e_1^i + \cdots + p_k e_k^i$ dollars to be used in the purchase of allocation $\mathbf{x}^i$.

General Equilibrium: Competitive Markets

- ***Excess demand***:
  - Summing the Walrasian demand $\mathbf{x}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$ for good k of every individual in the economy, we obtain the *aggregate demand* for good k.
  - The difference between the aggregate demand and the aggregate endowment of good k yields the *excess demand of good* k:

$$z_k(\mathbf{p}) = \sum_{i=1}^{I} x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \sum_{i=1}^{I} e_k^i$$

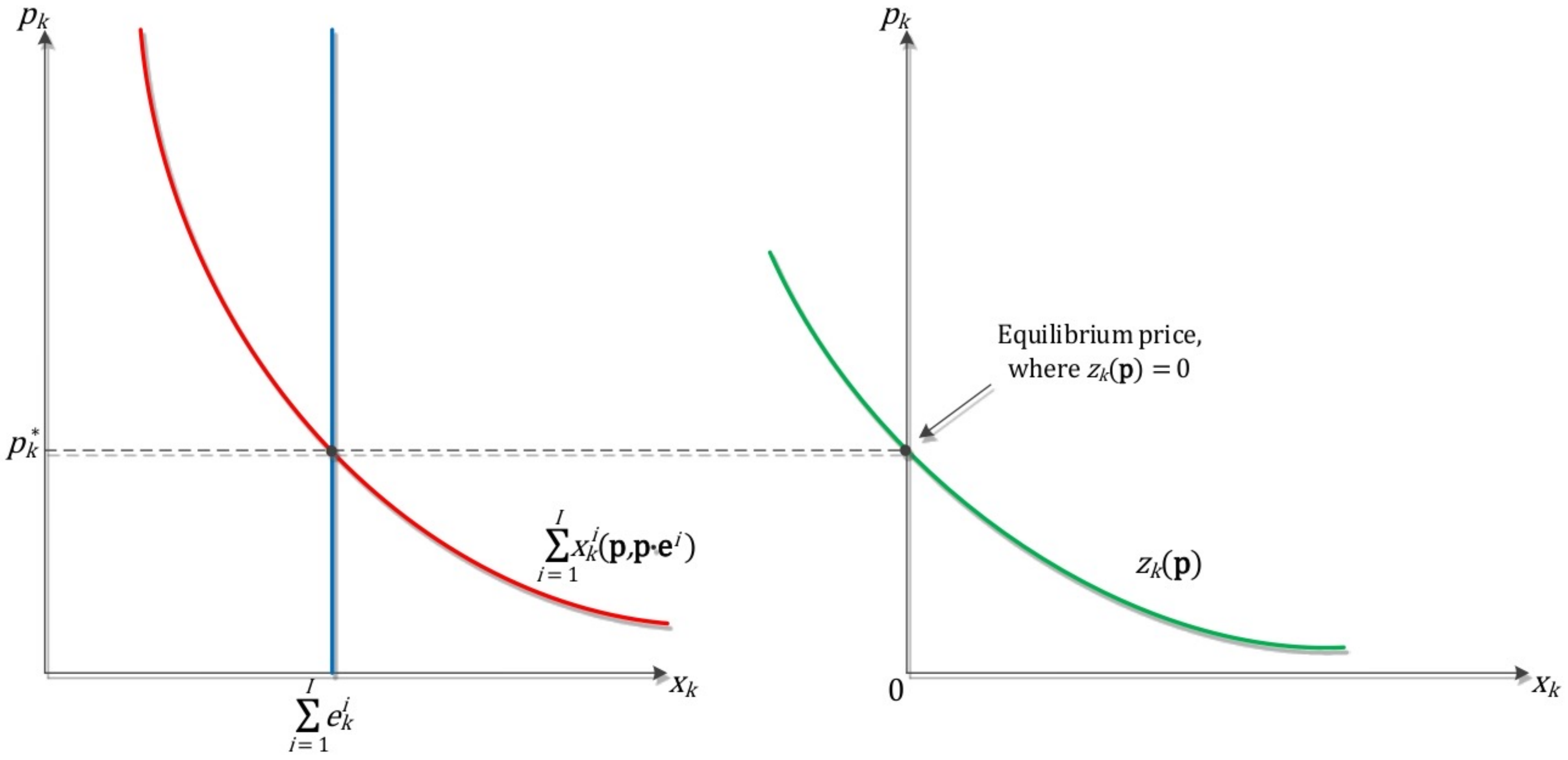
where $z_k(\mathbf{p}) \in \mathbb{R}$.

General Equilibrium: Competitive Markets

- When $z_k(\mathbf{p}) > 0$, the aggregate demand for good k exceeds its aggregate endowment.
  - Excess demand of good k
- When $z_k(\mathbf{p}) < 0$, the aggregate demand for good k falls short of its aggregate endowment.
  - Excess supply of good k

General Equilibrium: Competitive Markets

- Difference in demand and supply, and excess demand



General Equilibrium: Competitive Markets

- The excess demand function $\mathbf{z}(\mathbf{p}) \equiv \left(z_k(\mathbf{p}), z_k(\mathbf{p}), \dots, z_k(\mathbf{p})\right)$ satisfies following properties:

  1) Walras' law: $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0.$

  – Since every consumer $i \in I$ exhausts all his income,

$$\sum_{k=1}^{n} p_k \cdot x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) = \sum_{k=1}^{n} p_k e_k^i \iff$$
$$\sum_{k=1}^{n} p_k \left[x_k^i\left(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i\right) - e_k^i\right] = 0$$

General Equilibrium: Competitive Markets

– Summing over all individuals,

$$\sum_{i=1}^{I} \sum_{k=1}^{n} p_k \left[x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i\right] = 0$$

– We can re-write the above expression as

$$\sum_{k=1}^{n} \sum_{i=1}^{I} p_k \left[x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i\right] = 0$$

which is equivalent to

$$\sum_{k=1}^{n} p_k \underbrace{\left(\sum_{i=1}^{I} \left[x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)\right] - \sum_{i=1}^{I} e_k^i\right)}_{z_k(\mathbf{p})} = 0$$

– Hence,

$$\sum_{k=1}^{n} p_k \cdot z_k(\mathbf{p}) = \mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$$

General Equilibrium: Competitive Markets

– In a two-good economy, Walras' law implies
$$p_1 \cdot z_1(\mathbf{p}) = -p_2 \cdot z_2(\mathbf{p})$$

– *Intuition*: if there is excess demand in market 1, $z_1(\mathbf{p}) > 0$, there must be excess supply in market 2, $z_2(\mathbf{p}) < 0$.

– Hence, if market 1 is in equilibrium, $z_1(\mathbf{p}) = 0$, then so is market 2, $z_2(\mathbf{p}) = 0$.

– More generally, if the markets of $n - 1$ goods are in equilibrium, then so is the nth market.

General Equilibrium: Competitive Markets

2) ***Continuity***: $\mathbf{z}(\mathbf{p})$ is continuous at $\mathbf{p}$.
   - This follows from individual Walrasian demands being continuous in prices.

3) ***Homegeneity***: $\mathbf{z}(\lambda\mathbf{p}) = \mathbf{z}(\mathbf{p})$ for all $\lambda > 0$.
   - This follows from Walrasian demands being homogeneous of degree zero in prices.

- We now use excess demand to define a Walrasian equilibrium allocation.

General Equilibrium: Competitive Markets

- ***Walrasian equilibrium***:
  - A price vector $\mathbf{p}^* \gg 0$ is a Walrasian equilibrium if aggregate excess demand is zero at that price vector, $\mathbf{z}(\mathbf{p}^*) = 0$.
    - In words, price vector $\mathbf{p}^*$ clears all markets.
  - Alternatively, $\mathbf{p}^* \gg 0$ is a Walrasian equilibrium if:
    1) Each consumer solves his UMP, and
    2) Aggregate demand equals aggregate supply

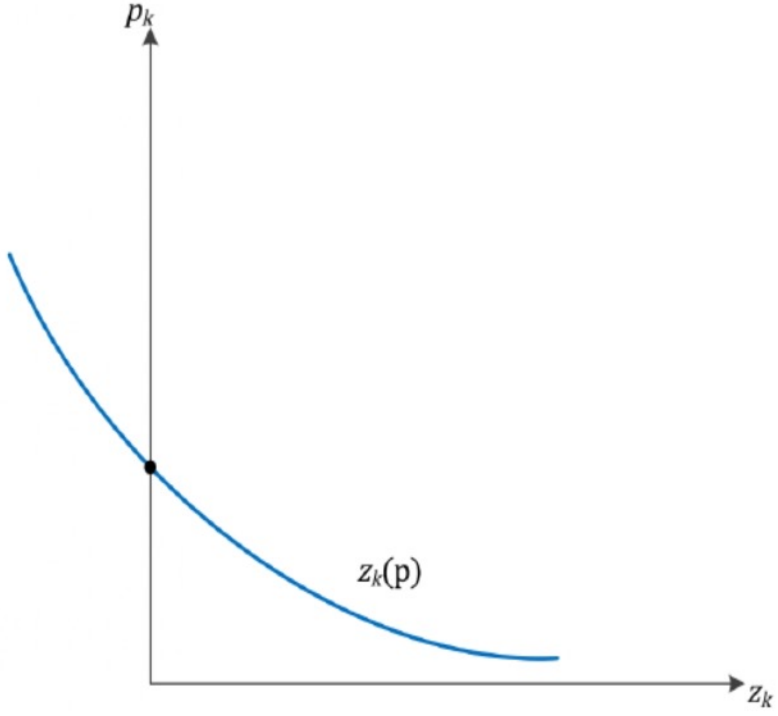
$$\sum_{i=1}^{I} x^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) = \sum_{i=1}^{I} \mathbf{e}^i$$

General Equilibrium: Competitive Markets

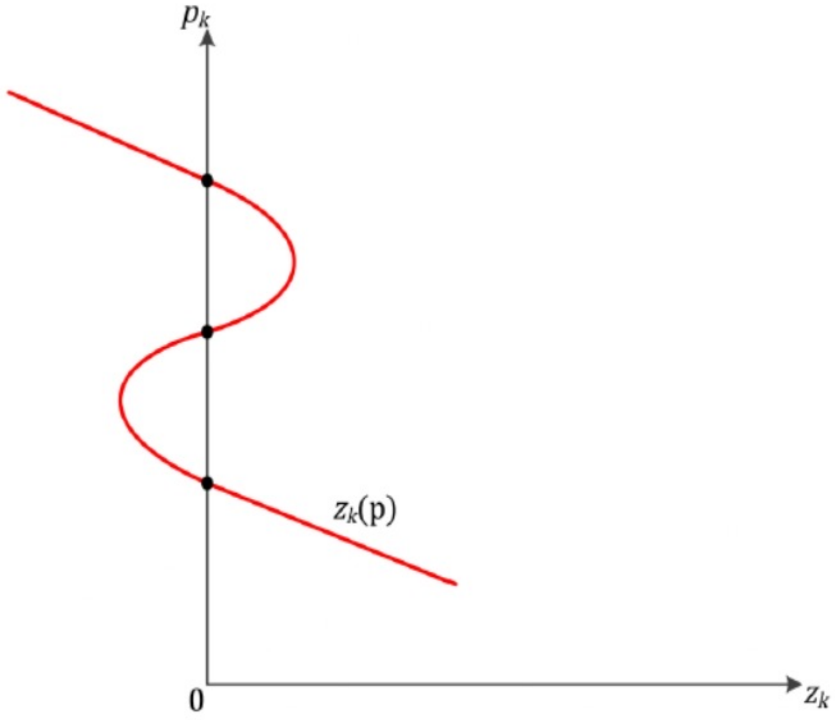
- ***Existence of a Walrasian equilibrium***:
  - A Walrasian equilibrium price vector $\mathbf{p}^* \in \mathbb{R}^n_{++}$, i.e., $\mathbf{z}(\mathbf{p}^*) = 0$, exists if the excess demand function $\mathbf{z}(\mathbf{p})$ satisfies *continuity* and *Walras' law* (Varian, 1992).

General Equilibrium: Competitive Markets

- ***Uniqueness of equilibrium prices***:



satisfied



violated

General Equilibrium: Competitive Markets

- ***Example 6.6*** (Walrasian equilibrium allocation):
  - In example 6.1, we determined that

$$MRS^A = MRS^B = \frac{p_1}{p_2}$$

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{p_1}{p_2}$$

  - Let us determine the Walrasian demands of each good for each consumer.
  - Rearranging the second equation above, we get

$$p_1 x_1^A = p_2 x_2^A$$

General Equilibrium: Competitive Markets

- ***Example 6.6*** (continued):
  - Substituting this into consumer A's budget constraint yields

$$p_1 x_1^A + p_1 x_1^A = p_1 \cdot 100 + p_2 \cdot 350$$

$$\text{or } \; x_1^A = 50 + 175 \frac{p_2}{p_1}$$

    which is consumer A's Walrasian demand for good 1.
  - Plugging this value back into $p_1 x_1^A = p_2 x_2^A$ yields

$$p_1 \left(50 + 175 \frac{p_2}{p_1}\right) = p_2 x_2^A$$

$$\text{or } \; x_2^A = 175 + 50 \frac{p_1}{p_2}$$

    which is consumer A's Walrasian demand for good 2.

General Equilibrium: Competitive Markets

- ***Example 6.6*** (continued):
  - We can obtain consumer B's demand in an analogous way. In particular, substituting $p_1 x_1^B = p_2 x_2^B$ into consumer B's budget constraint yields
$$p_1 x_1^B + p_1 x_1^B = p_1 \cdot 100 + p_2 \cdot 50$$
$$\text{or } x_1^B = 50 + 25\frac{p_2}{p_1}$$
which is consumer B's Walrasian demand for good 1.
  - Plugging this value back into $p_1 x_1^B = p_2 x_2^B$ yields
$$p_1\left(50 + 25\frac{p_2}{p_1}\right) = p_2 x_2^B$$
$$\text{or } x_2^B = 25 + 50\frac{p_1}{p_2}$$
which is consumer B's Walrasian demand for good 2.

General Equilibrium: Competitive Markets

- ***Example 6.6*** (continued):
  - For good 1, the feasibility constraint is

$$x_1^A + x_1^B = 100 + 100$$

$$\left(50 + 175\frac{p_2}{p_1}\right) + \left(50 + 25\frac{p_2}{p_1}\right) = 200$$

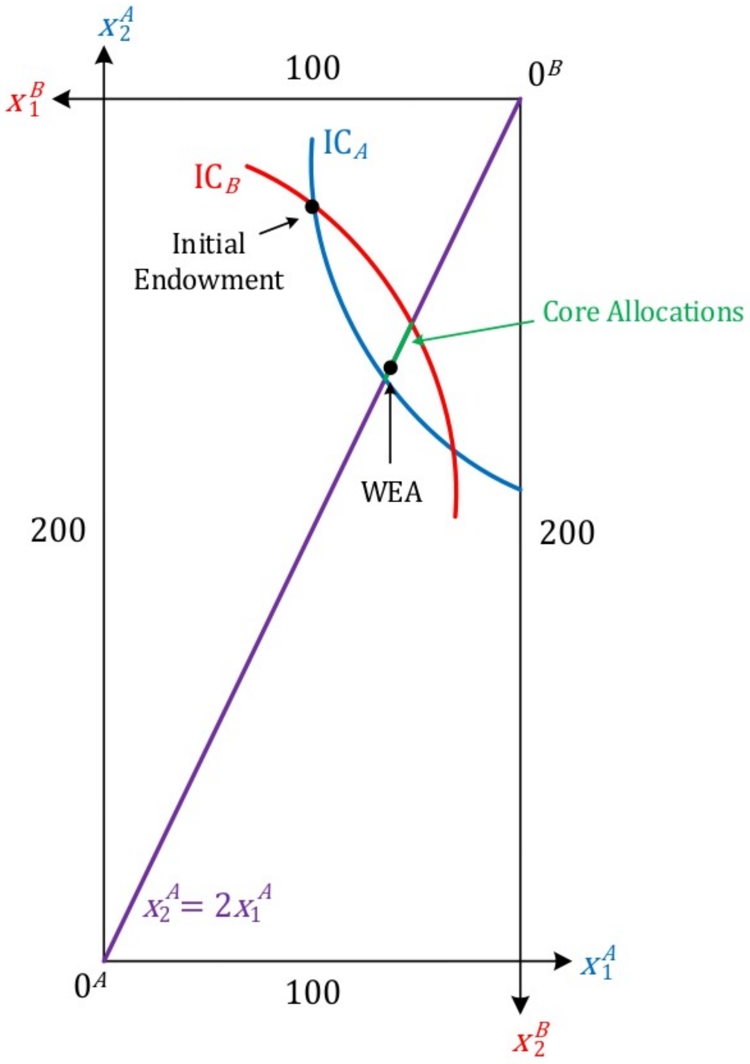
$$\frac{p_2}{p_1} = \frac{1}{2}$$

  - Plugging the relative prices into the Walrasian demands yields Walrasian equilibrium:

$$\left(x_1^{A,*}, x_2^{A,*}, x_1^{B,*}, x_2^{B,*}; \frac{p_1}{p_2}\right) = (137.5, 275, 62.5, 125; 2)$$

General Equilibrium: Competitive Markets

- ***Example 6.6*** (continued):
  - Initial allocation,
  - Core allocation, and
  - Walrasian equilibrium allocations (WEA).



General Equilibrium: Competitive Markets

- ***Equilibrium allocations must be in the Core***:
  - If each consumer's utility function is strictly increasing, then every WEA is in the Core, i.e., $W(\mathbf{e}) \subset C(\mathbf{e})$.
- *Proof* (by contradiction):
  - Take a WEA $\mathbf{x}(\mathbf{p}^*)$ with equilibrium price $\mathbf{p}^*$, but $\mathbf{x}(\mathbf{p}^*) \notin C(\mathbf{e})$.
  - Since $\mathbf{x}(\mathbf{p}^*)$ is a WEA, it must be feasible.
  - If $\mathbf{x}(\mathbf{p}^*) \notin C(\mathbf{e})$, we can find a coalition of individuals S and another allocation $\mathbf{y}$ such that

$$u^i(\mathbf{y}^i) \geq u^i(\mathbf{x}^i(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)) \text{ for all } i \in S$$

General Equilibrium: Competitive Markets

- *Proof* (continued):
  - The above expression:
    - holds with strict inequality for at least one individual in the coalition
    - is feasible for the coalition, i.e., $\sum_{i\in S} \mathbf{y}^i = \sum_{i\in S} \mathbf{e}^i$.
  - Multiplying both sides of the feasibility condition by $\mathbf{p}^*$ yields

$$\mathbf{p}^* \cdot \sum_{i\in S} \mathbf{y}^i = \mathbf{p}^* \cdot \sum_{i\in S} \mathbf{e}^i$$

  - However, the most preferable vector $\mathbf{y}^i$ must be more costly than $\mathbf{x}^i(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)$:

$$\mathbf{p}^*\mathbf{y}^i \geq \mathbf{p}^*\mathbf{x}^i\left(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i\right) = \mathbf{p}^* \cdot \mathbf{e}^i$$

    with strict inequality for at least one individual.

General Equilibrium: Competitive Markets

- *Proof* (continued):
  - Hence, summing over all consumers in the coalition S, we obtain

$$\mathbf{p}^* \cdot \sum_{i\in S} \mathbf{y}^i > \mathbf{p}^* \cdot \sum_{i\in S} \mathbf{x}^i(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i) = \mathbf{p}^* \cdot \sum_{i\in S} \mathbf{e}^i$$

    which contradicts $\mathbf{p}^* \cdot \sum_{i\in S} \mathbf{y}^i = \mathbf{p}^* \cdot \sum_{i\in S} \mathbf{e}^i$.

  - Therefor, all WEAs must be part of the Core, i.e.,

$$\mathbf{x}(\mathbf{p}^*) \in C(\mathbf{e})$$

General Equilibrium: Competitive Markets

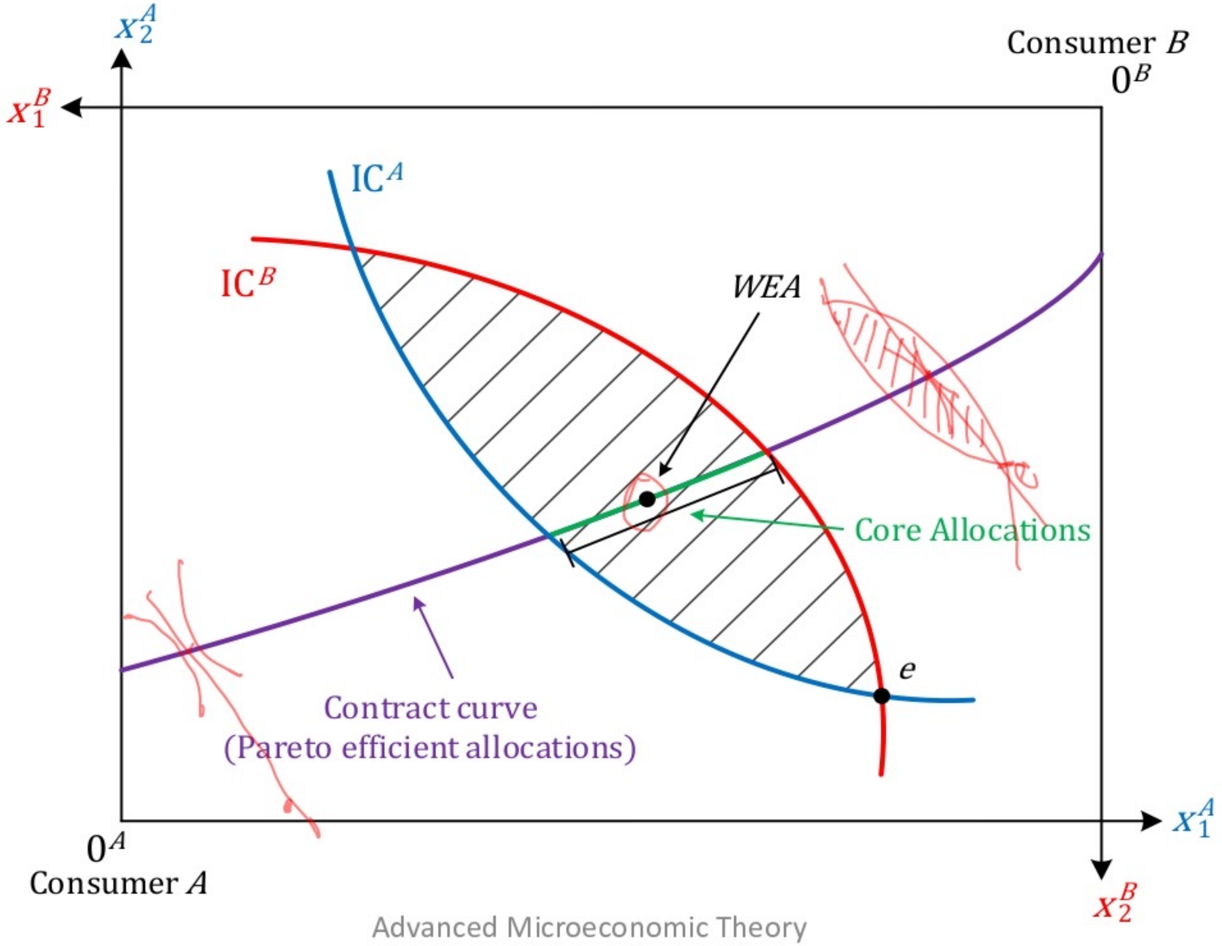
- *Remarks*:
  1) The Core $C(\mathbf{e})$ contains the WEA (or WEAs)
     - That is, the Core is nonempty.
  2) Since all core allocations are Pareto efficient (i.e., we cannot increase the welfare of one consumer without decreasing that of other consumers), then all WEAs (which are part of the Core) are also Pareto efficient.

General Equilibrium: Competitive Markets

- ***First Welfare Theorem***: Every WEA is Pareto efficient.
  - The WEA lies on the core (the segment of the contract curve within the lens-shaped area),
  - The core is a subset of all Pareto efficient allocations.

General Equilibrium: Competitive Markets

- First Welfare Theorem



General Equilibrium: Competitive Markets

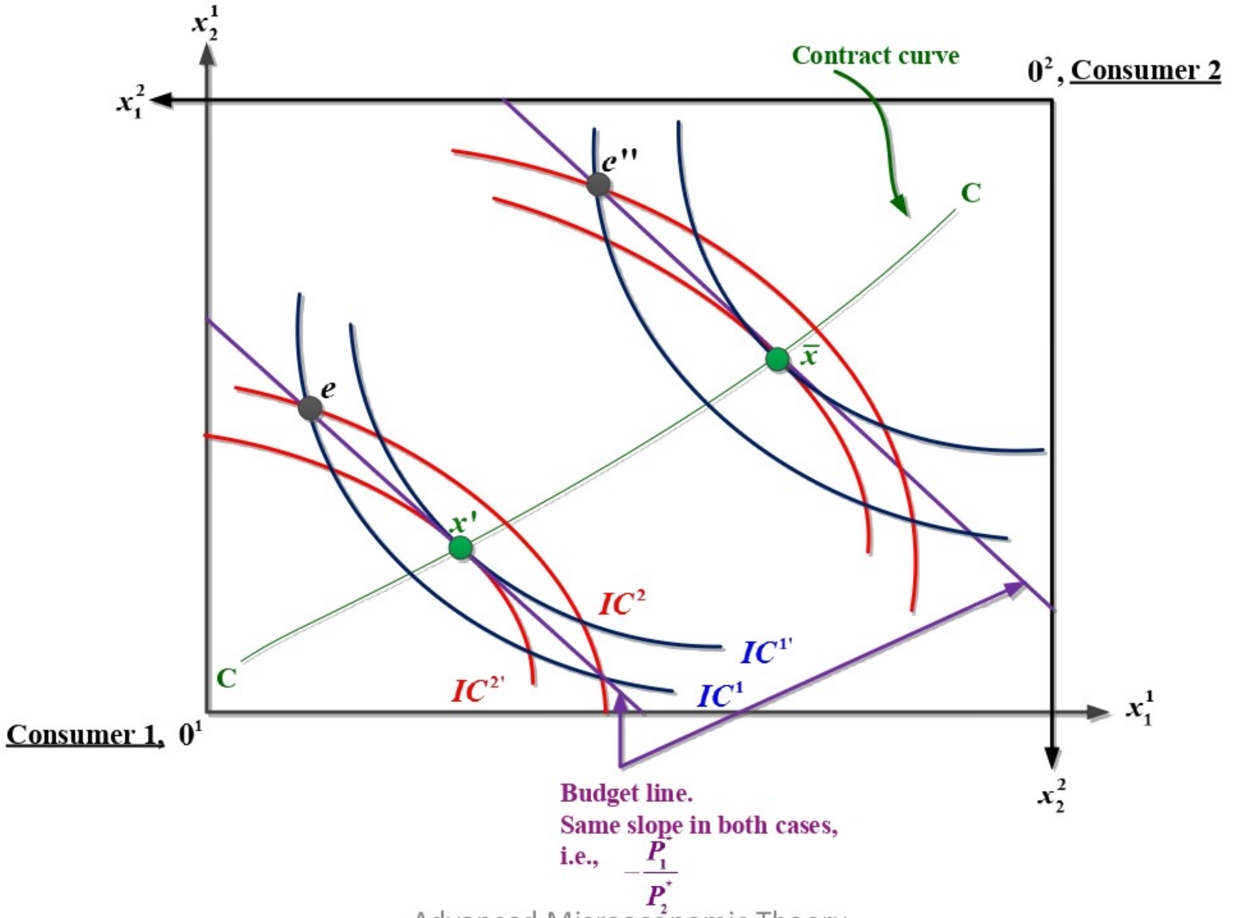
- ***Second Welfare Theorem***:
  - Suppose that $\bar{\mathbf{x}}$ is a Pareto-efficient allocation (i.e., it lies on the contract curve), and that endowments are redistributed so that the new endowment vector $\mathbf{e}^{*i}$ lies on the budget line, thus satisfying

$$\mathbf{p}^* \cdot \mathbf{e}^{*i} = \mathbf{p}^* \cdot \bar{\mathbf{x}}^i \text{ for every consumer } i$$

  - Then, the Pareto-efficient allocation $\bar{\mathbf{x}}$ is a WEA given the new endowment vector $\mathbf{e}^*$.

General Equilibrium: Competitive Markets

- Second welfare theorem



General Equilibrium: Competitive Markets

- ***Example 6.7*** (WEA and Second welfare theorem):
  - Consider an economy with utility functions $u^A = x_1^A x_2^A$ for consumer A and $u^B = \{x_1^B, x_2^B\}$ for consumer B.
  - The initial endowments are $\mathbf{e}^A = (3,1)$ and $\mathbf{e}^B = (1,3)$.
  - Good 2 is the numeraire, i.e., $p_2 = 1$.

General Equilibrium: Competitive Markets

- ***Example 6.7*** (continued):

  1) *Pareto Efficient Allocations*:

  – Consumer B's preferences are perfect complements. Hence, he consumes at the kink of his indifference curves, i.e.,

$$x_1^B = x_2^B$$

  – Given feasibility constraints

$$x_1^A + x_1^B = 4$$
$$x_2^A + x_2^B = 4$$

  substitute x_2^B for x_1^B in the first constraint to get

$$x_2^B = 4 - x_1^A$$

General Equilibrium: Competitive Markets

- ***Example 6.7*** (continued):
  1) *Pareto Efficient Allocations*:
     - Substituting the above expression in the second constraint yields
       $$x_2^A + \underbrace{(4 - x_1^A)}_{x_2^B} = 4 \iff x_2^A = x_1^A$$
     - This defines the contract curve, i.e., the set of Pareto efficient allocations.

General Equilibrium: Competitive Markets

- ***Example 6.7*** (continued):

  2) *WEA*:

  – Consumer A's maximization problem is

$$\max_{x_1^A, x_2^A} \quad x_1^A x_2^A$$
$$\text{s.t.} \quad p_1 x_1^A + x_2^A \leq p_1 \cdot 3 + 1$$

  – FOCs:

$$x_2^A - \lambda p_1 = 0$$
$$x_1^A - \lambda = 0$$
$$p_1 x_1^A + x_2^A = 3p_1 + 1$$

  where λ is the lagrange multiplier.

General Equilibrium: Competitive Markets

- ***Example 6.7*** (continued):

  2) *WEA*:

  – Combining the first two equations,

$$\lambda = \frac{x_2^A}{p_1} = x_1^A \quad \text{or} \quad p_1 = \frac{x_2^A}{x_1^A}$$

  – From Pareto efficiency, we know that $x_2^A = x_1^A$. Hence,

$$p_1 = \frac{x_2^A}{x_1^A} = 1$$

General Equilibrium: Competitive Markets

- ***Example 6.7*** (continued):
  - Substituting both the price and Pareto efficient allocation requirement into the budget constraint,

$$1 \cdot x_1^A + x_1^A = 1 \cdot 3 + 1$$
$$\text{or } x_1^{A*} = x_2^{A*} = 2$$

  - Using the feasibility constraint,

$$\underbrace{2}_{x_1^A} + x_1^B = 4 \text{ or } x_1^{B*} = x_2^{B*} = 2$$

  - Thus, the WEA is

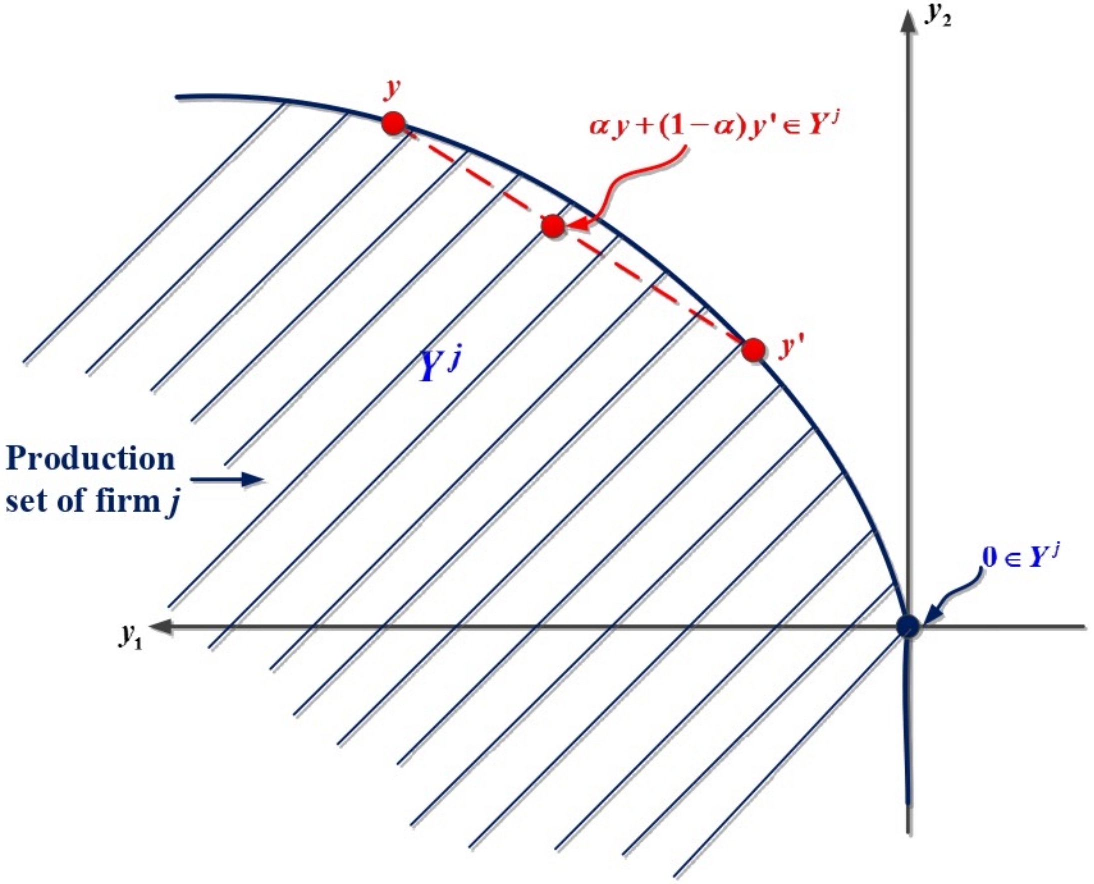
$$\left(x_1^{A*}, x_2^{A*}; x_1^{B*}, x_2^{B*}; \frac{p_1}{p_2}\right) = (2,2; 2,2; 1)$$

General Equilibrium: Production

- Let us now extend our previous results to setting where firms are also active.
- Assume J firms in the economy, each with production set Y^j, which satisfies:
  - Inaction is possible, i.e., $\mathbf{0} \in Y^j$.
  - Y^j is closed and bounded, so points on the production frontier are part of the production set and thus feasible.
  - Y^j is strictly convex, so linear combinations of two production plans also belong to the production set.

General Equilibrium: Production

- Production set Y^j for a representative firm



General Equilibrium: Production

- Every firm j facing a fixed price vector $\mathbf{p} \gg 0$ independently and simultaneously solves

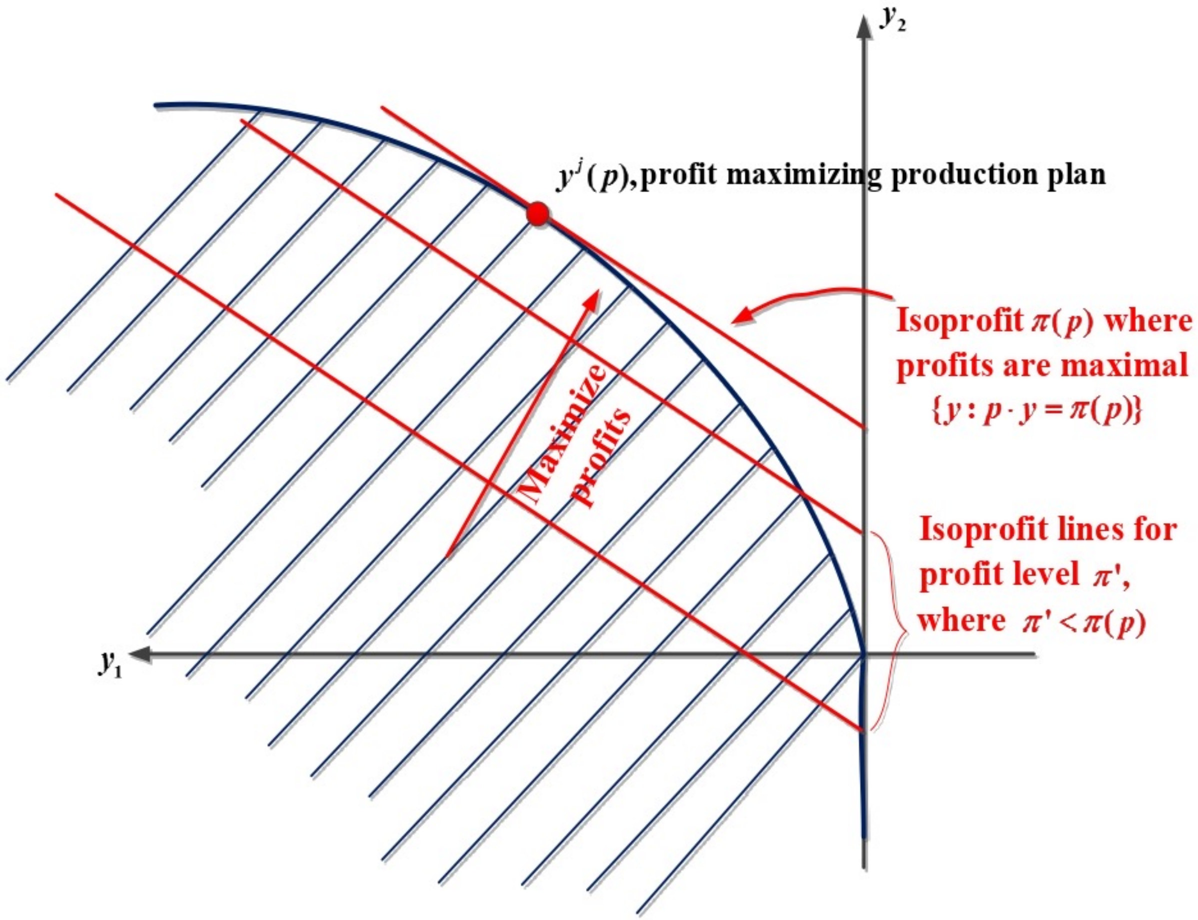
$$\max_{y_j \in Y^j} \quad \mathbf{p} \cdot y_j$$

- A profit-maximizing production plan $y_j(\mathbf{p})$ exists for every firm j, and it is unique.

- By the theorem of the maximum, both the argmax, $y_j(\mathbf{p})$, and the value function, $\pi_j(\mathbf{p}) \equiv \mathbf{p} \cdot y_j(\mathbf{p})$, are continuous in p.

General Equilibrium: Production

- $y^j(p)$ exists and is unique



General Equilibrium: Production

- ***Aggregate production set***:
  - The aggregate production set is the sum of all firms' production plans (either profit maximizing or not):

$$Y = \left\{\mathbf{y} | \mathbf{y} = \sum_{j=1}^{J} y_j \text{ where } y_j \in Y^j\right\}$$

  - A joint-profit maximizing production plan $\mathbf{y}(\mathbf{p})$ is the sum of each firm's profit-maximizing plan, i.e.,

$$\mathbf{y}(\mathbf{p}) = y_1(\mathbf{p}) + y_2(\mathbf{p}) + \cdots + y_J(\mathbf{p})$$

General Equilibrium: Production

- In an economy with J firms, each of them earning $\pi_j(\mathbf{p})$ profits in equilibrium, how are profits distributed?
  - Assume that each individual i owns a share θ^{ij} of firm j's profits, where $0 \leq \theta^{ij} \leq 1$ and $\sum_{i=1}^{I} \theta^{ij} = 1$.
  - This allows for multiple sharing profiles:
    - $\theta^{ij} = 1$: individual i owns all shares of firm j
    - $\theta^{ij} = 1/I$: every individual's share of firm j coincides

General Equilibrium: Production

– Consumer 's budget constraint becomes

$$\mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i + \sum_{j=1}^{J} \theta^{ij} \pi_j(\mathbf{p})$$

where $\sum_{j=1}^{J} \theta^{ij} \pi_j(\mathbf{p})$ is new relative to the standard budget constraint.

– Let us express the budget constraints as

$$\mathbf{p} \cdot \mathbf{x}^i \leq \underbrace{\mathbf{p} \cdot \mathbf{e}^i + \sum_{j=1}^{J} \theta^{ij} \pi_j(\mathbf{p})}_{m^i(\mathbf{p})}$$

$$\Rightarrow \mathbf{p} \cdot \mathbf{x}^i \leq m^i(\mathbf{p})$$

where $m^i(\mathbf{p}) > 0$ (given assumptions on Y^j).

General Equilibrium: Production

- ***Equilibrium with production***:
  - We start defining excess demand functions and use such a definition to identify the set of equilibrium allocations.
  - *Excess demand*: The excess demand function for good k is

    $$z_k(\mathbf{p}) \equiv \sum_{i=1}^{I} x_k^i(\mathbf{p}, m^i(\mathbf{p})) - \sum_{i=1}^{I} e_k^i - \sum_{j=1}^{J} y_k^j(\mathbf{p})$$

    where $\sum_{j=1}^{J} y_k^j(\mathbf{p})$ is a new term relative to the analysis of general equilibrium without production.

General Equilibrium: Production

– Hence, the aggregate excess demand vector is

$$\mathbf{z}(\mathbf{p}) = (z_1(\mathbf{p}), z_2(\mathbf{p}), \dots, z_n(\mathbf{p}))$$

– *WEA with production*: If the price vector is strictly positive in all of its components, $\mathbf{p}^* \gg 0$, a pair of consumption and production bundles $(\mathbf{x}(\mathbf{p}^*), \mathbf{y}(\mathbf{p}^*))$ is a WEA if:

1) Each consumer i solves his UMP, which becomes the ith entry of $\mathbf{x}(\mathbf{p}^*)$, i.e., $\mathbf{x}^i(\mathbf{p}^*, m^i(\mathbf{p}^*))$;
2) Each firm j solves its PMP, which becomes the jth entry of $\mathbf{y}(\mathbf{p}^*)$, i.e., $\mathbf{y}^j(\mathbf{p}^*)$;

General Equilibrium: Production

3) Demand equals supply

$$\sum_{i=1}^{I} \mathbf{x}^i(\mathbf{p}^*, m^i(\mathbf{p}^*)) = \sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j(\mathbf{p}^*)$$

which is the market clearing condition.

– *Existence*: Assume that
  - consumers' utility functions are continuous, strictly increasing and strictly quasiconcave;
  - every firm j's production set Y^j is closed and bounded, strictly convex, and satisfies inaction being possible;
  - every consumer is initially endowed with positive units of at least one good, so the sum $\sum_{i=1}^{I} \mathbf{e}^i \gg 0$.

Then, there is a price vector $\mathbf{p}^* \gg 0$ such that a WEA exisits, i.e., $z(\mathbf{p}^*) = 0$.

General Equilibrium: Production

- ***Example 6.8*** (Equilibrium with production):
  - Consider a two-consumer, two-good economy where consumer $i = \{A, B\}$ has utility function $u^i = x_1^i x_2^i$.
  - There are two firms in this economy, and each of them use capital (K) and labor (L) to produce one of the consumption goods each.
  - Firm 1 produces good 1 according to $y_1 = K_1^{0.75} L_1^{0.25}$.
  - Firm 2 produces good 2 according to $y_2 = K_2^{0.25} L_2^{0.75}$.
  - Consumer A is endowed with $(K^A, L^A) = (1,1)$, while consumer B is endowed with $(K^B, L^B) = (2,1)$.
  - Let us find a WEA in this economy with production.

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - ***UMPs***: Consumer i's maximization problem is

$$\max_{x_1^i, x_2^i} \quad x_1^i x_2^i$$
$$\text{s.t.} \quad p_1 x_1^i + x_2^i = rK^i + wL^i$$

    where r and w are prices for capital and labor, respectively.
  - FOC:

$$\frac{p_1}{p_2} = MRS_{1,2}^i \implies \frac{p_1}{p_2} = \frac{x_2^i}{x_1^i} \implies p_1 x_1^i = p_2 x_2^i$$

    for $i = \{A, B\}$.

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Taking the above equation for consumers A and B, and adding them together yields

$$p_1(x_1^A + x_1^B) = p_2(x_2^A + x_2^B)$$

    where $x_1^A + x_1^B$ is the left side of the feasibility condition $x_1^A + x_1^B = y_1 = K_1^{0.75}L_1^{0.25}$.
  - Substituting both feasibility conditions into the above expression, and re-arranging, yields

$$\frac{p_1}{p_2} = \frac{K_2^{0.25}L_2^{0.75}}{K_1^{0.75}L_1^{0.25}}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - ***PMPs***: Firm 1's maximization problem is

$$\max_{K_1, L_1} \; p_1 K_1^{0.75} L_1^{0.25} - rK_1 - wL_1$$

  - FOCs:

$$r = 0.75 p_1 K_1^{-0.25} L_1^{0.25}$$
$$w = 0.25 p_1 K_1^{0.75} L_1^{-0.75}$$

  - Combining these conditions gives the tangency condition for profit maximization

$$\frac{r}{w} = MRTS^1_{L,K} \implies \frac{r}{w} = 3\frac{L_1}{K_1}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Likewise, firm 2's PMP gives the following FOCs:

$$r = 0.25 p_2 K_2^{-0.75} L_2^{0.75}$$
$$w = 0.75 p_2 K_2^{0.25} L_2^{-0.25}$$

  - Combining these conditions gives the tangency condition for profit maximization

$$\frac{r}{w} = MRTS_{L,K}^{2} \implies \frac{r}{w} = \frac{1}{3}\frac{L_2}{K_2}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Combining both $MRTS$ yields,

$$3\frac{L_1}{K_1} = \frac{1}{3}\frac{L_2}{K_2} \implies \frac{K_1}{L_1} = 9\frac{K_2}{L_2}$$

    - *Intuition*: firm 1 is more capital intensive than firm 2, i.e., its capital to labor ratio is higher.
  - Setting both firm's price of capital, r, equal to each other yields

$$0.75p_1K_1^{-0.25}L_1^{0.25} = 0.25p_2K_2^{-0.75}L_2^{0.75}$$

$$\implies \frac{p_1}{p_2} = \frac{1}{3}\left(\frac{K_1}{L_1}\right)^{0.25}\left(\frac{K_2}{L_2}\right)^{-0.75}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  – Setting both firm's price of labor, w, equal to each other yields

$$0.25 p_1 K_1^{0.75} L_1^{-0.75} = 0.75 p_2 K_2^{0.25} L_2^{-0.25}$$

$$\Longrightarrow \frac{p_1}{p_2} = 3 \left(\frac{K_1}{L_1}\right)^{-0.75} \left(\frac{K_2}{L_2}\right)^{0.25}$$

  – Setting price ratio from consumers' UMP equal to the first price ratio from firms' PMP yields

$$\frac{K_2^{0.25} L_2^{0.75}}{K_1^{0.75} L_1^{0.25}} = \frac{1}{3} \left(\frac{K_1}{L_1}\right)^{0.25} \left(\frac{K_2}{L_2}\right)^{-0.75} \Longrightarrow K_1 = 3K_2$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - By the feasibility conditions, we know that $K_1 + K_2 = K^A + K^B = 3$ or $K_2 = 3 - K_1$.
  - Substituting the above expression into $K_1 = 3K_2$, we find the profit-maximizing demands for capital use by firms 1 and 2:

$$K_1 = 3(3 - K_1) \implies K_1^* = \frac{9}{4}$$

$$K_2^* = \frac{1}{3}K_1^* = \frac{3}{4}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Setting price ratio from consumers' UMP equal to the second price ratio from firms' PMP yields

$$\frac{K_2^{0.25}L_2^{0.75}}{K_1^{0.75}L_1^{0.25}} = 3\left(\frac{K_1}{L_1}\right)^{-0.75}\left(\frac{K_2}{L_2}\right)^{0.25} \Longrightarrow L_1 = \frac{1}{3}L_2$$

  - By the feasibility conditions, we know that $L_1 + L_2 = L^A + L^B = 2$ or $L_2 = 2 - L_1$.

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Substituting the above expression into $L_1 = \frac{1}{3} L_2$, we find the profit-maximizing demands for labor use by firms 1 and 2:

$$L_1 = \frac{1}{3}(2 - L_1) \implies L_1^* = \frac{1}{2}$$

$$L_2^* = 3L_1^* = \frac{3}{2}$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Substituting the capital and labor demands for firm 1 and 2 into the price ratio from consumers' UMP yields

$$\frac{p_1}{p_2} = \frac{\left(\frac{3}{4}\right)^{0.25}\left(\frac{3}{2}\right)^{0.75}}{\left(\frac{9}{4}\right)^{0.75}\left(\frac{1}{2}\right)^{0.25}} = 0.82$$

where normalizing the price of good 2, i.e., $p_2 = 1$, gives $p_1 = 0.82$.

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Furthermore, substituting our calculated values into the price of capital and labor yields

$$r^* = 0.75(0.82)\left(\frac{9}{4}\right)^{-0.25}\left(\frac{1}{2}\right)^{0.25} = 0.42$$

$$w^* = 0.25(0.82)\left(\frac{9}{4}\right)^{0.75}\left(\frac{1}{2}\right)^{-0.75} = 0.63$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Using consumer A's tangency condition, we know
$$x_2^A = \frac{p_1}{p_2} x_1^A \implies x_2^A = 0.82 x_1^A$$
  - Substituting this value into consumer A's budget constraint yields
$$p_1 x_1^A + p_2 (0.82 x_1^A) = rK^A + wL^A$$
  - Plugging in our calculated values and solving for x_1^A yields
$$x_1^{A,*} = 0.64$$
$$x_2^{A,*} = 0.82 x_1^{A,*} = 0.53$$

General Equilibrium: Production

- ***Example 6.8*** (continued):
  - Performing the same process with the tangency condition of consumer B yields
$$x_1^{B,*} = 0.90$$
$$x_2^{B,*} = 0.74$$
  - Thus, our WEA is
$$\left(x_1^A, x_2^A; x_1^B, x_2^B; \frac{p_1}{p_2}; L_1, L_2; K_1, K_2\right) =$$
$$\left(0.64, 0.53; 0.90, 0.74; 0.82; \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, \frac{3}{4}\right)$$

General Equilibrium: Production

- ***Equilibrium with production – Welfare***:
  - We extend the First and Second Welfare Theorems to economies with production, connecting WEA and Pareto efficient allocations.
  - *Pareto efficiency*: The feasible allocation $(\mathbf{x}, \mathbf{y})$ is Pareto efficient if there is no other feasible allocation $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ such that
$$u^i(\bar{\mathbf{x}}^i) \geq u^i(\mathbf{x}^i)$$
for every consumer $i \in I$, with $u^i(\bar{\mathbf{x}}^i) > u^i(\mathbf{x}^i)$ for at least one consumer.

General Equilibrium: Production

– In an economy with two goods, two consumers, two firms and two inputs (labor and capital), the set of Pareto efficient allocations solves

$$\max_{x_1^1, x_2^1, x_1^2, x_2^2, L_1, K_1, L_2, K_2 \geq 0} \quad u^1(x_1^1, x_2^1)$$

$$\text{s.t.} \quad u^2(x_1^2, x_2^2) \geq \bar{u}^2$$

$$\left.\begin{array}{l} x_1^1 + x_2^1 \leq F_1(L_1, K_1) \\ x_1^2 + x_2^2 \leq F_2(L_2, K_2) \end{array}\right\} \text{tech. feasibility}$$

$$\left.\begin{array}{l} L_1 + L_2 \leq \bar{L} \\ K_1 + K_2 \leq \bar{K} \end{array}\right\} \text{input feasibility}$$

General Equilibrium: Production

– The Lagrangian is

$$\mathcal{L} = u^1(x_1^1, x_2^1) + \lambda[u^2(x_1^2, x_2^2) - \bar{u}^2] + \mu_1[F_1(L_1, K_1) - x_1^1 - x_2^1] + \mu_2[F_2(L_2, K_2) - x_1^2 - x_2^2] + \delta_L[\bar{L} - L_1 - L_2] + \delta_K[\bar{K} - K_1 - K_2]$$

– In the case of interior solutions, the set of FOCs yield a condition for efficiency in consumption similar to barter economics:

$$MRS_{1,2}^1 = MRS_{1,2}^2$$

General Equilibrium: Production

- FOCs wrt L_j and K_j, where $j = \{1,2\}$, yield a condition for efficiency that we encountered in production theory

$$\frac{\frac{\partial F_1}{\partial L}}{\frac{\partial F_1}{\partial K}} = \frac{\frac{\partial F_2}{\partial L}}{\frac{\partial F_2}{\partial K}}$$

- That is, the $MRTS_{L,K}$ between labor and capital must coincide across firms.
- Otherwise, welfare could be increased by assigning more labor to the firm with the highest $MRTS_{L,K}$.

General Equilibrium: Production

– Combining the above two conditions for efficiency in consumption and production, we obtain

$$\frac{\frac{\partial U^i}{\partial x_1^i}}{\frac{\partial U^i}{\partial x_2^i}} = \frac{\frac{\partial F_2}{\partial L}}{\frac{\partial F_1}{\partial L}}$$

– That is, $MRS^i_{1,2}$ must coincide with the rate at which units of good 1 can be transformed into units of good 2, i.e., the marginal rate of transformation $MRT_{1,2}$.

General Equilibrium: Production

- If we move labor from firm 2 to firm 1, the production of good 2 increases by $\frac{\partial F_2}{\partial L}$ while that of good 1 decreases by $\frac{\partial F_1}{\partial L}$. Hence, in order to increase the total output of good 1 by one unit we need $\frac{\partial F_2}{\partial L} / \frac{\partial F_1}{\partial L}$ units of good 2.
- *Intuition*: for an allocation to be efficient we need that the rate at which consumers are willing to substitute goods 1 and 2 coincides with the rate at which good 1 can be transformed into good 2.

General Equilibrium: Production

- ***First Welfare Theorem with production***: if the utility function of every individual i, u^i, is strictly increasing, then every WEA is Pareto efficient.

- *Proof* (by contradiction):
  - Suppose that $(\mathbf{x}, \mathbf{y})$ is a WEA at prices $\mathbf{p}^*$, but is *not* Pareto efficient.
  - Since $(\mathbf{x}, \mathbf{y})$ is a WEA, then it must be feasible

$$\sum_{i=1}^{I} \mathbf{x}^i = \sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j$$

General Equilibrium: Production

– Because $(\mathbf{x}, \mathbf{y})$ is *not* Pareto efficient, there exists some other feasible allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ such that

$$u^i(\hat{\mathbf{x}}^i) \geq u^i(\mathbf{x}^i)$$

for every consumer $i \in I$, with $u^i(\hat{\mathbf{x}}^i) > u^i(\mathbf{x}^i)$ for at least one consumer.

- That is, the alternative allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ makes at least one consumer strictly better off than WEA .

– But this implies that bundle $\hat{\mathbf{x}}^i$ is more costly than $\mathbf{x}^i$,

$$\mathbf{p}^* \cdot \hat{\mathbf{x}}^i \geq \mathbf{p}^* \cdot \mathbf{x}^i$$

for every individual i (with at least one strictly inequality).

General Equilibrium: Production

– Summing over all consumers yields

$$\mathbf{p}^* \cdot \sum_{i=1}^{I} \hat{\mathbf{x}}^i > \mathbf{p}^* \cdot \sum_{i=1}^{I} \mathbf{x}^i$$

which can be re-written as

$$\mathbf{p}^* \cdot \left(\sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \hat{\mathbf{y}}^j\right) > \mathbf{p}^* \cdot \left(\sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j\right)$$

or re-arranging

$$\mathbf{p}^* \cdot \sum_{j=1}^{J} \hat{\mathbf{y}}^j > \mathbf{p}^* \cdot \sum_{j=1}^{J} \mathbf{y}^j$$

– However, this result implies that $\mathbf{p}^* \cdot \hat{\mathbf{y}}^j > \mathbf{p}^* \cdot \mathbf{y}^j$ for some firm j.

General Equilibrium: Production

- This indicates that production plan $\mathbf{y}^j$ was not profit-maximizing and, as a consequence, it cannot be part of a WEA.
- We therefore reached a contradiction.
- This implies that the original statement was true: if an allocation $(\mathbf{x}, \mathbf{y})$ is a WEA, it must also be Pareto efficient.

General Equilibrium: Production

- ***Example 6.9*** (WEA and PE with production):
  - Consider the setting described in example 6.8.
  - The set of Pareto efficient allocations must satisfy

$$MRS^A_{1,2} = MRS^B_{1,2} \text{ and } MRTS^1_{L,K} = MRTS^2_{L,K}$$

  - We can show that

$$MRS^A_{1,2} = \frac{x^A_2}{x^A_1} = \frac{0.53}{0.64} = 0.82$$

$$MRS^B_{1,2} = \frac{x^B_2}{x^B_1} = \frac{0.74}{0.90} = 0.82$$

which implies that $MRS^A_{1,2} = MRS^B_{1,2}$.

General Equilibrium: Production

- ***Example 6.9*** (continued):
  - We can also show that

$$MRTS^1_{L,K} = 3\,\frac{L_1}{K_1} = 3\left(\frac{1}{2}/\frac{9}{4}\right) = \frac{2}{3}$$

$$MRTS^2_{L,K} = 3\,\frac{L_2}{K_2} = 3\left(\frac{3}{2}/\frac{3}{4}\right) = \frac{2}{3}$$

  which implies that $MRTS^1_{L,K} = MRTS^2_{L,K}$.

  - Since both of these conditions hold, our WEA from example 6.8 is Pareto efficient.

General Equilibrium: Production

- ***Second Welfare Theorem with production***:
  - Consider the assumptions on consumers and producers described above.
  - Then, for every Pareto efficient allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ we can find:
    a) a profile of income transfers $(T_1, T_2, \ldots, T_I)$ redistributing income among consumers, i.e., satisfying $\sum_{i=1}^{I} T_i = 0$;
    b) a price vector $\bar{\mathbf{p}}$,

    such that:

General Equilibrium: Production

1) Bundle $\hat{\mathbf{x}}^i$ solves the UMP

$$\max_{\mathbf{x}^i} \quad u^i(\mathbf{x}^i)$$

$$\text{s.t.} \quad \bar{\mathbf{p}} \cdot \mathbf{x}^i \leq m^i(\bar{\mathbf{p}}) + T_i \text{ for every } i \in I$$

where individual i's original income $m^i(\bar{\mathbf{p}})$ is increased (decreased) if the transfer T_i is positive (negative).

2) Production plan $\hat{\mathbf{y}}^j$ solves the PMP

$$\max_{y^j} \quad \bar{\mathbf{p}} \cdot \mathbf{y}^j$$

$$\text{s.t.} \quad \mathbf{y}^j \in Y^j \text{ for every } j \in J$$

General Equilibrium: Production

- ***Example 6.10*** (Second Welfare Theorem with production):
  - Consider an alternative allocation in the set of Pareto efficient allocations identified in example 6.9.
  - Such as, $\left(\hat{x}_1^A, \hat{x}_2^A; \hat{x}_1^B, \hat{x}_2^B\right) = (0.82{,}1; 0.79{,}0.65)$.
  - Consumer A's budget constraint becomes

$$p_1\hat{x}_1^A + p_2\hat{x}_2^A = rK^A + wL^A + T_1$$

  - Recall that

$$(p_1, p_2; K^A, L^A; r, w) = (0.82{,}1; 1{,}1; 0.42{,}0.63)$$

    remains unchanged.

General Equilibrium: Production

– Substituting these values into consumer A's budget constraint

$$0.82\hat{x}_1^A + \hat{x}_2^A = 1.05 + T_1$$

– Recall that

$$\frac{p_1}{p_2} = \frac{\hat{x}_2^A}{\hat{x}_1^A} \implies \hat{x}_2^A = 0.82\hat{x}_1^A$$

– Substituting

$$2(0.82)\underbrace{(0.75)}_{\hat{x}_1^A} = 1.05 + T_1 \implies T_1 = 0.17$$

General Equilibrium: Production

– Likewise for consumer B, his budget constraint becomes

$$p_1\hat{x}_1^B + p_2\hat{x}_2^B = rK^B + wL^B + T_2$$

– Substituting the unchanged values $(p_1, p_2; K^A, L^A; r, w) = (0.82,1; 1,1; 0.42,0.63)$,

$$0.82\hat{x}_1^B + \hat{x}_2^B = 1.47 + T_2$$

– Recall that

$$\frac{p_1}{p_2} = \frac{\hat{x}_2^B}{\hat{x}_1^B} \Longrightarrow \hat{x}_2^B = 0.82\hat{x}_1^B$$

General Equilibrium: Production

– Substituting

$$2(0.82)\underbrace{(0.79)}_{\hat{x}_1^B} = 1.47 + T_1 \implies T_1 = -0.17$$

– Clearly, $T_1 + T_2 = 0$

– Thus these transfers will allow for the new allocation to be a WEA.