

Problem Set 3

1. Peter has no initial wealth and the utility function $u(\cdot)$. He is indifferent between receiving 90€ with certainty and participating in a lottery that pays 200€ with probability 0.4 and 50€ with probability 0.6. If Peter conforms to the vN-M axioms, write down the condition that reflects his indifference, and propose appropriate utility values for $u(200)$, $u(90)$ and $u(50)$. What is Peter's risk premium for choosing the gamble?

2. For each of the following statements, argue whether they are true, false or uncertain.

- a) Two consumers with identical utility functions have the same absolute risk aversion, and hence, their behaviour in risky situations should be identical.
- b) vN-M utility functions allow only positive linear transformations.
- c) Regret theory is compatible with expected utility theory.

3. Peter and Alison have utility functions $u_P(w) = w^{1/2}$ and $u_A(w) = \ln(w)$ respectively.

- a) Calculate these persons' Arrow-Pratt measure of risk aversion and show that both Peter and Alison are risk averse. What condition must hold for Peter to be more risk averse than Alison?
- b) Suppose that Peter's wealth is w_P and that he is asked to participate (for free) in a lottery that will either multiply his wealth by 4 or diminish it to $1/4$ of its original level with probabilities 20% and 80% respectively. Show that Peter will choose not to take this gamble and calculate the probability premium.

4. Frieda is risk-averse and wants to send a package worth x monetary units. If she uses the national postal service, there is a chance π that the package will be lost or destroyed in transit. An insurance company offers full insurance against this eventuality for a premium of y . Calculate y if the insurance is actuarially fair and show that Frieda will prefer to insure.

5. If Mary's utility function is $u(\cdot)$, write down the condition that must hold if she denies a gamble where she wins 500€ with probability 0.6 or she loses 500€ with probability 0.4, and calculate the probability premium. Suppose Mary can take the gamble twice (the two different trials of gamble being independent). Under what condition would she prefer to take the gamble twice rather than once?

6. Jack faces a lottery that gives 1€ or 25€ with equal probabilities.

- a) What is the minimum amount you could give to Jack if his utility function is $u(x) = x^{1/2}$ (supposing he conforms to the vN-M axioms) for not taking this gamble? Assume that Jack has no initial wealth. What is Jack's risk premium?
- b) Calculate the probability premium for Jack of (a) and the above lottery.