LECTURE 4

- **MICROECONOMIC THEORY CONSUMER THEORY**
- **Choice under Uncertainty**
- **(MWG chapter 6, sections A-C, and Cowell chapter 8)**

- **Lecturer: Georgia Kaplanoglou**

Introduction

- Contents
  - Expected utility theory
  - Measures of risk aversion
  - Measures of risk

Introduction

- Until now, we have been concerned with choices under certainty.
  - If I choose A, the outcome is with certainty C_A and my utility is with certainty $u(C_A)$.
  - If I choose B, the outcome is with certainty C_B and my utility is with certainty $u(C_B)$.
  - $A \succeq B \quad \Leftrightarrow \quad u(C_A) \geq u(C_B)$
- But life is full of **uncertainty**! You often have to decide between choices that each lead to an uncertain outcome.
- Today's goal: represent preferences over uncertain outcomes.
- What if A and B are not certainties, but distributions over outcomes?

Introduction

EXAMPLE

- Suppose we are considering two different uncertain alternatives, each of which offers a different distribution over three outcomes:
  - I buy you a trip to Bermuda
  - you pay me $500
  - you do all my undergraduate tutorials of micro

Introduction

- The probability of each outcome under alternatives A and B are the following:

	Bermuda	-$500	Do micro tutorials
A	0.3	0.4	0.3
B	0.2	0.7	0.1

- We would like to express your utility for these two alternatives in terms of the utility you assign to each individual outcome and the probability that they occur

Introduction

- Suppose you assign:
  - Value u_B to the trip to Bermuda
  - Value u_m to paying me the money
  - Value u_t to doing the tutorials
- And we know:
  - Probability p_B to the trip to Bermuda
  - Probability p_B to paying me the money
  - Probability p_t to doing the tutorials

Expected Utility

- It would be very nice if we could express your utility for each alternative by multiplying each of these numbers by the probability of the outcome occurring, and summing up.

  That is:

$$U(A) = 0.3u_B + 0.4u_m + 0.3u_t$$

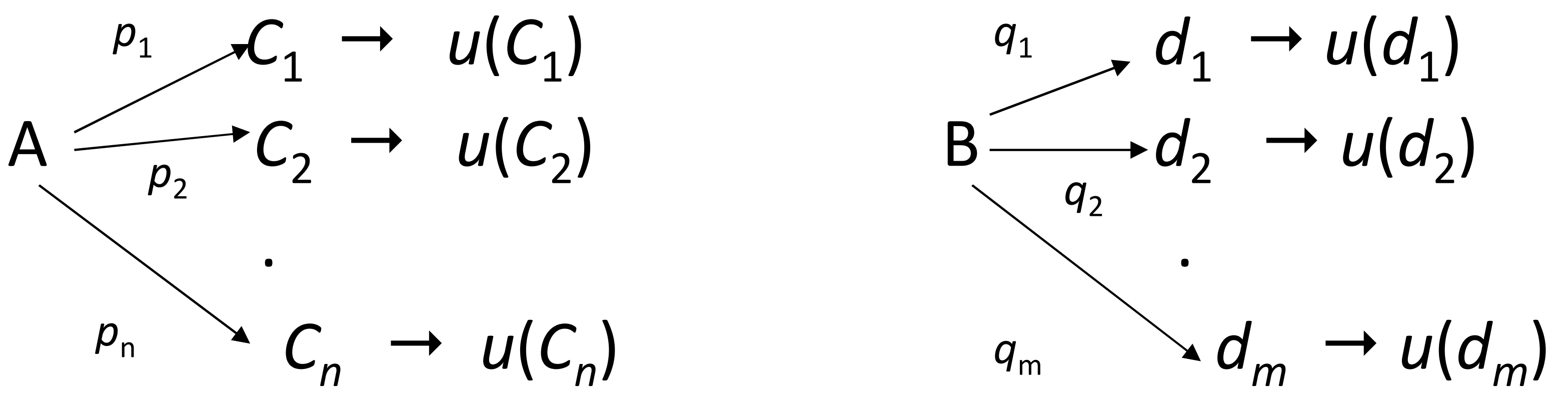
$$U(B) = 0.2u_B + 0.7u_m + 0.1u_t.$$

Or, in general, the expected utility of an alternative would be

$$EU(A) = p_B u_B + p_m u_m + p_t u_t$$

Expected Utility Principle

- More generally:



$p_1+p_2+\ldots+p_n=1$ $\qquad$ $q_1+q_2+\ldots+q_m=1$

Do I choose A or B?

Expected Utility Principle: $A \succeq B \Leftrightarrow EU(A) \geq EU(B)$

$$EU\,(A) = p_1u(C_1) + p_2u(C_2) + \ldots + p_nu(C_n)$$

$$EU\,(B) = q_1u(d_1) + q_2u(d_2) + \ldots + q_nu(d_m)$$

Expected Utility Principle

- The only difference is that we maximize expected utility.
- In ch. 3 of MWG we based our analysis on the assumption that a consumer has rational preferences
- However, the assumption of rational preferences over uncertain outcomes is not sufficient to represent these preferences by a utility function that has the expected utility form
- To be able to do so, we have to place additional structure on preferences
- Then we show how utility functions of the expected utility form can be used to study behavior under uncertainty, and draw testable implications

Expected Utility

- Suppose outcome 1 gives you utility u_1, outcome 2 u_2, and so on. What is your utility from lottery $L = [p_1, p_2, ..., p_n]$?
- Natural answer: $p_1 u_1 + p_2 u_2 + ... + p_n u_n$, which is the L's (von Neumann-Morgenstern) **expected utility**.
- But even if the u_i's represent your preferences over outcomes, expected utility may not represent your preferences over lotteries.
- Example: Fido prefers chicken (outcome 1) over pears (outcome 2) over apples (outcome 3). Assigning $u_1 = 2$, $u_2 = 1$ and $u_3 = 0$ would represent its preferences over these sure outcomes.
- But suppose Fido prefers $[0.4, 0, 0.6]$ over $[0, 1, 0]$. With the above utilities, does expected utility represent Fido's preferences over lotteries?
- So it's important to assign the right utility to each outcome - not just the order matters (ordinal utility), but size matters too (cardinal utility).

Assumptions on preferences: 1. Rationality

- Given preferences over lotteries, it's not always possible to find utilities over outcomes $u_1, u_2, \dots, u_n$ such that expected utility represents the said preferences over lotteries.
- Just as you needed assumptions on preferences over outcomes to build a utility function representing them, you need **assumptions on preferences over lotteries** to build an expected utility function representing them.
- There are four required axioms. The first two are the same as the axioms needed on preferences over outcomes, but now applied to lotteries:

  1. **Completeness:** For any lotteries L and L', either $L \succ L'$, $L \sim L'$, or $L \prec L'$.
  2. **Transitivity:** If $L \succsim L'$ and $L' \succsim L''$, then $L \succsim L''$.

Assumptions on preferences
2. Reduction of compound lotteries

2. Reduction of compound lotteries

(or consequentialist preferences)

A lottery is a probability distribution over a set of possible outcomes. A simple lottery is a vector $L=(p_1,p_2,\ldots,p_N)$ such that $p_n \geq 0$ for all n and $\Sigma_n p_n=1$.

In a compound lottery an outcome may itself be a simple lottery

Assumptions on preferences
Reduction of compound lotteries

Definition (MWG 6.B.2): Given K lotteries $L_k = (p_1^k, \ldots, p_N^k)$, $k = 1, \ldots, K$, and probabilities $\alpha_k \geq 0$ with $\Sigma_k \alpha_k = 1$, the compound lottery $(L_1,\ldots,L_K; \alpha_1,\ldots, \alpha_K)$ is the risky alternative that yields the simple lottery L_k with probability α_k.

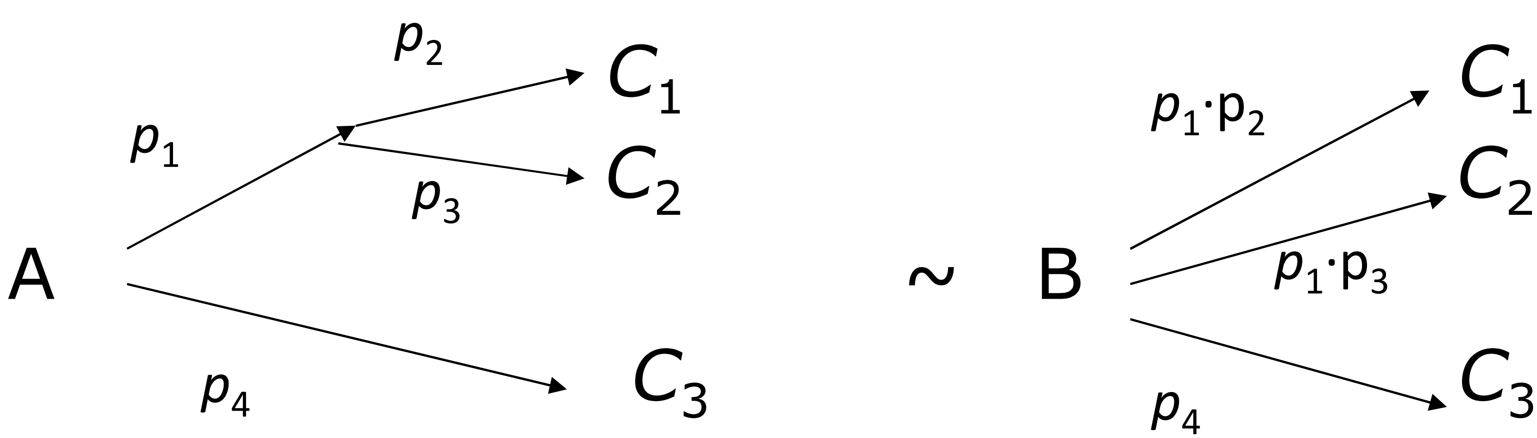
- for any compound lottery $(L_1,\ldots,L_K; \alpha_1,\ldots, \alpha_K)$, we can calculate a corresponding *reduced lottery* as the simple lottery $L = (p_1, \ldots, p_N)$ that generates the same ultimate distribution over outcomes:

$$L = \alpha_1 L_1 + \ldots + \alpha_K L_K \in \Delta$$

Assumptions on preferences Reduction of compound lotteries

- Reduction of compound lotteries (consequentialist preferences): Consumers care only about the distribution over final outcomes, not whether this distribution comes about as a result of a simple lottery, or a compound lottery. In other words, the consumer is indifferent between any two compound lotteries that can be reduced to the same simple lottery. This property is often called reduction of compound lotteries.
- Because of the reduction property, we can confine our attention to the set of all simple lotteries.

Assumptions on preferences
Reduction of compound lotteries



- Another way to think about the reduction property is that we're assuming there is no process-oriented utility. Consumers do not enjoy the process of the gamble, only the outcome, eliminating the "fun of the gamble" in settings like casinos.
- Only the outcome matters, not the process.

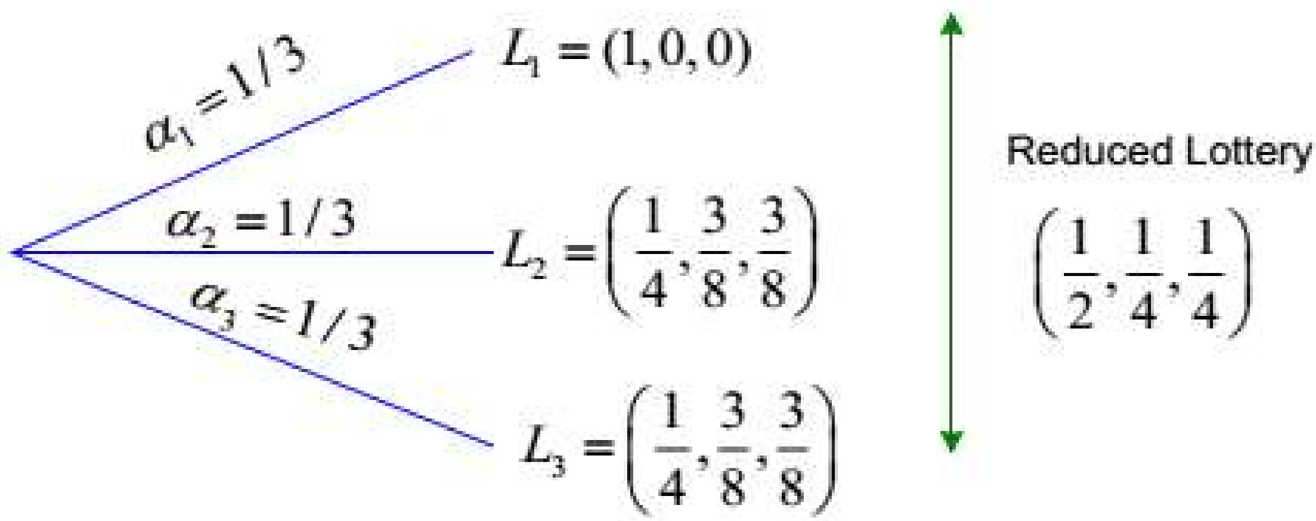
Compound and Reduced Lotteries: Example 1

All three lotteries are equally likely

$$P(\text{outcome } 1) = \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{2}$$

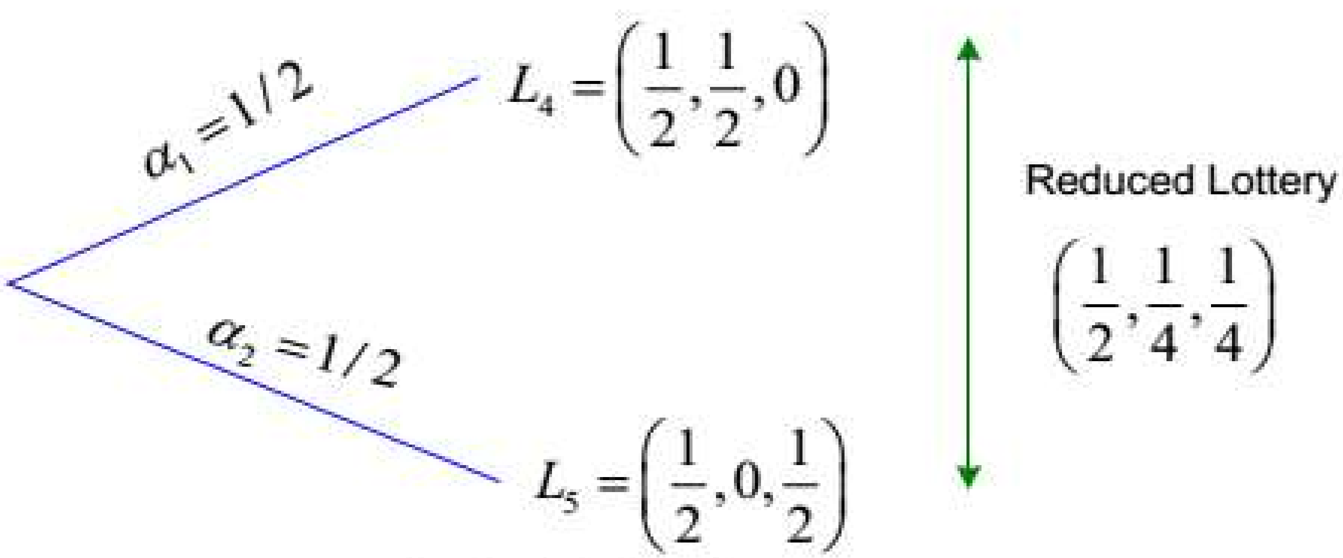
$$P(\text{outcome } 2) = \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot\frac{3}{8} + \frac{1}{3}\cdot\frac{3}{8} = \frac{1}{4}$$

$$P(\text{outcome } 3) = \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot\frac{3}{8} + \frac{1}{3}\cdot\frac{3}{8} = \frac{1}{4}$$



Compound and Reduced Lotteries: Example 2

Both lotteries are equally likely



Outcome 1 $\rightarrow \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

Outcome 2 $\rightarrow \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 0 = \frac{1}{4}$

Outcome 3 $\rightarrow \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Assumptions on preferences
3. The independence axiom

Independence Axiom (MWG Def 6.B.4): The preference relation $\succsim$ on the space of simple lotteries $\mathscr{L}$ satisfies the *independence axiom* if for any L,L',L'' $\in \mathscr{L}$ and $\alpha \in (0,1)$ we have

$$L \succsim L' \text{ if and only if } \alpha L + (1-\alpha)L'' \succsim \alpha L' + (1-\alpha)L''.$$

- i.e. if we mix each of two lotteries with a third one, then the preference ordering does not change,
  - i.e. is *independent* of the third lottery

Assumptions on preferences
3. The independence axiom

- The independence axiom says that I prefer *L* to *L′*, I'll also prefer the possibility of *L* to the possibility of *L′*, given that the other possibility in both cases is some *L″*.
- *OR in other words, more intuitively*

  If we mix each of two lotteries, *L* and L′, with a third one (*L″*), then the preference ordering of the two resulting compound lotteries is independent of the particular third lottery

Assumptions on preferences
The independence axiom

- Suppose that I offer you the choice between the following two alternatives:

  L : $5 with probability 1/5, 0 with probability 4/5

  L' : $12 with probability 1/10, 0 with probability 9/10

- Suppose you prefer L to L'. Now consider the following alternative. I flip a coin. If it comes up heads, I offer you the choice between L and L'. If it comes up tails, you get nothing. What the independence axiom says is that if I ask you to choose either L or L' before I flip the coin, your preference should be the same as it was when I didn't flip the coin.
- So, if you prefer L to L', then you should also prefer $\frac{1}{2}L + \frac{1}{2}0 \succsim \frac{1}{2}L' + \frac{1}{2}0$

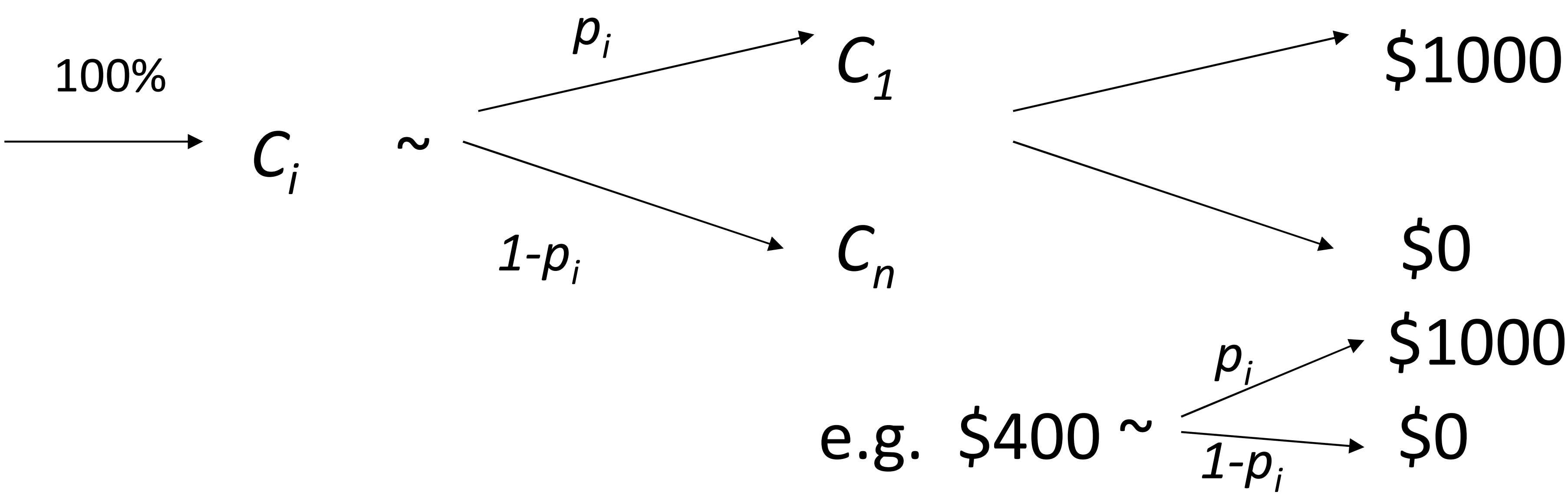
- Independence of irrelevant alternatives

Assumptions on preferences
4. Continuity

Suppose C_1 is the best outcome

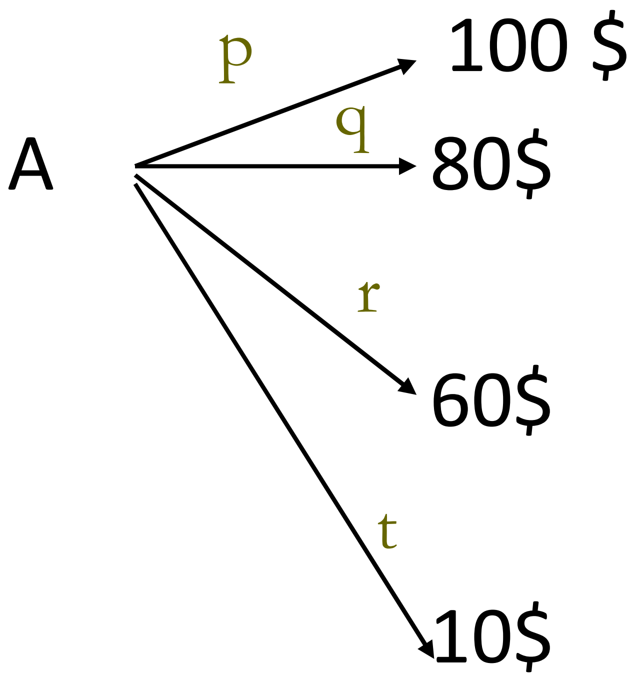
Suppose C_n is the worst outcome

For any outcome C_i between best and worst, there will be some probability p_i, such that we are indifferent between:

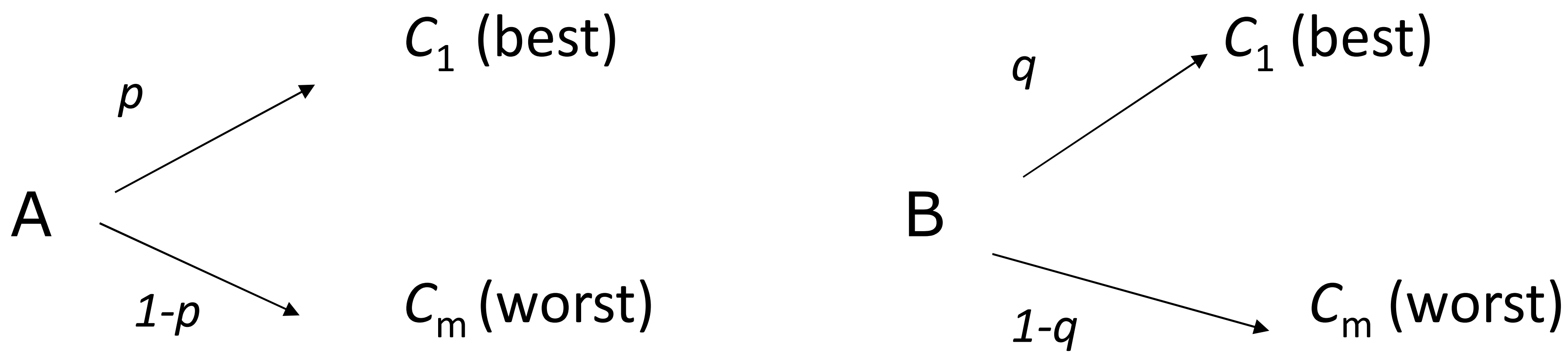


Assumptions on preferences

- The above assumptions combined mean that any lottery may be written as a simpler lottery that only involves the best and the worst outcome.
- Example:



Assumptions on preferences
5. Monotonicity



A $\succeq$ B iff $p \geq q$

Expected Utility Theorem

- The expected utility theorem says that if a consumer's preferences over simple lotteries are rational, continuous, and exhibit the reduction and independence properties, then there is a utility function of the expected utility form that represents those preferences.

$$EU(L) = \sum_n p_n u_n$$

That is, there are numbers $u_1, \ldots, u_N$ such that

$U(L) = \sum_{n=1}^{N} p_n^L u_n$, *and for any two lotteries,*

$$L \succsim L' \text{ if and only if } U(L) \geq U(L')$$

Expected Utility Theorem: example

- Take umbrella/Not take umbrella
- Preferences of the individual:

  u (no umbrella, sunny) = 10 = u_1

  u (umbrella, rains) = 8 = u_2

  u (umbrella, sunny) = 6 = u_3

  u (no umbrella, rains) = 0 = u_4

1) If you know it's going to rain : $u_2 > u_4 \rightarrow$ UMB
2) If you know it's not going to rain: $u_1 > u_3 \rightarrow$ no UMB

Expected Utility Theorem: example

- Take umbrella/Not take umbrella
- Preferences of the individual:

  u (no umbrella, sunny) = 10 = u_1

  u (umbrella, rains) = 8 = u_2

  u (umbrella, sunny) = 6 = u_3

  u (no umbrella, rains) = 0 = u_4

3) Utility maximization if $P_{rain} = 0.6$.

EU(UMB) = 0.6*8+0.4*6 = 7.2

EU(NO UMB) = 0.6*0+.4*10 = 4

I take an umbrella because EU(UMB) > EU (NO UMB)

Expected Utility Theorem: example

- Am I allowed to use ordinal utility functions when I work with uncertainty?

Expected Utility Theorem: example

- Am I allowed to use ordinal utility functions when I work with uncertainty?

NO

Expected Utility Theorem: example

- Take umbrella/Not take umbrella
- Preferences of the individual:

u (no umbrella, sunny) = 100 = u_1

u (umbrella, rains) = 8 = u_2

u (umbrella, sunny) = 6 = u_3

u (no umbrella, rains) = 0 = u_4

This is still a utility function preserving the order of preferences (ordinal)

Expected Utility Theorem: example

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This is still a utility function preserving the order of preferences (ordinal)

3) Utility maximization if $P_{rain} = 0.6$.

EU(UMB) = 0.6*8+0.4*6 = 7.2

EU(NO UMB) = 0.6*0+.4*100 = 40

Don't take an umbrella because EU(UMB) < EU (NO UMB) !!!

Von-Neumann-Morgenstern UF (vN-M)

- The Expected Utility Form is preserved only by positive linear transformations. If U (.) and V (.) are utility functions representing $\succeq$, and U () has the expected utility form, then V (.) also has the expected utility form if and only if there are numbers $a > 0$ and b such that:

  $$U(L) = aV(L) + b.$$

  In other words, the expected utility property is preserved by positive linear transformations, but any other transformation of U (.) does not preserve this property.
- We will call the utility function of the expected utility form a von-Neumann-Morgenstern (vNM) utility function.

Von-Neumann-Morgenstern UF (vN-M)

- In the previous example

  u (no umbrella, sunny) = 10 = u_1

  u (umbrella, rains) = 8 = u_2

  u (umbrella, sunny) = 6 = u_3

  u (no umbrella, rains) = 0 = u_4

Could be transformed to

  u (no umbrella, sunny) = 100 = v_1

  u (umbrella, rains) = 80 = v_2

  u (umbrella, sunny) = 60 = v_3

  u (no umbrella, rains) = 0 = v_4

Where $v(.)=\alpha * u(.)+\beta$
with $\alpha = 10$ and $\beta = 0$

Von-Neumann-Morgenstern UF (vN-M)

- In the previous example

  u (no umbrella, sunny) = 10 = u_1

  u (umbrella, rains) = 8 = u_2

  u (umbrella, sunny) = 6 = u_3

  u (no umbrella, rains) = 0 = u_4

Alternatively, I could set top utility $v_1 = 100$, worst utility $v_4 = 5$ and calculate what the other two utilities should be so that I have a linear transformation, $100 = a*10+\beta$ and $5 = \alpha*0+\beta$, so that $\alpha = 9.5$ and $\beta = 5$.

  u (no umbrella, sunny) = 100 = v_1

  u (umbrella, rains) = $9.5*8+5 = v_2$

  u (umbrella, sunny) = $9.5*6+5 = v_3$

  u (no umbrella, rains) = 5 = v_4

Von-Neumann-Morgenstern UF (vN-M)

- The numbers assigned by the vN-M utility function have cardinal significance.
- Suppose u(A) = 30

  u(B) = 20

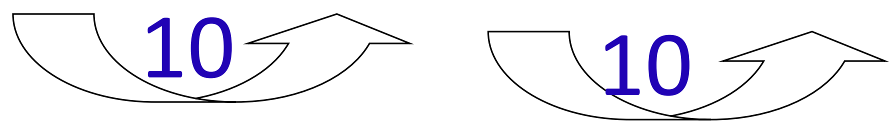
  u(C) =10

Is A three times better than C?

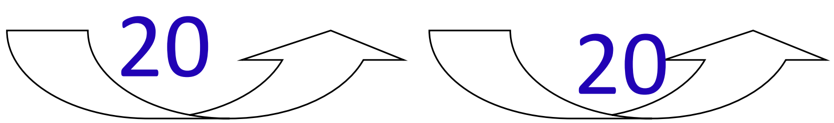
Answer: NO (a >0)

Von-Neumann-Morgenstern UF (vN-M)

- The vN-M utility function is cardinal in the sense that utility differences **are** preserved.
- For example start with u(A) = 30 , u(B) = 20, u(C) =10



- Apply linear transformation with α=2 and β=1, v(A) = 61 , v(B) = 41, v(C) =21



- A is preferred to B as much as B is preferred to C.

Von-Neumann-Morgenstern UF (vN-M) - example

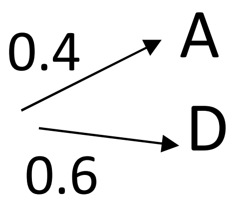
Exercise: assume an individual with preferences $A \succ B \succ C \succ D$. This individual is indifferent between *B* and the lottery (*A*, *D*;0.4, 0.6). Also she is indifferent between *C* and the lottery (*B*, *D*; 0.2, 0.8). Construct a set of vN-M utility numbers for the four situations.

Von-Neumann-Morgenstern UF (vN-M) - example

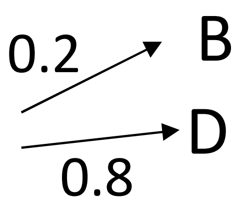
Exercise: assume an individual with preferences $A \succ B \succ C \succ D$. This individual is indifferent between *B* and the lottery (*A*, *D*;0.4, 0.6). Also she is indifferent between *C* and the lottery (*B*, *D*; 0.2, 0.8). Construct a set of vN-M utility numbers for the four situations.

Answer

By definition of the utility function $U_A > U_B > U_C > U_D$.

B ~ 

, therefore $U_B = 0.4*U_A + 0.6*U_D$

C ~ 

, therefore $U_C = 0.2*U_B + 0.8*U_D$

Von-Neumann-Morgenstern UF (vN-M) - Example

- Choose just two utility levels (α u(.) + β): gives us two degrees of freedom. So U_A = 1 and U_D=0, then solve for U_B and U_C

Objections with the theory of expected utility: Allais paradox

- Allais (1953): present participants with two different experiments.
- First experiment: choose between A and B

Would you choose A or B?

Objections with the theory of expected utility: Allais paradox

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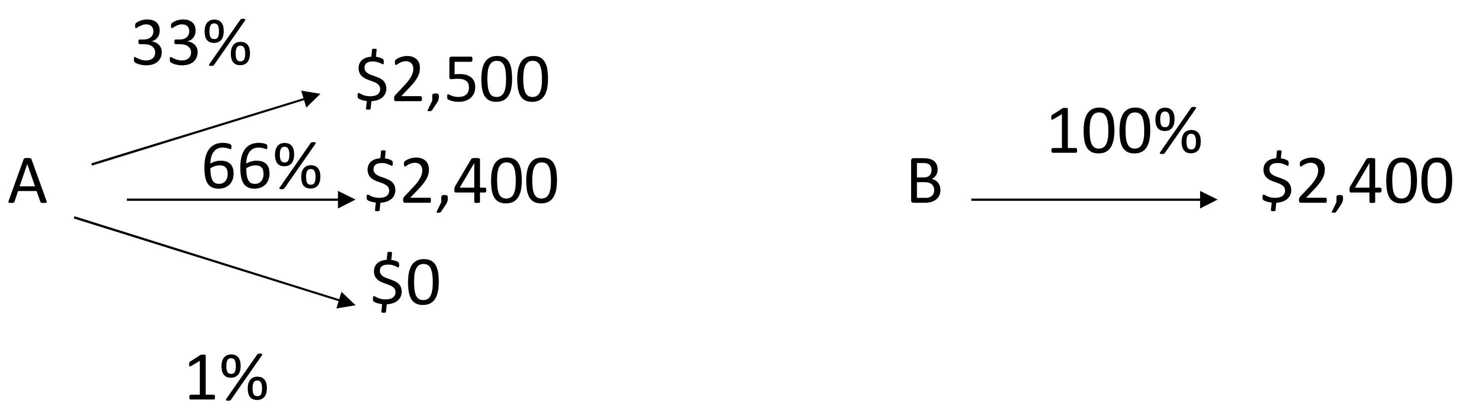


Most participants choose B, therefore they must consider

U(2,400) > 0.33 U(2,500) + 0.66U(2,400) + 0.01 U(0)

Objections with the theory of expected utility: Allais paradox

- Allais (1953): present participants with two different experiments.
- Second experiment: choose between C and D

Objections with the theory of expected utility: Allais paradox

- Allais (1953): present participants with two different experiments.
- Second experiment: choose between C and D

Most participants choose C, therefore they must consider

0.33*U(2,500) + 0.67*U(0) > 0.34 U(2,400) + 0.66 U(0)

Objections with the theory of expected utility: Allais paradox

- These choices do not accord with the Expected utility theory.

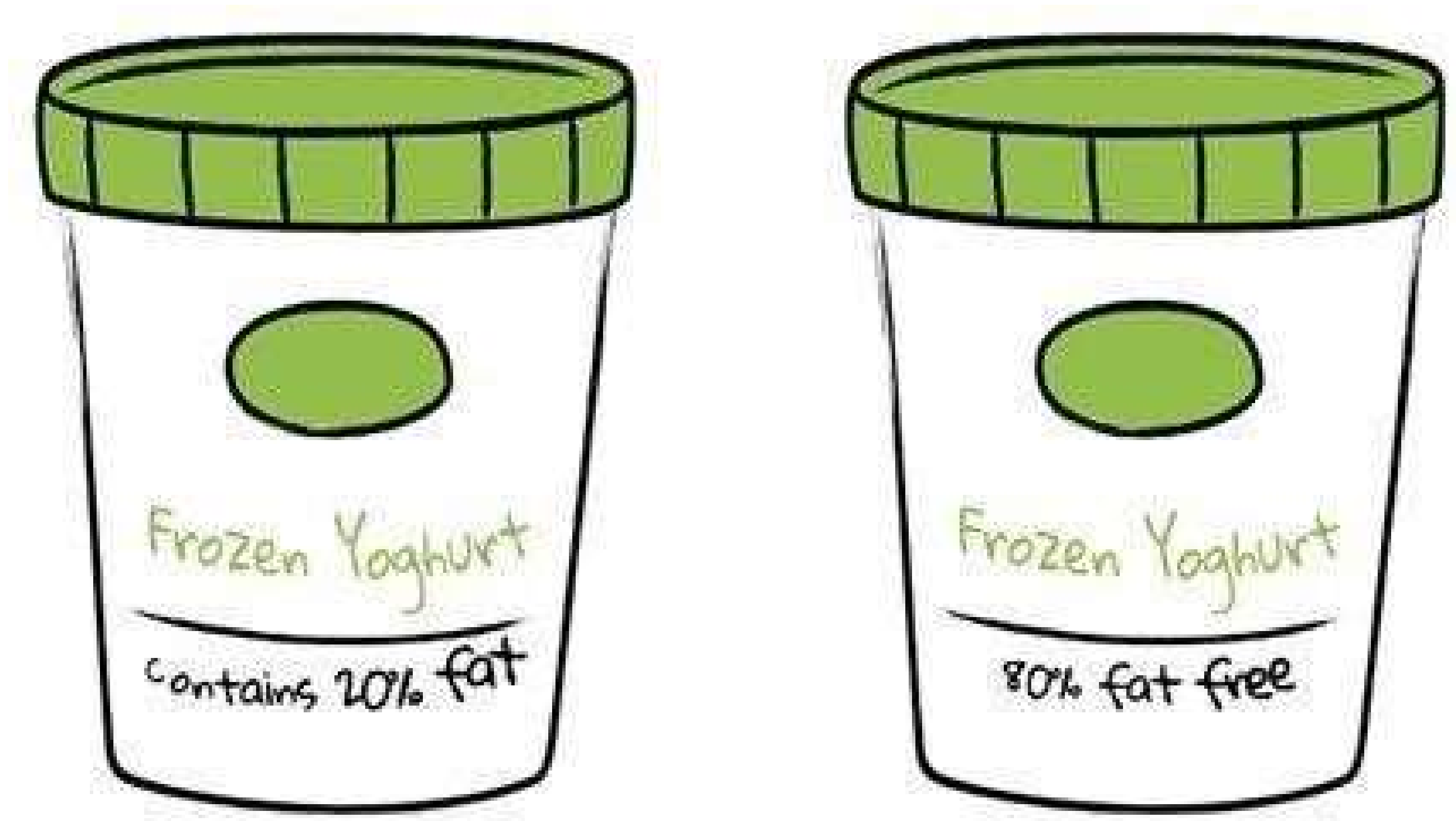
$U(2{,}400) > 0.33\ U(2{,}500) + 0.66\ U(2{,}400) + 0.01\ U(0)$ (1)

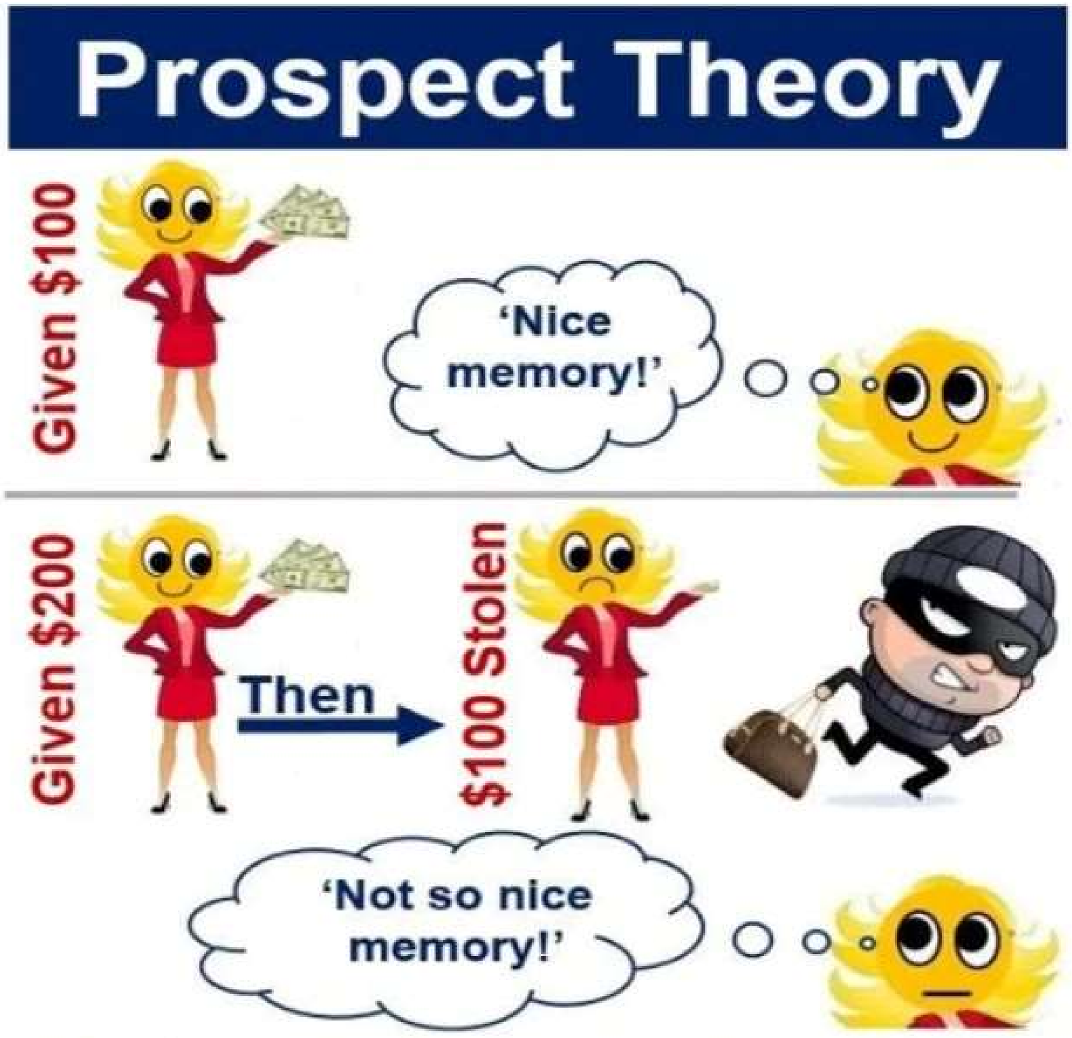
$0.33 * U(2{,}500) + 0.67 * U(0) > 0.34\ U(2{,}400) + 0.66\ U(0)$ (2)

These two say exactly the opposite thing!

Objections with the theory of expected utility: Allais paradox

- Possible explanations
  - U(0 / when $2,500 is also available) ≠ U(0 / if the top price is $10) because of the regret you would feel.
  - People are not rational
  - People cannot process very small/high probabilities
  - "framing effect": *equivalent descriptions of a decision problem lead to systematically different decisions*


Frozen Yoghurt
Contains 20% fat
Frozen Yoghurt
80% fat free


Prospect Theory
Given $100
'Nice memory!'
Given $200
Then
$100 Stolen
'Not so nice memory!'

Objections with the theory of expected utility: the “framing effect”

More on the framing effect

Objects described in terms of a positively valenced proportion are generally evaluated more favorably than objects described in terms of the corresponding negatively valenced proportion. For example, in one study, beef described as “75% lean” was given higher ratings than beef described as “25% fat” (Levin and Gaeth 1988)

Objections with the theory of expected utility: the "framing effect"

- The best-known risky choice framing problem is the so-called "Asian Disease Problem" (Tversky and Kahneman 1981).

  In it, subjects first read the following background blurb:

  - Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. One possible program to combat the disease has been proposed. Assume that the exact scientific estimate of the consequences of this program is as follows:

Some subjects are then presented with options A and B:

- A: If this program is adopted, 200 people will be saved.
- B: If this program is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no people will be saved.

Objections with the theory of expected utility: the “framing effect”

- Other subjects are presented with options C and D:
  - C: If this program is adopted, 400 people will die.
  - D: If this program is adopted, there is a one-third probability that nobody will die and a two-thirds probability that 600 people will die.

The robust experimental finding is that subjects tend to prefer the sure thing when given options A and B, but tend to prefer the gamble when given options C and D. Note, however, that options A and C are equivalent, as are options B and D. Subjects thus appear to be risk-averse for gains and risk-seeking for losses, a central tenet of prospect theory.

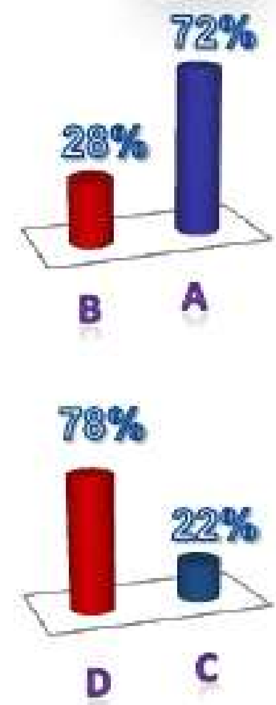
Framing Effect Experiment

Scenario 1

- Option A saves 200 people's lives
- Option B has a 33% chance of saving all 600 people and a 66% possibility of saving no one

Scenario 2

- If option C is taken, then 400 people die
- If option D is taken, then there is a 33% chance that no one will die and a 66% probability that all 600 will die



- **Scenario 1 = Scenario 2**
- **Different phrasing impacted participants' responses to a question about a disease prevention strategy**

Framing effect: real world examples

- Political jargon:

– “Tax Cut” vs. “Tax Relief”

– “Global Warming” vs. “Climate Change”

- Medical prognoses:

– 90% death rate vs. 10% survival rate

- Employee Satisfaction Surveys:

– Is the company inflexible with respect to family responsibilities

vs.

– Is the company flexible with respect to family responsibilities?

Used in marketing…

Use Of Loss Aversion For Retention

- An Italian telecom company managed to increase the acceptance rate of an offer made to customers when they called to cancel their service
- Originally, a script informed them that they would receive 100 free calls if they kept their plan
- The script was reworded to read - "We have already credited your account with 100 calls — how could you use those?"
- Many customers did not want to give up free talk time they felt they already owned

Objections with the theory of expected utility: justifications

- People make decisions based on justifications rather than utilities (Shafir)
  - Imagine that you've just taken finals, and it's the end of the fall semester. **You're feeling tired and run-down, but you find out that you passed all your classes with good grades.** Now you have the opportunity to buy a very attractive 5 day Christmas vacation package in Hawaii at a really low price. The offer expires tomorrow. Would you:
    - a) Buy the vacation package
    - b) Not buy it

  Most people say they would buy it (54%)

Objections with the theory of expected utility: justifications

- People make decisions based on justifications rather than utilities (Shafir)
  - Imagine that you've just taken finals, and it's the end of the fall semester. **You're feeling tired and run-down, and you find out that you've failed all your classes.** Now you have the opportunity to buy a very attractive 5 day Christmas vacation package in Hawaii at a really low price. The offer expires tomorrow. Would you:
    - a) Buy the vacation package
    - b) Not buy it

Still, most people say they would buy it (57%)

Objections with the theory of expected utility: justifications

- People make decisions based on justifications rather than utilities (Shafir)
  - Imagine that you've just taken finals, and it's the end of the fall semester. **You're feeling tired and run-down, and you won't find out if you passed your exams until next month.** Now you have the opportunity to buy a very attractive 5 day Christmas vacation package in Hawaii at a really low price. The offer expires tomorrow. Would you:
    - a) Buy the vacation package
    - b) Not buy it

Fewer people say they would buy it (32%)

Objections with the theory of expected utility: justifications

- Passing or failing does not affect purchase of vacation as much as **knowing the outcome** of the exam.

- This can't be explained by utility theory.

- These results make sense, *if* decisions depend on justifications:
  - Passing justification:
    - You need a vacation to celebrate!
  - Failing justification:
    - You need a vacation to make yourself feel better
  - Don't know the results:
    - No basis for justifying the vacation

What affects justifications?

- Emotional factors
  - Most people would trade in an old car for a brand new one.
  - Fewer people would trade in an old wedding ring for a brand new one.
- Regret
  - People are highly motivated to avoid mistakes they will regret later
  - (this is a component of loss-aversion)
- Fairness
  - “fair” decisions favored over “unfair” ones

Money lotteries and risk aversion

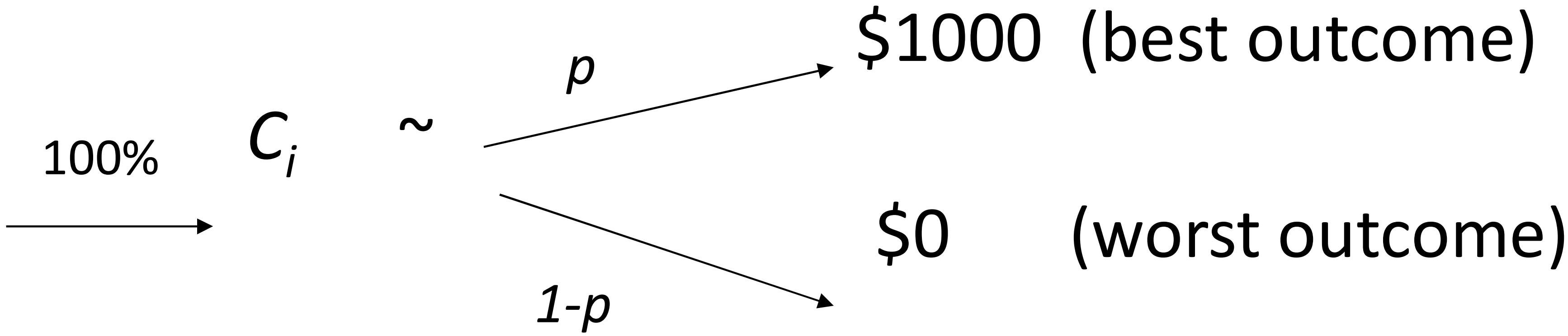
- Concept of “risk aversion”

Suppose you face the following lottery

If you prefer B to A, then you are “risk averse”, since for you, $u(4) > 0.5*u(10) + 0.5*u(0)$

Money lotteries and risk aversion

For any outcome C_i between best and worst, there will be some probability p_i, such that:



For any C_i (e.g. $2, or $10) between best and worst outcome, there will be some probability p_i, such that we are indifferent between the certainty of C_i and the gamble between best and worst.

Choice under uncertainty

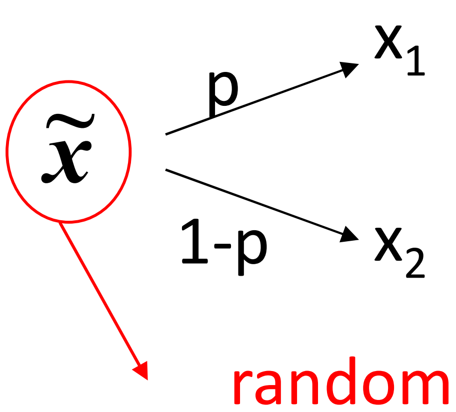
- Expected utility principle: (*u* is cardinal and we work with uncertainty)

  $A \succeq B \iff Eu(A) \geq Eu(B)$

(Recall that without uncertainty *u* was ordinal and we had

$A \succeq B \iff u(A) \geq u(B)$)

Suppose that an individual faces the following lottery (gamble):



What is the expected value of this gamble?

Choice under uncertainty

- $E(\tilde{x}) = px_1 + (1-p)x_2 = \bar{x}$, where $\bar{x}$ is the mean value of the gamble (what I would gain on average).
- Suppose that you also have the choice

$$\xrightarrow{100\%} \quad px_1 + (1\text{-}p)x_2$$

The gamble and the sure bet have exactly the same expected value. Which one would you choose? The one that gives you greater utility. So, let’s compare the utility levels of the gamble and of the sure bet.

Choice under uncertainty

- Suppose that the utility from x_1 is $u(x_1)$ and the utility from x_2 is $u(x_2)$.
- Expected utility of the gamble:

$$Eu(\tilde{x}) = pu(x_1) + (1-p)u(x_2)$$

- Utility of the sure bet

$$u(\bar{x}) = u(E(\tilde{x})) = u(px_1 + (1-p)x_2)$$

Risk aversion

- We define an individual as risk-avert if he prefers gaining the expected value with certainty than incurring some risk but gaining the same value on average.

$$\bar{x} \succeq \tilde{x} \quad \text{iff} \quad u(px_1 + (1\text{-}p)\, x_2) \geq pu(x_1) + (1\text{-}p)\, u(x_2)$$

certain amount gamble

Risk aversion

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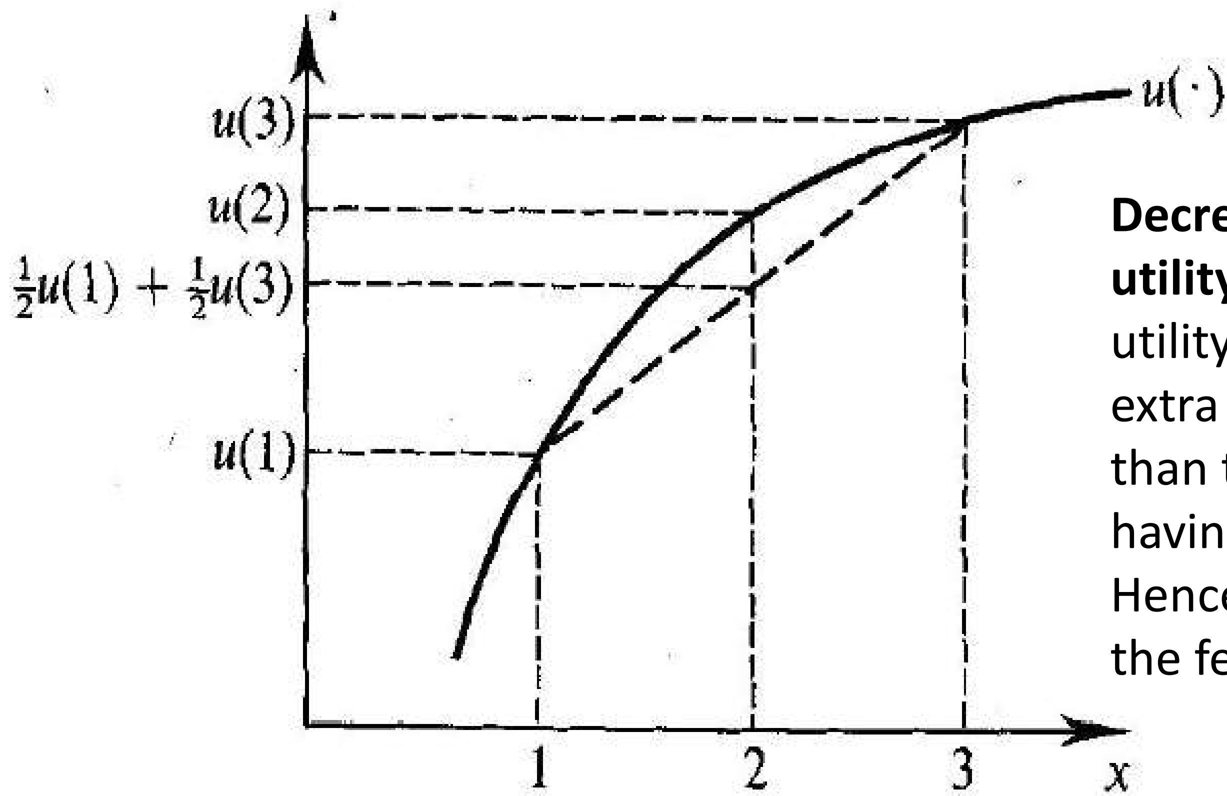
$$\bar{x} \succeq \tilde{x} \quad \text{iff} \quad u(px_1 + (1-p)\,x_2) \geq pu(x_1) + (1-p)\,u(x_2)$$

certain amount ($\bar{x}$) — gamble ($\tilde{x}$)

But this happens when u(.) is a concave function

Risk aversion

- strict concavity: marginal utility of money is decreasing
  - hence utility gain from 1 extra Euro lower than utility loss of 1€ less
  - hence equal probability risk of gaining or loosing 1€ not worth taking



Decreasing marginal utility of income. The utility gain from an extra euro is lower than the utility loss of having a euro less. Hence risk aversion = the fear of losing

Risk-loving attitude

- Risk-loving attitude

$\tilde{x} \succeq \bar{x}$ iff $u(px_1 + (1-p)\,x_2) \le pu(x_1) + (1-p)\,u(x_2)$

certain amount

gamble

Risk-loving attitude

- Risk-loving attitude

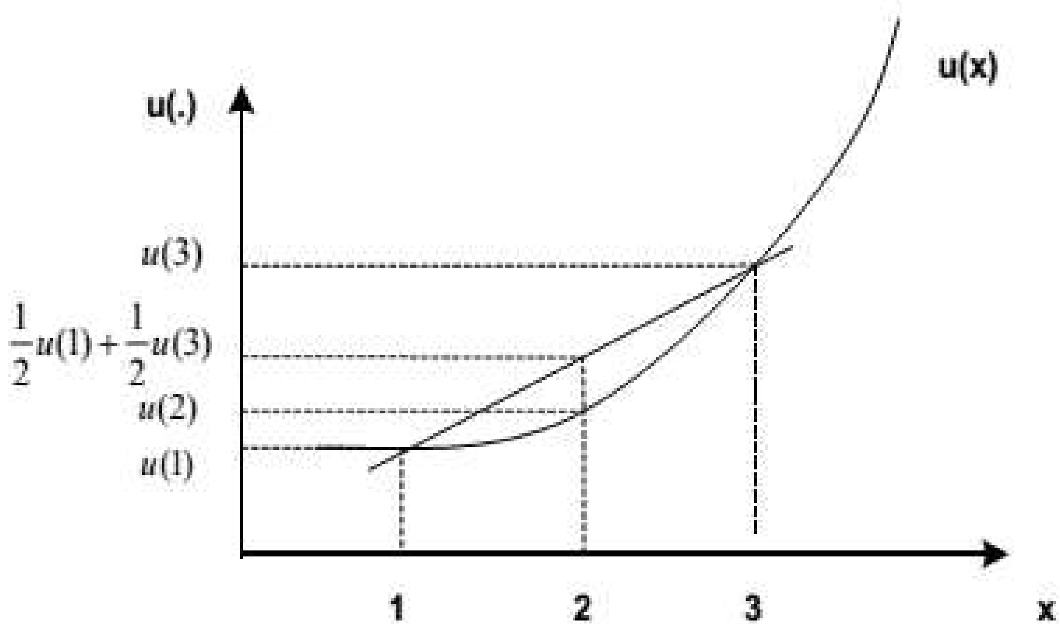
$$\bar{x} \succeq \tilde{x} \quad \text{iff} \quad u(px_1 + (1-p)\, x_2) \geq pu(x_1) + (1-p)\, u(x_2)$$

certain amount   gamble

But this happens when u(.) is a convex function

Risk-loving attitude

Utility from the expected value of the lottery, $u(2)$, is **lower** than the expected utility from playing the lottery, $\frac{1}{2}u(1) + \frac{1}{2}u(3)$.



Risk neutrality

- We define an individual as risk-neutral

$\bar{x} \sim \tilde{x}$ iff $u(px_1 + (1-p)\, x_2) = pu(x_1) + (1-p)\, u(x_2)$

certain amount ($\bar{x}$) gamble ($\tilde{x}$)

Risk neutrality

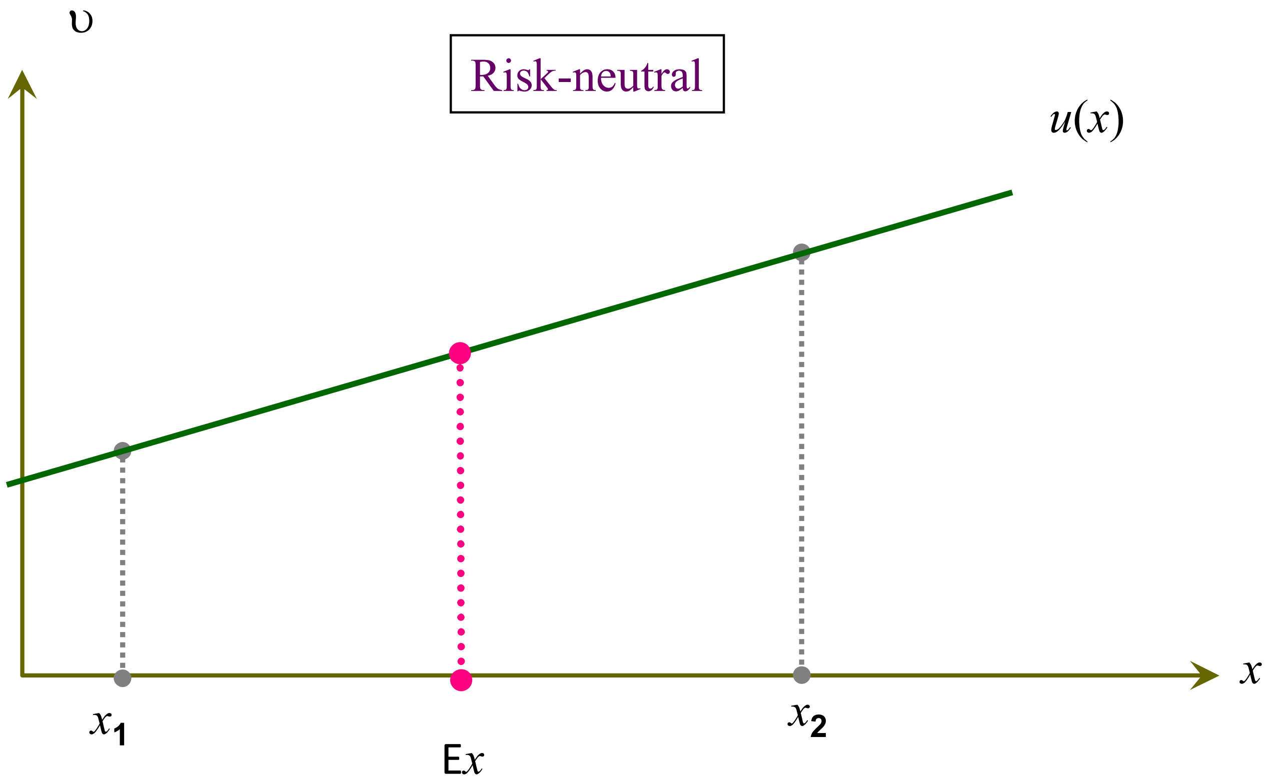
- We define an individual as risk-neutral

$\bar{x}$ ~ $\tilde{x}$ iff $u(px_1 + (1\text{-}p)\, x_2) = pu(x_1) + (1\text{-}p)\, u(x_2)$

certain amount gamble

This means that u(.) is a linear function

Risk neutrality



Example

- Suppose that you face a choice between A and B, where

If you prefer B to A, can we say that you are risk-averse?

Example

- Suppose that you face a choice between A and B, where

If you prefer B to A, can we say that you are risk-averse?

Answer: YES. Because for you $u(4) > 0.5\, u(10) + 0.5\, u(0)$

Summary so far

- If a person has a concave utility function, he is risk-averse.

  That is $u(px_1 + (1\text{-}p)\, x_2) \geq pu(x_1) + (1\text{-}p)\, u(x_2)$

- If a person has a convex utility function, he is a risk-lover.

  That is $u(px_1 + (1\text{-}p)\, x_2) \leq pu(x_1) + (1\text{-}p)\, u(x_2)$

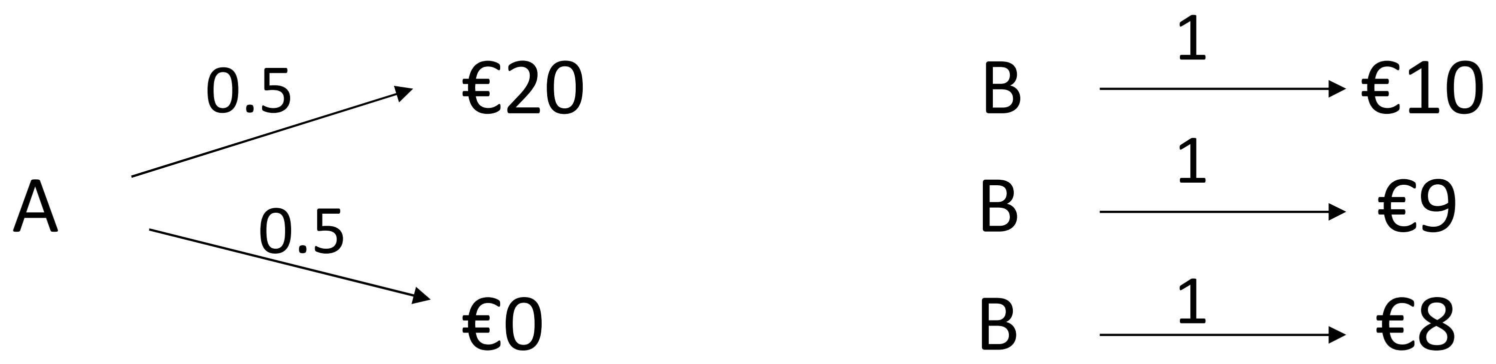
- If a person has a linear utility function, he is a risk-neutral person.

  That is $u(px_1 + (1\text{-}p)\, x_2) = pu(x_1) + (1\text{-}p)\, u(x_2)$

Certainty equivalent

- Again consider a choice between A and B.

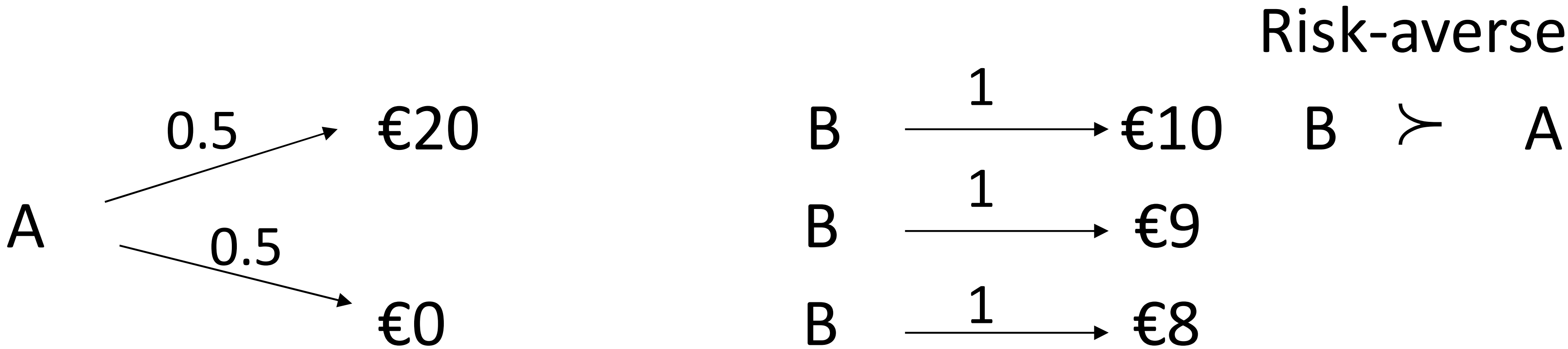
Risk-averse



The expected value of the lottery is 10€.

Certainty equivalent

- Again consider a choice between A and B.



Certainty equivalent

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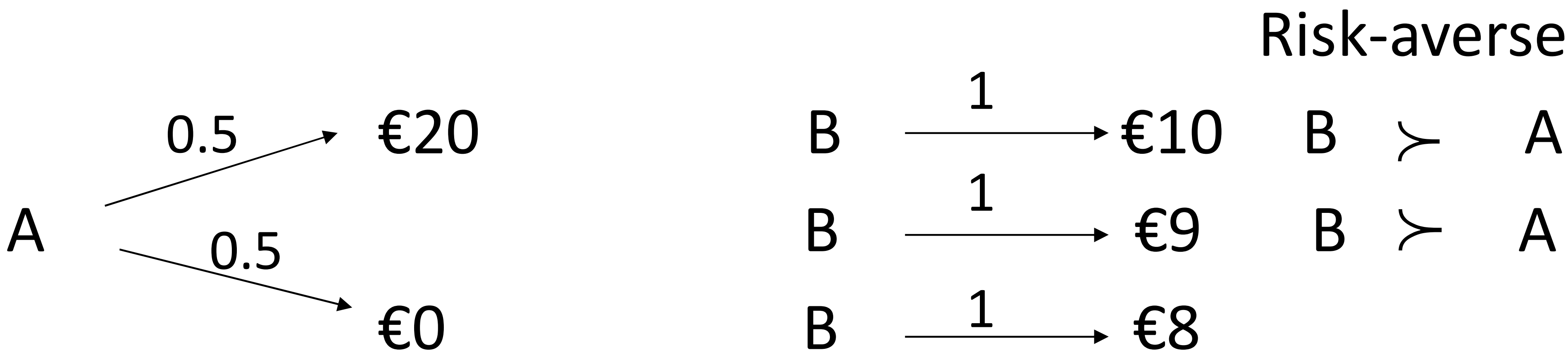

Certainty equivalent

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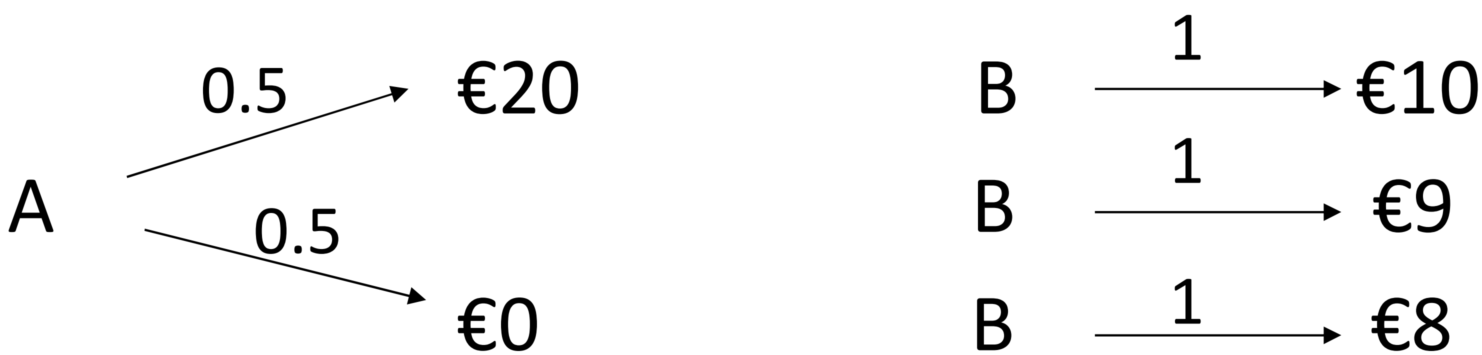


Risk-averse

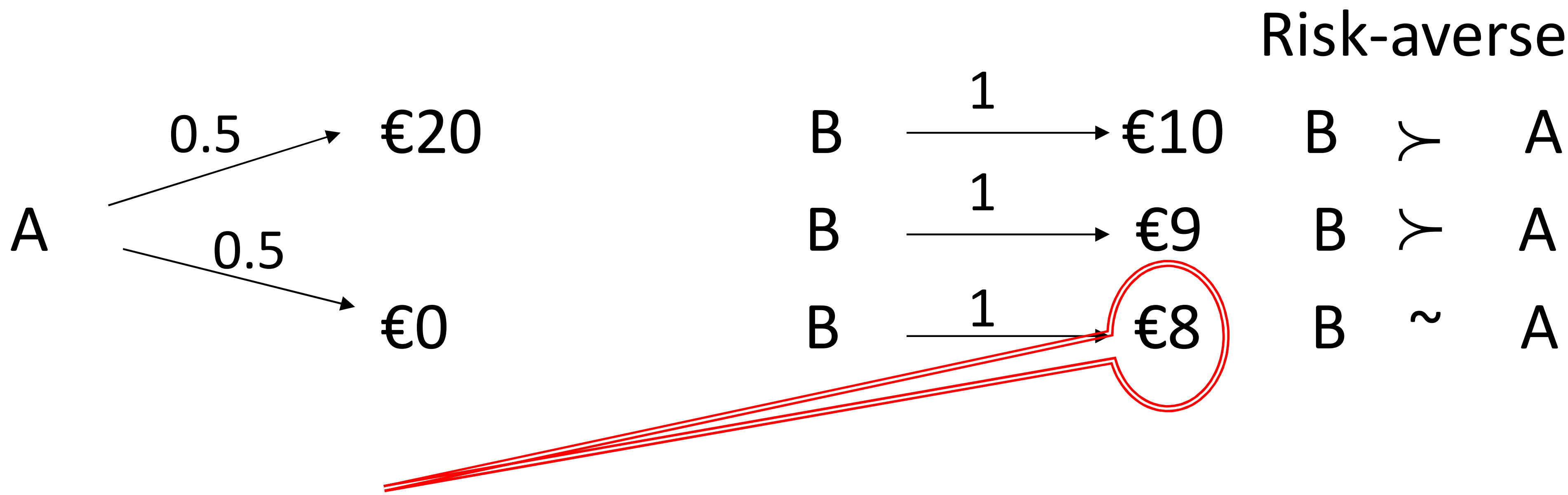
$B \succ A$

$B \succ A$

$B \sim A$

Certainty equivalent

- Again consider a choice between A and B.



Certainty equivalent (let's denote it *y*): it is the amount that makes me indifferent between the gamble and the sure amount. It is the amount of money that, if gained with certainty, provides the same utility as the gamble.

Risk premium

- Again consider a choice between A and B.

Risk-averse

A: 0.5 → €20; 0.5 → €0

B $\xrightarrow{1}$ €10　　B $\succ$ A

B $\xrightarrow{1}$ €9　　B $\succ$ A

B $\xrightarrow{1}$ €8　　B ~ A

Risk premium (π): the difference between the expected value of the gamble and the certainty equivalent
(: 10€ -8€ = 2€).

Risk premium (π)

- The risk premium is the money I abandon in order to have more safety.

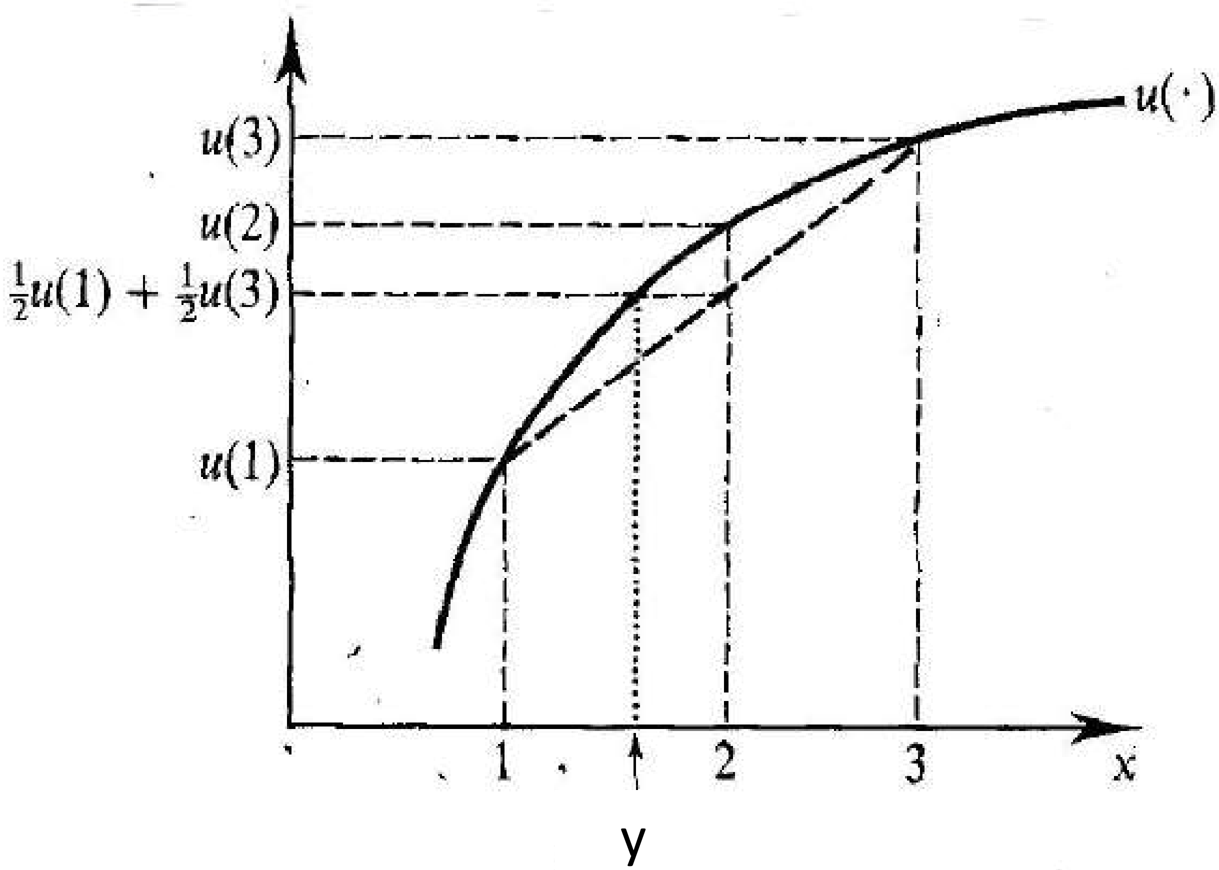
Or, in other words, the loss of income that can be conceded in order to get rid of the risk (and obtain the certainty equivalent).

It measures the gap between the expected value of the gamble and the certainty equivalent. It is "positively correlated" with risk aversion.

In summary, $u(\bar{x} - \pi) = u(y) = Eu(\tilde{x})$ and $y + \pi = \bar{x}$

Graphical representation (similar from MWG)

- certainty equivalent for even probability game between 1 and 3 Euro



- risk premium: difference between expected value of lottery and certainty equivalent

The probability premium

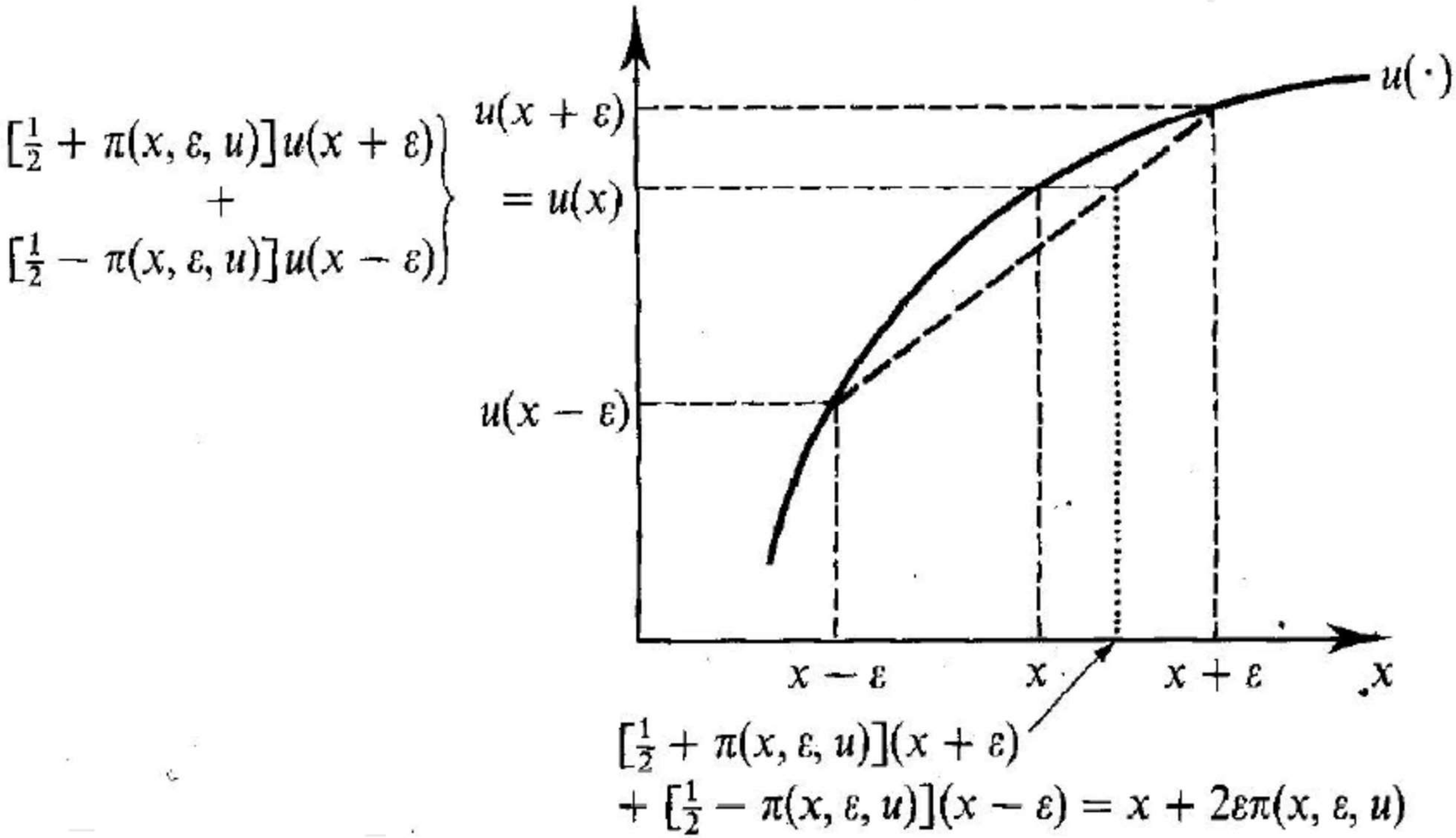
- For any fixed amount of money x and positive number ε, the probability premium denoted by $\pi(x, \varepsilon, u)$, is the excess in winning probability over fair odds that makes the individual indifferent between the certain outcome x and a gamble between the two outcomes $x+\varepsilon$ and $x-\varepsilon$. That is

$$u(x) = (½ + \pi(x, \varepsilon, u))*u(x+\varepsilon) + (½ - \pi(x, \varepsilon, u))*u(x-\varepsilon)$$

Intuition: Better than fair odds must be given for the individual to accept the risk.

The probability premium graphically

probability premium



Example

- Suppose you have to pay \$2 for a ticket to enter a competition. The prize is \$19 and the probability that you win is 1/3. You have an expected utility function with $u(x)=\log x$ and your current wealth is \$10.
- 1. What is the certainty equivalent of this competition?
- 2. What is the risk premium?
- 3. Should you enter the competition?

Measuring risk aversion

- How can we compare degrees of risk aversion?
- It must have something to do with the concavity of the utility function. More concave functions should correspond to more risk aversion. The higher the distance between u(x) and Eu(x).
- U'' is a measure of concavity, but it is not suitable because if we linearly transform u to $au + b$, a>0, the second derivative of u is u'', while the second derivative of $au + b$ is au''.
- Solution: standardize with u'(.)
- But u'(.) will be negative for risk averse persons, so put a minus sign in front in order to get an coefficient of risk aversion.

The Arrow-Pratt measure of absolute risk aversion

We can define:

Definition (Absolute risk aversion coefficient)

The absolute risk aversion coefficient (also called the ***Arrow-Pratt coefficient of absolute risk aversion****) is:*

$$r_A(x, u) = -\frac{u''(x)}{u'(x)} > 0$$

It is a concavity index for $u(.)$ which is invariant to positive linear transformations of $u(.)$.

The Arrow-Pratt measure of absolute risk aversion

Note that:

1.

$$r_A(x)\begin{cases} >0 & \text{for risk - averse decison maker} \\ =0 & \text{for risk - neutral decison maker} \\ <0 & \text{for risk - loving decison maker} \end{cases}$$

2. r_A (*x*) is a function of *x*, where *x* can be thought of as the consumer's current level of wealth. Thus we can admit the situation where the consumer is risk averse, risk loving, or risk neutral for different levels of initial wealth.

The Arrow-Pratt measure of absolute risk aversion

3. We can also think about how the decision maker's risk aversion changes with her wealth. How do you think this should go? Do you become more or less likely to accept a gamble that offers 100 with probability ½ and −50 with probability ½ as your wealth increases?

The Arrow-Pratt measure of absolute risk aversion

3. We can also think about how the decision maker’s risk aversion changes with her wealth. How do you think this should go? Do you become more or less likely to accept a gamble that offers 100 with probability ½ and −50 with probability ½ as your wealth increases?

Hopefully, you answered more. This means that you become less risk averse as wealth increases, and this is how we usually think of people, as having non-increasing absolute risk aversion.

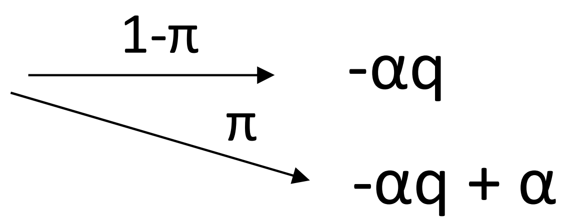
The Arrow-Pratt measure of absolute risk aversion

4. The AP measure is called a measure of absolute risk aversion because it says how you feel about lotteries that are defined over absolute numbers of dollars. A gamble that offers to increase or decrease your wealth by a certain percentage is a relative lottery, since its prizes are defined relative to your current level of wealth. We also have a measure of relative risk aversion,

$$r_R(x) = -\frac{xu''(x)}{u'(x)}.$$

Application: Insurance

- A consumer has initial wealth *w*.
- With probability *π*, the consumer suffers damage of *D*.
- Thus, in the absence of insurance, the consumer's final wealth is *w* − *D* with probability *π*, and w with probability 1 − *π*.
- Suppose insurance is available. Each unit of insurance costs *q*, and pays 1 dollar in the event of a loss. Suppose the person buys *α* units of insurance.
- Cost of insurance



Suppose that the insurance is "actuarially fair" if its expected cost is zero.

Application: Insurance

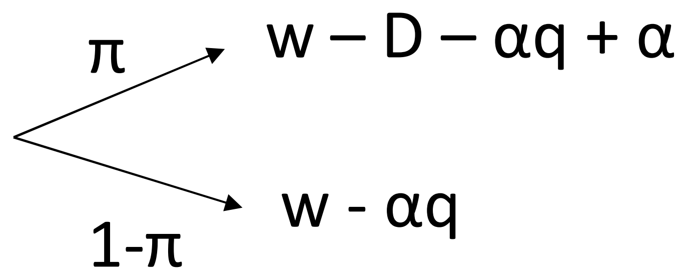
- Exp. Cost $=(1-\pi).(-\alpha q) + \pi(-\alpha q+\alpha) = 0$

  $\rightarrow \quad q = \pi$

(the cost to the consumer of 1 euro of insurance is just the expected cost of providing that coverage)

How many units of insurance should the consumer buy if the insurance is actuarially fair? Find α to max expected utility.

Application: Insurance



Max over α : $Eu = \pi u(w\text{-}D\text{-}\alpha q\text{+}\alpha) + (1\text{-}\pi)u(w\text{-}\alpha q)$

First derivative w.r.t. α:

$$\pi u'(w\text{-}D\text{-}\alpha q\text{+}\alpha)(\text{-q}+1) + (1\text{-}\pi)u'(w\text{-}\alpha q)(\text{-q}) = 0$$

or $$u'(w\text{-}D\text{-}\alpha q\text{+}\alpha) = u'(w\text{-}\alpha q)$$

If the consumer is risk averse, then u'(.) is strictly decreasing, so that

$$w\text{-}D\text{-}\alpha q\text{+}\alpha = w\text{-}\alpha q$$

Or $$a^* = D$$ (full insurance)