

# LECTURE 1

## MICROECONOMIC THEORY

### CONSUMER THEORY

#### Preference and Choice

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## Why Study Micro Theory?

- Basic tools for all economics
  - Microeconomic theory itself doesn't say very much. You use theory as a tool so that you can say things about economy.
  - Tools are basic and general.
  - Tools are useful for building models
  - Much of my own research uses the tools I will teach you.

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## What is the Aim of Micro Theory?

- It models economic activity.
- It views economic activity as an interaction of individual economic agents.
- It assumes that agents pursue their own private interests.
- Therefore, we will study microeconomic theory as an analysis of individual decision making.

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## Microeconomic Theory on Consumption Decisions

- Neoclassical Preference and Choice: how individuals make decisions.
  - Probably the weakest part of microeconomics. Complete rationality assumed.
  - Little psychology, little empirical evidence.
  - Source of preferences or reasons for choices are not explained.
  - Consistency in preference or choice is required.
  - Theory of demand.

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## Is that what we do in real life?

- Rational Decision Making Process
  - define the problem
  - identify the decision criteria
  - weight the identified decision making criteria
  - generate possible alternatives
  - rate each alternative against the dm criteria
  - compute the optimal decision

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## Assumptions of the Rational Decision-Making Model

- **Problem Clarity-**
  - The problem is clear and unambiguous.
- **Known Options-**
  - The decision-maker can identify all relevant criteria and viable alternatives.
- **Clear Preferences-**
  - Rationality assumes that the criteria and alternatives can be ranked and weighted.
- **Constant Preferences-**
  - Specific decision criteria are constant and that the weights assigned to them are stable over time.
- **No Time or Cost Constraints-**
  - Full information is available because there are no time or cost constraints.
- **Maximum Payoff-**
  - The choice alternative will yield the highest perceived value.

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## How decisions are actually made...

- most decisions don't result from the rational decision making model.
- **Issues:**
  - bounded rationality
  - intuition
  - problem identification
  - making choices

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## Bounded Rationality

- limited capability of information processing
- simplify complex problems
- choose first solution that is good enough (I.e. satisfactory and sufficient).

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## Making Choices

- **Sources of bias:**
  - heuristics (judgmental shortcuts to make a decision faster)
  - availability (information readily available)
    - representatives (analogies between a current issue and a previous one).

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## Microeconomic Theory on Consumption Decisions

- Modern fields, behavioral economics and behavioural finance, use psychological concepts, rather than a simple presumption of economic rationality.
  - Founders: Daniel Kahneman & Amos Tversky, among others.
  - Psychologists who changed economics.
- Modern decision theory also reflects these recent changes.
  - Psychological topics, e.g. temptation, are frequently modeled.

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## Microeconomic Theory on Consumption Decisions

Classical economists did pay attention to psychology.

Adam Smith wrote *The Theory of Moral Sentiments*, which proposed psychological explanations of individual behavior

Bentham wrote extensively on the psychological underpinnings of utility.

In neo-classical economics, economists sought to reshape the discipline as a natural science, deducing economic behavior from assumptions about the nature of economic agents.

*Homo economicus has a fundamentally rational psychology.*

Great book on this: Mirowski - *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics (Historical Perspectives on Modern Economics)*

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## Course Textbooks

- ▣ Mas-Colell, Whinston and Green, *Microeconomic Theory*, 1995, Oxford.
- ▣ F. Muñoz-Garcia, *Advanced Microeconomic Theory*, 2017, MIT Press.
- ▣ F.A. Cowell, *Microeconomics: Principles and Analysis*, 2005, Oxford.

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## Course Outline

- ▣ Lecture 1: Preference and Choice
  - Preference-based approach to individual behaviour / Choice-based approach to individual behaviour
  - (Ms-Colell, Whinston and Green - Chapter 1, Muñoz-Garcia - Chapter 1, Cowell - Chapter 4, )
- ▣ Lecture 2: Consumer Choice
  - Consumer demand from the choice-based perspective to individual behaviour
  - (Mas-Colell, Whinston and Green - Chapter 2, Muñoz-Garcia - Chapters 1&2, Cowell - Chapter 4)

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## Course Outline

- ▣ Lecture 3: Classical Demand Theory
  - Consumer demand from the preference-based perspective
  - (Ms-Colell, Whinston and Green - Chapter 3, Sections A-E, G, H and J, Muñoz-Garcia - Chapter 2, Cowell - Chapter 4)
- ▣ Lecture 4: Choice and Uncertainty
  - Lotteries / Expected utility theory / Attitudes to risk / Alternative theories
  - (Ms-Colell, Whinston and Green - Chapter 6, Sections A-C, Muñoz-Garcia - Chapter 5, Cowell - Chapter 8 )

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## The basic problem

- ▣ How do individuals choose from a set of opportunities?
- ▣ What can we conclude from observed choices?
- ▣ Objective: formulation of a general theory that may be applied to a host of conceivable choice problems.

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## Starting point

- ▣ The starting point for any individual decision problem is *a set of possible (mutually exclusive) alternatives* from which the individual must choose.
- ▣ This set can be anything.
- ▣ E.g. in case of career path {go to law school, study economics, ..., become a rock star}

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## Definition of Choice Set ("Consumption Set")

### Notations

Consider an individual (agent) facing a *choice set*  $X$ .

### Definition (Choice set, "Consumption set")

$X$  is a set of mutually exclusive choices.

#### Ex.1

$X = \{\text{NKUA MPhil, another PhilEcon, other studies, stop studying}\}$

#### Ex.2

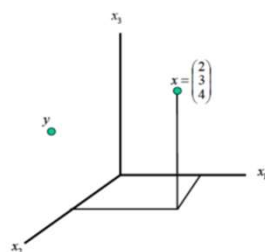
$X = \{\text{study advanced micro, watch TV, go to the cinema, do nothing}\}$

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## Definition of Choice Set ("Consumption Set")

### □ Ex. 3 In the case of three commodities

- Commodity space: each point is a bundle of goods.



$x \in X$  is a vector of quantities of each commodity in existence.

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## Preference and Choice

### □ We can approach the decision making process of a consumer in two ways:

- Preference-based approach: how does an individual use his preferences to choose an element from the set of alternatives  $X$ . We will impose rationality assumptions on preferences. (preferences  $\rightarrow$  choices)
- Choice-based approach: study the actual choices an individual makes when he is called to choose an element from the set of possible alternatives. We will impose consistency conditions on choices. (choices  $\rightarrow$  preferences)

Question: what is the relationship between the two approaches? Is there an equivalence? Under which conditions rational preferences imply a consistent choice behaviour? Under which conditions the opposite relationship holds?

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## In summary:

Two approaches to model consumer behavior:

### A preference-based approach (dominant model)

Assumes the decision maker has a preference relation over the set of possible choices that satisfies a rationality axiom.

The decision maker's tastes (summarized in his preference relation) are the primitive characteristic of the individual.

The theory is developed by first imposing rationality axioms on the decision maker's preferences and then analyzing the consequences of these preferences for his choice behaviour (i.e. on the decisions made).

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## In summary:

Two approaches to model consumer behavior:

### A choice-based approach

Focuses directly on the choice behavior imposing consistency restrictions (parallels the rationality axiom of the preference-based approach).

This approach treats the individual's choice behaviour as the primitive feature.

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## In summary:

### □ The choice-based approach has several attractive features:

- It leaves room for more general forms of individual behaviour
- It makes assumptions about objects that are directly observable (i.e. choice behaviour) rather than things that are not (i.e. preferences)
- It makes clear that the theory of individual decision making need not be based on introspection, but can be given an entirely behavioural foundation.

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## Preferences

- Preferences are psychological entities.
- Most aspects of preferences are usually ignored by economists.
  - Origin ignored
  - Causes ignored
  - Intensity ignored
  - Dynamics ignored
    - What causes preferences to change?
    - What are the effects of changing preferences?
    - Equilibria of preferences.

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### Preference relations. Main properties.

The agent has a preference relation over  $X$ .  
 We impose rationality axioms on these preferences.  
 What are the consequences for the agent's choices?

**Definition (Preference relation)**

A preference relation (denoted  $\succeq$ ) is a binary relation on  $X$  which compares couples  $x, y \in X$ .

$x \succeq y$  reads "x is preferred over or equivalent to y".  
 "x is at least as good as y".

→ Two other relations:

- The *strict-preference* relation  $\succ$
- The *indifference* relation  $\sim$

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### Preference relations. Main properties.

**Definition (Strict-preference relation)**

A preference relation (denoted  $\succ$ ) is defined as follows:

$$x \succ y \Leftrightarrow x \succeq y \text{ when not } y \succeq x$$

"x is (strictly) preferred over y".

"x provides more well-being than y".

**Definition (Indifference relation)**

An indifference relation (denoted  $\sim$ ) is defined as follows:

$$x \sim y \Leftrightarrow x \succeq y \text{ and } y \succeq x$$

"x and y are indifferent ("the individual is...")".

"x and y provide the same well-being"

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### Preference relations. Main properties.

A person is defined to be rational if his preferences are complete and transitive

**Definition (Rational preference (rationality axiom))**

A preference relation  $\succeq$  is **rational** if it is:

- (i) **complete**:

$$\forall x, y \in X, x \succeq y \text{ and / or } y \succeq x$$

- (ii) **transitive**:

$$\forall x, y, z \in X, x \succeq y \text{ and } y \succeq z \Rightarrow x \succeq z$$

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### Preference relations. Main properties.

- **COMPLETENESS**: an individual has well-defined preferences between ANY two possible alternatives.

In terms of ex.2

$X = \{\text{study advanced micro (x), watch TV (y), go to the cinema (z), do nothing (w)}\}$

Example of complete preferences:

$$(x \succeq z, x \succeq y, x \succeq w, y \succeq z, y \succeq w, z \succeq w)$$

All possible pairs are compared.

\* Note that I cannot say "I love x but hate y"

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### Preference relations. Main properties

By asking:	We impose the assumption:
Tick one box (i.e., not refrain from answering)	<b>Completeness</b> : individuals must compare any two alternatives, even the ones they don't know.
Tick only one box	The individual is capable of comparing any pair of alternatives.
Don't add any new box in which the individual says, "I love x and hate y"	We don't allow the individual to specify the intensity of his preferences.

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### Preference relations. Main properties.

**Proposition**: if the weak preference relation  $\succeq$  of a consumer is rational, then

- (i) the strong relation is transitive:

$$\text{for all } x, y, z \in X, \text{ we have that if } x \succ y \text{ and } y \succ z, \text{ then } x \succ z.$$

- (ii) the relation  $\sim$  is transitive:

$$\text{for all } x, y, z \in X, \text{ we have that if } x \sim y \text{ and } y \sim z, \text{ then } x \sim z.$$

- (iii) if  $x \succ y$  and  $y \succeq z$ , then  $x \succ z$ .

$\succ$  is irreflexive ( $x \succ x$  never holds)

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## Example of rational preferences

- Let  $A = \{\text{London; Paris; Rome}\}$ , a set of holiday destinations. If you would rather go to London than to Paris, rather to Paris than to Rome, but rather to Rome than to London, this is inconsistent with any ranking of the alternatives.
- If you don't know whether you would prefer Rome or London, completeness is violated. If you prefer London to Rome, then you have a rational preference over holiday destinations, according to the definition.
- Why require transitivity? If you compare alternatives in pairs, the winner should not be determined by sequencing. But with the intransitive preferences, you favor London if you first compare Paris to Rome (and then London to Paris), whereas you select Paris if you first compare London to Rome (and then Paris to Rome).

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## Preference relations.

### Discussion of main rationality assumptions.

- Completeness: Obviously false for real people:
  - People don't know characteristics of most goods (e.g. menu in thai restaurant).
  - People don't know how characteristics will affect them.
  - We neglect the (time) costs of comparing alternatives.
  - Comparing alternatives can be difficult if we have little experience with them (e.g. climate change)
  - Worse: people make decisions without knowing their preferences.

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## Preference relations.

### Discussion of main rationality assumptions.

- Transitivity – and (apparent) violations
  - problem of *just perceptible differences*
    - agent may be indifferent between just perceptible differences of colors for painting a room.
    - However, as we repeat this the agent may prefer starting to final color

◦ similar shades of gray paint



◦ milligrams of sugar in your coffee



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## Preference relations.

### Discussion of main rationality assumptions.

- *framing* (manner of presenting alternatives matters for choice)
  - prices in store 1: €125 for stereo and €15 for calculator
  - salesman tells you that one of them costs €5 less in store 2, which is located 20 minutes away
  - in experiments, fraction that would travel to other store is much higher, if discount is on calculator
- by contrast, the same individuals express indifference to the following question
  - Because of a stock out you must travel to the other store to get the two items, but you will receive €5 off on either item as compensation. Do you care on which item the rebate is given?
- this violates transitivity (see next slide)

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## Preference relations.

### Discussion of main rationality assumptions.

- x: travel to other store and €5 discount on calculator  
 y: travel to other store and €5 discount on stereo  
 z: buy both items at first store
- first two choices reveal:  $x \succ z$  and  $z \succ y$
  - third choice reveals:  $x \sim y$ 
    - but: maybe we have misspecified the choice alternatives
    - individuals do also care about making good bargains, often understood as price reductions in %
    - perception for first two choices: discount on individual product
    - perception for third choice: discount on bundle of goods
  - framing very important when outcomes are uncertain

(Kahneman and Tversky 1984)

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## Preference relations.

### Discussion of main rationality assumptions.

- Aggregation of several individual preferences might violate transitivity.
- Consider  $X = \{\text{MIT, WSU, Home University}\}$
- When considering which university to attend, you might compare:
  - a) Academic prestige (criteria #1)  
 $\succ_1: \text{MIT} \succ_1 \text{WSU} \succ_1 \text{Home Univ.}$
  - b) City size/congestion (criteria #2)  
 $\succ_2: \text{WSU} \succ_2 \text{Home Univ.} \succ_2 \text{MIT}$
  - c) Proximity to family and friends (criteria #3)  
 $\succ_3: \text{Home Univ.} \succ_3 \text{MIT} \succ_3 \text{WSU}$

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## Preference relations.

### Discussion of main rationality assumptions.

- By majority of these considerations:

MIT  $\succsim$  WSU  $\succsim$  Home Univ  $\succsim$  MIT  
criteria 1 & 3                      criteria 1 & 2                      criteria 2 & 3

- Transitivity is violated due to a cycle.
- A similar argument can be used for the aggregation of individual preferences in *group decision-making*:
  - Every person in the group has a different (transitive) preference relation but the group preferences are not necessarily transitive ("**Condorcet paradox**").

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## Preference relations.

### Discussion of main rationality assumptions.

- aggregation of preferences

- we often take households as the primitive of our analysis
  - preferences of mom:  $x \succ y \succ z$
  - preferences of dad:  $y \succ z \succ x$
  - preferences of child:  $z \succ x \succ y$
- majority-rule votes produces cyclical household preferences (Condorcet Paradox):  $x \succ y \succ z \succ x$

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## Preference relations.

### Discussion of main rationality assumptions.

#### Intransitivity due to a *change in preferences*

- When you start smoking  
One cigarette  $\succeq$  No smoking  $\succeq$  Smoking heavily
- By transitivity,  
One cigarette  $\succeq$  Smoking heavily
- Once you started  
Smoking heavily  $\succeq$  One cigarette  $\succeq$  No smoking
- By transitivity,  
Smoking heavily  $\succeq$  One cigarette
- But this contradicts the individual's past preferences when he started to smoke.

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## Preference relations.

### Discussion of main rationality assumptions.

- changes in taste

x: smoke 1 cigarette a day

y: abstinence (initial situation)

z: heavy smoking

- preferences in initial situation:  $x \succ y \succ z$
- once the individual has started smoking, preferences change to  $z \succ x \succ y$ 
  - intransitivity when neglecting preference change:  $x \succ z \succ x$
- change-of-taste models important for analyzing addictive behavior

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## Utility functions

- Preference relations can be described by a *utility function*.
  - A utility function  $u(x)$  assigns a numerical value to each element in  $X$ , ranking the elements of  $X$  in accordance with the individual's preferences.
- E.g. according to  $u_1$ ,  $x \rightarrow 10$ ,  $y \rightarrow 9$ ,  $z \rightarrow 8$ ,  $w \rightarrow 7$   
 according to  $u_2$ ,  $x \rightarrow 3$ ,  $y \rightarrow 0$ ,  $z \rightarrow -10$ ,  $w \rightarrow -50$   
 according to  $u_3$ ,  $x \rightarrow 100$ ,  $y \rightarrow 3$ ,  $z \rightarrow 2$ ,  $w \rightarrow 0$

A list is possible when I have completeness and transitivity  
 The three utility functions give me the same information, concerning the preference order, not intensity.

ORDINAL UTILITY FUNCTIONS

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## Utility functions

#### Definition (Utility function)

Consider a function  $u : X \rightarrow \mathbb{R}$

$u$  represents the preference relation  $\succsim$  if  $\forall x, y \in X$ :

$x \succsim y \Leftrightarrow u(x) \geq u(y)$

$u(x)$  is not unique. Let  $f$  be a strictly increasing function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , then  $v(x) = f(u(x))$  is a new utility function representing the same preferences as  $u(x)$ .

- Ordinal utility. Increases in utility have no meaning, since the scale of measurement may be transformed by any positive monotonic transformation.

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## Utility functions

Rationality is a necessary condition that a given weak preference relation can be represented by a utility function.

Proof: we have to show that if there is a utility function that represents preferences  $\succeq$ , then  $\succeq$  must be complete and transitive.

Completeness: get the example of two choices ( $x$  and  $y$ )

We have  $u(x) \geq u(y)$  or  $u(y) \geq u(x)$  or both (since they are real numbers)

Since  $u$  represents preferences this must imply that

$u(x) \geq u(y)$  implies that  $x \succeq y$ ,  $u(y) \geq u(x)$  implies that  $y \succeq x$  or both.

Hence preferences are complete.

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## Utility functions

Transitivity. Assume  $y \succ x$ ,  $x \succ z$ . Then, a utility function representing the preference relation must have  $u(y) \geq u(x)$  and  $u(x) \geq u(z)$ , which requires  $x \succ z$  and hence transitivity.

So, if a utility function exists, then preferences must be rational.

If preferences are rational, does a utility function always exist? In general, no. If  $X$  is finite, then a rational preference relation can always be represented by a utility function.

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## Summary so far...

- Someone is rational as long as his  $\succsim$  is complete and transitive.
- ORDINAL FUNCTION:

$$u: X \rightarrow \mathbb{R}$$

$$x \succsim y \Leftrightarrow u(x) \geq u(y)$$

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## Choice-based approach

- In the **choice-based approach** we focus on the actual choices made by the individual, not on the process of introspection by which the individual discovers his own preferences by systematically comparing different alternatives.
- **Rationale:** economists like to think that “you cannot get inside a person’s head” so that utility and preferences are fundamentally unobservable.
- Therefore, we need to develop a system for inferring utility and preferences from a person’s behavior in the market place.

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## Choice-based approach

### Choice rules

Choice may be conceived of as implying two things.

- (i) A set of alternatives to choose from.
- (ii) A rule that prescribes the preferred choice depending on available alternatives.

- Towards (i):

- Let  $B \subset X$  be a budget set.
- Let  $\mathcal{B}$  be the family (set) of all budget sets. Changes of prices, income, regulation, etc. lead to different budget sets within this family  $\mathcal{B}$ .

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## Choice-based approach

### □ Examples of sets B:

- In consumer theory, set  $B$  can be understood as a particular set of all the affordable bundles for a consumer, given his wealth and market prices.
- $B$  could be a particular list of all universities where you were admitted, among all universities in the scope of your imagination  $X$ , i.e.  $B \subset X$ .

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## Choice-based approach

- Towards (ii):
  - $C(\cdot)$  assigns a set of chosen elements  $C(B) \subset B$  for every budget set  $B \subset \mathbb{B}$ .
  - If  $C(B)$  identifies a single element, then this element is chosen.
  - If  $C(B)$  identifies several alternatives, one of these alternatives (set of acceptable alternatives) is chosen.
- A choice structure may be defined as the set of budget sets and a choice rule:  $(\mathbb{B}, C(\cdot))$ .

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## Choice-based approach

Example 1:

$$X = \{x, y, z, w\}$$

$$B_1 = \{x, y, w\} \quad C(\{x, y, w\}) = \{x, y\}$$

Example 2:

$$X = \{x, y, z, w\}$$

$$B_1 = \{x, y, w\} \quad C(B_1) = \{w\}$$

$$B_2 = \{y, w\} \quad C(B_2) = \{y, w\}$$

$$B = \{B_1, B_2\}$$

For any subset of  $X$  that belongs to  $\mathbb{B}$  I need a choice rule to give me what I choose from each one of these sets.  
 $C(\dots)$  can't be none.

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## Choice-based approach

### □ Summary of Choice structure

- $X$  is a set of possible alternatives.
- I get some subsets of  $X$ ,  $\mathbb{B} = \{B_1, B_2, \dots, B_n\}$ .  
 $\mathbb{B}$  contains all the possible situations I might be involved and have to make a choice.
- $C(\cdot)$  takes a budget set  $B$  as its argument and returns another set which contains all the elements I choose from  $B$ .
- $C(\{x, y, w\}) = \{x\}$  means that "I was faced with a choice between  $x, y, w$  and I chose  $x$ "

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## Consistency of choices: the WARP

- The choices of individuals must display a certain amount of consistency (in the same way as preferences were assumed to be rational).
- We consider the actual choices of an individual to be consistent if they satisfy the **Weak Axiom of Revealed Preference (WARP)**
- The WARP parallels the rationality axiom in the preference-based approach.

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## Consistency of choices: the WARP

Definition. The choice structure  $(\mathbb{B}, C(\cdot))$  satisfies the **weak axiom of revealed preferences** if the following property holds:

If for some  $B \in \mathbb{B}$  with  $x, y \in B$  we have that  $x \in C(B)$ , then for any  $B' \in \mathbb{B}$  with  $x, y \in B'$  and  $y \in C(B')$ , we must also have that  $x \in C(B')$ .

- In words, if  $x$  is ever chosen when  $y$  is available, there can be no budget set containing both alternatives for which  $y$  is chosen and  $x$  is not.

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## Consistency of choices: the WARP

Example 1.

Let  $B_1$  be  $\{\text{apple, orange juice, opera ticket}\}$  and  $B_2 = \{\text{apple, orange juice, strawberry}\}$ .

If  $C_1 = \{\text{apple}\}$  and  $C_2 = \{\text{orange juice}\}$  this is a violation of the weak axiom of revealed preferences.

The apple was revealed as being "at least as good as" the juice. But with the changed budget set the ranking of the apple and the juice has been changed. While the juice is in the set of most preferred alternatives, the apple is not.

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## Consistency of choices: the WARP

Example 2.

$$B = (\{x,y\}, \{x,z\}, \{x,y,z\})$$

$$C(\{x,y\}) = \{y\}$$

$$C(\{x,z\}) = \{x\}$$

$$C(\{x,y,z\}) = \{x,y\}$$

Are these choices consistent? Do they satisfy the WARP?

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## Consistency of choices: the WARP

Answer : **NO!**

When faced with  $B = \{x,y,z\}$ ,  $x \in C(B)$ . This means that I found  $x$  to be at least as good as  $y$ .

This means that  $C(\{x,y\}) = \{y\}$  contradicts the previous intuition. I cannot choose  $y$  without choosing  $x$  as well. Because if I only choose  $y$ , this would mean that I find  $y$  better than  $x$ !

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## Consistency of choices: the WARP

Example 4. (Cowell, page 75, mini problem 4)

- Each day I buy one piece of fruit for my lunch. On Monday apples and bananas are available, but no oranges: I buy an apple. On Tuesday bananas and oranges are available, but no apples: I buy a banana. On Wednesday apples and oranges are available (sorry we have no bananas): I buy an orange. Am I consistent?

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## Preferences and Choices: Comparison

- We would like to have an equivalence
- Two basic questions:
  - If my preferences are complete and transitive, will the corresponding choices satisfy the WARP? **YES**
  - If my choices satisfy the WARP, will the implied preferences be complete and transitive? **NOT NECESSARILY**

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## Comparison

- **Question 1:** If my preferences are complete and transitive, will the corresponding choices satisfy the WARP?
- By "corresponding choices" I mean a special choice rule according to which "I choose what I prefer".
- Suppose that an individual has a rational preference relation  $\succeq$  on  $X$ . If this individual faces a non-empty set of alternatives  $B \subset X$ , his preference-maximising behaviour is to choose any one of the elements in the set:

$$C^*(B, \succeq) = \{x \in B: x \succeq y \text{ for every } y \in B\}$$

The elements of set  $C^*(B, \succeq)$  are the individual's most preferred alternatives in  $B$ .

We say that the rational preference relation  $\succeq$  *generates* the choice structure  $(B, C^*(\cdot, \succeq))$ .

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## Comparison

- **Question 1:** If my preferences are complete and transitive, will the corresponding choices satisfy the WARP?
- We have assumed that  $\succeq$  is rational. We will show that this implies that the choice structure generated by this preference relation, i.e.  $(B, C^*(\cdot, \succeq))$  satisfies the WARP.

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## Comparison

Proof:

Assume for some  $B$  that  $x, y \in B$  and  $x \in C^*(B, \succeq)$ . From the definition of  $C^*$ ,  $x \succeq y$ .

Now assume that for some  $B'$  with  $x, y \in B'$  we have  $y \in C^*(B', \succeq)$ . This implies  $y \succeq z$  for all  $z \in B'$ . At the same time we already know that  $x \succeq y$ . Hence, by transitivity,  $x \succeq z$  and  $x \in C^*(B', \succeq)$ . This is what the weak axiom of revealed preference requires.

- Implication: once we assume rational preference ordering, we can conclude that the decision maker "weakly reveals" her preferences.

So, if behaviour is generated by rational preferences, then it satisfies the consistency requirements embodied in the W.A.

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## Comparison

- Question 2: If my choices satisfy the WARP, will the implied preferences be complete and transitive?
- By "implied preferences" I mean  $x \succeq^* y$ , i.e. "x is revealed to be at least as good as y".
- This means that we can interpret an individual's choices as if he were a preference maximizer.
- There is some  $B \in \mathcal{B}$ ,  $x, y \in B$ ,  $x \in C(B)$ .
- If  $\{B, C(\cdot)\}$  satisfies the WARP, is  $x \succeq^* y$  rational?
- Answer:  $\succeq^*$  isn't necessarily rational.

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## Comparison

- Counter example:

$$\begin{array}{l} C(\{x, y\}) = \{x\} \rightarrow x \succ^* y \\ C(\{y, z\}) = \{y\} \rightarrow y \succ^* z \rightarrow \text{violates} \\ C(\{x, z\}) = \{z\} \rightarrow z \succ^* x \quad \text{transitivity} \\ \uparrow \end{array}$$

Ok with the WARP

So the WARP is not sufficient to ensure the existence of a rationalizing preference relation.

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## Comparison

- Given the choice structure complies with the weak axiom of revealed preferences, can we conclude that the underlying preferences are rational?
- Answer: yes, if an additional assumption about the set of budget sets is met.
- Proposition: If  $(B, C(\cdot))$  is a choice structure such that (i) the weak axiom is satisfied and (ii)  $B$  includes all subsets of  $X$  of up to three elements, then there is a rational preference relation  $\succeq$  that rationalizes  $C(\cdot)$  relative to  $B$ ; i.e.  $C(B) = C^*(B, \succeq)$  for all  $B \in \mathcal{B}$ .
- Proof omitted (see MWG, p. 13-14).

Implication: If choice is defined for all subsets of  $X$  and the weak axiom holds, then a theory based on observable choice is equivalent to a decision theory based on rational choice.

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## Main points

- Definition of rationality
  - 1<sup>st</sup> definition (in terms of preferences), preference relation should be complete and transitive
  - 2<sup>nd</sup> definition (in terms of choices) Choices should be consistent - WARP
- 1<sup>st</sup> definition  $\rightarrow$  2<sup>nd</sup> definition
- 2<sup>nd</sup> definition doesn't  $\rightarrow$  1<sup>st</sup> definition, unless  $B$  contains all the subsets of  $X$  up to 3 elements.

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