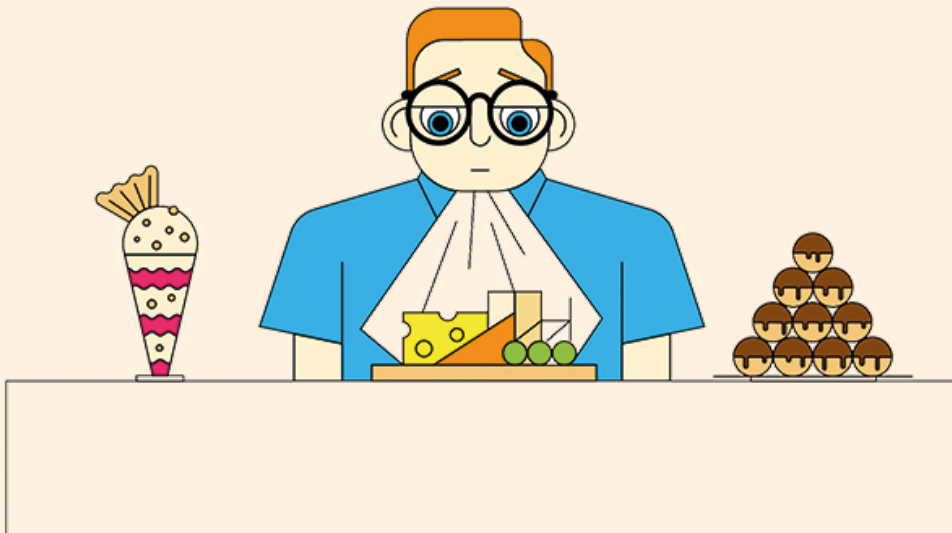**FT Magazine**

Society and the profiteroles paradox

'Can a society really want anything coherent? Arrow's impossibility theorem drives a stake through the very idea'

Undercover Economist

AN HOUR AGO by: **Tim Harford**

Ken is in a restaurant, pondering his choice of dessert. Ice cream, profiteroles or a cheese plate? He's about to request a scoop of ice cream when the waiter informs him that the profiteroles are off the menu. "I see," says Ken. "Well, I'll have the cheese, please."

Ken's behaviour is odd enough to be a piece of surrealist comedy. But what seems ludicrous from an individual is easy to imagine in an election. Think of George W Bush as ice cream, Ralph Nader as profiteroles and Al Gore as

the cheese plate. If Nader had not been on the menu in the 2000 US presidential election, then Gore would have been president instead of Bush. Since Nader himself was never a serious contender, it seems odd that his presence changed the result. But we've grown used to this sort of thing in politics.

Still, we might ask: is there a way to assemble individual preferences into social preferences without generating surreal outcomes? That was the first of many big problems studied by the great economist Kenneth Arrow (http://next.ft.com/content/114b4726-f90e-11e6-9516-2 d969e0d3b65), who died last month at the age of 95. His answer: no.

To understand Arrow's answer, imagine a society in which every individual has a ranking expressing their preferences over every possible outcome. Let's say that we can read minds, so we know what each person's ranking is. All we need is some system for combining those individual rankings into a social ranking that tells us what society as a whole prefers.

Arrow named this putative system a constitution. What properties would we like our constitution to have? It should be comprehensive, giving us an answer no matter what the individual rankings might be. And it wouldn't fall prey to the profiteroles paradox: if society prefers ice cream to cheese, then whether profiteroles are available or not shouldn't change that fact.

We want the constitution to reflect people's preferences in common-sense ways. If everyone expresses the same

preference, for example, the constitution should reflect that. And we shouldn't have a dictator — an individual who is a kind of swing voter, where the constitution reflects only her preferences and ignores everyone else.

None of these properties seem particularly stringent — which makes Arrow's discovery all the more striking. Arrow's "[impossibility theorem (http://next.ft.com/content/0cb9f868-f8e8-11e6-bd4e-68d53499ed71)](http://next.ft.com/content/0cb9f868-f8e8-11e6-bd4e-68d53499ed71)" proves that no constitution can satisfy all of them. Any comprehensive constitution will suffer the profiteroles paradox, or arbitrarily ignore individual preferences, or will simply install a dictator. How can this be?

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Let me now attempt the nerdiest move in more than 11 years of writing this column. Since there's no idea in economics more beautiful than Arrow's impossibility theorem, I'm going to try to outline a proof for you — very sketchily, but you may get the idea.

Imagine that our constitution must deliver a choice between ice cream, profiteroles and cheese.

Step one in the proof is to note that there must be a group whose preferences determine whether society as a whole prefers cheese or ice cream — if only because the constitution must respect a unanimous view on this. Call this group the Cheese Group. The Cheese Group might include everyone in society, but maybe it's a smaller group of swing voters.

The next step is to show that the Cheese Group doesn't

merely swing the decision between cheese and ice cream, but also over profiteroles and any other dessert we might add to the menu. We can show this by creating cases where it's impossible for the Cheese Group to express a preference between cheese and ice cream without profiteroles being caught in the middle.

This means that the Cheese Group actually gets to decide about everything, not just cheese and ice cream.

Finally, having established that the Cheese Group is all-powerful, we show that we can make it smaller without destroying its power. Specifically, we can keep dividing it into pairs of sub-groups, and show that at each division either one of the sub-groups is all-powerful, or the other one is.

In short: we prove that if any group of voters gets to decide one thing, that group gets to decide everything, and we prove that any group of decisive voters can be pared down until there's only one person in it. That person is the dictator. Our perfection constitution is in tatters.

That's Arrow's impossibility theorem. But what does it really tell us? One lesson is to abandon the search for a perfect voting system. Another is to question his requirements for a good constitution, and to look for alternatives. For example, we could have a system that allows people to register the strength of their feeling. What about the person who has a mild preference for profiteroles over ice cream but who loathes cheese? In Arrow's constitution there's no room for strong or weak

desires, only for a ranking of outcomes. Maybe that's the problem.

Arrow's impossibility theorem is usually described as being about the flaws in voting systems. But there's a deeper lesson under its surface. Voting systems are supposed to reveal what societies really want. But can a society really want anything coherent at all? Arrow's theorem drives a stake through the heart of the very idea.

People might have coherent preferences, but societies cannot. We will always find ourselves choosing ice cream, then switching to cheese because the profiteroles are off.

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