LECTURE 7

TAX POLICY Optimal Commodity Taxation

The definition of taxes

???
"...Taxes are unrequited in the sense that benefits provided by government to taxpayers are not normally in proportion to their payments..."

A tax is a compulsory, unrequited payment to general government. (OECD, 1996)

"...General government consists of supra-national authorities, the central administration and the agencies whose operations are under its effective control, state and local governments and their administrations, social security schemes and autonomous governmental entities, excluding public enterprises..."

"Optimality" criteria for judging tax systems

- Taxes have to be fair (although fairness means different things to different people)
- A good tax system is one which minimizes the resource cost involved in assessing, collecting and paying the taxes (administrative and compliance cost)
- A good tax system minimizes the efficiency cost of taxation, in terms of the distortions they cause in agents' behaviour

"Optimality" criteria for judging tax systems

- The point of departure of optimal tax theory has been the criterion of efficiency (3rd)
- The theory has gradually been extended to take into account distributional considerations (1st)
- Administrative costs have so far been ignored in the optimal taxation literature (or studied separately)
- □ Trade-offs between criteria???

- From an efficiency point of view an ideal tax system is one which is consistent with an efficient allocation of resources
- Solution : lump-sum taxes. Such taxes cannot be altered by action, so that there is no efficiency cost involved as a result of behavioural distortions. They are clearly neutral with respect to all marginal evaluations made by consumers and producers.
- Not a very helpful conclusion for the public finance economist. Lump-sum taxes can not be part of a permanent system.

- Even if lump-sum taxes are ruled out, there are still taxes which are consistent with Pareto optimality. For example it has been argued since Pigou (1920) that indirect taxes can be used to improve the efficiency of the market allocation of resources in the presence of externalities.
- But, can the public sector raise all its tax revenue from Pigouvian taxes?
- Thus the optimal taxation literature deals with the second-best problem of making the best of a necessarily distortionary system.

- The conclusions of any model on the optimal tax design depend on the set of tax instruments that the model allows to be used.
- The classical models on optimal commodity taxation solve the optimal tax problem assuming that commodity taxes are the only instrument the government can use to achieve its goals.

- **Two seminal articles:**
- The first analytical formulation is given by Ramsey (1927), concentrating on efficiency.
- Diamond and Mirrlees (1972) extended Ramsey's analysis to include distributional considerations.

FIGURE 2 – Indirect taxation as a share of GDP (OECD countries)



Source : OECD, Revenues Statistics (2019); OECD.Stat.



FIGURE 57: DEVELOPMENT OF AVERAGE STANDARD VAT RATE, EU-27, 2001-2023

Source: European Commission, DG Taxation and Customs Union, based on Taxes in Europe database.

Optimal commodity taxation

- The literature of optimal commodity taxation deals with the design of taxes on commodities (e.g. VAT, or alcohol excises).
- Goods and services are obvious things to tax
- But commodity taxation distorts consumer choices and causes inefficiency.
- Because the price of the taxed commodity rises, consumers alter their consumption choices.
- Historical example: window tax in the UK introduced in 1696 and lasted until 1851. The tax was paid on any house with more than six windows.
- □ Result?



Photo courtesy of Age Fotostock.

"The adage 'free as air' has become obsolete by Act of Parliament. Neither air nor light have been free since the imposition of the window-tax. We are obliged to pay for what nature lavishly supplies to all, at so much per window per year; and the poor who cannot afford the expense are stinted in two of the most urgent necessities of life." — Charles Dickens (1850, p. 461)

Optimal Commodity Taxation

- Commodity taxes are imposed upon purchases of goods
 - transactions are generally public information
 - the taxes drive a wedge between producer and consumer prices
 - this leads to inefficiency and reduces the level of welfare compared to using lump-sum taxes
 - this is the price of incentive-compatible taxation
- On the demand side of the market, income and substitution effects predicts the consequences of a price rise
- For producers the tax is a cost increase and they respond accordingly.

Lump-sum taxation does not cause any distortions

- a lump-sum tax is defined by the condition that no change in behavior can affect the level of the tax
- Commodity taxation does cause distortions
 - demand can shift from goods subject to high taxes to goods with low taxes
 - total consumption can be reduced by earning less or saving more.
- It is these substitution effects that are the tax-induced distortions
- The introduction of a commodity tax raises tax revenue but causes consumer welfare to be reduced.
- □ The deadweight loss of the tax is the extent to which the reduction in welfare exceeds the revenue raised.

Consider figure 14.1

- the price of the good is p before the tax is introduced
- the quantity consumed is X⁰
- consumer surplus is given by the triangle *abc*
- With a tax of amount t
 - the price rises to q = p + t
 - the quantity consumed falls to X¹
 - consumer surplus falls to aef
- The tax raises revenue equal to tX¹ which is area cdef
- The deadweight loss (DWL) is the triangle bde



- □ Triangle *ebd* is equal to [1/2]tdX where *dX* is the change in demand
- Using the elasticity of demand, $\varepsilon^{d} = \frac{p}{X} \frac{dX}{dp}$ we get $dX = \varepsilon^{d} \frac{X^{0}}{p} dp.$
- The change in demand $dX = X^0 X^2$ and the change in price dp = t
- □ The deadweight loss is

$$DWL = \frac{1}{2} |\varepsilon^d| \frac{X^0}{p} t^2$$

this is approximate because it assumes that the elasticity is constant

The deadweight loss is proportional to the square of the tax rate

rises rapidly as the tax rate is increased

Deadweight loss is proportional to the elasticity of demand

will be larger the more elastic is demand

Optimal Commodity Taxation

- Optimal commodity taxes attain the highest level of welfare possible whilst raising the revenue required by the government
 - consumers free to choose their most preferred consumption plans
 - firms choose production to maximise profits
- □ Welfare is measured using the government's objective function
- □ With a single consumer obtain an efficient tax system
- □ With many consumers obtain an equitable tax system

The Ramsey problem

- Problem set by Pigou to his 24 year-old student
- "A given revenue is to be raised by proportionate taxes on some or all uses of income, the taxes on different uses being possibly at different rates; how should these rates be adjusted in order that the decrement of utility may be a minimum?
- I propose to neglect altogether questions of distribution and considerations arising from the differences in the marginal utility of money to different people; and I shall deal only with a purely competitive system with no foreign trade." Ramsey (1927)

- The literature of optimal commodity taxation deals with the design of taxes on commodities (e.g. VAT, or alcohol excises).
- Ramsey (1927) did not look at the trade-off between equity and efficiency, but he analysed the problem of designing sales taxes to raise a given amount of revenue at the least possible distortionary cost in a single-person economy (or an economy with many identical people).
- The target is to minimize the loss in utility arising from taxation, or equivalently, to maximize social welfare subject to the revenue constraint.

- □ Suppose that there *n* commodities in the economy and a single form of labour, *l*.
- **\square** Producer prices are **p** and the wage rate faced by the consumer is *w*.
- The consumer faces consumer prices **q** and has a budget constraint $\mathbf{q}\mathbf{x} = wl$.
- The government must raise a given amount of revenue, \boldsymbol{R} by imposing unit taxes

$$\mathbf{t} = (t_1, t_2, \ldots, t_n).$$

where t_k is the difference between consumer price (q_k) and producer price (p_k) .

Assume producer prices to be fixed (constant returns to scale).

Selecting tax structure \equiv choosing a structure for consumer prices.

□ The preferences of the representative consumer are represented by the indirect utility function *V*, defined over prices, $U = V(\mathbf{q}, w)$.

Our problem then is to choose t_i s so as to maximise consumer utility subject to the revenue constraint of the government, that is

Maximise
$$V(\mathbf{q}, w)$$
, subject to $R(\mathbf{t}) = \sum_{k} t_{k} x_{k} \ge \overline{R}$ (1)
 \mathbf{t}

where x_k is the consumption of the *k*th good by the consumer.

The Lagrange function is:

$$L = V(\mathbf{q}, w) + \lambda [R(\mathbf{t}) - \mathbf{R}]$$
 (2)

■ Set the partial derivatives of *L* with respect to the tax rates equal to zero:

$$\frac{\partial L}{\partial t_i} \equiv \frac{\partial V}{\partial t_i} + \lambda \frac{\partial R(\mathbf{t})}{\partial t_i} = \mathbf{0} \qquad , i = 1, ..., n \quad (3)$$

Note that $\partial x_k / \partial q_i \equiv \partial x_k / \partial t_i$, so that (3) becomes:

$$\frac{\partial V}{\partial t_i} + \lambda \left(x_i + \sum_k t_k \frac{\partial x_k}{\partial q_i} \right) = \mathbf{0}$$

$$, i = 1, ..., n$$
 (4)

□ Using duality in consumer theory, Roy's identity gives:

$$\frac{\partial V}{\partial q_i} = -x_i \frac{\partial V}{\partial M} = -ax_i \tag{5}$$

where a is the marginal utility of income (M).

□ The Slutsky equation decomposes the change in demand due to a price change $(\partial x_k / \partial q_i)$ into an income and a symmetric substitution effect):

$$\frac{\partial \boldsymbol{x}_{k}}{\partial \boldsymbol{q}_{i}} = -\boldsymbol{x}_{i} \frac{\partial \boldsymbol{x}_{k}}{\partial \boldsymbol{M}} + \boldsymbol{s}_{ki} \qquad i, k = 1, \dots, n \qquad (6)$$

Where *M* is income and s_{ki} is the substitution effect $\left(\left(\frac{\partial x_k}{\partial q_i}\right)_{U_i}\right)$

or the utility-compensated change in demand for the *k*th good when the price of the *i*th good changes.

□ Substituting (5) and (6) into (4) and after rearranging and utilizing the fact that the substitution effects are symmetric $(s_{ik} = s_{ki})$, :

$$\frac{\left(\sum_{k} t_{k} s_{ik}\right)}{x_{i}} = -\mathcal{G}, \qquad \text{where} \qquad \mathcal{G} = 1 - \frac{\alpha}{\lambda} - \sum_{k} t_{k} \frac{\partial x_{k}}{\partial M} \qquad (7)$$

This is the Ramsey tax rule. Notice that \mathcal{G} is a positive number independent of *i*.

Intuitive explanation of the tax rule:

 $\sum_{k} t_{k} s_{ik}$ can be viewed as a first-order approximation of the compensated change in demand for the *i*th good resulting from the imposition of a vector of taxes, **t**.

The Ramsey rule can be interpreted as saying that the optimal tax rates should be such that the proportional reduction in compensated demand is the same for all commodities.

- □ Implications of the Ramsey rule:
 - Uniform taxes are not efficient from an efficiency point of view.
 - It is quantities that matter, not prices.
 - Prices are only important in so far as they determine demands.
 - The Ramsey rule directs taxation towards goods that are unresponsive to price changes, i.e. "necessities".

The Ramsey Rule

- □ The tax rates remain implicit in the Ramsey rule since it focuses on what happens to demand
- □ The rule suggests that those goods whose demand is unresponsive to price changes must bear higher taxes
- □ Goods that are unresponsive to price changes are typically necessities such as food and housing
- □ This tax system would bear most heavily on necessities
- Low income consumers pay proportionately larger fractions of income in taxes relative to rich consumers
- The inequitable nature of this is simply a reflection of the single consumer assumption: the optimisation does not involve equity and the solution reflects only efficiency criteria

The Ramsey Rule: another intuition (Corlett and Hague, 1953)

- Exploit the existence of an untaxed good, i.e, labour and its relation to leisure.
- The presence of the untaxed good allows the optimal tax formula to be expressed in terms of the complementarity or substitutability of the taxed commodities with the untaxed good.
- Corlett and Hague (1953) take an example with two consumption goods.

The Ramsey Rule: another intuition (Corlett and Hague, 1953)

- Result: if goods differ in their degree of complementarity or substitutability with leisure, efficiency can be improved by taxing more heavily the good that is most complementary with leisure.
- So impose a high tax on e.g. skiing equipment and a low tax on e.g. work uniforms or bus tickets.

- □ The general intuition behind the Ramsey rule is clear, but there is no explicit formula for the calculation of taxes.
- □ More precise tax rules can be achieved at the expense of additional assumptions.
- □ Assume all cross-price effects to be zero.
- □ Take equation (4) as the starting point:

$$\frac{\partial V}{\partial t_i} + \lambda \left(x_i + \sum_k t_k \frac{\partial x_k}{\partial q_i} \right) = \mathbf{0}$$

And replace Roy's identity

$$\frac{\partial V}{\partial q_i} = -x_i \frac{\partial V}{\partial M} = -ax_i$$

□ To get:

$$\alpha x_i = \lambda \left(x_i + \sum_k t_k \frac{\partial x_k}{\partial q_i} \right)$$
(8)

If demands are independent, the only non-zero effect at the sum in the right hand side of (8) is $t_i (\partial x_i / \partial q_i)$, so that (8) becomes:

$$\alpha x_i = \lambda \left(x_i + t_i \frac{\partial x_i}{\partial q_i} \right)$$
(9)

□ Rearranging and considering that the price elasticity of demand for good *i*, ε_i is $(\partial x_k/x_i) / (\partial q_i/q_i)$, (9) becomes:

$$\frac{t_i}{p_i + t_i} = \left(\frac{\alpha - \lambda}{\lambda}\right) \frac{1}{\varepsilon_i}$$

This is the well known inverse elasticities rule, which states that at the optimum, proportional rates of taxes should be inversely related to the price elasticity of demand of the good on which they are levied.

- These observations imply that necessities, which by definition have low elasticities of demand, should be highly taxed
- Luxuries with a high elasticity of demand should have a low rate of tax

- The inverse elasticity rule is derived by assuming there is a single consumer and that the demand for each good is dependent only upon its own price and the wage rate.
 - there are no cross-price effects between the taxed goods
 - the independence of demands is a strong assumption
- Since there is a single consumer the tax system derived is an efficient one
 - an equitable system may have different characteristics

- The extension of the Ramsey rule to the many-person economy (practically more relevant) is by Diamond and Mirrlees (1971).
- \Box The economy now consists of *H* individuals.
- □ The problem is still to maximize social welfare subject to the revenue constraint of the government.

- □ Suppose that there are *n* commodities in the economy and a single form of labour, *l*.
- **\square** Producer prices are **p** and the wage rate faced by the consumer is *w*.
- The consumer faces consumer prices **q** and has a budget constraint $\mathbf{q}\mathbf{x} = w^h l$.
- The government must raise a given amount of revenue, \boldsymbol{R} by imposing unit taxes

$$\mathbf{t} = (t_1, t_2, \dots, t_n).$$

where t_k is the difference between consumer price (q_k) and producer price (p_k) .

Assume producer prices to be fixed (constant returns to scale).

Selecting tax structure \equiv choosing a structure for consumer prices.

□ Individual welfare is determined in terms of the indirect utility function V^h , defined over prices, $U^h = V^h(\mathbf{q}, w^h)$.

Social welfare is determined by a Bergson-Samuelson social welfare function, defined over individual utilities:

$$W = W(V^1, V^2, ..., V^h, ..., V^H)$$
(10)

Total demand for commodity *i* is expressed as

$$X_i = \sum_h x_i^h$$

The optimization problem becomes

Maximize $W(V^1, V^2, \dots, V^h, \dots, V^H)$, t

subject to $R(\mathbf{t}) = \sum_{k} t_{k} X_{k} \ge \overline{R}$

$$L = W(V^1, V^2, \dots, V^H) + \lambda \left[\sum_k t_k X_k - \overline{R} \right]$$
(11)

■ Set the partial derivatives of *L* with respect to the tax rates equal to zero:

$$\frac{\partial L}{\partial q_i} = \sum_{h} \frac{\partial W}{\partial V^h} \frac{\partial V}{\partial q_i} + \lambda \left(X_i + \sum_{k} t_k \frac{\partial X_k}{\partial q_i} \right) = \mathbf{0} \quad (12)$$

□ Using duality in consumer theory, Roy's identity gives:

$$\frac{\partial V^{h}}{\partial q_{i}} = -x_{i} \frac{\partial V^{h}}{\partial M^{h}} = -a^{h} x_{i}$$

where a^h is the marginal utility of income (M^h) of individual h, we have:

$$\sum_{h} \frac{\partial W}{\partial V^{h}} \frac{\partial V^{h}}{\partial q_{i}} = -\sum_{h} \frac{\partial W}{\partial V^{h}} \alpha^{h} x_{i}^{h}$$
(13)

Define

$$\beta^{h} = \frac{\partial W}{\partial V^{h}} \frac{\partial V^{h}}{\partial M} = \frac{\partial W}{\partial V^{h}} \alpha^{h}$$
(14)

 β^h is very important and can be interpreted as the social marginal utility of income for individual *h*, that is the increase in social welfare resulting from a marginal increase in the income accruing to individual *h*.

\square Replacing (13) and (14) into (12) we get:

$$\sum_{h} \beta^{h} x_{i}^{h} = \lambda \left(X_{i} + \sum_{k} t_{k} \frac{\partial X_{k}}{\partial q_{i}} \right)$$
(15)

Substituting from the Slutsky equation as before, and after some algebraic manipulations (15) becomes:

$$\frac{\sum_{h}\sum_{k}t_{k}s_{ik}^{h}}{X_{i}} = -\left[1 - \sum_{h}\frac{b^{h}}{H}\frac{x_{i}^{h}}{x_{i}}\right]$$

where

b

$${}^{h} = \frac{\beta^{h}}{\lambda} + \sum_{k} t_{k} \frac{\partial x_{k}^{h}}{\partial M^{h}}$$

Remember that β^h is the social marginal utility of income for individual *h*.

$$b^{h} = \frac{\beta^{h}}{\lambda} + \sum_{k} t_{k} \frac{\partial x_{k}^{h}}{\partial M^{h}}$$

 b^h consists of two elements,

- (i) the welfare weights (β^h) which depend on the distributional value judgments of the government
- (ii) the marginal propensity to pay indirect taxes out of extra income $\mathbf{t}\partial \mathbf{x}^h / \partial M^h$.

□ Therefore, the tax should be lower

(i) the more the good is consumed by individuals with a high social valuation of income (reflecting equity criteria) and

(ii) the more the good is consumed by individuals with a high marginal propensity to consume taxed goods (reflecting efficiency considerations)

- If the demand structure is such that the rich (with low β^h) have a high propensity to spend their extra income on highly taxed goods at the margin, the two elements in b^h will move in opposite directions.
- This will make the spread of b^h lower than the distributional weights alone would imply.
- Explicit conflict between equity and efficiency criteria in the design of an optimal tax system.

- □ How do the two considerations balance?
- □ It depends on
 - The structure of the demand function (curvature of Engel curves)
 - The form of the differences among the population
 - Government's aversion to inequality
- If, for example, the government's aversion to inequality is low and the curvature of the Engel curve is large, efficiency criteria dominate in the determination of optimal tax rates.
- Again explicit calculations of optimal tax rates are not straightforward and additional assumptions have to be made in order to arrive at detailed results.

- Previous analysis essentially ignored production side of economy by assuming that producer prices are fixed.
- Diamond-Mirrlees AER 1971 tackle the optimal tax problem with endogenous production.
- D-M Result: even in an economy where first-best is unattainable (i.e. 2nd Welfare Thm breaks down), it is optimal to have production efficiency – that is, no distortions in production of goods..
- The result can also be stated as follows. Suppose there are two industries, x and y and two inputs, K and L. Then with the optimal tax schedule, production is efficient:

$$MRTS_{KL}^{\times} = MRTS_{KL}^{Y}$$

even though allocation is inefficient:

$$MRT_{xy} \neq MRS_{xy}$$

- The condition for production efficiency is that the marginal rate of technical substitution (*MRTS*) between any two inputs is the same for all firms
- Such a position of equality is attained in the absence of taxation by the profit maximisation of firms in competitive markets
 - each firm sets the marginal rate of technical substitution equal to the ratio of factor prices
 - factor prices are the same for all firms
 - this induces the necessary equality in the *MRTS*s.
- The same is true when there is taxation provided all firms face the same post-tax prices for inputs so input taxes are not differentiated between firms.

- One implication of the production efficiency theorem is that goods that enter into production processes, such as inputs and intermediate goods, should not be taxed.
- All firms should buy and sell at the at the same prices, in order for the whole production sector to be efficient.
- If different industries face different relative prices, MRTS between inputs will differ across industries.
- □ Then, in principle, it would be possible to reallocate inputs and have strictly more of one good while having no less of another.

Policy consequences of production efficiency (continued)

- No taxation of intermediate goods (goods that are neither direct inputs or direct outputs consumed by individuals).
- Goods transactions between firms should go untaxed because taxing these transactions would distort (aggregate) production and destroy production efficiency.
- Example: Computer produced by IBM but sold to other firms should be untaxed
 - but the same computer sold to direct consumers should be taxed.
- Government sales of publicly provided good (such as postal services) to firms should be untaxed
 - but government sales to individual consumers should be taxed.

Policy consequences of production efficiency

- Trade and Tariffs:
- In open economy, the production set is extended because it is possible to trade at linear prices (for a small country) with other countries.
- Diamond-Mirrlees result states that the small open economy should be on the frontier of the extended production set.
- Implies that no tariffs should be imposed on goods and inputs imported or exported by the production sector.
- Examples:
 - If IBM sells computers to other countries, that transaction should be untaxed.
 - If the oil companies buy oil from other countries, that should be untaxed.
 - If US imports cars from Japan, there should be no special tariff but should bear same commodity tax as cars made in US.

- The principle of not taxing intermediate goods is heavily dependent on the following assumptions:
 - No pure private sector profits
 - Perfect competition
 - The possibility to tax all final goods
- □ What happens if not all final goods can be taxed?
- □ The production efficiency theorem no longer applies

□ Newbery (1986) showed that:

- If the output of one firm cannot be taxed for some reason (e.g. administrative feasibility), then it may be desirable to tax its inputs.
- This implies that you introduce an inefficiency in the production process (marginal rates of transformation between pairs of goods will be different across producers).
- Inefficiency is balanced against the gains from surrogate taxation of the final good.
- In developing countries administrative feasibility often directs taxation towards a few easily taxable targets.

- The production efficiency theorem also implies that we should not impose any tariffs on inputs into production and that all final goods should be taxed the same regardless of whether the source of origin is domestic or foreign.
- □ EU has abolished tariffs within the unified market.

Production Efficiency

- In summary, the Diamond-Mirrlees lemma provides a persuasive argument for
 - the non-taxation of intermediate goods
 - the non-differentiation of input taxes between firms
- □ The result is of immediate practical importance
 - it provides a basic property that an optimal tax system must possess
- Value Added Taxation satisfies this property
 - taxes paid on inputs can be reclaimed
 - only final consumers pay tax

Applications of optimal commodity taxation

- The fundamental motive of the analysis is to provide practical policy recommendations.
- The results so far derived provide some valuable insights, e.g.
 - The need for production efficiency
 - The non-uniformity of commodity taxes

Applications of optimal commodity taxation

- Results from two well-known studies:
 - Atkinson (1972) calculated optimal tax rates for UK and concluded that sole efficiency considerations lead to high taxes on goods like food and rent and low taxes on durables. In any case, the optimal indirect tax system is not uniform.
 - Deaton (1977) concludes that in general optimal tax rates move further from uniformity as equity considerations become more important.

Applications of optimal commodity taxation

- Limitations of applied optimal commodity taxation studies:
 - In order to calculate the welfare weights, you need to know both the private utility functions and the social welfare function.
 - You need to know the complete demand system of the consumer.
 - In general, simpler practically relevant prescriptions can be obtained only at the expense of additional assumptions (e.g. absence of cross-price effects)

Main conclusions

- An efficient tax system places the burden of taxation primarily on necessities.
- □ Such a system is very damaging to the poor.
- □ When equity is introduced, it directs taxes towards luxuries.
- Equity-efficiency trade-off in the design of taxes.

Main conclusions

- The production efficiency theorem states implies that there should be no taxes on intermediate goods.
- □ Actual value-added tax systems satisfy this property.

Main references

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