

# Lecture 4

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## Measuring inequality

Spring 2026

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## Organization of the lecture

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- ⊗ Why study income distribution?
- ⊗ Inequality of what among whom? Definitional issues
- ⊗ Measuring inequality
  - Charting inequality
  - Inequality measures
  - Rankings
  - Inequality measures based on welfare functions

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## Why study income distribution?

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- People are concerned with justice, fairness, the distribution of rewards, ethics about the worst-off in our society.
- Many policy issues are at heart distributional
- Policy makers are interested (indicators used by European Union, Millennium Development Goals, etc.)

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## Why study income distribution?

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- We've seen the welfare-economics basis for redistribution as a public-policy objective
- How to assess the impact and effectiveness of such policy?
- We need appropriate criteria for comparing distributions of income and personal welfare
- This requires a treatment of issues in distributional analysis.

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## Functional versus personal distribution of income

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- Functional distribution of income (Ricardo):
  - Wages
  - Profits
  - Rent
- Personal distribution of income
  - + Earnings of Giorgos
  - + Earnings of Irini
  - + Interest on savings
  - + Pensions of Irini's mother
  - + Rent on mother's house

NATIONAL INCOME

typical household today has some capital (assets),  
 transfer payments from pension rights  
 property

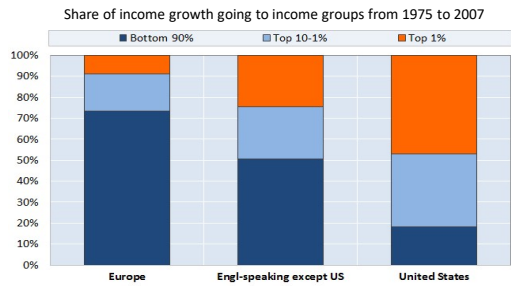
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## Inequality of **what**,

among **whom**?

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## Η οικονομική ανάπτυξη έχει ωφελήσει δυσανάλογα τους πολύ πλούσιους...



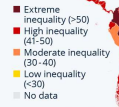
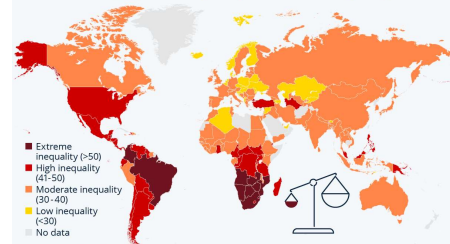
Source: OECD 2014, Focus on Top Incomes and Taxation in OECD Countries: Was the Crisis a Game Changer? (<http://www.oecd.org/els/topincomes/OECD-2014-FocusOnTopIncomes.pdf>). Based on World Top Income Database. Note: Incomes refer to pre-tax incomes, excluding capital gains.

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## The State of Global Income Inequality

Level of income inequality by country according to the Gini index (0=perfect equality, 100=total inequality)\*



\* Latest available Gini indexes per country  
Source: The World Bank - Poverty and Inequality Platform (Sep. 2024)

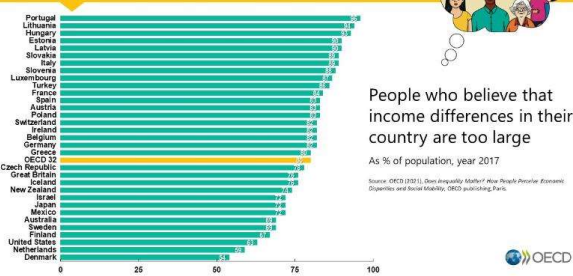


statista

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## Does Inequality Matter?

How People Perceive Economic Disparities and Social Mobility



People who believe that income differences in their country are too large

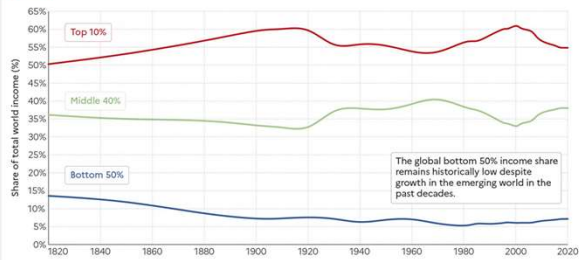
As % of population, year 2017

Source: OECD (2017), Does Inequality Matter? How People Perceive Economic Disparities and Social Mobility, OECD Publishing, Paris.



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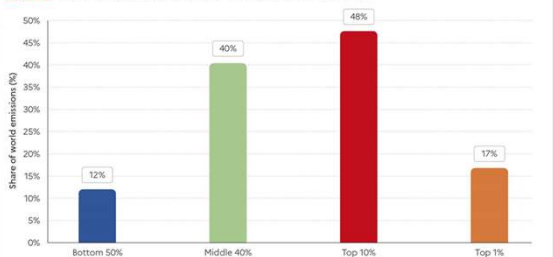
Figure 7 Global income inequality, 1820-2020



Interpretation: The share of global income going to top 10% highest incomes at the world level has fluctuated around 50-60% between 1820 and 2020 (50% in 1820, 60% in 1910, 56% in 1980, 61% in 2000, 55% in 2020), while the share going to the

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Figure 14 Global carbon inequality, 2019. Group contribution to world emissions (%)

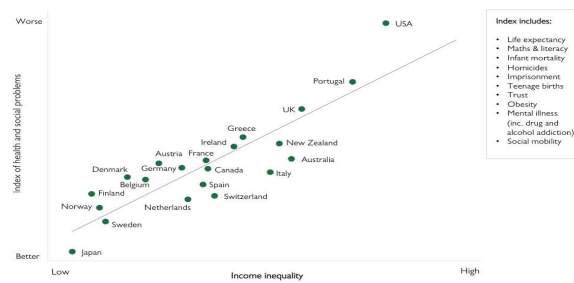


Interpretation: Personal carbon footprints include emissions from domestic consumption, public and private investments as well as imports and exports of carbon embedded in goods and services traded with the rest of the world. Modelled estimates based on the systematic combination of tax data, household surveys and input-output tables. Emissions split equally within households. Sources and series: wri2022.world/methodology and Chancel (2021).

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## Κόστος της ανισότητας

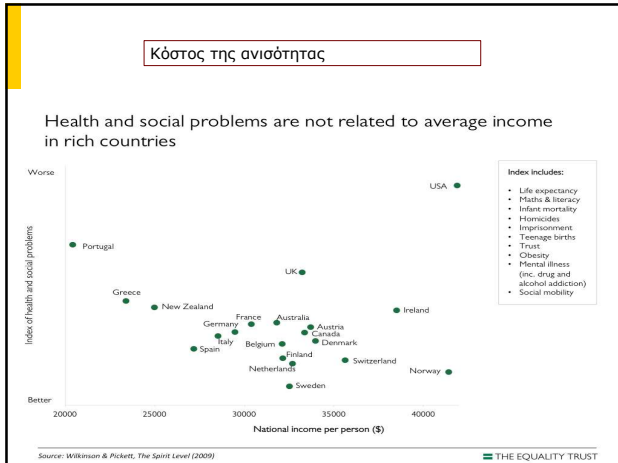
Health and social problems are worse in more unequal countries



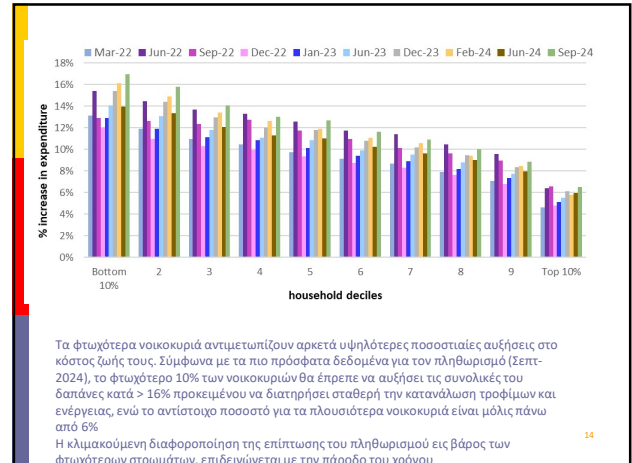
Source: Wilkinson & Pickett, The Spirit Level (2009)

THE EQUALITY TRUST

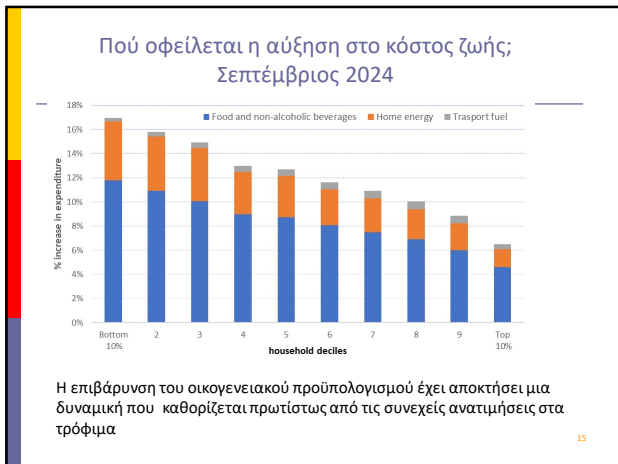
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### Basic questions about income

- Is it unique?
- How comprehensive should it be?
- What is the relevant receiving unit?
- Is it comparable between persons?

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### Income: Uniqueness?

- Should we use univariate or multivariate analysis?
  - income and expenditure?
  - income and wealth?
  - income over time? Lifetime income? You can count this only if a person is dead! Alternative: **anticipated** lifetime income.
- Several definitions may be relevant?
  - gross income?
  - disposable income?
  - other concepts? Value of goods provided by the state (e.g. public libraries, parks, healthcare)?

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### Income: Comparability?

- Price adjustment
  - Normalise by price indices
- Adjustment for needs and household size
  - Usual approach is to introduce equivalence scales
- The equivalence transformation is
 
$$x = \chi(y, \mathbf{a})$$

Equivalised income (x), nominal income (y), and personal attributes (a).

  - Usually a simplifying assumption is made.
  - Write transformation as an income-independent equivalence scale:
 
$$x = y / v(\mathbf{a})$$

Number of equivalent adults (v(a)).
  - Where does the function  $\chi$  come from?

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## Equivalence Scales

- We will assume that there is an agreed method of determining equivalence scales.
- But there is a variety of possible sources of information for equivalence scales:
  - From official government sources
  - From international bodies such as OECD
  - From econometric models of household budgets

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## Example: the modified OECD equivalence scale

- 1 for head of household
- 0,5 for each additional adult
- 0,3 for each child

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## Απόσπασμα από το Σχέδιο Νόμου που ψηφίστηκε στη Βουλή στις 7/11/12

- **I) Ενιαίο επίδομα στήριξης τέκνων**
- Θεσπίζεται ενιαίο επίδομα στήριξης τέκνων, το οποίο αντικαθιστά τα καταργούμενα με τις υποπεριπτώσεις 12 και 14 της παρούσας διάταξης οικογενειακά επιδόματα.
- Το ενιαίο επίδομα στήριξης τέκνων καταβάλλεται λαμβάνοντας υπόψη τον αριθμό των εξαρτώμενων τέκνων, την κλίμακα ισοδυναμίας, το ισόδυναμο εισόδημα και την εισοδηματική κατηγορία.
- Ως κλίμακα ισοδυναμίας ορίζεται το σταθμισμένο άθροισμα των μελών της οικογένειας. Ο πρώτος γονέας έχει στάθμιση 1, ο δεύτερος γονέας έχει στάθμιση 1/3 και κάθε εξαρτώμενο τέκνο έχει στάθμιση 1/6. Ως ισόδυναμο εισόδημα ορίζεται το καθαρό, ετήσιο, οικογενειακό εισόδημα (φορολογητέο εισόδημα) διαιρεμένο με την κλίμακα ισοδυναμίας.

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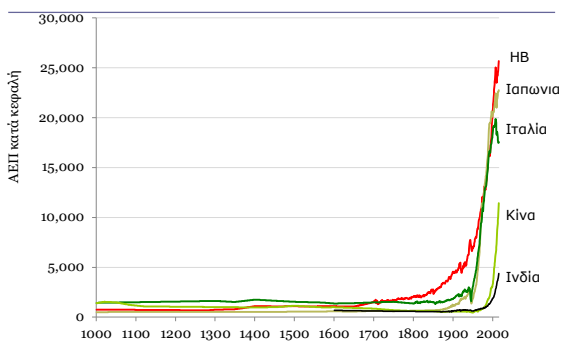
## Άρθρο 214 του ν. 4512/2018 με θέμα: «Καθορισμός της διαδικασίας χορήγησης επιδόματος παιδιού»

Άρθρο 214 ν. 4512/17-1-2018

1. Θεσπίζεται επίδομα παιδιού, το οποίο αντικαθιστά τα καταργούμενα με την παράγραφο 15 επιδόματα.
2. Το επίδομα παιδιού καταβάλλεται λαμβάνοντας υπόψη τον αριθμό των εξαρτώμενων τέκνων, το ισόδυναμο οικογενειακό εισόδημα και την κατηγορία ισόδυναμου οικογενειακού εισοδήματος.
3. Ως ισόδυναμο οικογενειακό εισόδημα ορίζεται το συνολικό, πραγματικό ή τεκμαρτό, εισόδημα από κάθε πηγή ημεδαπής και αλλοδαπής προέλευσης προ φόρων, μετά την αφαίρεση των εισφορών για κοινωνική ασφάλιση, εξαιρουμένων των επιδομάτων που δεν προσμετρώνται στο φορολογητέο εισόδημα, όλων των μελών της οικογένειας, διαιρούμενο με την κλίμακα ισοδυναμίας.
4. Η κλίμακα ισοδυναμίας, για τους σκοπούς του παρόντος άρθρου, προκύπτει από το σταθμισμένο άθροισμα των μελών της οικογένειας, σύμφωνα με την ακόλουθη στάθμιση: α) πρώτος γονέας: στάθμιση 1, β) δεύτερος γονέας: στάθμιση 1/2, γ) κάθε εξαρτώμενο τέκνο: στάθμιση 1/4.  
Ειδικά για τις μονογονεϊκές οικογένειες, το πρώτο εξαρτώμενο τέκνο έχει στάθμιση 1/2 και κάθε επόμενο εξαρτώμενο τέκνο 1/4.

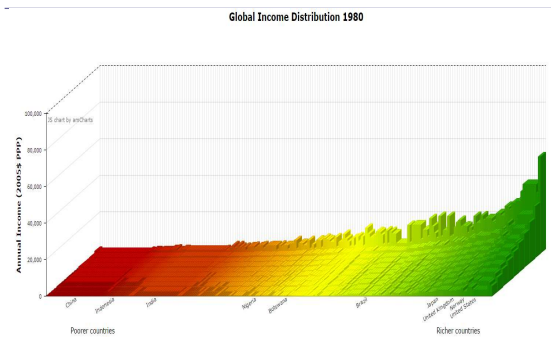
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Κατά κεφαλή ΑΕΠ σε πέντε χώρες (1000-2015). Μπαστούνι του Χοκεϋ.

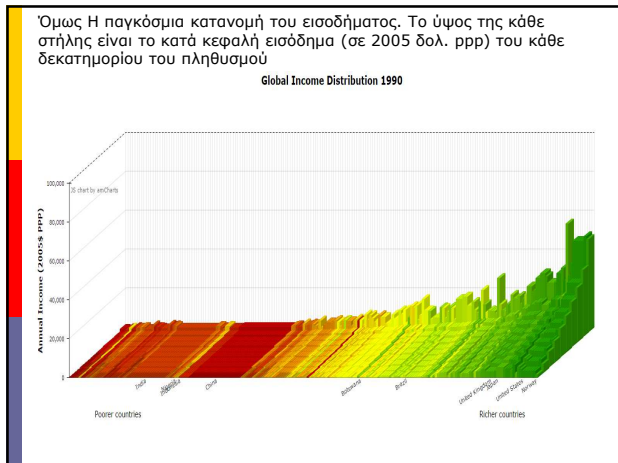


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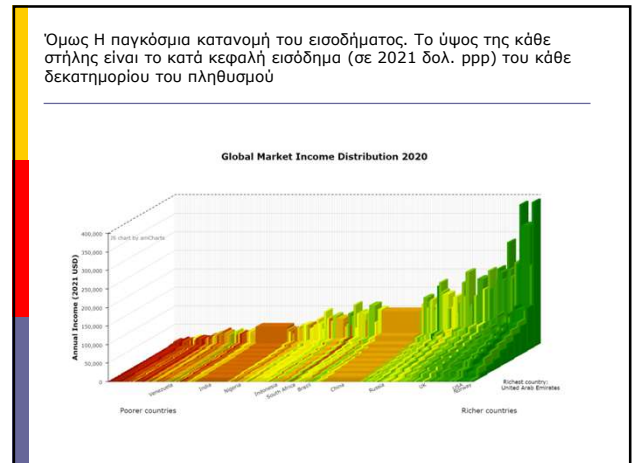
Όμως η παγκόσμια κατανομή του εισοδήματος. Το ύψος της κάθε στήλης είναι το κατά κεφαλή εισόδημα (σε 2005 δολ. ppp) του κάθε δεκατημορίου του πληθυσμού



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### Measuring inequality: introduction

- Representing an income distribution and presenting inequality in a large heterogeneous group of people is not simple.

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### Measuring inequality: introduction

- Three methods of appraising the complicated information that is contained in an income distribution:
  - Diagrams
  - Inequality measures
  - Rankings

These methods can be applied to any variable, whose distribution we want to appraise (e.g. per capita energy consumption)

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### Charting Inequality

- We put information about the income distribution into diagrammatic form (we put inequality in pictures). One of the most common ways:
  - Lorenz curve

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### The Lorenz curve

- Introduced by Lorenz in 1905.
- Again line up everybody in ascending order of income and let them parade by.
- Once point C is reached, everybody has passed by, so  $F(y) = 1$ .
- As each person passes, hand him his share of the «cake», i.e. the proportion of total income that he receives.
- When the parade reaches people with income  $y$ , let us suppose that a proportion  $\Phi(y)$  of the cake has gone. So of course when  $F(y) = 0$ ,  $\Phi(y)$  is also 0 (no cake gone);
- and when  $F(y) = 1$ ,  $\Phi(y)$  is also 1 (all the cake has been handed out).
- $\Phi(y)$  is measured on the vertical scale in Figure 2.4, and the graph of  $\Phi$  plotted against  $F$  is the Lorenz curve.

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## Lorenz curve

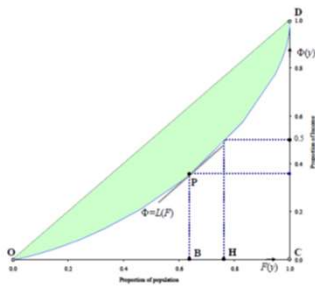


Figure 2.4: Lorenz Curve of Income. Source: as for Figure 2.1

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## Lorenz curve

- Properties
  - It is always convex toward the point C. Suppose that the first 10% ( $F(y_1) = 0.1$ ) have been given 4% of the cake ( $\Phi(y_1) = 0.04$ ). Then by the time the next 10% of the people go by ( $F(y_2) = 0.2$ ), you must have handed out at least 8% of the cake ( $\Phi(y_2) = 0.08$ ). Why? Because we arranged the parade in ascending order of cake-receivers.
  - If the Lorenz curve lays along OD, we would have a state of perfect equality, for along that line the first 5% get 5% of the cake, the first 10% get 10% ... and so on.

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## Inequality measures

- The graphical ways of presenting the income distribution are used to introduce some conventional inequality measures.

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## Inequality measures

- Range  $R$
- Relative Mean Deviation  $M$
- Variance  $V$
- Coefficient of variation  $c$
- Gini coefficient  $G$

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## Inequality measures - Range

- Range ( $R$ ) is the distance
- $R = y_{max} - y_{min}$   
 where  $y_{max}$  and  $y_{min}$  are the maximum and minimum values of income in the parade.  
 We can also standardize by considering  $R/y_{min}$ .

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## Inequality measures - Range

- Problems
  - In large heterogeneous populations, minimum and maximum income can only be guessed.
  - Highly sensitive to estimates of the two extreme values.
- Possible solution:  $R = y_{bottom\ 5\%} - y_{top\ 5\%}$
- More serious problem: What happens to  $R$  if  $y_{max}$  and  $y_{min}$  remains the same and everybody else's income is levelled to some equal intermediate income?

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## Inequality measures - Range

- $R$  remains the same!

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## Inequality measures – Relative mean deviation ( $M$ )

- Relative mean deviation ( $M$ ): the average absolute distance of everyone's income from the mean, expressed as a proportion of the mean.

$$M = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_i}{\bar{y}} - 1 \right|$$

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## Inequality measures – Gini coefficient ( $G$ )

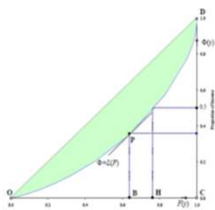


Figure 2.4: Lorenz Curve of Income. Source: as for Figure 2.1

The Gini coefficient ( $G$ ) is derived from the Lorenz curve. It is the ratio of the shaded area to the area OCD.

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## Inequality measures – Gini coefficient ( $G$ )

- In mathematical terms,  $G$  is the average difference between all possible pairs of incomes in the population, expressed as a proportion of total income:

$$\text{Gini} = \frac{1}{2n^2 \bar{y}} \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j|$$

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## Inequality measures – Gini coefficient - Disadvantage

- Main Problem
  - It places different relative value in transfers taking place in different parts of the distribution:
  - An income transfer from a relatively rich person to a person with £  $x$  less has a much greater effect on  $G$  if the two persons are near the middle rather than at either end of the parade.
  - *Transfer effect:*

$$\frac{F(y_j) - F(y_i)}{n \bar{y}}$$

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## Inequality measures – Gini coefficient - Disadvantage

- Main Problem
  - So, consider transferring \$ 1 from a person with \$ 10,100 to a person with \$ 10,000. This has a much greater effect on reducing  $G$  than transferring \$ 1 from a person with \$ 1,100 to one with \$ 1,000 or than transferring £ 1 from a person with \$ 100,100 to a person with \$ 100,000.
  - This valuation may be desirable, but it is not obvious that it is desirable.

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## Inequality measures – Variance ( $V$ )

- Consider the frequency distribution and its log transformation.
- Use tools from statistics: Measure inequality as the **dispersion** of the frequency distribution

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## Inequality measures – Variance ( $V$ )

- Assume there are  $n$  people. Define *variance* ( $V$ ) as :

$$V = \frac{1}{n} \sum_{i=1}^n [y_i - \bar{y}]^2$$

- Measure the distance between individual's income  $y_i$  and mean income  $\bar{y}$ , square this (why?), and then find the average of the resulting quantity in the whole population.

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## Inequality measures – Variance ( $V$ ) - Problem

- If we double everybody's income (so also double mean income and essentially leave the distribution unchanged),  $V$  quadruples.
- Way out: Standardise  $V$ .

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## Inequality measures – Coefficient of Variation ( $c$ )

Coefficient of variation ( $c$ ):

$$c = \frac{\sqrt{V}}{\bar{y}}$$

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## Scalar inequality (use inequality indices)

- Inequality measure (simple definition): a scalar numerical representation of the interpersonal differences in income within a given population.
- “scalar” means that all difference features of inequality are compressed into a single number

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## Scalar inequality

- Advantages:
  - If we want a multi-number representation of inequality, we can do this by using different inequality indices ( $I_1, I_2$ )
  - We can answer the question of “whether inequality has increased or decreased” with a straight “yes” or “no”.

Problem: If we make the concept of inequality multi-dimensional, we may come up with ambiguous answers. See example below:

Starting off from point B, which reveals that we have an amount of  $I_1$  of type-1 inequality and an amount of  $I_2$  of type-2 inequality, how do I compare B and D, or B and E?

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## Problems using scalar inequality

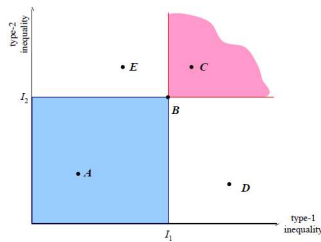


Figure 1.1: Two Types of Inequality

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## Rankings

- Ways of comparing whole distributions, even if we get ambiguous results:
  - e.g. Lorenz rankings (based on Lorenz curves)

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## Lorenz comparisons

- What happens to the share of income accruing to different groups of the population over time (or as a result of the redistributive action of government policy)?

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## Straightforward case: Lorenz curves do not cross

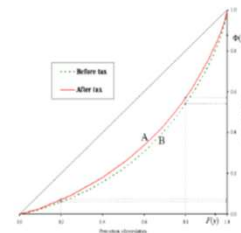


Figure 2.10: Ranking by Shares. UK 1994/5 Incomes before and after tax. Source: as for Figure 2.1

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## Straightforward case: Lorenz curves do not cross

- B: Before tax income distribution
- A: After tax income distribution
- A lies everywhere inside B. What does this mean?
- E.g. people in the bottom 20 percent would have received a larger slice of the after-tax cake (curve A) than they used to get in B.
- Also those in the bottom 80 percent received a larger proportionate slice of the A-cake than their proportionate slice of the B-cake (which of course is equivalent to saying that the richest 20 percent gets a smaller proportionate slice in A than it received in B).

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## Straightforward case: Lorenz curves do not cross

- Whatever "bottom proportion" of people  $F(y)$  is selected, this group gets a larger share of the cake ( $y$ ) in A than in B.
- Thus, A dominates B, and leads to lower inequality by almost all inequality measures.

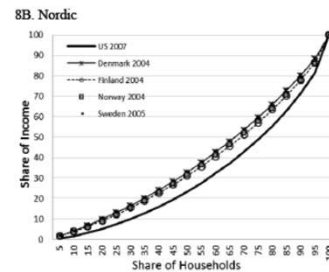
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## Inequality ranking: Summary

- Lorenz dominance equivalent to ranking by shares.
- Where Lorenz-curves intersect unambiguous inequality orderings are not possible.
- This makes inequality measures especially interesting.

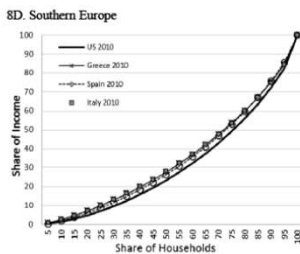
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## Lorenz curves (Morelli et al 2014)



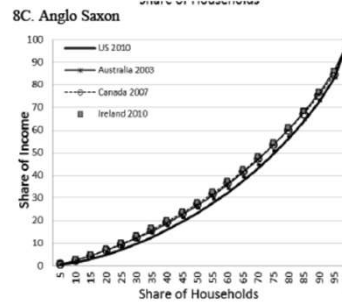
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## Lorenz curves (Morelli et al 2014)



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## Lorenz curves (Morelli et al 2014)



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## Inequality measures examined so far: basic problem

- Essentially arbitrary
  - Does not mean that CV or Gini is a bad index
  - But what is the basis for it?
- What is the relationship with social welfare?
  - Examine the **welfare-inequality** relationship directly

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## Inequality indices based on Social welfare functions

- Basic tool is a *social welfare function* (SWF)
  - Maps set of distributions into the real line
  - I.e. for each distribution we get one specific number
  - All distributions can be ranked
- Use a simple framework to list some of the basic axioms
  - Assume a fixed population of size  $n$ .
  - Assume that individual utility can be measured by  $x$
  - Income normalised by equivalence scales
  - Rules out utility interdependence
  - Welfare is just a function of the vector  $\mathbf{x} := (x_1, x_2, \dots, x_n)$

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## Social welfare functions: properties

- Five desirable characteristics:
- 1. The SWF is *individualistic* and *nondecreasing*, if the welfare level in any state A, denoted by a number  $W_A$ , can be written:  
$$W_A = W(y_{1A}, y_{2A}, \dots, y_{nA})$$
and, if  $y_{iB} \geq y_{iA}$  for all  $i$  implies, ceteris paribus, that  $W_B \geq W_A$ , which in turn implies that state B is at least as good as state A.

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## Social welfare functions: properties

- This property simply states that the welfare numbers should be related to individual incomes (or wealth, etc.) so that if any person's income goes up social welfare cannot go down.
- The idea that welfare is non-decreasing in income is perhaps not very innocent: it rules out for example the idea that if one disgustingly rich person gets richer still whilst everyone else's income stays the same, the effect on inequality is so awful that social welfare actually goes down.

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## Social welfare functions: properties

- 2. The SWF is *symmetric* if it is true that, for any state,  
 $W(y_1, y_2, \dots, y_n) = W(y_2, y_1, \dots, y_n) = \dots = W(y_n, y_2, \dots, y_1)$ ;  
This means that the function  $W$  treats individual incomes anonymously: the value of  $W$  does not depend on the particular assignment of labels to members of the population.

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## Social welfare functions: properties

- Given that we treat these standardised incomes  $y_i$  as a measure that puts everyone in the population on an equal footing as regards needs and desert, the second property (symmetry) naturally follows.
- There is no reason why welfare should be higher or lower if any two people simply swapped incomes.

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## Social welfare functions: properties

- The SWF is *additive* if it can be written

$$W(y_1, y_2, \dots, y_n) = \sum_{i=1}^n U_i(y_i) = U_1(y_1) + U_2(y_2) + \dots + U_n(y_n),$$

where  $U_i$  is a function of  $y_i$  alone, and so on.

If the above properties are satisfied, we can write the SWF as:

$$W(y_1, y_2, \dots, y_n) = \sum_{i=1}^n U(y_i) = U(y_1) + U(y_2) + \dots + U(y_n),$$

Where  $U$  is the same function for each person and where  $U(y_i)$  increases with  $y_i$ .

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## Social welfare functions: properties

- This is a very strong assumption and is independent from assumptions 1 and 2.
- It implies that if we want to measure the increase in welfare between states A and B (and so by calculating the difference  $W_B - W_A$ ), what matters is only the incomes that have changed, not what the rest of the income distribution looks like.

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## Social welfare functions: properties

- Example:
- Suppose the only change is an increase in person 1's income from \$ 20,000 to \$ 21,000. Then the additivity assumption states that the effect of this change alone (increasing person 1's income from \$ 20,000 to \$ 21,000) is  $(W'_B - W'_A)$  and is just the same for this particular change, regardless of whether everyone else had \$ 1 or \$ 100,000.

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## Social welfare functions: properties

- Let us call  $U(y_1)$  the *social utility* of person 1. The rate at which this index increases is

$$U'(y_1) = \frac{dU(y_1)}{dy_1}$$

which can be thought of as the *social marginal utility* of, or the *welfare weight*, for person 1. This tells me how much social welfare increases if I give one more euro to person 1.

Because of the first property, none of the welfare weights can be negative.

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## Social welfare functions: properties

- 4. The SWF is *strictly concave* if the welfare weight always decreases as  $y_i$  increases.
- The notion of *social marginal utility* (or *welfare weight* is very useful). Consider a government programme which brings about a (small) change in everyone's income:  $\Delta y_1, \Delta y_2, \dots, \Delta y_n$ . What is the change in social welfare?

$$dW = U'(y_1)\Delta y_1 + U'(y_2)\Delta y_2 + \dots + U'(y_n)\Delta y_n$$

- So  $U'$  act as a system of weights when summing the effects of the programme over the whole population.

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## Social welfare functions: properties

- How should the weights be fixed? The strict concavity assumption tells us that the higher a person's income, the lower the social weight he is given.
- If we are averse to inequality this seems reasonable: a small redistribution from rich to poor should lead to a socially-preferred state.

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## Social welfare functions: properties

- 5. The SWF has *constant elasticity*, or *constant relative inequality aversion* if  $U(y_i)$  can be written

$$U(y_i) = \frac{y_i^{1-\varepsilon} - 1}{1-\varepsilon}$$

(or in a cardinally equivalent form), where  $\varepsilon$  is the inequality aversion parameter, which is non-negative

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## SWF-based inequality measures

Introduce the concept of equally distributed equivalent level of income ( $Y_e$ ) as the per capita amount of the smallest total income which if equally distributed offers the same level of welfare as the original distribution, so that

$$W[U_1(Y_e), U_2(Y_e), \dots, U_n(Y_e)] = W[U_1(Y_1), U_2(Y_2), \dots, U_n(Y_n)]$$

Then the Atkinson index is

$$AI = 1 - (Y_e / \bar{Y})$$

where  $Y_e < \bar{Y}$

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## SWF-based inequality measures

- In the isoelastic case, this becomes

$$AI = 1 - \left[ \frac{1}{n} \sum_{i=1}^n (Y_i / \bar{Y})^{1-\epsilon} \right]^{1/(1-\epsilon)}$$

- If  $AI = 0.3$ , we can say that, if income were equally distributed, we would need only  $(100-30)\% = 70\%$  of present national income to achieve the same level of total welfare.

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## Additional statistical data, poverty and inequality

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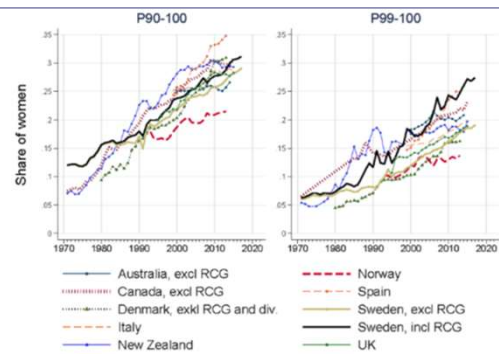
Table 2: The gender divide in the labour market in advanced economies

	% of women in the top 10% income group	% of women in the top 1% income group	Share of unpaid care work done by women (latest year)
Spain 2010	33%	22%	63%
Denmark 2013	31%	16%	57%
Canada 2013	30%	22%	61%
New Zealand 2013	29%	19%	65%
Italy 2014	29%	20%	75%
UK 2013	28%	18%	65%
Australia 2012	25%	22%	64%
Norway 2013	22%	14%	57%

Source: <http://www.lse.ac.uk/InternationalInequalities/pdf/III-Working-Paper-5-Atkinson.pdf> and OECD stat Employment: Time spent in paid and unpaid work, by sex

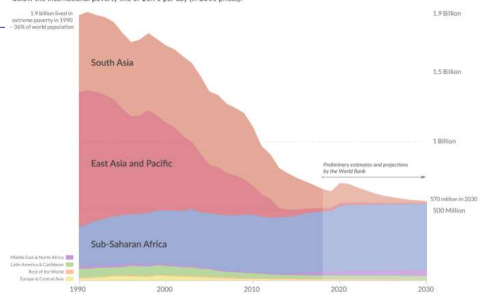
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Share of women in top income groups in various countries, 1971-2017 (Roine and Boschini, 2020)



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The number of people in extreme poverty – including projections to 2030. This data is adjusted for inflation and for differences in the cost of living between countries. Extreme poverty is defined as living below the international poverty line of \$1.90 per day (in 2015 prices).



Note: 1 billion is the same as the population of the United States in 2015. This data is adjusted for inflation and for differences in the cost of living between countries. Extreme poverty is defined as living below the international poverty line of \$1.90 per day (in 2015 prices). From 2010 onwards, data refers to projections based on the growth forecasts from the June 2020 Global Economic Prospects publication. For the period 1990-2010, the data is based on the most recent household survey data for each of the regions and is subject to minor revisions. Source: Lakner et al. (2020) Institute for Global Development. OurWorldInData.org. Research and data to make progress against the world's largest problems. Licensed under CC BY by the author, Jan Hanel.

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