

LECTURE 1b

TOOLS OF NORMATIVE ANALYSIS

1

Basic questions that all economic systems must answer

- What is to be produced and in what quantities ?
- How is the desired output to be produced ?
- How is the desired output to be distributed ?
- How does the economy provide for cyclical stability ?
- How does the economy sustain economic growth overtime ?

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Basic questions that all economic systems must answer

- What is to be produced and in what quantities ?
- How is the desired output to be produced ?



RESOURCE ALLOCATION QUESTIONS

3

Basic questions that all economic systems must answer

- How is the desired output to be distributed ?



DISTRIBUTION QUESTION

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Basic questions that all economic systems must answer

- How does the economy provide for cyclical stability ?
- How does the economy sustain economic growth overtime ?



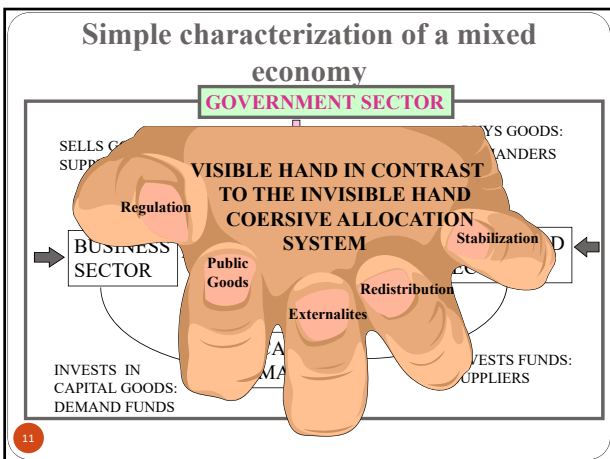
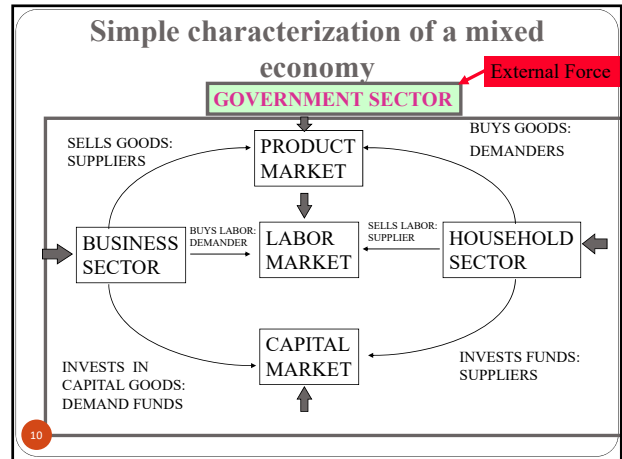
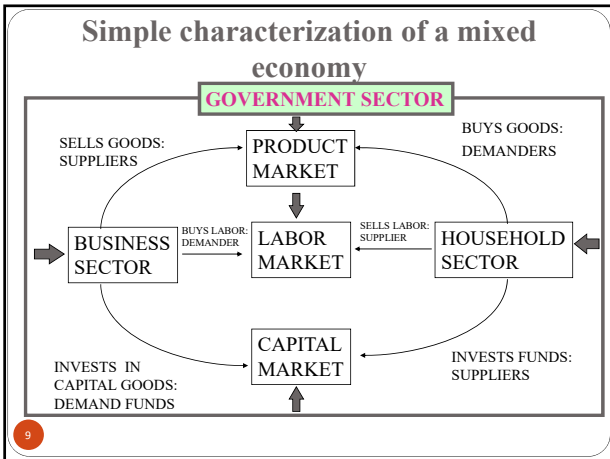
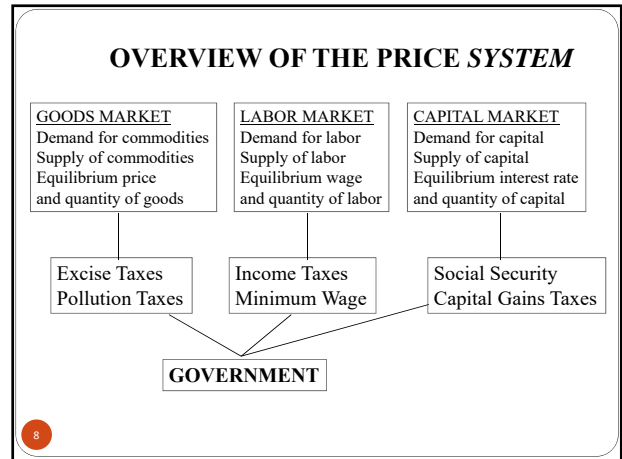
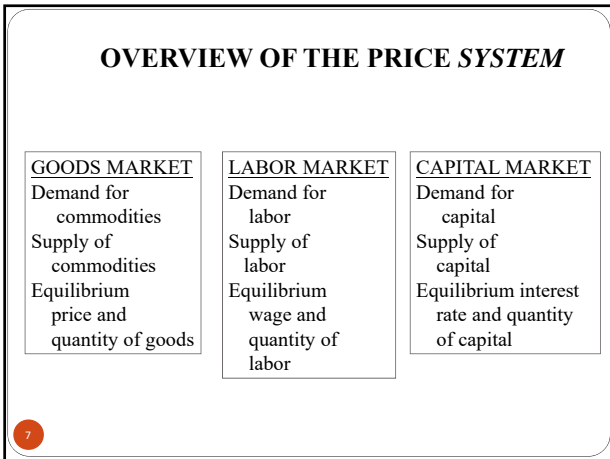
STABILIZATION QUESTIONS

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MARKET SYSTEM

- A price system is a social economic organization based on **individual choices** and **property rights**.
- Understanding the price system is important because:
 - 1 The market is the alternative to government intervention and control.
 - 2 Tax and expenditure policies impact decisions in the private markets.
 - 3 The concept of economic efficiency needs to be defined more specifically.

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Efficiency criterion

- **Positive Economics** is the scientific view of economic events. It tries to find cause and effect, predictive relationships.
- **Normative Economics** is based on value judgments. It tries to formulate recommendations as to what should be.
- The **efficiency criterion** is satisfied when resources are used over a given period of time in such away as to make it impossible to increase the well-being of any one person without reducing the well-being of any other person. This situation is referred to as a **Pareto Optimum** state

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Efficiency criterion

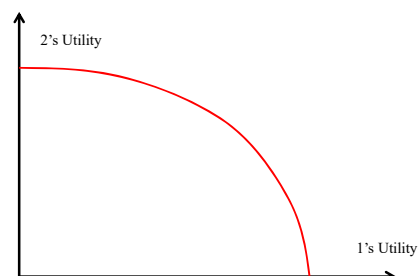
Definition: An *allocation* of resources is *Pareto Efficient* if it is not possible to reallocate resources to make everyone better off.

How do we measure better off?

We use *Utility* to measure welfare/happiness.

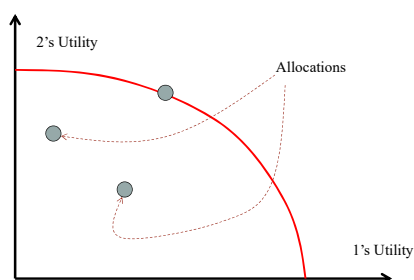
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Utility Possibilities: What is Feasible



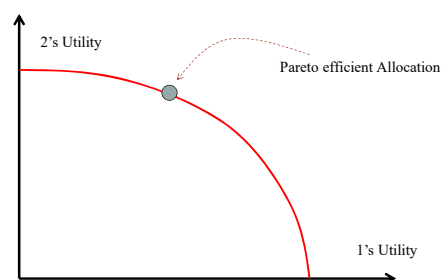
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Utility Possibilities: What is Feasible



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Pareto efficiency: There is no waste



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Optimality conditions

- *Marginal Condition for Exchange.*
- To attain a **Pareto Maximum**, the marginal rate of substitution (MRS) between any pair of goods must be the same for all individuals who consumer both goods.

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Optimality conditions

- *Marginal Condition for Factor Substitution.*
- To attain **Pareto Maximum**, the marginal rate of technical substitution (MRTS) between any pair of inputs must be the same for all producers who use both inputs.

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Optimality conditions

- *Marginal Condition for Product Substitution.*
- To attain a **Pareto Maximum**, the marginal rate of transformation (MRT) in production must equal the marginal rate of substitution in consumption for every pair of commodities and for every individual who consumes both.

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Optimality conditions

- *Corollary Proposition.*
- If the political organization of a society is such to accord paramount importance to its individual members -- mechanistic approach to government -- social welfare will be maximized if every consumer, every firm, and every input **market** is perfectly competition.

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CONSTRAINED UTILITY MAXIMIZATION

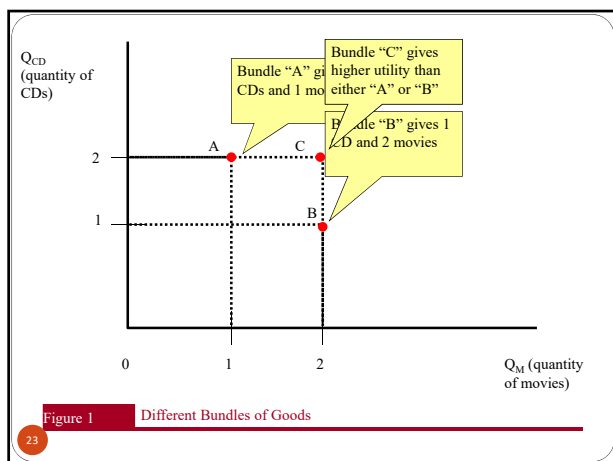
- *Constrained utility maximization* means that all decisions are made in order to maximize the well-being of the individual, subject to his available resources.
- Utility maximization involves *preferences* and a *budget constraint*.
- One of the key assumptions about preferences is *non-satiation*—that “more is preferred to less.”

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Constrained Utility Maximization Preferences and indifference curves

- *Figure 1* illustrates some preferences over movies (on the x-axis) and CDs (on the y-axis).
- Because of non-satiation, bundles *A* and *B* are both inferior to bundle *C*.

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Constrained Utility Maximization: Preferences and indifference curves

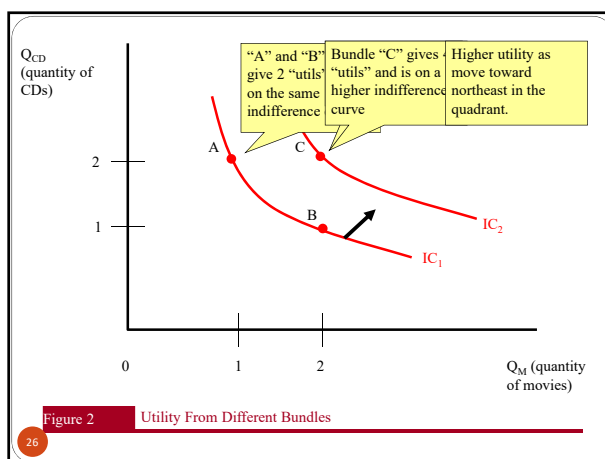
- A *utility function* is a mathematical representation
- $U = f(X_1, X_2, X_3, \dots)$
 - Where X_1, X_2, X_3 and so on are the *goods* consumed by the individual,
 - And $f(\bullet)$ is some mathematical function.

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Constrained Utility Maximization: Preferences and indifference curves

- One formulation of a utility function is $U(Q_M, Q_C) = Q_M Q_C$, where Q_M = quantity of movies and Q_C = quantity of CDs.
- The combinations $\{1, 2\}$ (bundle A) and $\{2, 1\}$ (bundle B) both give 2 “utils.”
- The combination $\{2, 2\}$ (bundle C) gives 4 “utils.”
- With these preferences, *indifferent* to A or B.
- **Figure 2** illustrates this.

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Constrained Utility Maximization: Utility mapping of preferences

- How are indifference curves derived?
- Set utility equal to a constant level and figure out the bundles of goods that get that utility level.
- For $U = Q_M Q_C$, how would we find the bundles for the indifference curve associated with 25 utils?
 - Set $25 = Q_M Q_C$,
 - Yields $Q_C = 25/Q_M$,
 - Or bundles like $\{1, 25\}$, $\{1.25, 20\}$, $\{5, 5\}$, etc.

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Constrained Utility Maximization: Marginal utility

- **Marginal utility** is the additional increment to utility from consuming an additional unit of a good.
- **Diminishing marginal utility** means each additional unit makes the individual less happy than the previous unit.

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Constrained Utility Maximization: Marginal utility

- With the utility function given before, $U = Q_M Q_C$, the marginal utility is:

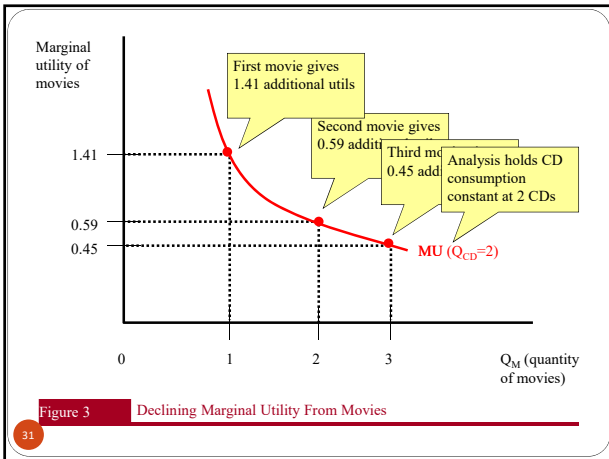
$$MU_{Q_M} = \frac{\partial U}{\partial Q_M} = Q_C$$
- Take the partial derivative of the utility function with respect to Q_M to get the marginal utility of movies.

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Constrained Utility Maximization: Marginal utility

- Evaluating the utility function $U = (Q_M Q_C)^{1/2}$, at $Q_C = 2$ allows us to plot a relationship between marginal utility and movies consumed.
- **Figure 3** illustrates this.

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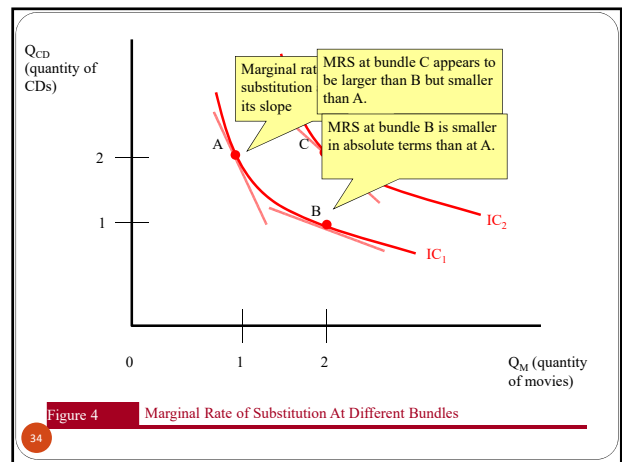


Constrained Utility Maximization: Marginal utility

- Why does diminishing marginal utility make sense?
- Most consumers order consumption of the goods with the highest utility first.

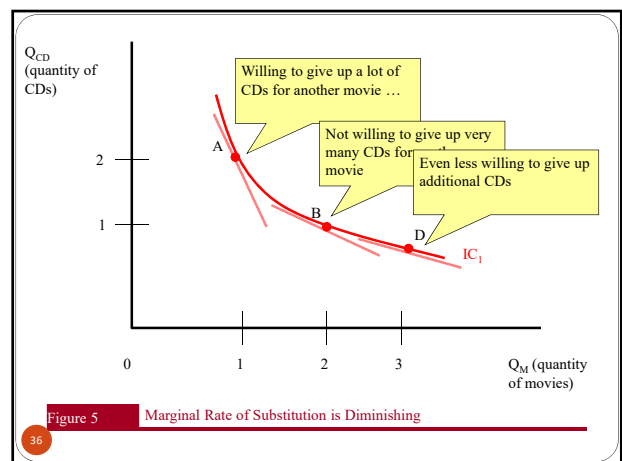
Constrained Utility Maximization: Marginal rate of substitution

- **Marginal rate of substitution**—slope of the indifference curve is called the **MRS**, and is the rate at which consumer is willing to trade off the two goods.
- Returning to the (CDs, movies) example.
- **Figure 4** illustrates this.



Constrained Utility Maximization: Marginal rate of substitution

- **MRS** is diminishing (in absolute terms) as we move along an indifference curve.
- This means that Andrea is willing to give up fewer CD's to get more movies when she has more movies (bundle B) than when she has less movies (bundle A).
- **Figure 5** illustrates this.



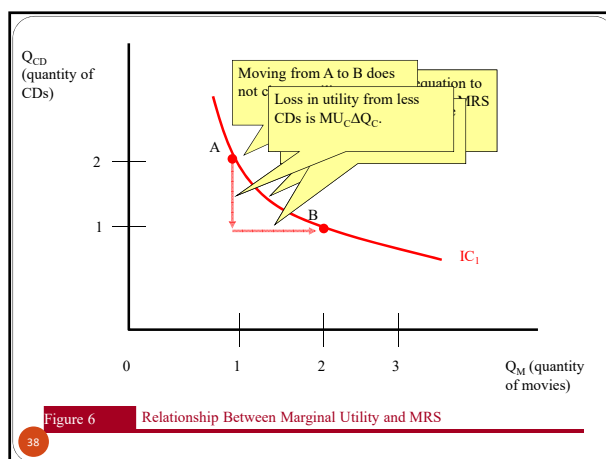
Constrained Utility Maximization Marginal rate of substitution

- Direct relationship between *MRS* and marginal utility.

$$MRS = -\frac{MU_M}{MU_C}$$

- *MRS* shows how the relative marginal utilities evolve over the indifference curve.
- Straightforward to derive this relationship graphically, as well.
- Consider the movement from bundle *A* to bundle *B*. **Figure 6** illustrates this.

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Constrained Utility Maximization: Budget constraints

- The **budget constraint** is a mathematical representation of the combination of goods the consumer can afford to buy with a given income.
- Assume there is no saving or borrowing.
- In the example, denote:
 - Y = Income level
 - P_M = Price of one movie
 - P_C = Price of one CD

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Constrained Utility Maximization: Budget constraints

- The expenditure on movies is:

$$P_M Q_M$$

- While the expenditure on CDs is:

$$P_C Q_C$$

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Constrained Utility Maximization: Budget constraints

- Thus, the total amount spent is:

$$P_M Q_M + P_C Q_C$$

- This must equal income, because of no saving or borrowing.

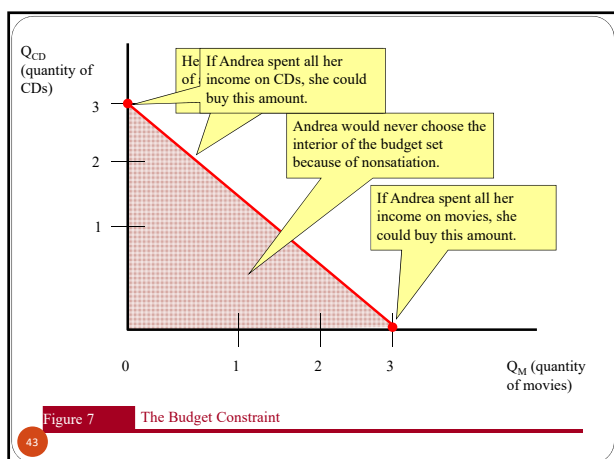
$$Y = P_M Q_M + P_C Q_C$$

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Constrained Utility Maximization: Budget constraints

- This budget constraint is illustrated in the next figure.
- **Figure 7** illustrates this.

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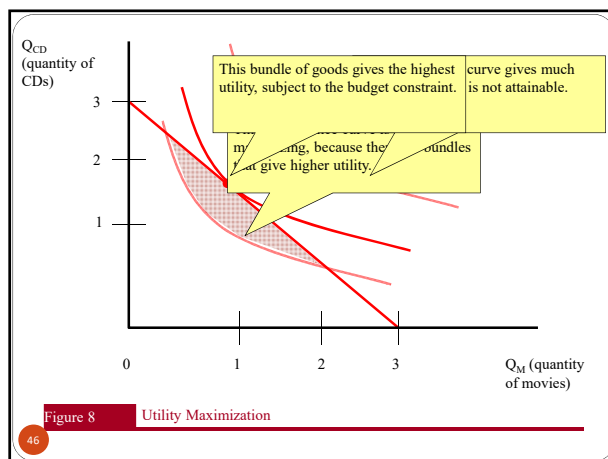
Constrained Utility Maximization: Budget constraints

- The slope of the budget constraint is:

$$-\frac{P_M}{P_C}$$
- It is thought that government actions can change a consumer's budget constraint, but that a consumer's preferences are fixed.

Constrained Utility Maximization: Putting it together: Constrained choice

- What is the highest indifference curve that an individual can reach, given a budget constraint?
- Preferences tells us what a consumer wants, and the budget constraint tells us what a consumer can actually purchase.
- This leads to utility maximization, shown graphically, in **Figure 8**.



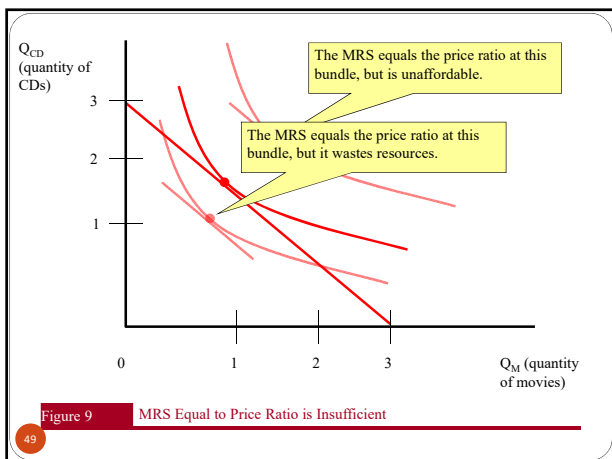
Constrained Utility Maximization: Putting it together: Constrained choice

- In this figure, the utility maximizing choice occurs where the indifference curve is *tangent* to the budget constraint.
- This implies that the slope of the indifference curve equals the slope of the budget constraint.

Constrained Utility Maximization: Putting it together: Constrained choice

- Thus, the marginal rate of substitution equals the ratio of prices:

$$MRS = -\frac{MU_M}{MU_C} = -\frac{P_M}{P_C}$$
- At the optimum, the ratio of the marginal utilities equals the ratio of prices. But this is not the only condition for utility maximization.
- Figure 9** illustrates this.



Constrained Utility Maximization: Putting it together: Constrained choice

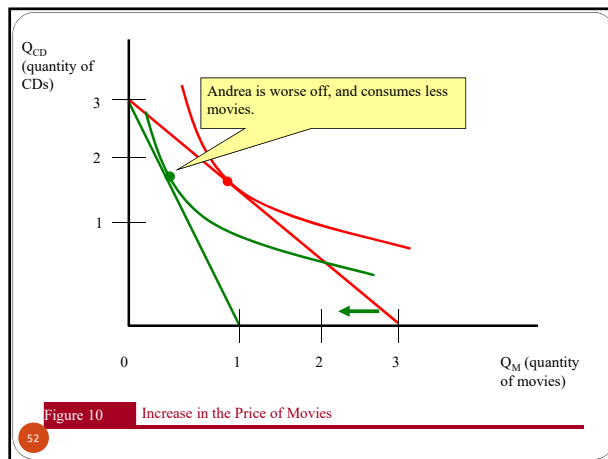
- Thus, the second condition is that all of the consumer's money is spent:

$$Y = P_M Q_M + P_C Q_C$$

- These two conditions are used for utility maximization.

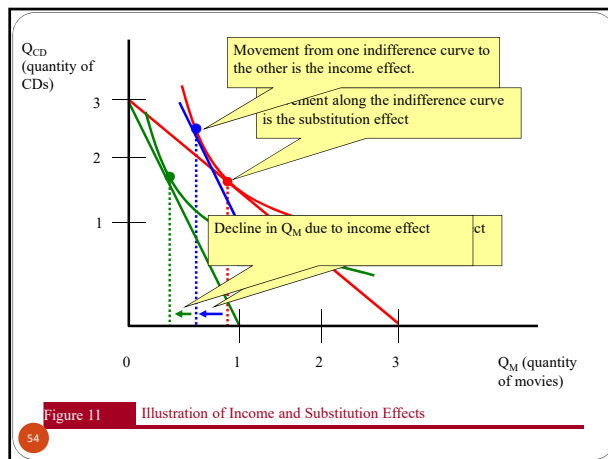
The Effects of Price Changes: Substitution and income effects

- Consider a typical price change in our framework:
- Increase the price of movies, P_M .
- This rotates the budget constraint inward along the x-axis.
- **Figure 10** illustrates this.



The Effects of Price Changes: Substitution and income effects

- A change in price consists of two effects:
- **Substitution effect**—change in consumption due to change in relative prices, *holding utility constant*.
- **Income effect**—change in consumption due to feeling “poorer” after price increase.
- **Figure 11** illustrates this.



EQUILIBRIUM AND SOCIAL WELFARE

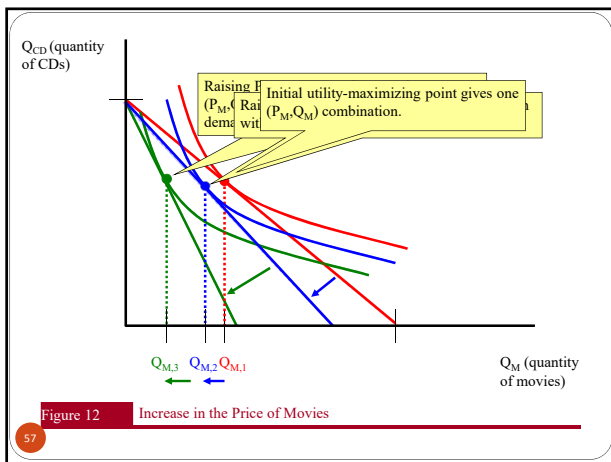
- **Welfare economics** is the study of the determinants of well-being, or welfare, in society. It depends on:
 - Determinants of social efficiency, or size of the economic “pie.”
 - Redistribution.

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EQUILIBRIUM AND SOCIAL WELFARE Demand curves

- **Demand curve** is the relationship between the price of a good and the quantity demanded.
- Derive demand curve from utility maximization problem, as shown in **Figure 12**.

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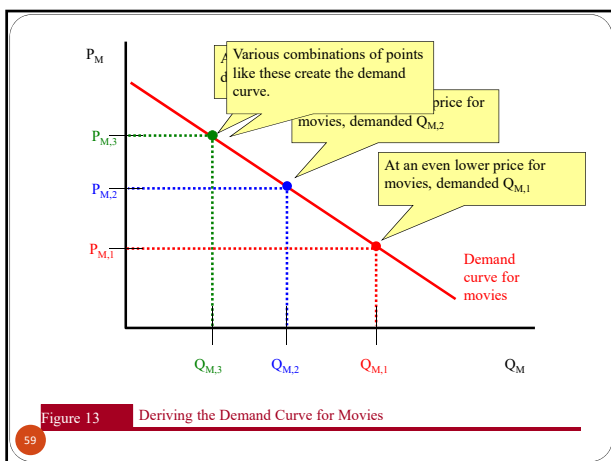


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EQUILIBRIUM AND SOCIAL WELFARE Demand curves

- This gives various (P_M, Q_M) combinations that can be mapped into price/quantity space.
- This gives us the demand curve for movies.
- **Figure 13** illustrates this.

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EQUILIBRIUM AND SOCIAL WELFARE Elasticity of demand

- A key feature of demand analysis is the **elasticity of demand**. It is defined as:

$$\epsilon_D = \frac{\Delta Q_D / Q_D}{\Delta P / P}$$

- That is, the percent change in quantity demanded divided by the percent change in price.

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EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- For example, an increase in the price of movies from € 8 to €12 is a 50% rise in price.
- If the number of movies purchased fell from 6 to 4, there is an associated 33% reduction in quantity demanded.
 - The demand elasticity is therefore -0.67.
- Demand elasticities features:
 - Typically negative number.
 - Not constant along the demand curve (for a linear demand curve).

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EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- For a vertical demand curve
 - Elasticity of demand is zero—quantity does not change as price goes up or down.
 - **Perfectly inelastic**
- For a horizontal demand curve
 - Elasticity of demand is negative infinity—quantity changes infinitely for even a small change in price.
 - **Perfectly elastic**
- **Figure 14** illustrates this.

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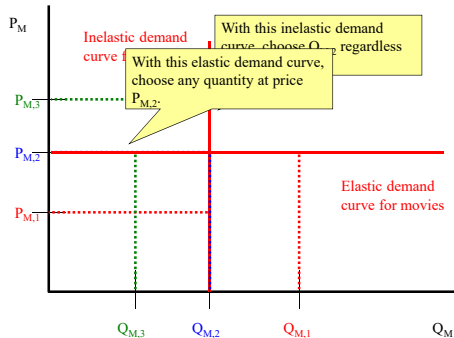


Figure 14 Perfectly Elastic and Perfectly Inelastic Demand

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EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- More generally, an **elasticity** divides the percent change in a dependent variable by the percent change in an independent variable:

$$\varepsilon = \frac{\Delta Y / Y}{\Delta X / X}$$

- For example, Y is often the quantity demanded or supplied, while X might be own-price, cross-price, or income.

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EQUILIBRIUM AND SOCIAL WELFARE

Supply curves

- **Supply curve** is the relationship between the price of a good and the quantity supplied.
 - Derive supply curve from profit maximization problem.
- The firm's **production function** measures the impact of a firm's input use on output levels.

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EQUILIBRIUM AND SOCIAL WELFARE

Supply curves

- Assume two inputs, labor (L) and capital (K). Firm's production function for movies is, in general:

$$Q_M = f(L_M, K_M)$$

- That is, the quantity of movies produced is related to the amount of labor and capital devoted to movie production.
- Similarly, there would be a production function for CDs.

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EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- One specific production function is:

$$Q_M = \sqrt{L_M K_M}$$

- From a production function like this, we can figure out the *marginal productivity* of an input by taking the derivative with respect to it.

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Equilibrium and Social Welfare: Supply curves

- For example, the marginal productivity of labor is:

$$\frac{\partial Q_M}{\partial L_M} = \frac{1}{2} \sqrt{\frac{K_M}{L_M}} > 0$$

- This is the partial derivative of Q with respect to L . The marginal product is positive.

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Equilibrium and Social Welfare: Supply curves

- Taking the second derivative yields:

$$\frac{\partial^2 Q_M}{\partial L_M^2} = -\frac{1}{4} \sqrt{\frac{K_M}{L_M^3}} < 0$$

- This second derivative is negative, meaning that the production function features *diminishing marginal productivity*.

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EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- *Diminishing marginal productivity* means that holding all other inputs constant, increasing the level of one input (such as labor) yields less and less additional output.

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EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- The total costs of production are given by:

$$TC = rK + wL$$

- In this case, r and w are the input prices of capital and labor, respectively.

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EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- If we assume capital is fixed in the short-run, the cost function becomes:

$$TC = r\bar{K} + wL$$

- Thus, only labor can be varied in the short run. The *marginal cost* is the incremental cost of producing one more unit of Q , or the product of the wage rate and amount of labor used to produce that unit.

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EQUILIBRIUM AND SOCIAL WELFARE

Supply curves

- Diminishing marginal productivity implies rising marginal costs.
- Since each additional unit, Q , means calling forth less and less productive labor at the same wage rate, costs of production rise.

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EQUILIBRIUM AND SOCIAL WELFARE

Supply curves

- **Profit maximization** means maximizing the difference between total revenue and total costs.
- This occurs at the quantity where **marginal revenue** equals **marginal costs**.

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EQUILIBRIUM AND SOCIAL WELFARE

Equilibrium

- In a perfectly competitive market, the marginal revenue is the market price. Thus, the firm produces until:
 - $P = MC$.
- Thus, the MC curve is the supply curve.

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EQUILIBRIUM AND SOCIAL WELFARE

Equilibrium

- In equilibrium, we *horizontally sum* individual demand curves to get aggregate demand.
- We also *horizontally sum* individual supply curves to get aggregate supply.
- Competitive equilibrium represents the point at which both consumers and suppliers are satisfied with the price/quantity combination.
- **Figure 15** illustrates this.

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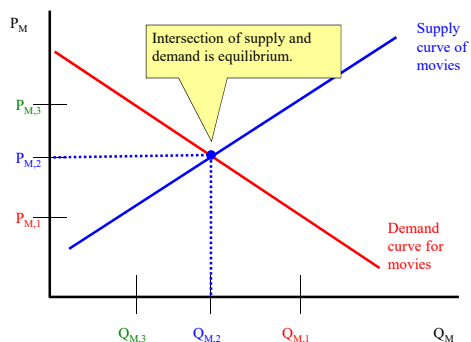


Figure 15 Equilibrium with Supply and Demand

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EQUILIBRIUM AND SOCIAL WELFARE

Social efficiency

- Measuring *social efficiency* is computing the potential size of the economic pie. It represents the net gain from trade to consumers and producers.

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EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- **Consumer surplus** is the benefit that consumers derive from a good, beyond what they paid for it.
- Each point on the demand curve represents a “willingness-to-pay” for that quantity.
- **Figure 16** illustrates this.

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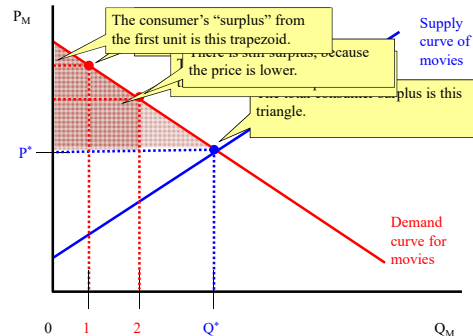


Figure 16 Deriving Consumer Surplus

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EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- Consumer surplus is determined by market price and the elasticity of demand:
 - With inelastic demand, demand curve is more vertical, so surplus is higher.
 - With elastic demand, surplus is lower.
- **Figure 17** illustrates this.

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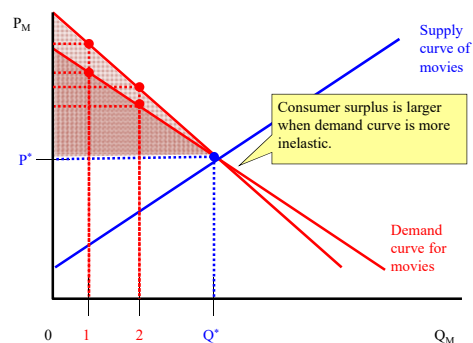


Figure 17 Consumer Surplus and Inelastic Demand

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EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- **Producer surplus** is the benefit derived by producers from the sale of a unit above and beyond their cost of producing it.
- Each point on the supply curve represents the marginal cost of producing it.
- **Figure 18** illustrates this.

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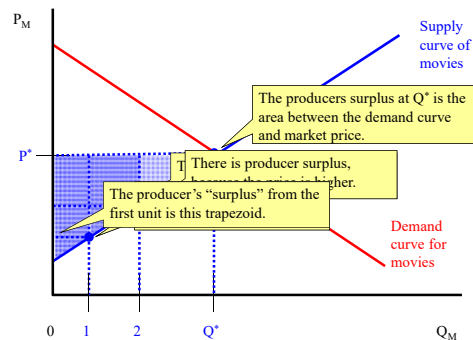


Figure 18 Producer Surplus

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EQUILIBRIUM AND SOCIAL WELFARE

Social efficiency

- Similar to consumer surplus, producer surplus is determined by market price and the elasticity of supply:
 - With inelastic supply, supply curve is more vertical, so producer surplus is higher.
 - With elastic supply, producer surplus is lower.

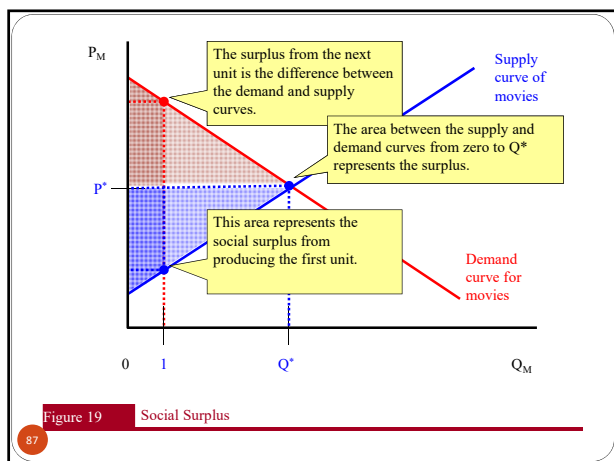
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EQUILIBRIUM AND SOCIAL WELFARE

Social efficiency

- The **total social surplus**, also known as “social efficiency,” is the sum of the consumer’s and producer’s surplus.
- **Figure 19** illustrates this.

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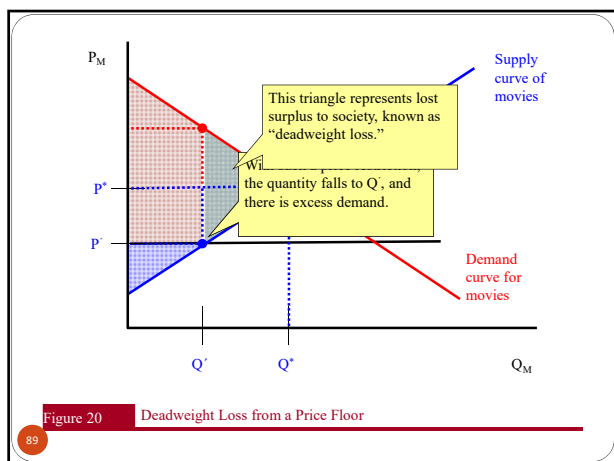
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EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

- The **First Fundamental Theorem of Welfare Economics** states that the competitive equilibrium, where supply equals demand, maximizes social efficiency.
- Any quantity other than Q^* reduces social efficiency, or the size of the “economic pie.”
- Consider restricting the price of the good to $P' < P^*$.
- **Figure 20** illustrates this.

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EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

- A policy like price controls creates **deadweight loss**, the reduction in social efficiency by restricting quantity below the competitive equilibrium.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

- Societies usually care not only about how much surplus there is, but also about how it is distributed among the population.
- *Social welfare* is determined by both criteria.
- The *Second Fundamental Theorem of Welfare Economics* states that society can attain any efficient outcome by a suitable redistribution of resources and free trade.
- In reality, society often faces an equity-efficiency tradeoff.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

- Society's tradeoffs of equity and efficiency are models with a *Social Welfare Function*.
- This maps individual utilities into an overall social utility function.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

- The *utilitarian* social welfare function is:

$$SWF = \sum_i U_i$$

- The utilities of all individuals are given equal weight.
- Implies that government should transfer from person 1 to person 2 as long as person 2's gain is bigger than person 1's loss in utility.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

- Utilitarian SWF is defined in terms of utility, not euros.
- Society not indifferent between giving €1 of income to rich and poor; rather indifferent between one *util* to rich and one util to poor.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

- Utilitarian SWF is maximized when the marginal utilities of everyone are equal:

$$MU_1 = MU_2 = \dots = MU_i$$

- Thus, society should redistribute from rich to poor if the marginal utility of the next euro is higher to the poor person than to the rich person.

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EQUILIBRIUM AND SOCIAL WELFARE

The role of equity

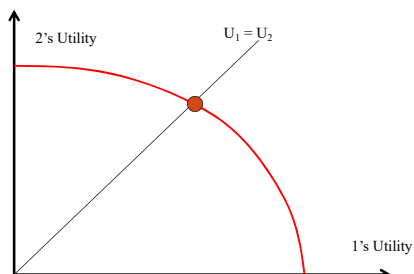
- The *Rawlsian* social welfare function is:

$$SWF = \min(U_1, U_2, \dots, U_N)$$

- Societal welfare is maximized by maximizing the well-being of the worst-off person in society.
- Generally suggests more redistribution than the utilitarian SWF.

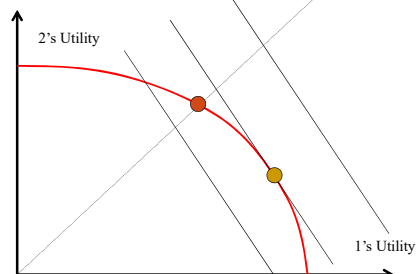
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Equity: equal shares



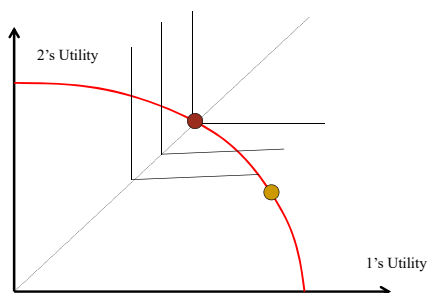
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Utilitarianism: Maximize $U(1)+U(2)$



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Rawls: Maximize $\min\{U(1),U(2)\}$



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Recap of Theoretical Tools

- Utility maximization
- Labor supply example
- Efficiency
- Social welfare functions

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Welfare maximization in general equilibrium

- In the preceding analysis we analyzed the efficiency conditions in a partial equilibrium framework.
- In other words, we assumed that in the market there was one good, and one factor of production.
- Reality, however is different. We live in a world with many goods and many factors of production.
- In a general equilibrium framework, with many goods and factors of production the conditions for welfare maximization are presented below.

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Requirements for welfare maximization

- **Marginal rate of substitution** between every pair of goods must be the same for all consumers. In a pure market setting, this occurs when consumers equate the MRS's to the common market determined output ratio.

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Requirements for welfare maximization

- **Marginal rate of technical substitution** between every pair of inputs must be the same for all producers . in a pure market setting, this occurs when producers maximize profit by equating MRTS's to the common market determined input price ratio.

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Requirements for welfare maximization

- **Marginal rate of transformation** must be equal to the marginal rate of substitution in consumption for each pair of goods. In a pure market setting, this condition occurs when producers set marginal cost (MC) equal to the output price.

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Conditions for welfare maximisation

$$MRS_{XY}^A = MRS_{XY}^B = MRT_{XY}$$

$$MRT_{XY} = \frac{P_X}{P_Y} = \frac{MC_X}{MC_Y}$$

$$MRT_{XY} = \frac{P_X}{P_Y} = MRS_{XY}$$

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Conditions for welfare maximisation

Competitive equilibrium Maximisation of consumer welfare	implies	Pareto efficiency Efficiency in exchange
$MRS_{XY}^A = \frac{P_X}{P_Y}$		$MRS_{XY}^A = MRS_{XY}^B$
$MRS_{XY}^B = \frac{P_X}{P_Y}$		

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Conditions for welfare maximisation

Competitive equilibrium Cost minimization	Implies	Pareto Efficiency Efficiency in production
$MRTS_{LK}^X = \frac{w}{r}$		$MRTS_{KL}^X = MRTS_{KL}^Y$
$MRTS_{LK}^Y = \frac{w}{r}$		

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Conditions for welfare maximisation

Competitive equilibrium Profit maximisation	Implies	Pareto Efficiency Overall efficiency
$P_X = MC_X$ $P_Y = MC_Y$		$MRT_{XY} = \frac{MC_X}{MC_Y}$ $= \frac{P_X}{P_Y} = MRS_{XY}$

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The First Fundamental Theorem of Welfare Economics

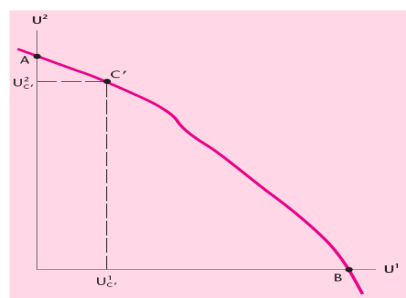
A competitive economy can achieve a Pareto optimal allocation of resources

Necessary conditions for a Pareto optimum:

1. **Consumption:** Marginal rates of substitution between X & Y must be equal for 1 & 2
2. **Production:** Marginal rates of technical substitution between K & L must be equal for production of X & Y
3. **Consumption-production:** Marginal rates of substitution between X & Y must also equal Marginal rates of transformation between X & Y

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Every point on the Utility possibilities frontier is Pareto efficient



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Efficiency and equity

- In the above diagram the distribution of utility is very unequal.
- If society is interested in a more equal distribution of utility can this be achieved through the free markets mechanism?
- The answer is given by the second fundamental theorem of welfare economics

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The Second Fundamental Theorem of Welfare Economics

- Second welfare theorem says that a new Pareto-optimal outcome can be achieved given existing resources, without government intervention.
- Any point on the UPF can be achieved through the functioning of decentralized markets, by an appropriate initial distribution of resources.

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Review Questions

- What will happen in our two goods, two-person world if **prices** do not reflect true marginal benefits and all increment costs to society are not included in **marginal costs** ?
- The market will still generate an equilibrium but it will not be Pareto optimal.
- True marginal benefits will not equal marginal costs or vice versa.
- When the market or price system gets the wrong signals we say that there has been a **market failure**.

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Market failures

- Imperfect competition
- Public goods
- Externalities
- Incomplete markets
- Imperfect information
- Unemployment, inflation and other macroeconomic disturbances

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Theory of second best

- Basic question:
- What happens to Pareto optimality when one efficiency condition is violated? Should we continue sticking to the rest of the efficiency conditions?
- Generally, the answer is No.
- Consider an economy with three goods, X_1, X_2, X_3 . X_1 is controlled by the government, in X_2 there is a distortion (i.e. $P \neq MC$), and X_3 is a composite good that includes all other goods, with $price=MC$.

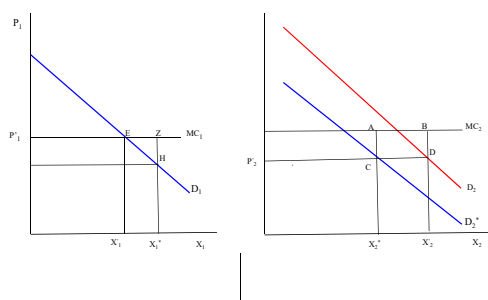
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Theory of second best

- Suppose an economy with 3 goods : Good X_1 is produced by a state company, good X_2 in which there is a distortion and the price is not equal to marginal cost, and a good X_3 , a composite good, which includes all other goods with a price equal to marginal cost.
- Good X_3 is the numeraire, which implies that its price is equal to 1.

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Theory of second best



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Theory of second best

Good X_2 could be the urban transportation system buses, and X_1 is metro. The price of X_2 is P'_2 and is lower than the marginal cost because the government thinks that this lower price encourages people to take the buses instead of their cars, and thus reduces congestion and pollution. So, the buses will be used above their optimal level.

This is depicted in the above diagram where we assume constant marginal costs.

The demand curves D_1 and D_2 reflect the demand that would exist when the price of the metro was $P'_1=MC_1$ for X_1 and $P'_2 < MC_2$ for X_2 and the quantities demanded would be X'_1 and X'_2 .

Suppose that the distortion between P_2 and MC_2 is fixed. We also assume that all other prices do not change and incomes remain unchanged.

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Theory of second best

- As we said earlier in good 2 there is overemployment of resources when in sector 1 price is equal to marginal cost. It is therefore, possible to improve social welfare by moving resources from sector 2 to other sectors.
- In our example X_1 and X_2 are substitutes and a reduction in the price of X_1 to $P'_1 < MC_1$, will shift curve D_2 to the left, to D'_2 . With the distortion between P_2 and MC_2 constant and constant marginal costs, P_2 does not change and the demand for X_2 is reduced from X'_2 to X''_2 . Also in sector 1 the quantity demanded increases from X'_1 to X''_1 .
- From these changes there is a change in welfare that can be measured as follows:

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Theory of second best

- In sector 1 the increase in the cost of resources because of the reduction in the price is $X'_1EZX''_1$, while the increase in welfare, i.e. of the consumer surplus is $X''_1EHX''_1$.
- Hence, we have a reduction in welfare in sector 1 equal to triangle EZH.
- In sector 2 we have the following changes. With the fall in demand the cost is reduced by the area $X'_2ABX''_2$. Also the total benefit is reduced by the area $X'_2CDX''_2$. Thus, the net benefit from sector 2 is the area ABDC. If ABDC larger than EZH, the reduction in price leads to an increase in social welfare. The second best price is the one that maximizes the difference between ABDC and EZH.

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Theory of second best A mathematical treatment

- The social welfare function is

$$U=U(X_1, X_2, X_3)$$

- Differentiation yields

$$dU = \frac{\partial U_1}{\partial X_1} dX_1 + \frac{\partial U_2}{\partial X_2} dX_2 + \frac{\partial U_3}{\partial X_3} dX_3$$

$$dU = MU_1 dX_1 + MU_2 dX_2 + MU_3 dX_3$$

- X_3 is the numeraire, and its price is set equal to one

$$q_3 = 1$$

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Theory of second best A mathematical treatment

- With q being the consumer price of the good, we have

$$\frac{MU_1}{MU_2} = \frac{q_1}{q_2}$$

- Dividing by we U_3 get

$$dW = \frac{dU}{U_3} = q_1 dX_1 + q_2 dX_2 + q_3 dX_3$$

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Theory of second best A mathematical treatment

- Suppose now that in there is a fixed per unit distortion of d_2 on good X_2 , and as a result

$$q_2 = p_2 + d_2$$

- Where p_2 is the producer price without distortion and is equal to marginal cost MC_2

- Since the government controls X_1 , it can give a subsidy or impose a tax t_1 , so that

$$q_1 = p_1 + t_1$$

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Theory of second best A mathematical treatment

- With $q_3 = p_3$

- We have

$$dW = \sum_1^3 p_i dX_i + d_2 dX_2 + t_1 dX_1$$

- On the production side we have the transformation function

$$F(X_1, X_2, X_3) = 0$$

- Total differentiation yields

$$\frac{\partial F_1}{\partial X_1} dX_1 + \frac{\partial F_2}{\partial X_2} dX_2 + \frac{\partial F_3}{\partial X_3} dX_3 = 0$$

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Theory of second best A mathematical treatment

- With $\partial F/\partial X = p =$ marginal cost, we get

$$p_1 dX_1 + p_2 dX_2 + p_3 dX_3 = 0$$

- and

$$dW = t_1 dX_1 + d_2 dX_2$$

- Since we have assumed that is d_2 fixed, there will be a change in welfare, if government changes t_1 .

$$dW = [t_1 (\frac{\partial X_1}{\partial t_1}) + d_2 (\frac{\partial X_2}{\partial t_1})] dt_1$$

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Theory of second best A mathematical treatment

- Maximization of welfare requires that $dW/dt_1 = 0$. Thus

$$t_1^* = -d_2 \frac{\frac{\partial X_2}{\partial t_1}}{\frac{\partial X_1}{\partial t_1}} \quad dp = 0 \text{ και } dq_1 = dt_1$$

- In the case of constant marginal costs,

$$t_1^* = -d_2 \frac{\frac{\partial X_2}{\partial q_1}}{\frac{\partial X_1}{\partial q_1}}$$

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Theory of second best A mathematical treatment

- With $d_2 > 0$, which implies that $q_2 > p_2$ in the distorted sector, then $t^* > 0$, if $(\partial X_2 / \partial q_1) < 0$, that is X_1 and X_2 are substitutes.
- On the contrary $t^* < 0$ when $(\partial X_2 / \partial q_1) > 0$, that is X_1 and X_2 are complements.
- The above results change when $d_2 < 0$.
- The preceding analysis can be generalised for N goods.

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Theory of second best A mathematical treatment

- When in the distorted sector $P < MC$, then the theory of second best suggests that the price in the controlled sector is higher than the MC, if the goods are complementary, and smaller than MC if the goods are substitutes.
- When in the distorted sector $P > MC$, then the theory of second best suggests that the price in the controlled sector is smaller than the MC if the goods are complementary, and greater than MC if the goods are substitutes.
- If the two goods are not related with each other then the price must be equal to MC.

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