# 8.1 Introduction

An externality is a link among economic agents that lies outside the price system of the economy. Everyday examples include the pollution from a factory that harms a local fishery and the envy that is felt when a neighbor proudly displays a new car. Such externalities are not controlled directly by the choices of those affected—the fishery cannot choose to buy less pollution nor can you choose to buy your neighbor a worse car. This prevents the efficiency theorems described in chapter 2 from applying. Indeed the demonstration of market efficiency was based on the following two presumptions:

- The welfare of each consumer depended solely on her own consumption decision.
- The production of each firm depended only on its own input and output choices.

In reality these presumptions may not be met. A consumer or a firm may be directly affected by the actions of other agents in the economy; that is, there may be external effects from the actions of other consumers or firms. In the presence of such externalities the outcome of a competitive market is unlikely to be Pareto-efficient because agents will not take account of the external effects of their (consumption/production) decisions. Typically the economy will generate too great a quantity of "bad" externalities and too small a quantity of "good" externalities.

The control of externalities is an issue of increasing practical importance. Global warming and the destruction of the ozone layer are two of the most significant examples, but there are numerous others, from local to global environmental issues. Some of these externalities may not appear immediately to be economic problems, but economic analysis can expose why they occur and investigate the effectiveness of alternative policies. Economic analysis can generate surprising conclusions and challenge standard policy prescriptions. In particular, it shows how government intervention that induces agents to internalize the external effects of their decisions can achieve a Pareto improvement.

The starting point for the chapter is to provide a working definition of an externality. Using this, it is shown why market failure arises and the nature of the resulting inefficiency. The design of the optimal set of corrective, or *Pigouvian*, taxes is then addressed and related to missing markets for externalities. The use of taxes is contrasted with direct control through tradable licenses. Internalization as a solution to externalities is

considered. Finally these methods of solving the externality problem are set against the claim of the Coase theorem that efficiency will be attained by trade even when there are externalities.

## 8.2 Externalities Defined

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An externality has already been described as an effect on one agent caused by another. This section provides a formal statement of this description, which is then used to classify the various forms of externalities. The way of representing these forms of externalities in economic models is introduced.

There have been several attempts at defining externalities and of providing classifications of various types of externalities. From among these the following definition is the most commonly adopted. Its advantages are that it places the emphasis on recognizing externalities through their effects and it leads to a natural system of classification.

**Definition 8.1** (Externality) An externality is present whenever some economic agent's welfare (utility or profit) is "directly" affected by the action of another agent (consumer or producer) in the economy.

By "directly" we exclude any effects that are mediated by prices. That is, an externality is present if a fishery's productivity is affected by the river pollution of an upstream oil refinery but not if the fishery's profitability is affected by the price of oil (which may depend on the oil refinery's output of oil). The latter type of effect (often called a *pecuniary externality*) is present in any competitive market but creates no inefficiency (since price mediation through competitive markets leads to a Pareto-efficient outcome). We will present later an illustration of a pecuniary externality.

This definition of an externality implicitly distinguishes between two broad categories. A *production externality* occurs when the effect of the externality is on a profit relationship and a *consumption externality* whenever a utility level is affected. Clearly, an externality can be simultaneously both a consumption and a production externality. For example, pollution from a factory may affect the profit of a commercial fishery and the utility of leisure anglers.

Using this definition of an externality, it is possible to move on to how they can be incorporated into the analysis of behavior. Denote, as in chapter 2, the consumption levels of the households by  $x = \{x^1, \ldots, x^H\}$  and the production plans of the firms by  $y = \{y^1, \ldots, y^m\}$ . It is assumed that consumption externalities enter the utility

functions of the households and that production externalities enter the production sets of the firms. At the most general level, this assumption implies that the utility functions take the form

$$U^{h} = U^{h}(x, y), \qquad h = 1, \dots, H,$$
(8.1)

and the production sets are described by

$$Y^{j} = Y^{j}(x, y), \qquad j = 1, \dots, m.$$
 (8.2)

In this formulation the utility functions and the production sets are potentially dependent on the entire arrays of consumption and production levels. The expressions in (8.1) and (8.2) represent the general form of the externality problem, and in some of the discussion below a number of further restrictions will be employed.

It is immediately apparent from (8.1) and (8.2) that the actions of the agents in the economy will no longer be independent or determined solely by prices. The linkages via the externality result in the optimal choice of each agent being dependent on the actions of others. Viewed in this light, it becomes apparent why competition will generally not achieve efficiency in an economy with externalities.

## 8.3 Market Inefficiency

It has been accepted throughout the discussion above that the presence of externalities will result in the competitive equilibrium failing to be Pareto-efficient. The immediate implication of this fact is that incorrect quantities of goods, and hence externalities, will be produced. It is also clear that a non–Pareto-efficient outcome will never maximize welfare. This provides scope for economic policy to improve the outcome. The purpose of this section is to demonstrate how inefficiency can arise in a competitive economy. The results are developed in the context of a simple two-consumer model, since this is sufficient for the purpose and also makes the relevant points as clear as possible.

Consider a two-consumer, two-good economy where the consumers have utility functions

$$U^{1} = x^{1} + u_{1}(z^{1}) + v_{1}(z^{2})$$
(8.3)

and

$$U^{2} = x^{2} + u_{2}(z^{2}) + v_{2}(z^{1}).$$
(8.4)

The externality effect in (8.3) and (8.4) is generated by consumption of good z by the consumers. The externality will be *positive* if  $v_h(\cdot)$  is increasing in the consumption level of the other consumer and *negative* if it is decreasing.

To complete the description of the economy, it is assumed that the supply of good x comes from an endowment  $\omega_h$  to consumer h, whereas good z is produced from good x by a competitive industry that uses one unit of good x to produce one unit of good z. Normalizing the price of good x at 1, the structure of production ensures that the equilibrium price of good z must also be 1. Given this, all that needs to be determined for this economy is the division of the initial endowment into quantities of the two goods.

Incorporating this assumption into the maximization decision of the consumers, the competitive equilibrium of the economy is described by the equations

$$u'_h(z^h) = 1, \qquad h = 1, 2,$$
(8.5)

$$x^{h} + z^{h} = \omega^{h}, \qquad h = 1, 2,$$
(8.6)

and

$$x^{1} + z^{1} + x^{2} + z^{2} = \omega^{1} + \omega^{2}.$$
(8.7)

It is equation (8.5) that is of primary importance at this point. For consumer *h* these state that the private marginal benefit from each good, determined by the marginal utility, is equated to the private marginal cost. The external effect does not appear directly in the determination of the equilibrium. The question we now address is whether this competitive market equilibrium is efficient.

The Pareto-efficient allocations are found by maximizing the total utility of consumers 1 and 2, subject to the production possibilities. The equations that result from this will then be contrasted to (8.5). In detail, a Pareto-efficient allocation solves

$$\max_{\{x^h, z^h\}} U^1 + U^2 = \left[x^1 + u_1(z^1) + v_1(z^2)\right] + \left[x^2 + u_2(z^2) + v_2(z^1)\right],\tag{8.8}$$

subject to

$$\omega^{1} + \omega^{2} - x^{1} - z^{1} - x^{2} - z^{2} \ge 0.$$
(8.9)

The solution is characterized by the conditions

$$u_1'(z^1) + v_2'(z^1) = 1$$
(8.10)

and

$$u_2'(z^2) + v_1'(z^2) = 1.$$
(8.11)

In (8.10) and (8.11) the externality effect can be seen to affect the optimal allocation between the two goods via the derivatives of utility with respect to the externality. If the externality is positive, then  $v'_h > 0$  and the externality effect will raise the value of the left-hand terms. It will decrease their value if there is a negative externality, so  $v'_h < 0$ . It can then be concluded that at the optimum with a positive externality the marginal utilities of both consumers are below their value in the market outcome. The converse is true with a negative externality. The externality leads to a divergence between the private valuations of consumption given by (8.5) and the corresponding social valuations in (8.10) and (8.11). This observation has the implication that the market outcome is not Pareto-efficient.

In general, it can also be concluded that if the externality is positive then more of good z will be consumed at the optimum than under the market outcome. The converse holds for a negative externality. This situation is illustrated in figure 8.1. The market outcome is represented by equality between the private marginal benefit of the good (*PMB*) and its marginal cost (*MC*). The social marginal benefit (*SMB*) of the good is the sum of the private marginal benefit,  $u'_h(z^h)$ , and the marginal external effect,  $v'_{\tilde{h}}(z^h)$ . When  $v'_{\tilde{h}}(z^h)$  is positive, *SMB* is above *PMB*. The converse holds when  $v'_{\tilde{h}}(z^h)$  is negative. The Pareto-efficient outcome equates the social marginal benefit to marginal cost. The market failure is characterized by too much consumption of a good



**Figure 8.1** Deviation of private from social benefits

causing a negative externality and too little consumption of a good generating a positive externality.

### 8.4 Externality Examples

The previous section has discussed externalities at a somewhat abstract level. We now consider more concrete examples of externalities. Some of the examples are very simple because of the binary nature of the choice and the assumption of identical individuals. This modeling choice was widely used by Schelling to achieve an extremely simple exposition that brings out the line of the argument very clearly. In addition it will illustrate the range of situations that fall under the general heading of externalities.

## 8.4.1 River Pollution

This example, from Louis Gevers, is one of the simplest examples that can be described using only two agents. Assume that two firms are located along the same river. The upstream firm u pollutes the river, which reduces the production (e.g., the output of fish) of the downstream firm d. Both firms produce the same output, which they sell at a constant unit price of 1 so that total revenue coincides with production.

Labor and water are used as inputs. Water is free, but the equilibrium wage w on the competitive labor market is paid for each unit of labor. The production technologies of the firms are given by  $F^u(L^u)$  and  $F^d(L^d, L^u)$ , with  $\frac{\partial F^d}{\partial L^u} < 0$  to reflect that the pollution reduces downstream output. Decreasing returns to scale are assumed with respect to own labor input. Each firm acts independently and seeks to maximize its own profit  $\pi^i = F^i(\cdot) - wL^i$ , taking prices as given.

The equilibrium is illustrated in figure 8.2. The total stock of labor is allocated between the two firms. The labor input of the upstream firm is measured from the left, that of the downstream from the right. Each point on the horizontal axis represents a different allocation between the firms. The upstream firm's profit maximization process is represented in the upper part of the diagram and the downstream firm's in the lower part. As the input of the upstream firm increases, the production function of the downstream firm moves progressively toward the horizontal axis. Given the profit-maximizing input level of the upstream firm, denoted  $L^{u*}$ , the downstream firm can do no better than choose  $L^{d*}$ . At these choices the firms earn profits  $\pi^{u}$  and  $\pi^{d}$  respectively. This is the competitive equilibrium. We now show that this is inefficient and that reallocating labor between the firms can increase total profit and reduce pollution.





Consider starting at the competitive equilibrium and make a small reduction in the labor input to the upstream firm. Since the choice was optimal for the upstream firm, the change has no effect on profit for the upstream firm (recall that  $\frac{\partial \pi^u}{\partial L^u} = 0$ ). However, it leads to an outward shift of the downstream firm's production function. This raises its profits. Hence the change raises aggregate profit. This demonstrates that the competitive equilibrium is not efficient and that the externality results in the upstream firm using too much labor and the downstream too little. Shifting labor to the downstream firm raises total production and reduces pollution.

# 8.4.2 Traffic Jams

The next example considers the externalities imposed by drivers on each other. Let there be N commuters who have the choice of commuting by train or by car. Commuting by train always takes 40 minutes regardless of the number of travelers. The commuting



Figure 8.3 Choice of commuting mode

time by car increases as the number of car users increases. This congestion effect, which raises the commuting time, is the externality for travelers. Individuals must each make decisions to minimize their own transportation time.

The equilibrium in the choice of commuting mode is depicted in figure 8.3. The number of car users will adjust until the travel time by car is exactly equal to the travel time by train. For the travel time depicted in the figure, the equilibrium occurs when 40 percent of commuters travel by car. The optimum occurs when the aggregate time saving is maximized. This occurs when only 20 percent of commuters use a car.

The externality in this situation is that the car drivers take into account only their own travel time but not the fact that they will increase the travel time for all other drivers. As a consequence too many commuters choose to drive.

# 8.4.3 Pecuniary Externality

Consider a set of students each of whom must decide whether to be an economist or a lawyer. Being an economist is great when there are few economists, and not so great when the labor market becomes crowded with economists (due to price competition). If the number of economists grows high enough, they will eventually earn less than their lawyer counterparts. Suppose that each person chooses the profession with the best earnings prospects. The externality (a pecuniary one!) comes from the fact that when one more person decides to become an economist, he lowers all other economists' incomes (through competition), imposing a cost on the existing economists. When making his decision, he ignores this external effect imposed on others. The question is whether the invisible hand will lead to the correct allocation of students across different jobs.

The equilibrium depicted in figure 8.4 determines the allocation of students between jobs. The number of economists will adjust until the earnings of an economist are exactly equal to the earnings of a lawyer. The equilibrium is given by the percentage of economists at point E. To the right of point E, lawyers would earn more and the number of economists would decrease. Alternatively, to the left of point E economists are relatively few in number and will earn more than lawyers, attracting more economists into the profession.

The laissez-faire equilibrium is efficient because the external effect is a change in price. The cost to an economists of a lower income is a benefit to employers. Since employers' benefits equals employees' costs, there is zero net effect. The policy implication is that there is no need for government intervention to regulate the access to professions. It follows that any public policy that aims to limit the access to some profession, like the *numerus clausus*, is not justified. Market forces will correctly allocate the right number of people to each of the different professions.



Percentage of economists

**Figure 8.4** Job choice

#### 8.4.4 The Rat Race Problem

The rat race problem is a contest for relative position as pointed out by George Akerlof. It can help explain why students work too hard when final marking takes the form of a ranking. It can also explain the intense competition for a promotion in the workplace when candidates compete with each other and only the best is promoted. We take the classroom example here. Assume that performance is judged not in *absolute* terms but in *relative* terms so that what matters is not how much is known but how much is known compared to what other students know.

In this situation an advantage over other students can only be gained by working harder than they do. Since this applies to all students, all must work harder. But since performance is judged in relative terms, all the extra effort cancels out. The result of this is an inefficient rat race in which each student works too hard to no ultimate advantage. If all could agree to work less hard, the same grades would be obtained with less work. Such an agreement to work less hard cannot be self-supporting, since each student would then have an incentive to cheat on the agreement and work harder.

A simple variant of the rat race with two possible effort levels is shown in figure 8.5. In this figure,  $c, 0 < c < \frac{1}{2}$ , denotes the cost of effort. For both students high effort is a dominant strategy. In contrast, the Pareto-efficient outcome is low effort. This game is an example of the prisoners' dilemma whereby a Pareto improvement could be made if the players could make a commitment to the low-effort strategy.

Another example of rat race is the use of performance-enhancing drugs by athletes. In the absence of effective drug regulations, many athletes will feel compelled to enhance their performance by using anabolic steroids, and the failure to use steroids might seriously reduce their success in competition. Since the rewards in athletics are

		Low	High
Player 1	Low	1/2	1 – <i>c</i>
		1/2	0
	High	0	1/2 - c
		1 - c	1/2 - c

Player 2

Figure 8.5 Rat race

determined by performance relative to others, anyone that uses such drugs to increase their chance of winning must necessarily reduce the chances of others (an externality effect). The result is that when the stakes are high in the competition, unregulated contests almost always lead to a race for using more and more performance-enhancing drugs. However, when everyone does so, the use of such drugs yields no real benefits for the contestants as a whole: the performance-enhancing actions cancel each other. At the same time the race imposes substantial risks. Anabolic steroids have been shown to cause cancer of the liver and other serious health problems. Given what is at stake, voluntary restraint is unlikely to be an effective solution, and public intervention now requires strict drug testing of all competing athletes.

The rat race problem is present in almost every contest where something important is at stake and rewards are determined by relative position. In an electoral competition race, contestants spend millions on advertising, and governing bodies have now put strict limits on the amount of campaign advertising. Similarly a ban on cigarette advertising has been introduced in many countries. Surprisingly enough, this ban turned out to be beneficial to cigarette companies. The reason is that the ban helped them out of the costly rat race in defensive advertising where a company had to advertise because the others did.

#### 8.4.5 The Tragedy of the Commons

The *tragedy of the commons* arises from the common right of access to a resource. The inefficiency to which it leads results again from the divergence between the individual and social incentives that characterizes all externality problems.

Consider a lake that can be used by fishermen from a village located on its banks. The fishermen do not own boats but instead can rent them for daily use at a cost c. If B boats are hired on a particular day, the number of fish caught by each boat will be F(B), which is decreasing in B. A fisherman will hire a boat to fish if they can make a positive profit. Let w be the wage if they choose to undertake paid employment rather than fish, and let p = 1 be the price of fish so that total revenue coincide with fish catch F(B). Then the number of boats that fish will be such as to ensure that profit from fishing activity is equal to the opportunity cost of fishing, which is the forgone wage w from the alternative job (if profit were greater, more boats would be hired and the converse if it were smaller). The equilibrium number of boats,  $B^*$ , then satisfies

$$\pi = F(B^*) - c = w. \tag{8.12}$$

The optimal number of boats for the community,  $B^{\circ}$ , must be that which maximizes the total profit for the village, net of the opportunity cost from fishing. Hence  $B^{\circ}$  satisfies

$$\max_{\{B\}} B[F(B) - c - w].$$
(8.13)

This gives the necessary condition

$$F\left(B^{\circ}\right) - c - w + BF'\left(B^{\circ}\right) = 0.$$
(8.14)

Since an increase in the number of boats reduces the quantity of fish caught by each,  $F'(B^{\circ}) < 0$ . Therefore contrasting (8.12) and (8.14) shows that  $B^{\circ} < B^*$ , so the equilibrium number of boats is higher than the optimal number. This situation is illustrated in figure 8.6.

The externality at work in this example is that each fisherman is concerned only with their own profit. When deciding whether to hire a boat, they do not take account of the fact that they will reduce the quantity of fish caught by every other fisherman. This negative externality ensures that in equilibrium too many boats are operating on the lake. Public intervention can take two forms. There is the price-based solution consisting of a tax per boat so as to internalize the external effect of sending a boat on the lake. As indicated in the figure, a correctly chosen tax will reduce the number of boats so as to restore the optimal outcome. Alternatively, the quantity-based solution consists of setting a quota of fishing equal to the optimal outcome.



**Figure 8.6** Tragedy of the commons

#### 8.4.6 Bandwagon Effect

The bandwagon effect studies the question of how standards are adopted and, in particular, how it is possible for the wrong standard to be adopted. The standard application of this is the choice of arrangement for the keys on a keyboard.

The current standard, Qwerty, was designed in 1873 by Christopher Scholes in order to deliberately slow down the typist by maximizing the distance between the most used letters. The motivation for this was the reduction of key-jamming problems (remember this would be for mechanical typewriters in which metal keys would have to strike the ink ribbon). By 1904 the Qwerty keyboard was mass produced and became the accepted standard. The key-jamming problem is now irrelevant, and a simplified alternative keyboard (Dvorak's keyboard) has been devised that reduces typing time by 5 to 10 percent.

Why has this alternative keyboard not been adopted? The answer is that there is a switching cost. All users are reluctant to switch and bear the cost of retraining, and manufacturers see no advantage in introducing the alternative. It has therefore proved impossible to switch to the better technology.

This problem is called a *bandwagon effect* and is due to a *network externality*. The decision of a typist to use the Qwerty keyboard makes it more attractive for manufacturers to produce Qwerty keyboards, and hence for others to learn Qwerty. No individual



**Figure 8.7** Equilibrium keyboard choice

has any incentive to switch to Dvorak. The nature of the equilibrium is displayed in figure 8.7. This shows the intertemporal link between the percentage using Qwerty at time t and the percentage at time t + 1. The natural advantage of Dvorak is captured in the diagram by the fact that the number of Qwerty users will decline over time starting from a position where 50 percent use Qwerty at time t. There are three equilibria. Either all will use Qwerty or Dvorak or else a proportion  $p^*$ ,  $p^* > 50$  percent, will use Qwerty and  $1 - p^*$  Dvorak However, this equilibrium is unstable, and any deviation from it will lead to one of the corner equilibria. The inefficient technology, Qwerty, can dominate in equilibrium if the initial starting point is to the right of  $p^*$ .

# 8.5 Pigouvian Taxation

The description of market inefficiency has shown that its basic source is the divergence between social and private benefits (or between social and private costs). This fact has been reinforced by the examples. A natural means of eliminating such divergence is to employ appropriate taxes or subsidies. By modifying the decision problems of the firms and consumers these can move the economy closer to an efficient position.

To see how a tax can enhance efficiency, consider the case of a negative consumption externality. With a negative externality the private marginal benefit of consumption is always in excess of the social marginal benefit. These benefits are depicted by the *PMB* and *SMB* curves respectively in figure 8.8. In the absence of intervention, the equilibrium occurs where the *PMB* intersects the private marginal cost (*PMC*). This gives a level of consumption  $x^m$ . The efficient consumption level equates the *PMC* with the *SMB*; this is at point  $x^o$ . As already noted, with a negative externality the market outcome involves more consumption of the good than is efficient. The market outcome can be improved by placing a tax on consumption. What it is necessary to do is to raise the *PMC* so that it intersects the *SMB* vertically above  $x^o$ . This is what happens for the curve *PMC'*, which has been raised above *PMC* by a tax of value *t*. This process, often termed *Pigouvian taxation*, allows the market to attain efficiency for the situation shown in figure 8.8.

Based on arguments like that exhibited above, Pigouvian taxation has been proposed as a simple solution to the externality problem. The logic is that the consumer or firm causing the externality should pay a tax equal to the marginal damage the externality causes (or a subsidy if there is a marginal benefit). Doing so makes them take account of the damage (or benefit) when deciding how much to produce or consume. In many ways this is a compellingly simple conclusion.



**Figure 8.8** Pigouvian taxation

The previous discussion is informative but leaves a number of issues to be resolved. Foremost among these is the fact that the figure implicitly assumes there is a single agent generating the externality whose marginal benefit and marginal cost are exhibited and that there is a single externality. The single tax works in this case, but will it still do so with additional externalities and agents? This is an important question to be answered if Pigouvian taxation is to be proposed as a serious practical policy.

To address these issues, we use our example from the market failure section again. This example involved two consumers and two goods with the consumption of one of the goods, z, causing an externality. The optimal structure of Pigouvian taxes is determined by characterizing the social optimum and inferring from that what the taxes must be. Recall from (8.10) and (8.11) that the social optimum is characterized by the conditions

$$u_1'(z^1) + v_2'(z^1) = 1$$
(8.15)

and

$$u_2'(z^2) + v_1'(z^2) = 1. ag{8.16}$$

It is from contrasting these conditions to those for individual choice that the optimal taxes can be derived.

Utility maximization by consumer 1 will equate their private marginal benefit,  $u'_1(z^1)$ , to the consumer price  $q_1$ . Given that the producer price is equal to 1 in this example, (8.15) shows that efficiency will be achieved if the price,  $q_1$ , facing consumer 1 satisfies

$$q_1 = 1 - v_2'(z^1). \tag{8.17}$$

Similarly from (8.16) efficiency will be achieved if the price facing consumer 2 satisfies

$$q_2 = 1 - v_1'(z^2). \tag{8.18}$$

These identities reveal that the taxes that ensure the correct difference between consumer and producer prices are given by

$$t_1 = -v_2'(z^1) \tag{8.19}$$

and

$$t_2 = -v_1'(z^2). ag{8.20}$$

Therefore the tax on consumer 1 is the negative of the externality effect their consumption of good z inflicts on consumer 2. Hence, if the good causes a negative externality  $(v'_2(z^1) < 0)$ , the tax is positive. The converse holds if it causes a positive externality. The same construction and reasoning can be applied to the tax facing consumer 2,  $t_2$ , to show that this is the negative of the externality effect caused by the consumption of good z by consumer 2. The argument is now completed by noting that these externality effects will generally be different, and so the two taxes will generally not be equal. Another way of saying this is that efficiency can only be achieved if the consumers face personalized prices that fully capture the externalities that they generate.

So what does this say for Pigouvian taxation? Put simply, the earlier conclusion that a single tax rate could achieve efficiency was misleading. In fact the general outcome is that there must be a different tax rate for each externality-generating good for each consumer. Achieving efficiency needs taxes to be differentiated across consumers and goods. Naturally this finding immediately shows the practical difficulties involved in implementing Pigouvian taxation. The same arguments concerning information that were placed against the Lindahl equilibrium for public good provision with personalized pricing are all relevant again here. In conclusion, Pigouvian taxation can achieve efficiency but needs an unachievable degree of differentiation.

If the required degree of differentiation is not available, for instance, information limitations require that all consumers must pay the same tax rate, then efficiency will not be achieved. In such cases the chosen taxes will have to achieve a compromise. They cannot entirely correct for the externality but can go some way toward doing so. Since the taxes do not completely offset the externality, there is also a role for intervening in the market for goods related to that causing the externality. For instance, pollution from car use may be lessened by subsidizing alternative mode of transports. These observations are meant to indicate that once the move is made from full efficiency, many new factors become relevant, and there is no clean and general answer as to how taxes should be set.

A final comment is that the effect of the tax or subsidy is to put a price (respectively positive or negative) on the externality. This leads to the conclusion, which will be discussed in detail below, that if there are competitive markets for the externalities, efficiency will be achieved. In other words, efficiency does not require intervention but only the creation of the necessary markets.

# 8.6 Licenses

The reason why Pigouvian taxation can raise welfare is that the unregulated market will produce incorrect quantities of externalities. The taxes alter the cost of generating an externality and, if correctly set, will ensure that the optimal quantity of externality is produced. An apparently simpler alternative is to control externalities directly by the use of licenses. This can be done by legislating that externalities can only be generated up to the quantity permitted by licenses held. The optimal quantity of externality can then be calculated and licenses totaling this quantity distributed. Permitting these licenses to be traded will ensure that they are eventually used by those who obtain the greatest benefit.

Administratively, the use of licenses has much to recommend it. As was argued in the previous section, the calculation of optimal Pigouvian taxes requires considerable information. The tax rates will also need to be continually changed as the economic environment evolves. The use of licenses only requires information on the aggregate quantity of externality that is optimal. Licenses to this value are released and trade is permitted. Despite these apparently compelling arguments in favor of licenses, when the properties of licenses and taxes are considered in detail, the advantage of the former is not quite so clear.

The fundamental issue involved in choosing between taxes and licenses revolves around information. There are two sides to this. The first is what must be known to calculate the taxes or determine the number of licenses. The second is what is known when decisions have to be taken. For example, does the government know costs and benefits for sure when it sets taxes or issues licenses?

Taking the first of these, although licenses may appear to have an informational advantage this is not really the case. Consider what must be known to calculate the Pigouvian taxes. The construction of section 8.5 showed that taxation required the knowledge of the preferences of consumers and, if the model had included production, the production technologies of firms. Such extensive information is necessary to achieve the personalization of the taxes. But what of licenses? The essential feature of licenses is that they must total to the optimal level of externality. To determine the optimal level requires precisely, the same information as is necessary for the tax rates. Consequently taxes and licenses are equivalent in their informational demands.

Now consider the issue of the information that is known when decisions must be made. When all costs and benefits are known with certainty by both the government and individual agents, licenses and taxation are equivalent in their effects. This result is easily seen by reconsidering figure 8.8. The optimal level of externality is  $x^o$ , which was shown to be achievable with tax t. The same outcome can also be achieved by issuing  $x^o$  licenses. This simple and direct argument shows there is equivalence with certainty.

In practice, it is more likely that the government must take decisions before the actual costs and benefits of an externality are known for sure. Such uncertainty brings with it the question of timing: Who chooses what and when? The natural sequence of events is the following. The government must make its policy decision (the quantity of licenses or the tax rate) before costs and benefits are known. In contrast, the economic agents can act after the costs and benefits are known. For example, in the case of pollution by a firm, the government may not know the cost of reducing pollution for sure when it sets the tax rate but the firm makes its abatement decision with full knowledge of the cost.

The effect of this difference in timing is to break the equivalence between the two policies. This can be seen by considering figure 8.9, which illustrates the pollution abatement problem for an uncertain level of cost. In this case the level of private marginal cost takes one of two values,  $PMC_L$  and  $PMC_H$ , with equal probability. Benefits are known for sure. When the government chooses its policy, it is not known whether private marginal cost is high or low, so it must act on the expected value,  $PMC_E$ . This leads to pollution abatement  $z^*$  being required (which can be supported by licenses equal in quantity to present pollution less  $z^*$ ) or a tax rate  $t^*$ .

Under the license scheme, the level of pollution abatement will be  $z^*$  for sure there is no uncertainty about the outcome. With the tax, the level of abatement will





depend on the realized level of cost since the firm chooses abatement after this is known. Therefore, if the cost turns out to be  $PMC_L$ , so that the cost of abatement is low, the firm will be willing to undertake abatement up to level  $z_L$ . If the realized cost is  $PMC_H$ , so abatement cost is high, the firm will choose to undertake the reduced level of abatement  $z_H$ . This is shown in figure 8.9. Two observations emerge from this. First, the claim that licenses and taxation will not be equivalent when there is uncertainty is confirmed. Second, when cost is realized to be low, taxation leads to abatement in excess of  $z^*$ . The converse holds when cost is high.

The analysis of figure 8.9 may be taken as suggesting that licenses are better, since they do not lead to the variation in abatement that is inherent in taxation. However, it should also be realized that the choices made by the firm in the tax case are responding to the actual cost of abatement, so there is some justification for what the firm is doing. In general, there is no simple answer to the question of which of the two policies is better.

### 8.7 Internalization

Consider the example of a beekeeper located next door to an orchard. The bees pollinate the trees and the trees provide food for the bees, so a positive production externality runs in both directions between the two producers. According to the theory developed above, the producers acting independently will not take account of this externality. This leads to too few bees being kept and too few trees being planted. The externality problem could be resolved by using taxation or insisting that both producers raise their quantities. Although both these would work, there is another simpler solution. Imagine the two producers merging and forming a single firm. If they were to do so, profit maximization for the combined enterprise would naturally take into account the externality. By so doing, the inefficiency is eliminated. The method of controlling externalities by forming single units out of the parties affected is called *internalization*, and it ensures that private and social costs become the same. It works for both production and consumption externalities whether they are positive or negative.

Internalization seems a simple solution, but it is not without its difficulties. To highlight the first of these, consider an industry in which the productive activity of each firm causes an externality for the other firms in the industry. In this situation the internalization argument would suggest that the firms become a single monopolist. If this were to occur, welfare loss would then arise due to the ability of the single firm to exploit its monopoly position, and this may actually be greater than the initial loss due to the externality. Although this is obviously an extreme example, the internalization argument always implies the construction of larger economic units and a consequent increase in market power. The welfare loss due to market power then has to be offset against the gain from eliminating the effect of the externality.

The second difficulty is that the economic agents involved may simply not wish to be amalgamated into a single unit. This objection is particularly true when applied to consumption externalities. That is, if a household generates an externality for their neighbor, it is not clear that they would wish to form a single household unit, particularly if the externality is a negative one.

In summary, internalization will eliminate the consequences of an externality in a very direct manner by ensuring that private and social costs are equated. However, it is unlikely to be a practical solution when many distinct economic agents contribute separately to the total externality, and it has the disadvantage of leading to increased market power.

### 8.8 The Coase Theorem

After identifying externalities as a source of market failure, this chapter has taken the standard approach of discussing policy remedies. In contrast to this, there has developed a line of reasoning that questions whether such intervention is necessary. The focal point for this is the Coase theorem, which suggests that economic agents may resolve externality problems themselves without the need for government intervention. This

conclusion runs against the standard assessment of the consequences of externalities and explains why the Coase theorem has been of considerable interest.

The Coase theorem asserts that if the market is allowed to function freely, then it will achieve an efficient allocation of resources. This claim can be stated formally as follows.

**Theorem 8.2** (Coase theorem) In a competitive economy with complete information and zero transaction costs, the allocation of resources will be efficient and invariant with respect to legal rules of entitlement.

The legal rules of entitlement, or property rights, are of central importance to the Coase theorem. Property rights are the rules that determine ownership within the economy. For example, property rights may state that all agents are entitled to unpolluted air or the right to enjoy silence (they may also state the opposite). Property rights also determine the direction in which compensation payments will be made if a property right is violated.

The implication of the Coase theorem is that there is no need for policy intervention with regard to externalities except to ensure that property rights are clearly defined. When they are, the theorem presumes that to eliminate any market failure, those affected by an externality will find it in their interest to reach private agreements with those causing it. These agreements will involve the payment of compensation to the agent whose property right is being violated. The level of compensation will ensure that the right price emerges for the externality and a Pareto-efficient outcome will be achieved. These compensation payments can be interpreted in the same way as the personalized prices discussed in section 8.5.

As well as claiming that the outcome will be efficient, the Coase theorem asserts that the equilibrium will be invariant to the how property rights are assigned. This is surprising since a natural expectation is, in the example, for the level of pollution under a polluter-pays system (i.e., giving property rights to pollutees) to be less than that under a pollutee-pays (i.e., giving property rights to the polluter). To show how the invariance argument works, consider the example of a factory that is polluting the atmosphere of a neighboring house. When the firm has the right to pollute, the householder can only reduce the pollution by paying the firm a sufficient amount of compensation to make it worthwhile to stop production or to find an alternative means of production. Let the amount of compensation the firm requires be C. Then the cost to the householder of the pollution, G, will either be greater than C, in which case the householder will be willing to compensate the firm and the externality will cease, or it will be less than C and the

externality will be left to continue. Now consider the outcome with the polluter-pays principle. The cost to the firm for stopping the externality now becomes C and the compensation required by the household is G. If C is greater than G, the firm will be willing to compensate the household and continue producing the externality; if it is less than G, it stops the externality. Considering the two cases, it can be seen the outcome is determined only by the value of G relative to C and not by the assignment of property rights, which is essentially the content of the Coase theorem.

There is a further issue before invariance can be confirmed. The change in property rights between the two cases will cause differences in the final distribution of income due to the direction of compensation payments. Invariance can only hold if this redistribution of income does not cause a change in the level of demand. This requires there to be no income effects, or to put it another way, the marginal unit of income must be spent in the same way by both parties.

When the practical relevance of the Coase theorem is considered, a number of issues arise. The first lies with the assignment of property rights in the market. With commodities defined in the usual sense, it is clear who is the purchaser and who is the supplier, and therefore the direction in which payment should be transferred. This is not the case with externalities. For example, with air pollution it may not be clear that the polluter should pay, with the implicit recognition of the right to clean air, or whether there is a right to pollute, with clean air something that should have to be paid for. This leaves the direction in which payment should go unclear. Without clearly specified property rights, the bargaining envisaged in the Coase theorem does not have a firm foundation: neither party would willingly accept that they were the party that should pay.

If the exchange of commodities would lead to mutually beneficial gains for two parties, the commodities will be exchanged unless the cost of doing so outweighs the benefits. Such transactions costs may arise from the need for the parties to travel to a point of exchange or from the legal costs involved in formalizing the transactions. They may also arise due to the search required to find a trading partner. Whenever they arise, transactions costs represent a hindrance to trade and, if sufficiently great, will lead to no trade at all taking place. The latter results in the economy having a missing market.

The existence of transactions costs is often seen as the most significant reason for the nonexistence of markets in externalities. To see how they can arise, consider the problem of pollution caused by car emissions. If the reasoning of the Coase theorem is applied literally, then any driver of a car must purchase pollution rights from all of the agents that are affected by the car emissions each time, and every time, that the car is used. Obviously this would take an absurd amount of organization, and since considerable time and resources would be used in the process, transactions costs would be significant. In many cases it seems likely that the welfare loss due to the waste of resources in organizing the market would outweigh any gains from having the market.

When external effects are traded, there will generally only be one agent on each side of the market. This thinness of the market undermines the assumption of competitive behavior needed to support the efficiency hypothesis. In such circumstances the Coase theorem has been interpreted as implying that bargaining between the two agents will take place over compensation for external effects and that this bargaining will lead to an efficient outcome. Such a claim requires substantiation.

Bargaining can be interpreted as taking the form of either a cooperative game between agents or as a noncooperative game. When it is viewed as cooperative, the tradition since Nash has been to adopt a set of axioms that the bargain must satisfy and to derive the outcomes that satisfy these axioms. The requirement of Pareto-efficiency is always adopted as one of the axioms so that the bargained agreement is necessarily efficient. If all bargains over compensation payments were placed in front of an external arbitrator, then the Nash bargaining solution would have some force as descriptive of what such an arbitrator should try and achieve. However, this is not what is envisaged by the Coase theorem, which focuses on the actions of markets free of any regulation. Although appealing as a method for achieving an outcome agreeable to both parties, the fact that Nash bargaining solution is efficient does not demonstrate the correctness of the Coase theorem.

The literature on bargaining in a noncooperative context is best divided between games with complete information and those with incomplete information, since this distinction is of crucial importance for the outcome. One of the central results of noncooperative bargaining with complete information is due to Rubinstein who considers the division of a single object between two players. The game is similar to the fundraising game presented in the public goods chapter. The players take turns to announce a division of the object, and each period an offer and an acceptance or rejection are made. Both players discount the future, so they are impatient to arrive at an agreed division. Rubinstein shows that the game has a unique (subgame perfect) equilibrium with agreement reached in the first period. The outcome is Pareto-efficient.

The important point is the complete information assumed in this representation of bargaining. The importance of information for the nature of outcomes will be extensively analyzed in chapter 10, and complete information is equally important for bargaining. In the simple bargaining problem of Rubinstein the information that must be known are the preferences of the two agents, captured by their rates of time discount. When these discount rates are private information, the attractive properties of the complete information bargain are lost, and there are many potential equilibria whose nature is dependent on the precise specification of the structure of bargaining.

In the context of externalities it seems reasonable to assume that information will be incomplete, since there is no reason why the agents involved in bargaining an agreement over compensation for an external effect should be aware of each other's valuations of the externality. When they are not aware, there is always the incentive to try to exploit a supposedly weak opponent or to pretend to be strong and make excessive demands. This results in the possibility that agreement may not occur even when it is in the interests of both parties to trade.

To see this more clearly, consider the following bargaining situation. There are two agents: a polluter and a pollutee. They bargain over the decision to allow or not the pollution. The pollutee cannot observe the benefit of pollution *B* but knows that it is drawn from a distribution F(B), which is the probability that the benefit is less than or equal to *B*. Similarly the polluter cannot observe the cost of pollution *C* but knows that it is drawn from a distribution G(C). Obviously the benefit is known to the polluter and the cost is known to the pollutee. Let us give the property rights to the pollutee so that he has the right to a pollution-free environment. Pareto-efficiency requires that pollution be allowed whenever  $B \ge C$ . Now the pollutee (with all the bargaining power) can make a take-it-or-leave-it offer to the polluter. What will be the bargaining outcome?

The pollutee will ask for compensation T > 0 (since C > 0) to grant permission to pollute. The polluter will only accept to pay T if his benefit from polluting exceeds the compensation he has to pay, so  $B \ge T$ . Hence the probability that the polluter will accept the offer is equal to 1 - F(T), that is, the probability that  $B \ge T$ . The best deal for the pollutee is to ask for compensation that maximizes her expected payoff defined as the probability that the offer is accepted times the net gain if the offer is accepted. Therefore the pollutee asks for compensation  $T^*$ , which solves

$$\max_{\{T\}} (1 - F(T)) [T - C].$$
(8.21)

Clearly, the optimal value,  $T^*$ , is such that

$$T^* > C. \tag{8.22}$$

But then bargaining can result (with strictly positive probability) in an inefficient outcome. This is the case for all realizations of *C* and *B* such that  $C < B < T^*$ , which implies that the offer is rejected (since the compensation demanded exceeds the benefit) and thus pollution is not allowed, while Pareto-efficiency requires permission to pollute to be granted (since its cost is less than its benefit). The efficiency thesis of the Coase theorem relies on agreements being reached on the compensation required for external effects. The results above suggest that when information is incomplete, bargaining between agents will not lead to an efficient outcome.

#### 8.9 Nonconvexity

One of the basic assumptions that supports economic analysis is that of convexity. Convexity gives indifference curves their standard shape, so consumers always prefer mixtures to extremes. It also ensures that firms have nonincreasing returns so that profit maximization is well defined. Without convexity, many problems arise with the behavior of the decisions of individual firms and consumers, and with the aggregation of these decisions to find an equilibrium for the economy.

Externalities can be a source of nonconvexity. Consider the case of a negative production externality. The panel at the left in figure 8.10 displays a firm whose output is driven to zero by an externality regardless of the level of other inputs. An example would be a fishery where sufficient pollution of the fishing ground by another firm can kill all the fish. In the panel at the right side of the figure, a zero output level is not reached, but output tends to zero as the level of the externality is increased. In both situations the production set of the firm is not convex.

In either case the economy will fail to have an equilibrium if personalized taxes are employed in an attempt to correct the externality. Suppose that the firm were to receive a subsidy for accepting externalities. Its profit-maximizing choice would be to produce



Figure 8.10 Nonconvexity

an output level of zero and to offer to accept an arbitrarily large quantity of externalities. Since its output is zero, the externalities can do it no further harm, so this plan will lead to unlimited profits. If the price for accepting externalities were zero, the same firm would not accept any. The demand for externalities is therefore discontinuous, and an equilibrium need not exist.

There is also a second reason for nonconvexity with externalities. It is often assumed that once all inputs are properly accounted for, all firms will have constant returns to scale, since behavior can always be replicated. That is, if a fixed set of inputs (i.e., a factory and staff) produces output y, doubling all those inputs must produce output 2y, since they can be split into two identical subunits (e.g., two factories and staff) producing an amount y each. Now consider a firm subject to a negative externality, and assume that it has constant returns to all inputs including the externality. From the perspective of society, there are constant returns to scale. Now let the firm double all its inputs, but with the externality held at a constant level. Since the externality is a negative one, it becomes diluted by the increase in other inputs, and output must more than double. The firm therefore faces private increasing returns to scale. With such increasing returns, the firm's profit-maximizing decision may not have a well-defined finite solution and market equilibrium may again fail to exist.

These arguments provide some fairly powerful reasons why an economy with externalities may not share some of the desirable properties of economies without. The behavior that follows from nonconvexity can prevent some of the pricing tools that are designed to attain efficiency from functioning in a satisfactory manner. At worst, nonconvexity can even cause there to be no equilibrium in the economy.

### 8.10 Conclusions

Externalities are an important feature of economic activity. They can arise at a local level between neighbors and at a global level between countries. The existence of externalities can lead to inefficiency if no attempt is made to control their level. The Coase theorem suggests that well-defined property rights will be sufficient to ensure that private agreements can resolve the externality problem. In practice, property rights are not well defined in many cases of externality. Furthermore the thinness of the market and the incomplete information of market participants result in inefficiencies that undermine the Coase theorem.

The simplest policy solution to the externality problem is a system of corrective Pigouvian taxes. If the tax rate is proportional to the marginal damage (or benefit) caused by the externality, then efficiency will result. However, for this argument to apply when there are many consumers and firms requires that the taxes be so differentiated between economic agents that they become equivalent to a system of personalized prices. The optimal system then becomes impractical due to its information limitations. An alternative policy response is the use of marketable licenses that limit the emission of externalities. Licenses have some administrative advantages over taxes and will produce the same outcome when costs and benefits are known with certainty. With uncertainty, licenses and taxes have different effects and combining the two can lead to a superior outcome.

# **Further Reading**

The classic analysis of externalities is in:

Meade, J. E. 1952. External economies and diseconomies in a competitive situation. *Economic Journal* 62: 54–76.

The externality analysis is carried further in a more rigorous and complete treatment in:

Buchanan, J. M. and Stubblebine, C. 1962. Externality. Economica 29: 371-84.

A persuasive argument for the use of corrective taxes is in:

Pigou, A. C. 1918. The Economics of Welfare. London: Macmillan.

The problem of social cost and the bargaining solution with many legal examples is developed in:

Coase, R. H. 1960. The problem of social cost. Journal of Law and Economics 3: 1-44.

An illuminating classification of externalities and nonmarket interdependences is in:

Bator, F. M. 1958. The anatomy of market failure. Quarterly Journal of Economics 72: 351-78.

A comprehensive and detailed treatment of the theory of externalities can be found in:

Lin, S., ed. 1976. Theory and Measurement of Economic Externalities. New York: Academic Press.

The efficient noncooperative bargaining solution with perfect information is in:

Rubinstein, A. 1982. Perfect equilibrium in a bargaining model. *Econometrica* 50: 97–110.

The general theory of bargaining with complete and incomplete information and many applications is in:

Muthoo, A. 1999. Bargaining Theory with Applications. Cambridge: Cambridge University Press.

An extremely simple exposition of the conflict between individual motives and collective efficiency is in:

Schelling, T. 1978. Micromotives and Macrobehavior. New York: Norton.

The bandwagon effect and technology adoption is in:

Arthur, B. 1988. Self-reinforcing mechanisms in economics. In P. Anderson, K. Arrow, and D. Pines, eds., *The Economy as an Evolving Complex System*. New York: Addison-Wesley.

David, P. 1985. Clio and the economics of Qwerty. American Economic Review 75: 332-37.

A summary of the arguments on the Tragedy of the Commons appears first in:

Hardin, G. 1968. The Tragedy of the Commons. Science 162: 1243-48.

The nonconvexity problem with externalities was first pointed out in:

Starrett, D. 1972. Fundamental non-convexities in the theory of externalities. *Journal of Economic Theory* 4: 180–99.

# Exercises

- **8.1** "Smoke from a factory dirties the local housing and poisons crops." Identify the nature of the externalities in this statement.
- 8.2 How would you describe the production function of a laundry polluted by a factory?
- 8.3 Let  $U = [x_1]^{\alpha} [x_2 y]^{1-\alpha}$ , where y is an externality. Is this externality positive or negative? How does it affect the demand for good 1 relative to the demand for good 2?
- 8.4 The two consumers in the economy have preferences  $U_1 = \left[x_1^1 x_1^2\right]^{\alpha} \left[x_2^1 x_2^2\right]^{1-\alpha}$  and  $U_2 = \left[x_1^2 x_1^1\right]^{\alpha} \left[x_2^2 x_2^1\right]^{1-\alpha}$ , where  $x_i^h$  is consumption of good *i* by consumer *h*. Show that the equilibrium is efficient despite the externality. Explain this conclusion.
- 8.5 Consider a group of *n* students. Suppose that each student *i* puts in  $h_i$  hours of work on her classes that involves a disutility of  $\frac{h_i^2}{2}$ . Her benefits depend on how she performs relative to her peers and take the form  $u(\frac{h_i}{\overline{h}})$  for all *i*, where  $\overline{h} = \frac{1}{n} \sum_i h_i$  denotes the average number of hours put in by all students in the class and  $u(\cdot)$  is an increasing and concave function.
  - a. Calculate the symmetric Nash equilibrium.
  - b. Calculate the Pareto-efficient level of effort.

c. Explain why the equilibrium involves too much effort compared to the Pareto-efficient outcome.

**8.6** There are 4 students registered for the class "Introduction to externalities" at the University of Life. The professor is lazy so decides to implement a simple grading system. No lectures are given, no exercises are set, but there is a final paper. The paper requests each student, *i*, to choose a single number  $z^i \ge 0$  and record the choice on the answer sheet. The next day the final grade  $G^i$ , with a maximum of 20 for each student, is released using the following rule (which is public information before the exam):  $G^i = 10 + \sqrt{z^i} - \frac{1}{2} \sum_{i=1}^4 z^i$ . The exam is passed if a grade of 10 out of 20, or more, is obtained.

a. State the maximization problem faced by each student.

b. If each student is maximizing his expected grade, what is the desired grade chosen?

c. If each student is maximizing his expected grade, what is the final grade received by each student? (Assume that the equilibrium is symmetric.)

d. Is the outcome Pareto efficient? If not, provide an example of an outcome that is a Pareto improvement.

- **8.7** Graduate student A smokes, but his office mate B hates smoking. A and B have the following utility functions:  $U^A = 100 + 10z 0.1z^2$  and  $U^B = 100 10z$ , where z is the number of cigarettes smoked by A (and  $U^A$  includes the cost of cigarettes). Determine:
  - a. The number of cigarettes smoked by A when the external effect on B is ignored.
  - b. The socially optimal level of cigarettes that should be smoked by A.
  - c. The optimal Pigouvian tax needed to decentralize the social optimum.
  - d. The outcome with Coasian bargaining when the property right is assigned to the smoker.

e. The outcome with Coasian bargaining when the property right is assigned to the nonsmoker.

**8.8** Consider two firms, X and Y, located on the same river bank. The two firms produce paper for a printing works, and the production of paper requires the use of chemicals. Firm X is located upstream from firm Y and has the production function  $x = 1,000 \, [\ell_X]^{1/2}$ , where  $\ell_X$  is the amount of labor used per day and x the amount of paper produced by firm X. Firm Y, located downstream, has a similar production function, but its output can be affected by the chemical waste produced by firm X. Hence firm Y has the following production function;

$$y = \begin{cases} 1000 \left[ \ell_Y \right]^{1/2} \left[ x - x_0 \right]^{-\alpha} & \text{if } x > x_0, \\ 1000 \left[ \ell_Y \right]^{1/2} & \text{if } x < x_0, \end{cases}$$

where  $x_0$  is the natural capacity for absorption of pollution by the river,  $\ell_Y$  the labor used, and *y* the amount of paper produced by the firm. The selling price of a unit of paper is p = 1 and the wage per unit of labor is w = 50. Each firm is a profit maximizer.

a. Assume  $\alpha = 0$ . Determine the quantity of paper produced by each firm in equilibrium.

b. Assume  $x_0 = 19,000$  and  $\alpha = 0.1$ . How does this modify the result in part a? What is the interpretation of  $\alpha$ ?

c. Assume that the two firms merge. Does the new firm has an incentive to reallocate some labor from X to Y? Explain the result.

8.9 There is a large number of commuters who decide to use either their car or the train. Commuting by train takes 70 minutes whatever the number of commuters taking the train. Commuting by car takes C(x) = 20 + 60x minutes, where x is the proportion of commuters taking their cars,  $0 \le x \le 1$ .

a. Plot the curves of the commuting time by car and the commuting time by train as a function of the proportion of car users.

b. What is the proportion of commuters who will take their car if everyone is taking her decision freely and independently so as to minimize her own commuting time?

c. What is the proportion of car users that minimizes the total commuting time?

d. Compare this with your answer given in part b. Interpret the difference. How large is the deadweight loss from the externality?

e. Explain how a toll could achieve the efficient allocation of commuters between train and car and be beneficial for everyone.

- 8.10 Re-do the previous problem by replacing the train by a bus and assuming that commuting time by bus is increasing with the proportion of commuters using car (traffic congestion). Let the commuting time by bus be B(x) = 40 + 20x and the commuting time by car be C(x) = 20 + 60x, where x is the proportion of commuters taking their car,  $0 \le x \le 1$ .
- **8.11** Consider a binary choice to allow or not the emission of pollutants. The cost to consumers of allowing the pollution is C = 2,000, but this cost is only observable to the consumers. The benefit for the polluter of allowing the externality is B = 2,300, and only the polluter knows this benefit. Clearly, optimality requires this externality to be allowed, since B > C. However, the final decision must be based on what each party chooses to reveal.

a. Construct a tax-subsidy revelation scheme such that it is a dominant strategy for each party to report truthfully their private information.

b. Show that this revelation scheme induces the optimal production of the externality.

c. Show that this revelation scheme is unbalanced in the sense that the given equilibrium reports the tax to be paid by the polluter is less than the subsidy paid to the pollutee.

- 8.12 How can licenses be used to resolve the tragedy of the commons?
- **8.13** If insufficient abatement is very costly, which of taxation or licenses is preferable?
- 8.14 Are the following statements true or false? Explain why.

a. If your consumption of cigarettes produces negative externalities for your partner (which you ignore), then you are consuming more cigarettes than is Pareto efficient.

- b. It is generally efficient to set an emission standard allowing zero pollution.
- c. A tax on cigarettes induces the market for cigarettes to perform more efficiently.
- d. A ban on smoking is necessarily efficient.
- e. A competitive market with a negative externality produces more output than is efficient.
- f. A snob effect is a negative (network) externality from consumption.
- 8.15 A chemical factory produces a product that is sold at the price of \$10 per ton. The cost of production is  $C^F(y) = 0.5y^2$ , where y is the number of tons of production. For each ton of production, the factory produces 1 kg of pollutants that are either dispersed into the atmosphere at a cost of \$0 to the factory or captured and stored at a cost of \$2 for each kg of pollutant. The amount of dispersant pollutant is denoted *e*. The pollution causes a nearby textile firm to have to wash its products twice with an additional cost of  $C^e = 0.5e^2$ , which does not depend on the amount of output, *m*, produced. The unit price of the output of the textile firm is \$5, and the cost of production  $C^T = 0.02m^2$ .

a. Determine the values of y, m, and e in the competitive equilibrium. How much does the chemical factory spend on capturing the pollutant? What level of environmental cost does the factory inflict on the textile firm?

- b. Why is the competitive equilibrium inefficient from a social perspective?
- c. What is the socially optimal level of *y*?

d. Assume that the externality is internalized. Determine the values of y, m, and e, the expenditure on capture, and the additional cost for the textile firm caused by the environmental pollution.

8.16 Consider two consumers with utility functions

$$U^{A} = \log(x_{1}^{A}) + x_{2}^{A} - \frac{1}{2}\log(x_{1}^{B}), \quad U^{B} = \log(x_{1}^{B}) + x_{2}^{B} - \frac{1}{2}\log(x_{1}^{A}).$$

Both consumers have income *M*, and the (before-tax) price of both goods is 1.

a. Calculate the market equilibrium.

b. Calculate the social optimum for a utilitarian social welfare function.

c. Show that the optimum can be sustained by a tax placed on good 1 (so that the after-tax price becomes 1 + t) with the revenue returned equally to the consumers in a lump-sum manner.

d. Assume now that preferences are given by

$$U^{A} = \rho^{A} \log(x_{1}^{A}) + x_{2}^{A} - \frac{1}{2} \log(x_{1}^{B}), \quad U^{B} = \rho^{B} \log(x_{1}^{B}) + x_{2}^{B} - \frac{1}{2} \log(x_{1}^{A}).$$

Calculate the taxes necessary to decentralize the optimum.

e. For preferences of part d and income M = 20, contrast the outcome when taxes can and cannot be differentiated between consumers.

8.17 A competitive refining industry releases one unit of waste into the atmosphere for each unit of refined product. The inverse demand function for the refined product is  $p^d = 20 - q$ , which represents the marginal benefit curve where q is the quantity consumed when the consumers pay price  $p^d$ . The inverse supply curve for refining is MPC = 2 + q, which represents the marginal private cost curve when the industry produces q units. The marginal external cost curve is MEC = 0.5q, where MEC is the marginal external cost when the industry releases q units of waste. Marginal social cost is given by MSC = MPC + MEC.

a. What are the equilibrium price and quantity for the refined product when there is no correction for the externality?

b. How much of the refined product should the market supply at the social optimum?

c. How large is the deadweight loss from the externality?

d. Suppose that the government imposes an emission fee of T per unit of emissions. How large must the emission fee be if the market is to produce the socially efficient amount of the refined product?

- **8.18** Discuss the following statement: "A tax is a fine for doing something right. A fine is a tax for doing something wrong."
- **8.19** Suppose that the government issues tradable pollution permits.

a. Is it better for economic efficiency to distribute the permits among polluters or to auction them?

b. If the government decides to distribute the permits, does the allocation of permits among firms matter for economic efficiency?

8.20 A chemical producer dumps toxic waste into a river. The waste reduces the population of fish, reducing profits for the local fishery industry by \$150,000 per year. The firm could eliminate the waste at a cost of \$100,000 per year. The local fishing industry consists of many small firms.

a. Apply the Coase theorem to explain how costless bargaining will lead to a socially efficient outcome, no matter to whom property rights are assigned (either to the chemical firm or the fishing industry).

b. Verify the Coase theorem if the cost of eliminating the waste is doubled to \$200,000 (with the benefit for the fishing industry unchanged at \$150,000).

c. Discuss the following argument: "A community held together by ties of obligation and mutual interest can manage the local pollution problems."

d. Why might bargaining not be costless?

**8.21** A firm, *S*, produces steel but also produces waste that contaminates a nearby river. Steel can be sold for \$10 per ton. The cost function of the steel firm is given by  $c_S(s) = s^2$ , where *s* is the output of steel in tons. The level of waste, *x*, is related to output by x = 0.1s. A fish farm, *F*, is located downstream and is negatively affected by the waste polluting the water. Every fish produced by the farm can be sold for \$2. The cost function of the fish farm is given by  $c_F(f; x) = f + x^2$ . The fish farm has a capacity constraint  $f \le 10$ .

a. Compute the optimal output of the steel firm if it makes its decision without any constraint. Assume now that the firm must compensate the fishing club members an amount q per unit of waste that is produced.

b. Compute the optimum value of q, the optimal steel output, and the profit levels of the steel firm and the fishery.

- c. Compare the solutions to parts a and b, and explain any differences.
- **8.22** It is often used as an objection to market-based policies of pollution abatement that they place a monetary value on cleaning up our environment. Economists reply that society implicitly places a monetary value on environmental cleanup even under command-and-control policies. Explain why this is true.
- **8.23** Use examples to answer whether the externalities related to common resources are generally positive or negative. Is the free-market use of common resources greater or less than the socially optimal use?
- 8.24 Why is there more litter along highways than in people's yards?
- **8.25** Evaluate the following statement: "Since pollution is bad, it would be socially optimal to prohibit the use of any production process that creates pollution."
- **8.26** Why is it not generally efficient to set an emissions standard allowing zero pollution?
- 8.27 Education is often viewed as a good with positive externalities.
  - a. Explain how education might produce positive external effects.

b. Suggest a possible action of the government to induce the market for education to perform more efficiently.