2 Equilibrium and Efficiency

2.1 Introduction

The link between competition and efficiency can be traced back, at least, to Adam Smith's eighteenth-century description of the working of the invisible hand. Smith's description of individually motivated decisions being coordinated to produce a socially efficient outcome is a powerful one that has found resonance in policy circles ever since. The expression of the efficiency argument in the language of formal economics, and the deeper understanding that comes with it, is a more recent innovation.

The focus of this chapter is to review what is meant by competition and to describe equilibrium in a competitive economy. The model of competition combines independent decision-making of consumers and firms into a complete model of the economy. Equilibrium is shown to be achieved in the economy by prices adjusting to equate demand and supply. Most important, the chapter employs the competitive model to demonstrate the efficiency theorems.

Surprisingly, equilibrium prices can always be found that simultaneously equate demand and supply for all goods. What is even more remarkable is that the equilibrium so obtained also has properties of efficiency. Why this is remarkable is that individual households and firms pursue their independent objectives with no concern other than their own welfare. Even so, the final state that emerges achieves efficiency solely through the coordinating role played by prices.

2.2 Economic Models

Prior to starting the analysis, it is worth reflecting on why economists employ models to make predictions about the effects of economic policies. Models are used essentially because of problems of conducting experiments on economic systems and because the system is too large and complex to analyze in its entirety. Moreover formal modeling ensures that arguments are logically consistent with all the underlying assumptions exposed.

The models used, while inevitably being simplifications of the real economy, are designed to capture the essential aspects of the problem under study. Although many different models will be studied in this book, there are important common features that apply to all. Most models in public economics specify the objectives of the individual agents (e.g., firms and consumers) in the economy, and the constraints they face, and then aggregate individual decisions to arrive at market demand and supply. The equilibrium of the economy is next determined, and in a policy analysis the effects of government choice variables on this are calculated. This is done with various degrees of detail. Sometimes only a single market is studied—this is the case of *partial equilibrium* analysis. At other times *general equilibrium* analysis is used with many markets analyzed simultaneously. Similarly the number of firms and consumers varies from one or two to very many.

An essential consideration in the choice of the level of detail for a model is that its equilibrium must demonstrate a dependence on policy that gives insight into the functioning of the actual economy. On the one hand, if the model is too highly specified, it may not be capable of capturing important forms of response. On the other hand, if it is too general, it may not be able to provide any clear prediction. The theory described in this book will show how this trade-off can be successfully resolved. Achieving a successful compromise between these competing objectives is the "art" of economic modeling.

2.3 Competitive Economies

The essential feature of competition is that the consumers and firms in the economy do not consider their actions to have any effect on prices. Consequently, in making decisions, they treat the prices they observe in the market place as fixed (or *parametric*). This assumption can be justified when all consumers and firms are truly negligible in size relative to the market. In such a case the quantity traded by an individual consumer or firm is not sufficient to change the market price. But the assumption that the agents view prices as parametric can also be imposed as a modeling tool, even in an economy with a single consumer and a single firm.

This defining characteristic of competition places a focus on the role of prices, as is maintained throughout the chapter. Prices measure values and are the signals that guide the decisions of firms and consumers. It was the exploration of what determined the relative values of different goods and services that led to the formulation of the competitive model. The adjustment of prices equates supply and demand to ensure that equilibrium is achieved. The role of prices in coordinating the decisions of independent economic agents is also crucial for the attainment of economic efficiency.

The secondary feature of the economies in this chapter is that all agents have access to the *same* information, or in formal terminology, that information is *symmetric*. This does not imply that there cannot be uncertainty, but only that when there is uncertainty, all agents are equally uninformed. Put differently, no agent is permitted to have an informational advantage. For example, by this assumption, the future profit levels of firms are allowed to be uncertain and shares in the firms to be traded on the basis of individual assessments of future profits. What the assumption does not allow is for the directors of the firms to be better informed than other shareholders about future prospects and to trade profitably on the basis of this information advantage.

Two forms of the competitive model are introduced in this chapter. The first form is an exchange economy in which there is no production. Initial stocks of goods are held by consumers and economic activity occurs through the trade of these stocks to mutual advantage. The second form of competitive economy introduces production. This is undertaken by firms with given production technologies who use inputs to produce outputs and distribute their profits as dividends to consumers.

2.4 The Exchange Economy

The exchange economy models the simplest form of economic activity: the trade of commodities between two parties in order to obtain mutual advantage. Despite the simplicity of this model, it is a surprisingly instructive tool for obtaining fundamental insights about taxation and tax policy. This will become evident as we proceed. This section presents a description of a two-consumer, two-good exchange economy. The restriction on the number of goods and consumers does not alter any of the conclusions that will be derived; they will all extend to larger numbers. What restricting the numbers does is allow the economy to be displayed and analyzed in a simple diagram.

Each of the two consumers has an initial stock, or *endowment*, of the economy's two goods. The endowments can be interpreted literally as stocks of goods, or less literally as human capital, and are the quantities that are available for trade. Given the absence of production, these quantities remain constant. The consumers exchange quantities of the two commodities in order to achieve consumption plans that are preferred to their initial endowments. The rate at which one commodity can be exchanged for the other is given by the market prices. Both consumers believe that their behavior cannot affect these prices. This is the fundamental assumption of competitive price-taking behavior. More will be said about the validity and interpretation of this in section 2.6.

A consumer is described by their endowments and their preferences. The endowment of consumer h is denoted by $\omega^h = \left(\omega_1^h, \omega_2^h\right)$, where $\omega_i^h \geq 0$ is h's initial stock of good i.

When prices are p_1 and p_2, a consumption plan for consumer h, $x^h = \left(x_1^h, x_2^h\right)$, is affordable if it satisfies the budget constraint

$$p_1 x_1^h + p_2 x_2^h = p_1 \omega_1^h + p_2 \omega_2^h. \tag{2.1}$$

The preferences of each consumer are described by their utility function. This function should be seen as a representation of the consumer's indifference curves and does not imply any comparability of utility levels between consumers—the issue of comparability is taken up in chapter 13. The utility function for consumer h is denoted by

$$U^h = U^h\left(x_1^h, x_2^h\right). \tag{2.2}$$

It is assumed that the consumers enjoy the goods (so the marginal utility of consumption is positive for both goods) and that the indifference curves have the standard convex shape.

This economy can be pictured in a simple diagram that allows the role of prices in achieving equilibrium to be explored. The diagram is constructed by noting that the total consumption of the two consumers must equal the available stock of the goods, where the stock is determined by the endowments. Any pair of consumption plans that satisfies this requirement is called a *feasible plan* for the economy. A plan for the economy is feasible if the consumption levels can be met from the endowments, so that

$$x_i^1 + x_i^2 = \omega_i^1 + \omega_i^2, \qquad i = 1, 2. \tag{2.3}$$

The consumption plans satisfying (2.3) can be represented as points in a rectangle with sides of length $\omega_1^1 + \omega_1^2$ and $\omega_2^1 + \omega_2^2$. In this rectangle the southwest corner can be treated as the zero consumption point for consumer 1 and the northeast corner as the zero consumption point for consumer 2. The consumption of good 1 for consumer 1 is then measured horizontally from the southwest corner and for consumer 2 horizontally from the northeast corner. Measurements for good 2 are made vertically.

The diagram constructed in this way is called an *Edgeworth box* and a typical box is shown in figure 2.1. It should be noted that the method of construction results in the endowment point, marked ω, being the initial endowment point for both consumers.

The Edgeworth box is completed by adding the preferences and budget constraints of the consumers. The indifference curves of consumer 1 are drawn relative to the southwest corner and those of consumer 2 relative to the northeast corner. From (2.1) it can be seen that the budget constraint for both consumers must pass through the endowment point, since consumers can always afford their endowment. The endowment point is

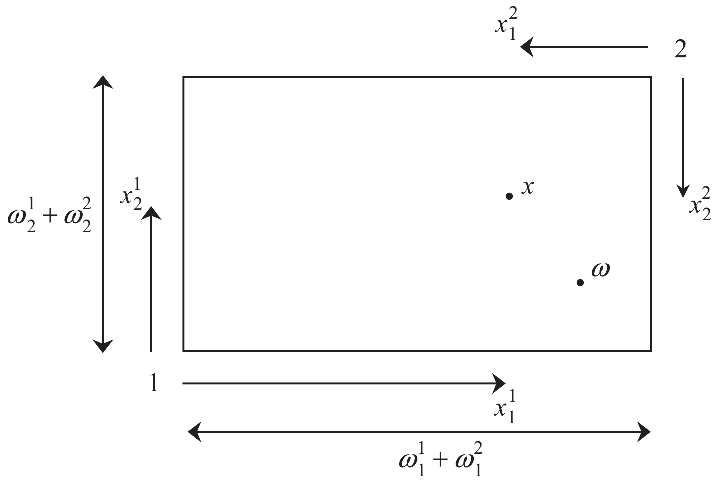

Figure 2.1
Typical Edgeworth box

common to both consumers, so a single budget line through the endowment point with gradient $-\frac{p_1}{p_2}$ captures the market opportunities of the two consumers. Thus, viewed from the southwest, it is the budget line of consumer 1, and viewed from the northeast, the budget line of consumer 2. Given the budget line determined by the prices p_1 and p_2, the utility-maximizing choices for the two consumers are characterized by the standard tangency condition between the highest attainable indifference curve and the budget line. This is illustrated in figure 2.2, where x^1 denotes the choice of consumer 1 and x^2 that of 2.

At an *equilibrium* of the economy, supply is equal to demand. This is assumed to be achieved via the adjustment of prices. The prices at which supply is equal to demand are called *equilibrium prices*. How such prices are arrived at will be discussed later. For the present the focus will be placed on the nature of equilibrium and its properties. The consumer choices shown in figure 2.2 do not constitute an equilibrium for the economy. This can be seen by summing the demands and comparing these to the level of the endowments. Doing this shows that the demand for good 1 exceeds the endowment but the demand for good 2 falls short. To achieve an equilibrium position, the relative prices of the goods must change. An increase in the relative price of good 1 raises the absolute value of the gradient $-p_1/p_2$ of the budget line, making the budget line steeper. It becomes flatter if the relative price of good 1 falls. At all prices the budget

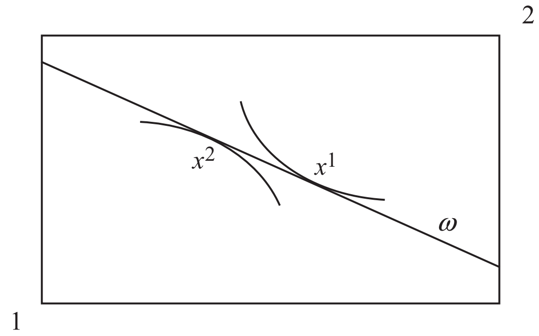

Figure 2.2
Preferences and demand

line continues to pass through the endowment point so that a change in relative prices sees the line pivot about the endowment point.

The effect of a relative price change on the budget constraint is shown in figure 2.3. In the figure the price of good 1 has increased relative to the price of good 2. This causes the budget constraint to pivot upward around the endowment point. As a consequence of this change the consumers will now select consumption plans on this new budget constraint.

The dependence of the consumption levels on prices is summarized in the consumers' demand functions. Taking the prices as given, the consumers choose their consumption plans to reach the highest attainable utility level subject to their budget constraints. The level of demand for good i from consumer h is $x_i^h = x_i^h(p_1, p_2)$. Using the demand functions, we see that demand is equal to supply for the economy when the prices are such that

$$x_i^1(p_1, p_2) + x_i^2(p_1, p_2) = \omega_i^1 + \omega_i^2, \qquad i = 1, 2. \tag{2.4}$$

A study of the Edgeworth box shows that an equilibrium is achieved when the prices lead to a budget line on which the indifference curves of the consumers have a point of common tangency. Such an equilibrium is shown in figure 2.4.

Having illustrated an equilibrium, we raise the question of whether an equilibrium is guaranteed to exist. As it happens, under reasonable assumptions, it will always do so. More important for public economics is the issue of whether the equilibrium has any desirable features from a welfare perspective. This is discussed in depth in section 2.6 where the Edgeworth box is put to substantial use.

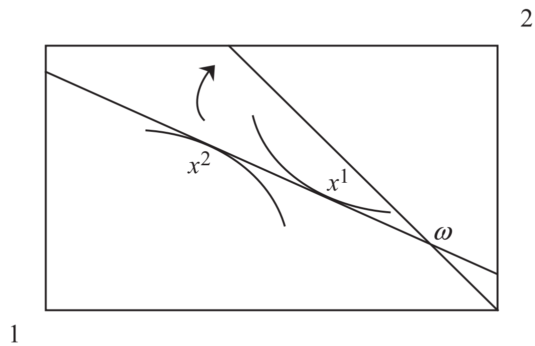

Figure 2.3
Relative price change

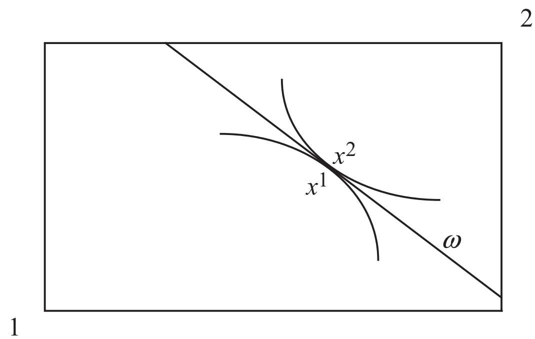

Figure 2.4
Equilibrium

Two further points now need to be made that are important for understanding the functioning of the model. These concern the number of prices that can be determined and the number of independent equilibrium equations. In the equilibrium conditions (2.4) there are two equations to be satisfied by the two equilibrium prices. It is now argued that the model can determine only the ratio of prices and not the actual prices. Accepting this, it would seem that there is one price ratio attempting to solve two equations. If this were the case, a solution would be unlikely, and we would be in the position of having a model that generally did not have an equilibrium. This situation is resolved by noting that the relationship between the two equilibrium conditions ensures that there is only one independent equation. The single price ratio then has to be solved by a single equation, making it possible for there to be always a solution.

The first point is developed by observing that the budget constraint always passes through the endowment point and its gradient is determined by the price ratio. The consequence of this is that only the value of p_1 relative to p_2 matters in determining demands and supplies rather than the absolute values. The economic explanation for this fact is that consumers are only concerned with the real purchasing power embodied in their endowment, and not with the level of prices. Since their nominal income is equal to the value of the endowment, any change in the level of prices raises nominal income just as much as it raises the cost of purchases. This leaves real incomes unchanged.

The fact that only relative prices matter is also reflected in the demand functions. If $x_i^h(p_1, p_2)$ is the level of demand at prices p_1 and p_2, then it must be the case that $x_i^h(p_1, p_2) = x_i^h(\lambda p_1, \lambda p_2)$ for $\lambda > 0$. A demand function having this property is said to be *homogeneous of degree* 0. In terms of what can be learned from the model, the homogeneity shows that only relative prices can be determined at equilibrium and not the level of prices. So, given a set of equilibrium prices, any scaling up or down of these prices will also be equilibrium prices because the change will not alter the level of demand. This is as it should be, since all that matters for the consumers is the rate at which they can exchange one commodity for another, and this is measured by the relative prices. This can be seen in the Edgeworth box. The budget constraint always goes through the endowment point so only its gradient can change, and this is determined by the relative prices.

In order to analyze the model, the indeterminacy of the level of prices needs to be removed. This is achieved by adopting a *price normalization*, which is simply a method of fixing a scale for prices. There are numerous ways to do this. The simplest way is to select a commodity as the *numéraire*, which means that its price is fixed at one, and measure all other prices relative to this. The numéraire chosen in this way can be thought of as the *unit of account* for the economy. This is the role usually played by money but, formally, there is no money in this economy.

The second point is to demonstrate the dependence between the two equilibrium equations. It can be seen that at the disequilibrium position shown in figure 2.2, the demand for good 1 exceeds its supply whereas the supply of good 2 exceeds demand. Considering other budget lines and indifference curves in the Edgeworth box will show that whenever there is an excess of demand for one good, there is a corresponding deficit of demand for the other. There is actually a very precise relationship between the excess and the deficit that can be captured in the following way: The level of *excess demand* for good i is the difference between demand and supply and is defined by $Z_i = x_i^1 + x_i^2 - \omega_i^1 - \omega_i^2$. By this definition, the value of excess demand can be calculated as

$$
\begin{aligned}
p_1 Z_1 + p_2 Z_2 &= \sum_{i=1}^{2} p_i \left[x_i^1 + x_i^2 - \omega_i^1 - \omega_i^2 \right] \\
&= \sum_{h=1}^{2} \left[p_1 x_1^h + p_2 x_2^h - p_1 \omega_1^h - p_2 \omega_2^h \right] \\
&= 0,
\end{aligned}
\tag{2.5}
$$

where the second equality is a consequence of the budget constraints in (2.1). The relationship in (2.5) is known as *Walras's law* and states that the value of excess demand is zero. This must hold for any set of prices, so it provides a connection between the extent of disequilibrium and prices. In essence, Walras's law is simply an aggregate budget constraint for the economy. Since all consumers are equating their expenditure to their income, so must the economy as a whole.

Walras's law implies that the equilibrium equations are interdependent. Since $p_1 Z_1 + p_2 Z_2 = 0$, if $Z_1 = 0$, then $Z_2 = 0$ (and vice versa). That is, if demand is equal to supply for good 1, then demand must also equal supply for good 2. Equilibrium in one market necessarily implies equilibrium in the other. This observation allows the construction of a useful diagram to illustrate equilibrium. Choose good 1 as the numéraire (so $p_1 = 1$) and plot the excess demand for good 2 as a function of p_2. The equilibrium for the economy is then found where the graph of excess demand crosses the horizontal axis. At this point excess demand for good 2 is zero, so by Walras's law, it must also be zero for good 1. An excess demand function is illustrated in figure 2.5 for an economy that has three equilibria. This excess demand function demonstrates

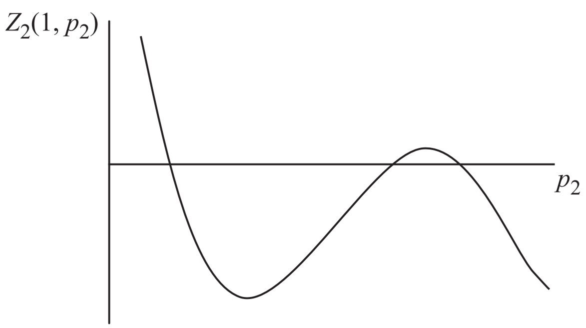

Figure 2.5
Equilibrium and excess demand

why at least one equilibrium will exist. As p_2 falls toward zero, demand will exceed supply (good 2 becomes increasingly attractive to purchase), making excess demand positive. Conversely, as the price of good 2 rises, it will become increasingly attractive to sell, resulting in a negative value of excess demand for high values of p_2. Since excess demand is positive for small values of p_2 and negative for high values, there must be at least one point in between where it is zero.

Finally, it should be noted that the arguments made above can be extended to include additional consumers and additional goods. Income, in terms of an endowment of many goods, and expenditure, defined in the same way, must remain equal for each consumer. The demand functions that result from the maximization of utility are homogeneous of degree zero in prices. Walras's law continues to hold, so the value of excess demand remains zero. The number of price ratios and the number of independent equilibrium conditions are always one less than the number of goods.

2.5 Production and Exchange

The addition of production to the exchange economy provides a complete model of economic activity. Such an economy allows a wealth of detail to be included. Some goods can be present as initial endowments (e.g., labor); others can be consumption goods produced from the initial endowments, while some goods, intermediates, can be produced by one productive process and used as inputs into another. The fully developed model of competition is called the *Arrow–Debreu economy* in honor of its original constructors.

An economy with production consists of consumers (or households) and producers (or firms). The firms use inputs to produce outputs with the intention of maximizing their profits. Each firm has available a production technology that describes the ways in which it can use inputs to produce outputs. The consumers have preferences and initial endowments as they did in the exchange economy, but they now also hold shares in the firms. The firms' profits are distributed as dividends in proportion to the shareholdings. The consumers receive income from the sale of their initial endowments (e.g., their labor time) and from the dividend payments.

Each firm is characterized by its production set, which summarizes the production technology it has available. A production technology can be thought of as a complete list of ways that the firm can turn inputs into outputs. In other words, it catalogs all the production methods of which the firm has knowledge. For firm j operating in an economy with two goods a typical production set, denoted Y^j, is illustrated in figure 2.6.

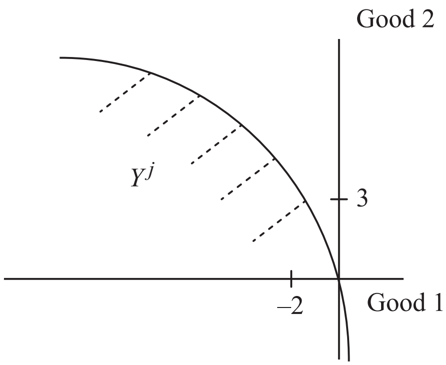

Figure 2.6
Typical production set

This figure employs the standard convention of measuring inputs as negative numbers and outputs as positive. The reason for adopting this convention is that the use of a unit of a good as an input represents a subtraction from the stock of that good available for consumption.

Consider the firm shown in figure 2.6 choosing the production plan $y_1^j = -2,\ y_2^j = 3$. When faced with prices $p_1 = 2,\ p_2 = 2$, the firm's profit is

$$\pi^j = p_1 y_1^j + p_2 y_2^j = 2 \times (-2) + 2 \times 3 = 2. \tag{2.6}$$

The positive part of this sum can be given the interpretation of sales revenue, and the negative part that of production costs. This is equivalent to writing profit as the difference between revenue and cost. Written in this way, (2.6) gives a simple expression of the relation between prices and production choices.

The process of profit maximization is illustrated in figure 2.7. Under the competitive assumption the firm takes the prices p_1 and p_2 as given. These prices are used to construct *isoprofit curves*, which show all production plans that give a specific level of profit. For example, all the production plans on the isoprofit curve labeled $\pi = 0$ will lead to a profit level of 0. Production plans on higher isoprofit curves lead to progressively larger profits, and those on lower curves to negative profits. Since doing nothing (which means choosing $y_1^j = y_2^j = 0$) earns zero profit, the $\pi = 0$ isoprofit curve always passes through the origin.

The profit-maximizing firm will choose a production plan that places it upon the highest attainable isoprofit curve. What restricts the choice is the technology that is available as described by the production set. In figure 2.7 the production plan that maximizes

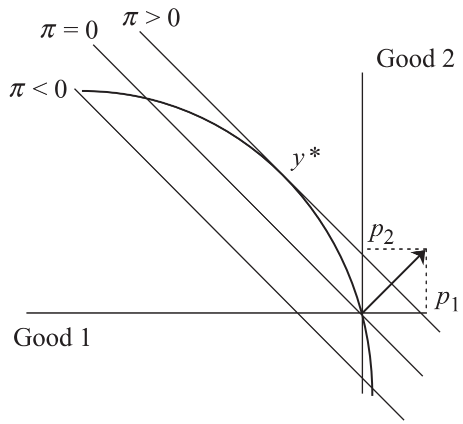

Figure 2.7
Profit maximization

profit is shown by y^*, which is located at a point of tangency between the highest attainable isoprofit curve and the production set. There is no other technologically feasible plan that can attain higher profit.

It should be noted how the isoprofit curves are determined by the prices. The geometry in fact is that the isoprofit curves are at right angles to the price vector. The angle of the price vector is determined by the price ratio, $\frac{p_2}{p_1}$, so a change in relative prices will alter the gradient of the isoprofit curves. The figure can be used to predict the effect of relative price changes. For instance, if p_1 increases relative to p_2, which can be interpreted as the price of the input (good 1) rising in comparison to the price of the output (good 2), then the price vector becomes flatter. This makes the isoprofit curves steeper, so the optimal choice must move round the boundary of the production set toward the origin. The use of input and the production of output both fall.

Now consider an economy with n goods. The price of good i is denoted p_i. Production is carried out by m firms. Each firm uses inputs to produce outputs and maximizes profits given the market prices. Demand comes from the H consumers in the economy. They aim to maximize their utility. The total supply of each good is the sum of the production of it by firms and the initial endowment of it held by the consumers.

Each firm chooses a production plan $y^j = \left(y_1^j, \ldots, y_n^j\right)$. This production plan is chosen to maximize profits subject to the constraint that the chosen plan must be in the production set. From this maximization can be determined firm j's supply function for good i as $y_i^j = y_i^j(p)$, where $p = (p_1, \ldots, p_n)$. The level of profit is $\pi^j = \sum_{i=1}^{n} p_i y_i^j(p) = \pi^j(p)$, which also depends on prices.

Aggregate supply from the production sector of the economy is obtained from the supply decisions of the individual firms by summing across the firms. This gives the aggregate supply of good i as

$$Y_i(p) = \sum_{j=1}^{m} y_i^j(p) \,. \tag{2.7}$$

Since some goods must be inputs, and others outputs, aggregate supply can be positive (the total activity of the firms adds to the stock of the good) or negative (the total activity of the firms subtracts from the stock).

Each consumer has an initial endowment of commodities and also a set of shareholdings in firms. The latter assumption makes this a *private ownership economy* in which the means of production are ultimately owned by individuals. In the present version of the model, these shareholdings are exogenously given and remain fixed. A more developed version would introduce a stock market and allow them to be traded. For consumer h the initial endowment is denoted ω^h and the shareholding in firm j is θ_j^h. The firms must be fully owned by the consumers, so $\sum_{h=1}^{H} \theta_j^h = 1$. That is, the shares in the firms must sum to one. Consumer h chooses a consumption plan x^h to maximize utility subject to the budget constraint

$$\sum_{i=1}^{n} p_i x_i^h = \sum_{i=1}^{n} p_i \omega_i^h + \sum_{j=1}^{m} \theta_j^h \pi^j. \tag{2.8}$$

This budget constraint requires that the value of expenditure be not more than the value of the endowment plus income received as dividends from firms. Since firms always have the option of going out of business (and hence earning zero profit), the dividend income must be nonnegative. The profit level of each firm is dependent on prices. A change in prices therefore affects a consumer's budget constraint through a change in the value of their endowment and through a change in dividend income. The maximization of utility by the consumer results in demand for good i from consumer h of the form $x_i^h = x_i^h(p)$. The level of aggregate demand is found by summing the individual demands of the consumers to give

$$X_i\,(p) = \sum_{h=1}^{H} x_i^h(p). \tag{2.9}$$

The same notion of equilibrium that was used for the exchange economy can be applied in this economy with production. That is, equilibrium occurs when supply is equal to demand. The distinction between the two is that supply, which was fixed in the exchange economy, is now variable and dependent on the production decisions of firms. Although this adds a further dimension to the question of the existence of equilibrium, the basic argument why such an equilibrium always exists is essentially the same as that for the exchange economy.

As already noted, the equilibrium of the economy occurs when demand is equal to supply or, equivalently, when excess demand is zero. Excess demand for good i, $Z_i(p)$, can be defined by

$$Z_i(p) = X_i(p) - Y_i(p) - \sum_{h=1}^{H} \omega_i^h. \tag{2.10}$$

Here excess demand is the difference between demand and the sum of initial endowment and firms' supply. The equilibrium occurs when $Z_i(p) = 0$ for all of the goods $i = 1, \ldots, n$. There are standard theorems that prove such an equilibrium must exist under fairly weak conditions.

The properties established for the exchange economy also apply to this economy with production. Demand is determined only by relative prices (so it is homogeneous of degree zero). Supply is also determined by relative prices. Together, these imply that excess demand is homogeneous of degree zero. To determine the equilibrium prices that equate supply to demand, a normalization must again be used. Typically one of the goods will be chosen as numéraire, and its price set to one. Equilibrium prices are then those that equate excess demand to zero.

2.6 Efficiency of Competition

Economics is often defined as the study of scarcity. This viewpoint is reflected in the concern with the efficient use of resources that runs throughout the core of the subject. Efficiency would seem to be a simple concept to characterize: if more cannot be achieved, then the outcome is efficient. This is certainly the case when an individual decision maker is considered. The individual will employ their resources to maximize utility subject to the constraints they face. When utility is maximized, the efficient outcome has been achieved.

Problems arise when there is more than one decision maker. To be unambiguous about efficiency, it is necessary to resolve the potentially competing needs of different decision makers. This requires efficiency to be defined with respect to a set of aggregate preferences. Methods of progressing from individual to aggregate preferences will be discussed in chapters 11 and 13. The conclusions obtained there are that the determination of aggregate preferences is not a simple task. There are two routes we can use to navigate around this difficulty. The first is to look at a single-consumer economy so that there is no conflict between competing preferences. But with more than one consumer some creativity has to be used to describe efficiency. The second route is met in section 2.6.2 where the concept of Pareto-efficiency is introduced. The trouble with such creativity is that it leaves the definition of efficiency open to debate. We will postpone further discussion of this until chapter 13.

Before we proceed further, some definitions are needed. A *first-best* outcome is achieved when only the production technology and the limited endowments restrict the choice of the decision maker. The first-best is essentially what would be chosen by an omniscient planner with complete command over resources. A *second-best* outcome arises whenever constraints other than technology and resources are placed on what the planner can do. Such constraints could be limits on income redistribution, an inability to remove monopoly power, or a lack of information.

2.6.1 Single Consumer

With a single consumer there is no doubt as to what is good and bad from a social perspective: the single individual's preferences can be taken as the social preferences. To do otherwise would be to deny the validity of the consumer's judgments. Hence, if the individual prefers one outcome to another, then so must society. The unambiguous nature of preferences provides significant simplification of the discussion of efficiency in the single-consumer economy. In this case the "best" outcome must be first-best because no constraints on policy choices have been invoked nor is there an issue of income distribution to consider.

If there is a single firm and a single consumer, the economy with production can be illustrated in a helpful diagram. This is constructed by superimposing the profit-maximization diagram for the firm over the choice diagram for the consumer. Such a model is often called the *Robinson Crusoe economy*. The interpretation is that Robinson acts as a firm carrying out production and as a consumer of the product of the firm. It is then possible to think of Robinson as a social planner who can coordinate the activities of the firm and producer. It is also possible (though in this case less compelling!) to think of Robinson as having a split personality and acting as a profit-maximizing firm on one side of the market and as a utility-maximizing consumer on the other. In the latter interpretation the two sides of Robinson's personality are reconciled through the prices on the competitive markets. The important fact is that these two interpretations lead to exactly the same levels of production and consumption.

The budget constraint of the consumer needs to include the dividend received from the firm. With two goods, the budget constraint is

$$p_1[x_1 - \omega_1] + p_2[x_2 - \omega_2] = \pi, \tag{2.11}$$

or

$$p_1\widetilde{x}_1 + p_2\widetilde{x}_2 = \pi, \tag{2.12}$$

where $\widetilde{x}_i$, the change from the endowment point, is the *net consumption* of good i. This is illustrated in figure 2.8 with good 2 chosen as the numéraire. The budget constraint (2.12) is always at a right angle to the price vector and is displaced above the origin by the value of profit. Utility maximization occurs where the highest indifference curve is reached given the budget constraint. This results in net consumption plan $\widetilde{x}^*$.

The equilibrium for the economy is shown in figure 2.9, which superimposes figure 2.7 onto 2.8. At the equilibrium the net consumption plan from the consumer must match the supply from the firm. The feature that makes this diagram work is the fact that the consumer receives the entire profit of the firm, so the budget constraint and the isoprofit curve are one and the same. The height above the origin of both is the level

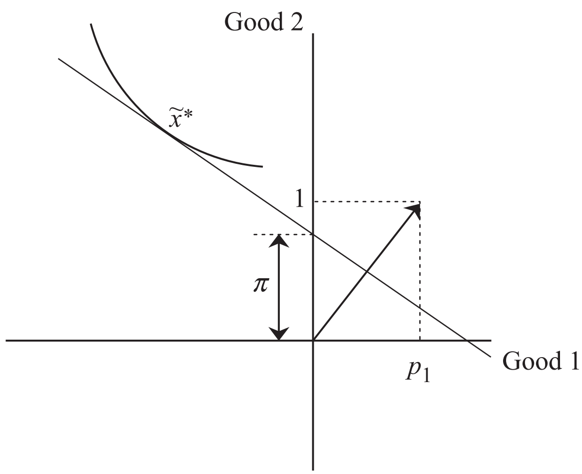

Figure 2.8
Utility maximization

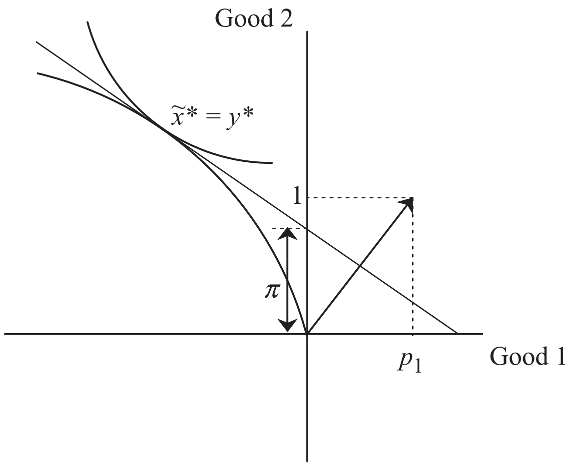

Figure 2.9
Efficient equilibrium

of profit earned by the firm and received by the consumer. Equilibrium can only arise when the point on the economy's production set that equates to profit maximization is the same as that of utility maximization. This is point $\widetilde{x}^* = y^*$ in figure 2.9.

It should be noted that the equilibrium is on the boundary of the production set so that it is efficient: it is not possible for a better outcome to be found in which more is produced with the same level of input. This captures the efficiency of production at the competitive equilibrium, about which much more is said soon. The equilibrium is also the first-best outcome for the single-consumer economy, since it achieves the highest indifference curve possible subject to the restriction that it is feasible under the technology. This is illustrated in figure 2.9 where $\widetilde{x}^*$ is the net level of consumption relative to the endowment point in the first-best and at the competitive equilibrium.

A simple characterization of this first-best allocation can be given by using the fact that it is at a tangency point between two curves. The gradient of the indifference curve is equal to the ratio of the marginal utilities of the two goods and is called the *marginal rate of substitution*. This measures the rate at which good 1 can be traded for good 2 while maintaining constant utility. The marginal rate of substitution is given by $MRS_{1,2} = \frac{U_1}{U_2}$, with subscripts used to denote the marginal utilities of the two goods. Similarly the gradient of the production possibility set is termed the *marginal rate of transformation* and denoted $MRT_{1,2}$. The $MRT_{1,2}$ measures the rate at which good 1 has to be given up to allow an increase in production of good 2. At the tangency point the two gradients are equal, so

$$MRS_{1,2} = MRT_{1,2}. \tag{2.13}$$

The reason why this equality characterizes the first-best equilibrium can be explained as follows: The *MRS* captures the marginal value of good 1 to the consumer relative to the marginal value of good 2, while the *MRT* measures the marginal cost of good 1 relative to the marginal cost of good 2. The first-best is achieved when the marginal value is equal to the marginal cost.

The market achieves efficiency through the coordinating role of prices. The consumer maximizes utility subject to their budget constraint. The optimal choice occurs when the budget constraint is tangential to the highest attainable indifference curve. The condition describing this is that ratio of marginal utilities is equal to the ratio of prices. Expressed in terms of the *MRS*, this is

$$MRS_{1,2} = \frac{p_1}{p_2}. \tag{2.14}$$

Similarly profit maximization by the firm occurs when the production possibility set is tangential to the highest isoprofit curve. Using the *MRT*, we write the profit-maximization condition as

$$MRT_{1,2} = \frac{p_1}{p_2}. \tag{2.15}$$

Combining these conditions, we find that the competitive equilibrium satisfies

$$MRS_{1,2} = \frac{p_1}{p_2} = MRT_{1,2}. \tag{2.16}$$

The condition in (2.16) demonstrates that the competitive equilibrium satisfies the same condition as the first-best and reveals the essential role of prices. By the competitive assumption, both the consumer and the producer are guided in their decisions by the same price ratio. Each optimizes relative to the price ratio; hence their decisions are mutually efficient.

There is one special case that is worth noting before moving on. When the firm has constant returns to scale, the efficient production frontier is a straight line through the origin. The only equilibrium can be when the firm makes zero profits. If profit was positive at some output level, then the constant returns to scale allows the firm to double profit by doubling output. Since this argument can be repeated, there is no limit to the profit that the firm can make. Hence we have the claim that equilibrium profit must be zero. Now the isoprofit curve at zero profit is also a straight line through the origin.

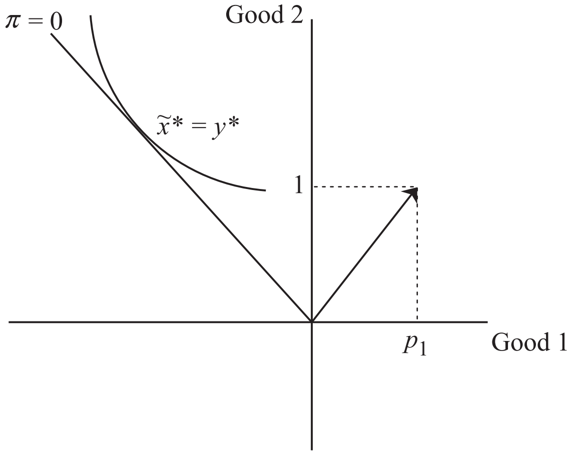

Figure 2.10
Constant returns to scale

The zero-profit equilibrium can only arise when this is coincident with the efficient production frontier. At this equilibrium the price vector is at right angles to both the isoprofit curve and the production frontier. This is illustrated in figure 2.10.

There are two further implications of constant returns. First, the equilibrium price ratio is determined by the zero-profit condition alone and is independent of demand. Second, the profit income of the consumer is zero, so the consumer's budget constraint also passes through the origin. As this is determined by the same prices as the isoprofit curve, the budget constraint must be coincident with the production frontier.

In this single-consumer context the equilibrium reached by the market simply cannot be bettered. Such a strong statement cannot be made when more consumers are introduced because issues of distribution among consumers then arise. However, what will remain is the finding that the competitive market ensures that firms produce at an efficient point on the frontier of the production set and that the chosen production plan is what is demanded at the equilibrium prices by the consumers. The key to this coordination are the prices that provide the signals guiding choices.

2.6.2 Pareto-Efficiency

When there is more than one consumer, the simple analysis of the Robinson Crusoe economy does not apply. Since consumers can have differing views about the success of an allocation, there is no single, simple measure of efficiency. The essence of the problem is that of judging among allocations with different distributional properties. What is needed is some process that can take account of the potentially diverse views of the consumers and separate efficiency from distribution.

To achieve this, economists employ the concept of *Pareto-efficiency*. The philosophy behind this concept is to interpret efficiency as meaning that there must be no unexploited economic gains. Testing the efficiency of an allocation then involves checking whether there are any such gains available. More specifically, Pareto-efficiency judges an allocation by considering whether it is possible to undertake a reallocation of resources that can benefit at least one consumer without harming any other. If it is possible to do so, then there will exist unexploited gains. When no improving reallocation can be found, then the initial position is deemed to be Pareto-efficient. An allocation that satisfies this test can be viewed as having achieved an efficient distribution of resources. For the present chapter this concept will be used uncritically. The interpretations and limitations of this form of efficiency will be discussed in chapter 13.

To provide a precise statement of Pareto-efficiency that applies in a competitive economy, it is first necessary to extend the idea of feasible allocations of resources that was used in (2.3) to define the Edgeworth box. When production is included, an allocation of consumption is feasible if it can be produced given the economy's initial endowments and production technology. Given the initial endowment, ω, the consumption allocation x is feasible if there is production plan y such that

$$x = y + \omega. \tag{2.17}$$

Pareto-efficiency is then tested using the feasible allocations. A precise definition follows.

Definition 2.1 A feasible consumption allocation $\hat{x}$ is Pareto-efficient if there does not exist an alternative feasible allocation $\overline{x}$ such that:

i. allocation $\overline{x}$ gives all consumers at least as much utility as $\hat{x}$, and
ii. allocation $\overline{x}$ gives at least one consumer more utility than $\hat{x}$.

These two conditions can be summarized as saying that allocation $\hat{x}$ is Pareto efficient if there is no alternative allocation (a move from $\hat{x}$ to $\overline{x}$) that can make someone better off without making anyone worse off. It is this idea of being able to make someone better off without making someone else worse off that represents the unexploited economic gains in an inefficient position.

It should be noted even at this stage how Pareto-efficiency is defined by the negative property of being unable to find anything better than the allocation. This is somewhat different from a definition of efficiency that looks for some positive property of the allocation. Pareto-efficiency also sidesteps issues of distribution rather than confronting them. This is why it works with many consumers. More will be said about this in chapter 13 when the construction of social welfare indicators is discussed.

2.6.3 Efficiency in an Exchange Economy

The welfare properties of the economy, which are commonly known as the *Two Theorems of Welfare Economics*, are the basis for claims concerning the desirability of the competitive outcome. In brief, the First Theorem states that a competitive equilibrium is Pareto-efficient and the Second Theorem that any Pareto-efficient allocation can be decentralized as a competitive equilibrium. Taken together, they have significant implications for policy and, at face value, seem to make a compelling case for the encouragement of competition.

The Two Theorems are easily demonstrated for a two-consumer exchange economy by using the Edgeworth box diagram. The first step is to isolate the Pareto-efficient allocations. Consider figure 2.11 and the allocation at point a. To show that a is not a Pareto-efficient allocation, it is necessary to find an alternative allocation that gives at least one of the consumers a higher utility level and neither consumer a lower level. In this case, moving to the allocation at point b raises the utility of both consumers when compared to point a—we say in such a case that b is *Pareto-preferred* to a. This

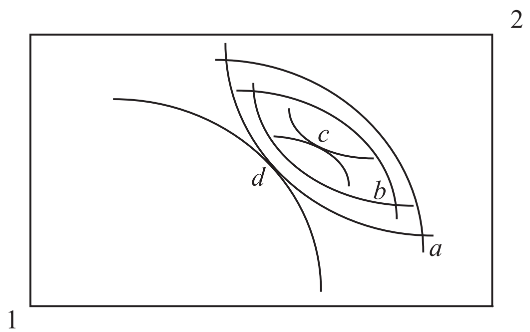

Figure 2.11
Pareto-efficiency

establishes that a is not Pareto-efficient. Although b improves on a, it is not Pareto-efficient either: the allocation at c provides higher utility for both consumers than b.

The allocation at c is Pareto-efficient. Beginning at c, any change in the allocation must lower the utility of at least one of the consumers. The special property of point c is that it lies at a point of tangency between the indifference curves of the two consumers. As it is a point of tangency, moving away from it must lead to a lower indifference curve for one of the consumers if not both. Since the indifference curves are tangential, their gradients are equal, so

$$MRS^1_{1,2} = MRS^2_{1,2}. \tag{2.18}$$

This equality ensures that the rate at which consumer 1 will want to exchange good 1 for good 2 is equal to the rate at which consumer 2 will want to exchange the two goods. It is this equality of the marginal valuations of the two consumers at the tangency point that results in there being no further unexploited gains and so makes c Pareto efficient.

The Pareto-efficient allocation at c is not unique. There are in fact many points of tangency between the two consumers' indifference curves. A second Pareto-efficient allocation is at point d in figure 2.11. Taken together, all the Pareto-efficient allocations form a locus in the Edgeworth box that is called the *contract curve*. This is illustrated in figure 2.12. With this construction it is now possible to demonstrate the First Theorem.

A competitive equilibrium is given by a price line through the initial endowment point, ω, that is tangential to both indifference curves at the same point. The common point of tangency results in consumer choices that lead to the equilibrium levels of demand. Such an equilibrium is indicated by point e in figure 2.12. As the equilibrium

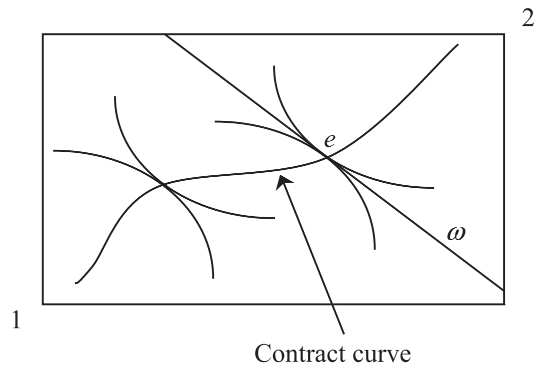

Figure 2.12
The First Theorem

is a point of tangency of indifference curves, it must also be Pareto-efficient. For the Edgeworth box, this completes the demonstration that a competitive equilibrium is Pareto-efficient.

The alternative way of seeing this result is to recall that each consumer maximizes utility at the point where their budget constraint is tangential to the highest indifference curve. Using the MRS, we can write this condition for consumer h as $MRS^h_{1,2} = p_1/p_2$. The competitive assumption is that both consumers react to the same set of prices, so it follows that

$$MRS^1_{1,2} = \frac{p_1}{p_2} = MRS^2_{1,2}. \tag{2.19}$$

Comparing this condition with (2.18) provides an alternative demonstration that the competitive equilibrium is Pareto-efficient. It also shows again the role of prices in coordinating the independent decisions of different economic agents to ensure efficiency.

This discussion can be summarized in the precise statement of the theorem.

Theorem 2.2 (First Theorem of Welfare Economics) The allocation of commodities at a competitive equilibrium is Pareto-efficient.

This theorem can be formally proved by assuming that the competitive equilibrium is not Pareto-efficient and by then deriving a contradiction. Assuming that the competitive equilibrium is not Pareto-efficient implies that there is a feasible alternative that is at least as good for all consumers and strictly better for at least one. Now take the consumer who is made strictly better off. Why did that consumer not choose the alternative consumption plan at the competitive equilibrium? The answer has to be because it was more expensive than the choice at the competitive equilibrium and not affordable with that consumer's budget. Similarly for all other consumers the new allocation has to be at least as expensive as their choice at the competitive equilibrium. (If it were cheaper, they could afford an even better consumption plan that made them strictly better off than at the competitive equilibrium.) Summing across the consumers, the alternative allocation has to be strictly more expensive than the competitive allocation. But the value of consumption at the competitive equilibrium must equal the value of the endowment. Therefore the new allocation must have greater value than the endowment, which implies it cannot be feasible. This contradiction establishes that the competitive equilibrium must be Pareto-efficient.

The theorem demonstrates that the competitive equilibrium is Pareto-efficient, but it is not the only Pareto-efficient allocation. Referring back to figure 2.12, we have that any point on the contract curve is also Pareto-efficient because all are defined by a tangency between indifference curves. The only special feature of e is that it is the allocation reached through competitive trading from the initial endowment point ω. If ω were different, then another Pareto-efficient allocation would be achieved. There is in fact an infinity of Pareto-efficient allocations. Observing these points motivates the Second Theorem of Welfare Economics.

The Second Theorem is concerned with whether any chosen Pareto-efficient allocation can be made into a competitive equilibrium by choosing a suitable location for the initial endowment. Expressed differently, can a competitive economy be constructed that has a selected Pareto-efficient allocation as its competitive equilibrium? In the Edgeworth box this involves being able to choose any point on the contract curve and turning it into a competitive equilibrium.

From the Edgeworth box diagram it can be seen that this is possible in the exchange economy if the households' indifference curves are convex. The common tangent to the indifference curves at the Pareto-efficient allocation provides the budget constraint that each consumer must face if they are to afford the chosen point. The convexity ensures that given this budget line, the Pareto-efficient point will also be the optimal choice of the consumers. The construction is completed by choosing a point on this budget line as the initial endowment point. This process of constructing a competitive economy to obtain a selected Pareto-efficient allocation is termed *decentralization*.

This process is illustrated in figure 2.13 where the Pareto-efficient allocation e' is made a competitive equilibrium by selecting ω' as the endowment point. Starting from ω',

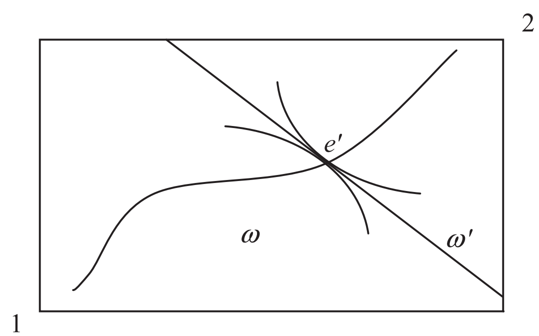

Figure 2.13
The Second Theorem

trading by consumers will take the economy to its equilibrium allocation e'. This is the Pareto-efficient allocation that was intended to be reached. Note that if the endowments of the households are initially given by ω and the equilibrium at e' is to be decentralized, it is necessary to redistribute the initial endowments of the consumers in order to begin from ω'.

The construction described above can be given a formal statement as the Second Theorem of Welfare Economics.

Theorem 2.3 (Second Theorem of Welfare Economics) With convex preferences, any Pareto-efficient allocation can be made a competitive equilibrium.

The statement of the Second Theorem provides a conclusion but does not describe the mechanism involved in the decentralization. The important step in decentralizing a chosen Pareto-efficient allocation is placing the economy at the correct starting point. For now it is sufficient to observe that behind the Second Theorem lies a process of redistribution of initial wealth. How this can be achieved is discussed later. Furthermore the Second Theorem determines a set of prices that make the chosen allocation an equilibrium. These prices may well be very different from those that would have been obtained in the absence of the wealth redistribution.

2.6.4 Extension to Production

The extension of the Two Theorems to an economy with production is straightforward. The major effect of production is to make supply variable: it is now the sum of the initial endowment plus the net outputs of the firms. In addition a consumer's income includes the profit derived from their shareholdings in firms.

Section 2.6.1 has already demonstrated efficiency for the Robinson Crusoe economy that included production. It was shown that the competitive equilibrium achieved the highest attainable indifference curve given the production possibilities of the economy. Since the single consumer cannot be made better off by any change, the equilibrium is Pareto-efficient and the First Theorem applies. The Second Theorem is of limited interest with a single consumer because there is only one Pareto-efficient allocation, and this is attained by the competitive economy.

When there is more than one consumer, the proof of the First Theorem follows the same lines as for the exchange economy. Given the equilibrium prices, each consumer is maximizing utility, so each consumer's marginal rate of substitution is equated to the same price ratio. This is true for all consumers and all goods, yielding

$$MRS^h_{i,j} = \frac{p_i}{p_j} = MRS^{h'}_{i,j} \tag{2.20}$$

for any pair of consumers h and h' and any pair of goods i and j. This is termed *efficiency in consumption*. In an economy with production this condition alone is not sufficient to guarantee efficiency; it is also necessary to consider production. The profit-maximization decision of each firm ensures that it equates its marginal rate of transformation between any two goods to the ratio of prices. For any two firms m and m',

$$MRT^m_{i,j} = \frac{p_i}{p_j} = MRT^{m'}_{i,j}, \tag{2.21}$$

a condition that characterizes *efficiency in production*. The price ratio also coordinates consumers and firms, giving the *top-level condition*

$$MRS^h_{i,j} = MRT^m_{i,j} \tag{2.22}$$

for any consumer and any firm for all pairs of goods. As for the Robinson Crusoe economy, the interpretation of this condition is that it equates the relative marginal values to the relative marginal costs. Since (2.20) through (2.22) are the conditions required for efficiency, this shows that the First Theorem extends to the economy with production.

The formal proof of this claim mirrors that for the exchange economy, except for the fact that the value of production must also be taken into account. Given this fact, the basis of the argument remains that since the consumers chose the competitive equilibrium quantities, anything that is preferred must be more expensive and hence can be shown not to be feasible.

The extension of the Second Theorem to include production is illustrated in figure 2.14. The set W describes the feasible output plans for the economy, with each point w in W being the sum of a production plan and the initial endowment; hence $w = y + \omega$. Set Z describes the quantities of the two goods that would allow a Pareto improvement (a re-allocation that makes neither consumer worse off and makes one strictly better off) over the allocation $\hat{x}^1$ to consumer 1 and $\hat{x}^2$ to consumer 2. If W and Z are convex, which occurs when firms' production sets and preferences are convex, then a common tangent to W and Z can be found. This makes $\hat{x}$ an equilibrium. Individual income allocations, the sum of the value of endowment plus profit income, can be placed anywhere on the budget lines tangent to the indifference curves at the individual allocations

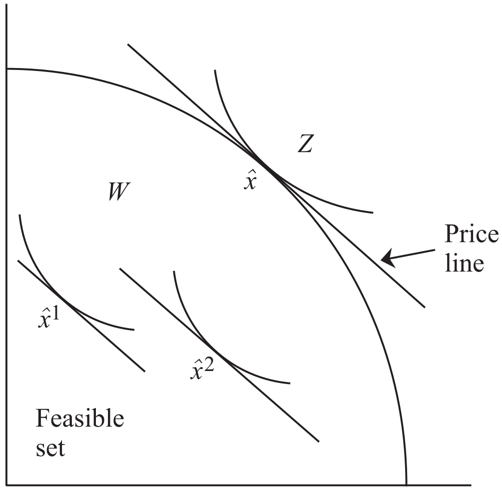

Figure 2.14
Proof of the Second Theorem

$\hat{x}_1$ and $\hat{x}_2$ provided that they sum to the total income of the economy. This decentralizes the consumption allocation $\hat{x}_1, \hat{x}_2$.

Before proceeding further, it is worth emphasizing that the proof of the Second Theorem requires more assumptions than the proof of the First, so there may be situations in which the First Theorem is applicable but the Second is not. The Second Theorem requires that a common tangent be found, which relies on preferences and production sets being convex. A competitive equilibrium can exist with some nonconvexity in the production sets of the individual firms or the preferences of the consumers, so the First Theorem will apply but the Second Theorem will not apply.

2.7 Lump-Sum Taxation

The discussion of the Second Theorem noted that it does not describe the mechanism through which the decentralization is achieved. It is instead implicit in the statement of the theorem that the consumers are given sufficient income to purchase the consumption plans forming the Pareto-efficient allocation. Any practical value of the Second Theorem depends on the government being able to allocate the required income levels. The way in which the theorem sees this as being done is by making what are called *lump-sum transfers* between consumers.

A transfer is defined as lump sum if no change in a consumer's behavior can affect the size of the transfer. For example, a consumer choosing to work less hard or reducing the consumption of a commodity must not be able to affect the size of the transfer. This differentiates a lump-sum transfer from other taxes, such as income or commodity taxes, for which changes in behavior do affect the value of the tax payment. Lump-sum transfers have a very special role in the theoretical analysis of public economics because, as we will show, they are the idealized redistributive instrument.

The lump-sum transfers envisaged by the Second Theorem involve quantities of endowments and shares being transferred among consumers to ensure the necessary income levels. Some consumers would gain from the transfers; others would lose. Although the value of the transfer cannot be changed, lump-sum transfers do affect consumers' behavior because their incomes are either reduced or increased by the transfers—the transfers have an income effect but do not lead to a substitution effect between commodities. Without recourse to such transfers, the decentralization of the selected allocation would not be possible.

The illustration of the Second Theorem in an exchange economy in figure 2.15 makes clear the role and nature of lump-sum transfers. The initial endowment point is denoted ω, and this is the starting point for the economy. It we assume that the Pareto-efficient allocation at point e is to be decentralized, then the income levels have to be modified to achieve the new budget constraint. At the initial point the income level of h is $\hat{p}\omega^h$ when evaluated at the equilibrium prices $\hat{p}$. The value of the transfer to consumer h that is necessary to achieve the new budget constraint is $M^h - \hat{p}\omega^h = \hat{p}\,\hat{x}^h - \hat{p}\omega^h$. One

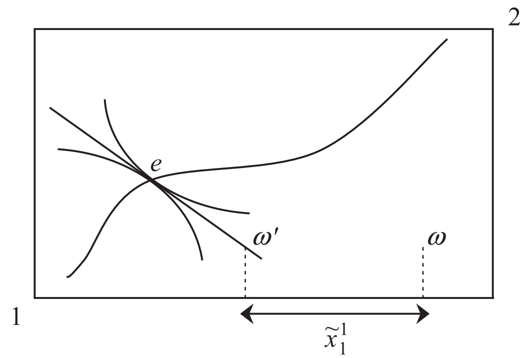

Figure 2.15
A lump-sum transfer

way of ensuring this is to transfer a quantity $\widetilde{x}_1^1$ of good 1 from consumer 1 to consumer 2. But any transfer of commodities with the same value would work equally well.

There is a problem, though, if we attempt to interpret the model this literally. For most people, income is earned almost entirely from the sale of labor so that their endowment is simply lifetime labor supply. This makes it impossible to transfer the endowment since one person's labor cannot be given to another. Responding to such difficulties leads to the reformulation of lump-sum transfers in terms of *lump-sum taxes*. Suppose that the two consumers both sell their entire endowments at prices $\hat{p}$. This generates incomes $\hat{p}\omega^1$ and $\hat{p}\omega^2$ for the two consumers. Now make consumer 1 pay a tax of amount $T^1 = \hat{p}\,\widetilde{x}_1^1$ and give this tax revenue to consumer 2. Consumer 2 therefore pays a negative tax (or, in simpler terms, receives a subsidy) of $T^2 = -\hat{p}\,\widetilde{x}_1^1 = -T^1$. The pair of taxes $\left(T^1, T^2\right)$ moves the budget constraint in exactly the same way as the lump-sum transfer of endowment. The pair of taxes and the transfer of endowment are therefore economically equivalent and have the same effect on the economy. The taxes are also lump sum because they are determined without reference to either consumers' behavior and their values cannot be affected by any change in behavior.

Lump-sum taxes have a central role in public economics due to their success in achieving distributional objectives. It should be clear from the discussion above that the economy's total endowment is not reduced by the application of the lump-sum taxes. This point applies to lump-sum taxes in general. As households cannot affect the level of the tax by changing their behavior, lump-sum taxes do not lead to any distortions in choice. There are also no resources lost due to the imposition of lump-sum taxes, so redistribution is achieved with no efficiency cost. In short, if they can be employed in the manner described they are the perfect taxes.

2.8 Discussion of Assumptions

The description of the competitive economy introduced a number of assumptions concerning the economic environment and how trade was conducted. These are important since they bear directly on the efficiency properties of competition. The interpretation and limitation of these assumptions is now discussed. This should provide a better context for evaluating the practical relevance of the efficiency theorems.

The most fundamental assumption was that of competitive behavior. This is the assumption that both consumers and firms view prices as fixed when they make their decisions. The natural interpretation of this assumption is that the individual economic agents are small relative to the total economy. When they are small, they naturally have no consequence. This assumption rules out any kind of market power such as monopolistic firms or trade unions in labor markets.

Competitive behavior leads to the problem of who actually sets prices in the economy. In the exchange model it is possible for equilibrium prices to be achieved via a process of barter and negotiation between the trading parties. However, barter cannot be a credible explanation of price determination in an advanced economic environment. One theoretical route out of this difficulty is to assume the existence of a fictitious "Walrasian auctioneer" who literally calls out prices until equilibrium is achieved. Only at this point trade is allowed to take place. Obviously this does not provide a credible explanation of reality. Although there are other theoretical explanations of price-setting, none is entirely consistent with the competitive assumption. How to integrate the two remains an unsolved puzzle.

The second assumption was symmetry of information. In a complex world there are many situations in which this does not apply. For instance, some qualities of a product, such as reliability (I do not know when my computer will next crash, but I expect it will be soon), are not immediately observable but are discovered only through experience. When it comes to re-sale, this causes an asymmetry of information between the existing owner and potential purchasers. The same can be true in labor markets where workers may know more about their attitudes toward work and potential productivity than a prospective employer. An asymmetry of information provides a poor basis for trade because the caution of those transacting prevents the full gains from trade being realized.

When any of the assumptions underlying the competitive economy fail to be met, and as a consequence efficiency is not achieved, we say that there is *market failure*. Situations of market failure are of interest to public economics because they provide a potential role for government policy to enhance efficiency. A large section of this book is in fact devoted to a detailed analysis of the sources of market failure and the scope for policy response.

As a final observation, notice that the focus has been on positions of equilibrium. Several explanations can be given for this emphasis. Historically economists viewed the economy as self-correcting so that, if it were ever away from equilibrium, forces exist that move it back toward equilibrium. In the long run, equilibrium would then always be attained. Although such adjustment can be justified in simple single-market contexts, both the practical experience of sustained high levels of unemployment and the theoretical study of the stability of the price adjustment process have shown that the self-adjusting equilibrium view is not generally justified. The present justifications for focusing on equilibrium are more pragmatic. The analysis of a model must begin somewhere, and the equilibrium has much merit as a starting point.

In addition, even if the final focus is on disequilibrium, there is much to be gained from comparing the properties of points of disequilibrium with those of the equilibrium. Finally, no positions other than those of equilibrium have any obvious claim to prominence.

2.9 Summary

This chapter described competitive economies and demonstrated the Two Theorems of Welfare Economics. To do this, it was necessary to introduce the concept of Pareto-efficiency. While Pareto-efficiency was simply accepted in this chapter, it will be considered very critically in chapter 13. The Two Theorems characterize the efficiency properties of the competitive economy and show how a selected Pareto-efficient allocation can be decentralized. It was also shown how prices are central to the achievement of efficiency through their role in coordinating the choices of individual agents. The role of lump-sum transfers or taxes in supporting the Second Theorem was highlighted. These transfers constitute the ideal tax system because they cause no distortions in choice and have no resource costs.

The subject matter of this chapter has very strong implications that are investigated fully in later chapters. An understanding of the welfare theorems, and of their limitations, is fundamental to appreciating many of the developments of public economics. Since claims about the efficiency of competition feature routinely in economic debate, it is important to subject it to the most careful scrutiny.

Further Reading

The two fundamental texts on the competitive economy are:

Arrow, K. J., and Hahn, F. H. 1971. *General Competitive Analysis*. Amsterdam: North-Holland.

Debreu, G. 1959. *The Theory of Value*. New Haven: Yale University Press.

A textbook treatment can be found in:

Ellickson, B. 1993. *Competitive Equilibrium: Theory and Applications*. Cambridge: Cambridge University Press.

The competitive economy has frequently been used as a practical tool for policy analysis. A survey of applications is in:

Shoven, J. B., and Whalley, J. 1992. *Applying General Equilibrium Theory*. Cambridge: Cambridge University Press.

A historical survey of the development of the model is given in:

Duffie, D., and Sonnenschein, H. 1989. Arrow and general equilibrium theory. *Journal of Economic Literature* 27: 565–98.

Some questions concerning the foundations of the model are addressed in:

Koopmans, T. C. 1957. *Three Essays on the State of Economic Science*. New York: McGraw-Hill.

The classic proofs of the Two Theorems are in:

Arrow, K. J. 1951. An extension of the basic theorems of welfare economics. In J. Neyman, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley: University of California Press, 507–532.

A formal analysis of lump-sum taxation can be found in:

Mirrlees, J. A. 1986. The theory of optimal taxation. In K. J. Arrow and M. D. Intrilligator, eds., *Handbook of Mathematical Economics*, vol. 3. Amsterdam: North-Holland, 1197–1249.

An extensive textbook treatment of Pareto-efficiency is:

Ng, Y.-K. 2003. *Welfare Economics*. Basingstoke, UK: Macmillan.

Exercises

2.1 Distinguish between partial equilibrium analysis and general equilibrium analysis. Briefly describe a model of each kind.

2.2 Keynesian models in macroeconomics are identified by the assumption of a fixed price for output. Are such models partial or general equilibrium?

2.3 You are requested to construct a model to predict the effect on the economy of the discovery of new oil reserves. How would you model the discovery? Discuss the number of goods that should be included in the model.

2.4 Let a consumer have preferences described by the utility function

$$U = \log(x_1) + \log(x_2),$$

and an endowment of 2 units of good 1 and 2 units of good 2.

a. Construct and sketch the consumer's budget constraint. Show what happens when the price of good 1 increases.

b. By maximizing utility, determine the consumer's demands.

c. What effect does increasing the endowment of good 1 have on the demand for good 2? Explain your finding.

2.5 How would you model an endowment of labor?

2.6 Let two consumers have preferences described by the utility function

$$U^h = \log\left(x_1^h\right) + \log\left(x_2^h\right), \qquad h = 1,\ 2,$$

and the endowments described below:

	Good 1	Good 2
Consumer 1	3	2
Consumer 2	2	3

a. Calculate the consumers' demand functions.

b. Selecting good 2 as the numéraire, find the equilibrium price of good 1. Hence find the equilibrium levels of consumption.

c. Show that the consumers' indifference curves are tangential at the equilibrium.

2.7 Consider an economy with two goods and two consumers with preferences

$$U^h = \min\left(x_1^h,\ x_2^h\right), \qquad h = 1,\ 2.$$

Assume that the endowments are as follows:

	Good 1	Good 2
Consumer 1	1	2
Consumer 2	2	1

a. Draw the Edgeworth box for the economy.

b. Display the equilibrium in the Edgeworth box.

c. What is the effect on the equilibrium price of good 2 relative to good 1 of an increase in each consumer's endowment of good 1 by 1 unit?

2.8 Consumer 1 obtains no pleasure from good 1, and consumer 2 obtains no pleasure from good 2. At the initial endowment point both consumers have endowments of both goods.

a. Draw the preferences of the consumers in an Edgeworth box.

b. By determining the trades that improve both consumers' utilities, find the equilibrium of the economy.

c. Display the equilibrium budget constraint.

2.9 Demonstrate that the demands obtained in exercise 2.4 are homogeneous of degree zero in prices. Show that doubling prices does not affect the graph of the budget constraint.

2.10 It has been argued that equilibrium generally exists on the basis that there must be a point where excess demand for good 2 is zero if excess demand is positive as the price of good 2 tends to zero and negative as it tends to infinity.

a. Select good 1 as the numéraire and show that these properties hold when preferences are given by the utility function

$$U^h = \log\left(x_1^h\right) + \log\left(x_2^h\right),$$

and the consumer's endowment of both goods is positive.

b. Show that they do not hold if the consumer has no endowment of good 2.

c. Consider the implications of the answer to part b for proving the existence of equilibrium.

2.11 Consider an economy with 2 consumers, A and B, and 2 goods, 1 and 2. The utility function of A is

$$U^A = \gamma \log\left(x_1^A\right) + [1-\gamma]\log\left(x_2^A\right),$$

where x_i^A is consumption of good i by A. A has endowments $\omega^A = \left(\omega_1^A, \omega_2^A\right) = (2, 1)$. For B,

$$U^B = \gamma \log\left(x_1^B\right) + [1-\gamma]\log\left(x_2^B\right) \quad \text{and} \quad \omega^B = (3, 2).$$

a. Use the budget constraint of A to substitute for x_2^A in U^A, and by maximizing over x_1^A, calculate the demands of A. Repeat for B.

b. Choosing good 2 as the numéraire, graph the excess demand for good 1 as a function of p_1.

c. Calculate the competitive equilibrium allocation by equating the demand for good 1 to the supply and then substituting for M^A and M^B. Verify that this is the point where excess demand is zero.

d. Show how the equilibrium price of good 2 is affected by a change in γ and in ω_1^A. Explain the results.

2.12 A firm has a production technology that permits it to turn 1 unit of good 1 into 2 units of good 2. If the price of good 1 is 1, at what price for good 2 will the firm just break even? Graph the firm's profit as a function of the price of good 2.

2.13 Consider the production process described by

$$F(x_1,\ x_2,\ x_3) \equiv x_1 - (-x_2)^{\beta}(-x_3)^{1-\beta} = 0,$$

where $x_1 \geq 0$ is the output, and $x_2 \leq 0$, $x_3 \leq 0$ are the inputs.

a. If good 1 is the numéraire, what prices of goods 2 and 3 are consistent with zero profit?

b. Discuss the observation that, in a model with constant returns to scale, equilibrium prices are determined by technology and not preferences.

2.14 How can the existence of fixed costs be incorporated into the production set diagram? After paying its fixed costs, a firm has constant returns to scale. Can it earn zero profits in a competitive economy?

2.15 Consider a two good exchange economy. Let the excess demand function for good 1 be given by

$$Z_1(p_1, p_2) = 3.426 + \delta - 10p_1 + 8(p_1)^2 - (p_1)^3 + \log(p_2).$$

a. Select good 2 as the numéraire and plot the excess demand function for $\delta = 0$. Show that the economy has two equilibria.

b. How many equilibria are there if $\delta = 0.1$? If $\delta = -0.1$? What can you conclude about the possiblity of observing an economy with an even number of equilibria?

c. Compute the excess demand function for good 2 using Walras's law. Can these excess demand functions result from utility maximization?

2.16 Consider a consumer with utility function $U = \log(x) + \log(1-\ell)$ and a firm with production function $x = (\ell)^{1/2}$, where x denotes output of a consumption good and ℓ denotes labor supply. Assume that the consumer receives the profit from the firm as a dividend and that both the firm and consumer act competitively. Choosing labor as the numéraire, find the maximized utility of the consumer and the maximized profit of the firm as functions of the price, p, of output. What value of p maximizes utility? What is special about this value of p?

2.17 Consider an economy with 2 goods, H consumers, and m firms. Each consumer, h, has an endowment of 2 units of good 1 and none of good 2, with the preferences described by $U^h = x_1^h x_2^h$, and a share $\theta_j^h = \frac{1}{H}$ in firms $j = 1, \ldots, m$. Each firm has a technology characterized by the production function $y_2^j = \left[-y_1^j\right]^{1/2}$.

a. Calculate a firm's profit-maximizing choices, a consumer's demands, and the competitive equilibrium of the economy.

b. What happens to $\frac{p_2}{p_1}$ as (i) m increases; (ii) H increases? Why?

c. Suppose that each consumer's endowment of good 1 increases to $2 + 2\delta$. Explain the change in relative prices.

d. What is the effect of changing

i. the distribution of endowments among consumers;

ii. the consumers' preferences to $U^h = \alpha \log\left(x_1^h\right) + \beta \log\left(x_2^h\right)$?

2.18 Reproduce the diagram for the Robinson Crusoe economy for a firm that has constant returns to scale. Under what conditions will it be efficient for the firm not to produce? What is the consumption level of the consumer in such a case? Provide an interpretation of this possibility.

2.19 After the payment of costs, fishing boat captains distribute the surplus to the owner and crew. Typically the owner receives 50 percent, the captain 30 percent, and the remaining 20 percent is distributed to crew according to status. (See *The Perfect Storm: A True Story of Man against the Sea* by Sebastian Junger Norton 1997.) Is this distribution Pareto-efficient? Is it equitable?

2.20 A box of chocolates is to be shared by two children. The box contains ten milk chocolates and ten dark chocolates. Neither child likes dark chocolates. Describe the Pareto-efficient allocations.

2.21 As economists are experts in resource allocation, you are invited by two friends to resolve a dispute about the shared use of a car. By applying Pareto-efficiency, how are you able to advise them? Are they impressed with your advice?

2.22 Two consumers have utility functions

$$U^h = \ln\left(x_1^h\right) + \ln\left(x_2^h\right).$$

a. Calculate the marginal rate of substitution between good 1 and good 2 in terms of consumption levels.

b. By equating the marginal rates of substitution for the two consumers, characterize a Pareto-efficient allocation.

c. Using the solution to part b, construct the contract curve for an economy with 2 units of good 1 and 3 units of good 2.

2.23 A university has a fixed sum of money to allocate in bonus payments between two professors. Each professor appreciates receiving the bonus but resents that the other professor recieves a bonus as well. Let the preferences of the two professors be given by

$$U_G = b_G - cb_J,$$

and

$$U_J = b_J - cb_G.$$

a. Describe the Pareto-efficient allocations when $c = 0$.

b. Is there a Pareto-efficient allocation when $0 < c < 1$?

c. What happens if $c \geq 1$?

2.24 A consumer views two goods as perfect substitutes.

a. Sketch the indifference curves of the consumer.

b. If an economy is composed of two consumers with these preferences, demonstrate that any allocation is Pareto-efficient.

c. If an economy has one consumer who views its two goods as perfect substitutes and a second that considers each unit of good 1 to be worth 2 units of good 2, find the Pareto-efficient allocations.

2.25 Consider an economy in which preferences are given by

$$U^1 = x_1^1 + x_2^1,$$

and

$$U^2 = \min\left\{x_1^2,\ x_2^2\right\}.$$

Given the endowments $\omega^1 = (1,\ 2)$ and $\omega^2 = (3,\ 1)$, construct the set of Pareto-efficient allocations and the contract curve. Which allocations are also competitive equilibria?

2.26 Take the economy in the exercise above, but change the preferences of consumer 2 to

$$U^2 = \max\left\{x_1^2,\ x_2^2\right\}.$$

Is there a Pareto-efficient allocation?

2.27 Consider an economy with two consumers, A and B, and two goods, 1 and 2. Using x_i^h to denote the consumption of good i by consumer h, assume that both consumers have the utility function $U^h = \min\left\{x_1^h,\ x_2^h\right\}$.

a. By drawing an Edgeworth box, display the Pareto-efficient allocations if the economy has an endowment of 1 unit of each good.

b. Display the Pareto-efficient allocations if the endowment is 1 unit of good 1 and 2 units of good 2.

c. What would be the competitive equilibrium prices for parts a and b?

2.28 Consider the economy in exercise 2.11.

a. Calculate the endowments required to make the equal-utility allocation a competitive equilibrium.

b. Discuss the transfer of endowment necessary to support this equilibrium.

2.29 Provide an example of a Pareto-efficient allocation that cannot be decentralized.

2.30 Let an economy have a total endowment of two units of the two available goods. If the two consumers have preferences

$$U^h = \alpha \log(x_1^h) + [1 - \alpha]\log(x_2^h),$$

find the ratio of equilibrium prices at the allocation where $U^1 = U^2$. Hence find the value of the lump-sum transfer that is needed to decentralize the allocation if the initial endowments are $\left(\frac{1}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{5}{4}\right)$.

2.31 Are the following statements true or false? Explain why in each case.

a. If one consumer gains from a trade, the other consumer involved in the trade must lose.

b. The gains from trade are based on comparative advantage, not absolute advantage.

c. The person who can produces the good with less input has an absolute advantage in producing this good.

d. The person who has the smaller opportunity cost of producing the good has a comparative advantage in producing this good.

e. The competitive equilibrium is the only allocation where the gains from trade are exhausted.