MICROECONOMICS

Principles and Analysis

A SIMPLE ECONOMY

THE SETTING…

- A closed economy
  - Prices determined internally
- A collection of natural resources
  - Determines incomes
- A variety of techniques of production
  - Also determines incomes
- A single economic agent
  - R. Crusoe

NOTATION AND CONCEPTS

$\mathbf{R} = (R_1, R_2,\ldots, R_n)$ ▪resources (available for consumption or production)

$\mathbf{q} = (q_1, q_2,\ldots, q_n)$ ▪net outputs (more on this soon...)

$\mathbf{x} = (x_1, x_2,\ldots, x_n)$ ▪consumption (just the same as before)

OVERVIEW…

Production in a multi-output, multi-process world

- Household Demand & Supply
- Structure of production
- The Robinson Crusoe problem
- Decentralisation
- Markets and trade

NET OUTPUT CLEARS UP PROBLEMS

- "Direction" of production
  - Get a more general notation
- Ambiguity of some commodities
  - Is paper an input or an output?
- Aggregation over processes
  - How do we add my inputs and your outputs?

APPROACHES TO OUTPUTS AND INPUTS

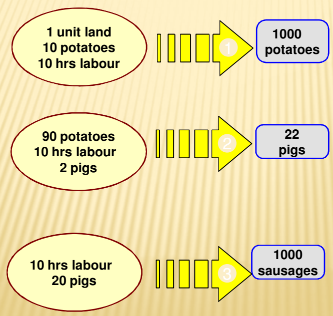
- *A standard "accounting" approach*
- *An approach using "net outputs"*
- *How the two are related*
- *A simple sign convention*

NET OUTPUTS	OUTPUT	INPUTS
q_1		z_1
q_2		z_2
…		…
q_{n-1}		z_m
q_n	q	

$$\begin{bmatrix} q_1 \\ q_2 \\ \cdots \\ q_{n-1} \\ q_n \end{bmatrix} = \begin{bmatrix} -z_1 \\ -z_2 \\ \cdots \\ -z_m \\ +q \end{bmatrix}$$

Outputs:	+	net additions to the stock of a good
Inputs:	–	reductions in the stock of a good
Intermediate goods:	0	your output and my input cancel each other out

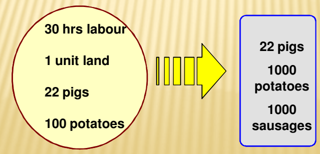
MULTISTAGE PRODUCTION



- *Process 1 produces a consumption good / input*
- *Process 2 is for a pure intermediate good*
- *Add process 3 to get 3 interrelated processes*

COMBINING THE THREE PROCESSES

	Process 1		Process 2		Process 3		Economy's net output vector
sausages	0		0		+1000		+1000
potatoes	+990		– 90		0		+900
pigs	0	+	20	+	–20	=	0
labour	–10		–10		–10		–30
land	–1		0		0		–1



MORE ABOUT THE POTATO-PIG-SAUSAGE STORY

- We have described just one technique
- What if more were available?
- What would be the concept of the isoquant?
- What would be the marginal product?
- What would be the trade-off between outputs?

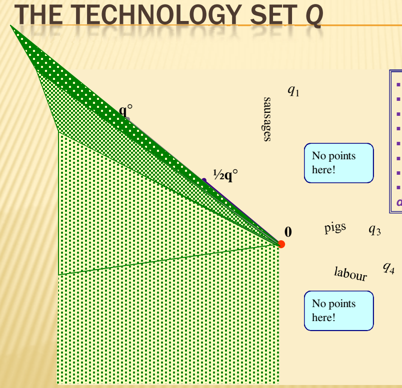
AN AXIOMATIC APPROACH

- Let *Q* be the set of all technically feasible net output vectors.
  - The *technology set.*
- "$\mathbf{q} \in Q$" means "**q** is technologically do-able"
- The shape of *Q* describes the nature of production possibilities in the economy.
- We build it up using some standard production axioms.

STANDARD PRODUCTION AXIOMS

- Possibility of Inaction
  - $\mathbf{0} \in Q$
- No Free Lunch
  - $Q \cap \mathrm{R}_+^n = \{\mathbf{0}\}$
- Irreversibility
  - $Q \cap (-Q) = \{\mathbf{0}\}$
- Free Disposal
  - If $\mathbf{q} \in Q$ and $\mathbf{q}' \leq \mathbf{q}$ then $\mathbf{q}' \in Q$
- Additivity
  - If $\mathbf{q} \in Q$ and $\mathbf{q}' \in Q$ then $\mathbf{q}+\mathbf{q}' \in Q$
- Divisibility
  - If $\mathbf{q} \in Q$ and $0 \leq t \leq 1$ then $t\mathbf{q} \in Q$

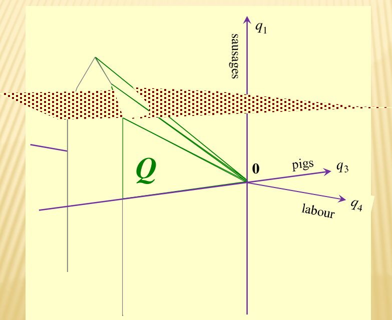
THE TECHNOLOGY SET *Q*



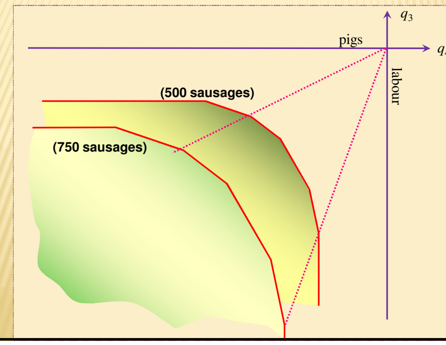
- *Possibility of inaction*
- *"No free lunch"*
- *Some basic techniques*
- *Irreversibility*
- *Free disposal*
- *Additivity*
- *Divisibility*
- *Implications of additivity and divisibility*

- *Additivity+Divisibility imply constant returns to scale.*
- *In this case Q is a cone.*

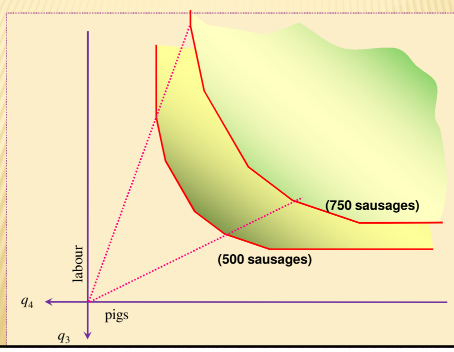
A "HORIZONTAL" SLICE THROUGH *Q*



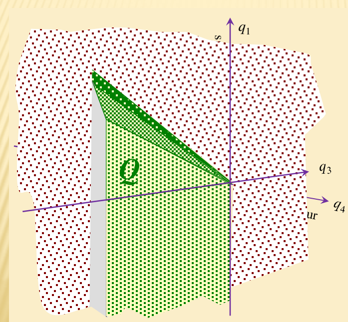
… TO GET THE TRADEOFF IN INPUTS



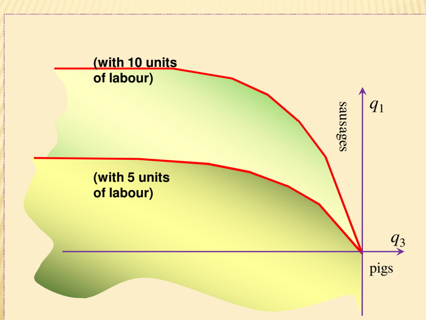
…FLIP THESE TO GIVE ISOQUANTS



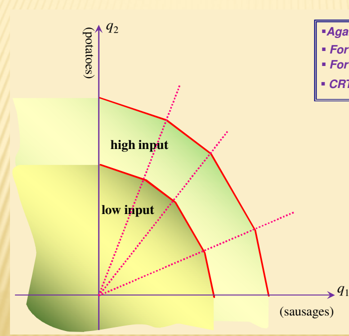
A "VERTICAL" SLICE THROUGH *Q*



THE PIG-SAUSAGE RELATIONSHIP



THE POTATO-SAUSAGE TRADEOFF



- *Again take slices through Q*
- *For low level of inputs*
- *For high level of inputs*
- *CRTS*

NOW REWORK A CONCEPT WE KNOW

- In earlier presentations we used a simple production function.
- A way of characterising technological feasibility in the 1-output case.
- Now we have defined technological feasibility in the many-input many-output case...
- ...using the set *Q*.
- So let's use this to define a production function for this general case...

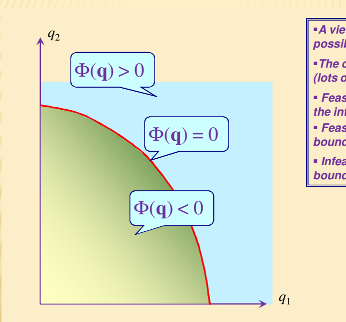
TECHNOLOGY SET AND PRODUCTION FUNCTION

- The technology set *Q* and the production function Φ are two ways of representing the same relationship:

$$\mathbf{q} \in Q \iff \Phi(\mathbf{q}) \leq 0$$

- Properties of Φ inherited from the properties with which *Q* is endowed.
- $\Phi(q_1, q_2, \ldots, q_n)$ is nondecreasing in each net output q_i.
- If *Q* is a convex set then Φ is a concave function.
- As a convention $\Phi(\mathbf{q}) = 0$ for efficiency, so...
- $\Phi(\mathbf{q}) \leq 0$ for feasibility.

THE SET *Q* AND THE FUNCTION Φ



- *A view of the set Q: production possibilities of two outputs.*
- *The case if Q has a smooth frontier (lots of basic techniques)*
- *Feasible but inefficient points in the interior*
- *Feasible and efficient points on the boundary*
- *Infeasible points outside the boundary*

HOW THE TRANSFORMATION CURVE IS DERIVED

- Do this for given stocks of resources.
- Position of transformation curve depends on technology and resources
- Changing resources changes production possibilities of consumption goods

OVERVIEW...

A simultaneous production-and-consumption problem

- Household Demand & Supply
  - Structure of production
  - The Robinson Crusoe problem
  - Decentralisation
  - Markets and trade

SETTING

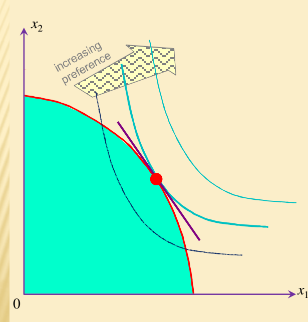
- A single isolated economic agent.
  - No market
  - No trade (as yet)
- Owns all the resource stocks **R** on an island.
- Acts as a rational consumer.
  - Given utility function
- Also acts as a producer of some of the consumption goods.
  - Given production function

THE CRUSOE PROBLEM (1)

- max $U(\mathbf{x})$ by choosing $\mathbf{x}$ and $\mathbf{q}$, subject to...
  - a joint consumption and production decision
- $\mathbf{x} \in X$
  - logically feasible consumption
- $\Phi(\mathbf{q}) \le 0$
  - technical feasibility: equivalent to "$\mathbf{q} \in Q$"
- $\mathbf{x} \le \mathbf{q} + \mathbf{R}$
  - materials balance: you can't consume more of any good than is available from net output + resources

The facts of life

CRUSOE'S PROBLEM AND SOLUTION



- Attainable set with $R_1 = R_2 = 0$
- Positive stock of resource 1
- More of resources 3,…,n
- Crusoe's preferences
- The optimum
- The FOC

- *Attainable set derived from technology and materials balance condition.*
- *MRS = MRT:*

$$\frac{U_i(\mathbf{x})}{U_j(\mathbf{x})} = \frac{\Phi_i(\mathbf{q})}{\Phi_j(\mathbf{q})}$$

THE NATURE OF THE SOLUTION

- From the FOC it seems as though we have two parts...
  1. A standard consumer optimum
  2. Something that looks like a firm's optimum
- Can these two parts be "separated out"...?
- A story:
  - Imagine that Crusoe does some accountancy in his spare time.
  - If there were someone else on the island ("Man Friday"?) he would delegate the production...
  - ...then use the proceeds from the production activity to maximise his utility.
- But to investigate this possibility we must look at the nature of income and profits

OVERVIEW...

The role of prices in separating consumption and production decision-making

- Household Demand & Supply
  - Structure of production
  - The Robinson Crusoe problem
  - **Decentralisation**
  - Markets and trade

THE NATURE OF INCOME AND PROFITS

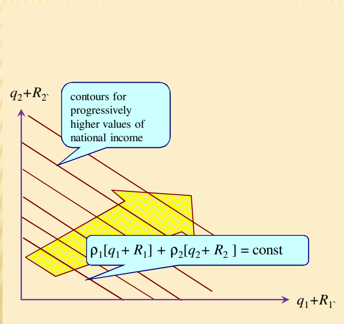
- The island is a closed and the single economic actor (Crusoe) has property rights over everything.
- Consists of "implicit income" from resources **R** and the surplus (Profit) of the production processes.
- We could use
  - the endogenous income model of the consumer
  - the definition of profits of the firm
- But there is no market and therefore no prices.
  - We may have to "invent" the prices.
- Examine the application to profits.

PROFITS AND INCOME AT SHADOW PRICES

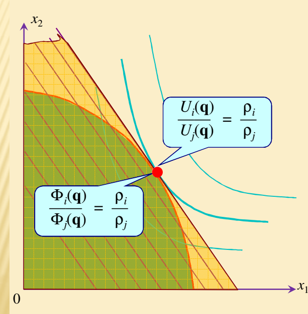
- We know that there is no system of prices.
- Invent some "shadow prices" for accounting purposes.
- Use these to value national income

$\rho_1 q_1 + \rho_2 q_2 + \ldots + \rho_n q_n$	profits
$\rho_1 R_1 + \rho_2 R_2 + \ldots + \rho_n R_n$	value of resource stocks
$\rho_1[q_1+R_1] + \ldots + \rho_n[q_n+R_n]$	value of national income

NATIONAL INCOME CONTOURS



“NATIONAL INCOME” OF THE ISLAND



- *Attainable set*
- *Iso-profit – income maximisaton*
- *The Island’s “budget set”*
- *Use this to maximise utility*

- *Using shadow prices* **ρ** *we’ve broken down the Crusoe problem into a two-step process*:

1. *Profit maximisation*
2. *Utility maximisation*

A SEPARATION RESULT

- By using “shadow prices” ρ ...
- ...a global maximisation problem
- ...is separated into sub-problems:

$$\max\ U(\mathbf{x}) \text{ subject to } \mathbf{x} \le \mathbf{q} + \mathbf{R},\quad \Phi(\mathbf{q}) \le 0$$

1. An income-maximisation problem.

$$\max \sum_{i=1}^{n} \rho_i [q_i+R_i] \text{ subj. to } \Phi(\mathbf{q}) \le 0$$

2. A utility maximisation problem

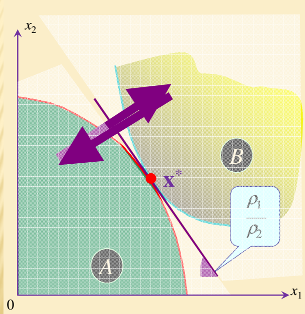
$$\max\ U(\mathbf{x}) \text{ subject to } \sum_{i=1}^{n} \rho_i x_i \le y$$

- Maximised income from 1 is used in problem 2

THE SEPARATION RESULT

- The result raises an important question…
- Can this trick *always* be done?
- It depends on the structure of the components of the problem.
- To see this let’s rework the Crusoe problem.
- Visualise it as a simultaneous value-maximisation and value minimisation.
- Then see if you can spot why the separation result works…

CRUSOE PROBLEM: ANOTHER VIEW



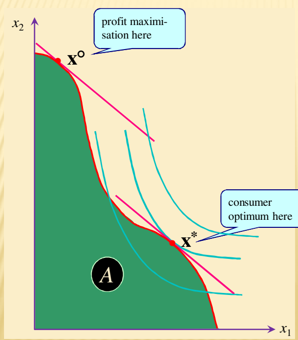
- *The attainable set*
- *The “Better-than-x* ” set*
- *The price line*
- *Decentralisation*

- $A = \{\mathbf{x}: \mathbf{x} \le \mathbf{q}+\mathbf{R},\ \Phi(\mathbf{q}) \le 0\}$
- $B = \{\mathbf{x}: U(\mathbf{x}) \ge U(\mathbf{x}^*)\}$
- *x* maximises income over A*
- *x* minimises expenditure over B*

THE ROLE OF CONVEXITY

- The “separating hyperplane” theorem is useful here.
  - Given two convex sets *A* and *B* in R^n with no points in common, you can pass a hyperplane between *A* and *B*.
  - In R^2 a hyperplane is just a straight line.
- In our application:
- *A* is the Attainable set.
  - Derived from production possibilities+resources
  - Convexity depends on divisibility of production
- *B* is the “Better-than” set.
  - Derived from preference map.
  - Convexity depends on whether people prefer mixtures.
- The hyperplane is the price system.

OPTIMUM CANNOT BE DECENTRALISED



- *A nonconvex attainable set*
- *The consumer optimum*
- *Implied prices: MRT=MRS*
- *Maximise profits at these prices*

- *Production responses do not support the consumer optimum.*
- *In this case the price system "fails"*

OVERVIEW...

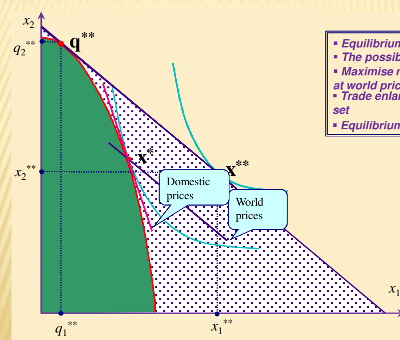
How the market simplifies the simple model

- Household Demand & Supply
- Structure of production
- The Robinson Crusoe problem
- Decentralisation
- Markets and trade

INTRODUCING THE MARKET AGAIN...

- Now suppose that Crusoe has contact with the world.
- This means that he is not restricted to "home production."
- He can buy/sell at world prices.
- This development expands the range of choice.
- ...and enters the separation argument in an interesting way.

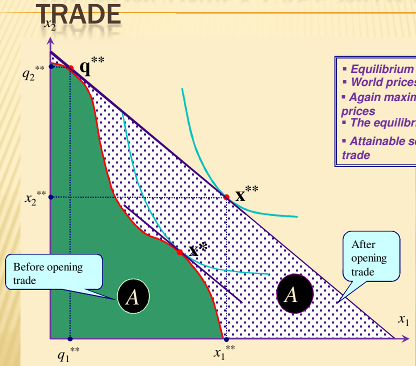
CRUSOE'S ISLAND TRADES



- *Equilibrium on the island*
- *The possibility of trade*
- *Maximise national income at world prices*
- *Trade enlarges the attainable set*
- *Equilibrium with trade*

- $\mathbf{x}^*$ *is the* *Autarkic equilibrium*: $x_1^* = q_1^*; x_2^* = q_2^*$
- *World prices imply a revaluation of national income.*
- *In this equilibrium the gap between* $\mathbf{x}^{**}$ *and* $\mathbf{q}^{**}$ *is bridged by imports & exports*

THE NONCONVEX CASE WITH WORLD TRADE



- *Equilibrium on the island*
- *World prices*
- *Again maximise income at world prices*
- *The equilibrium with trade*
- *Attainable set before and after trade*

- *Trade "convexifies" the attainable set*

"CONVEXIFICATION"

- There is nothing magic about this.
- When you write down a conventional budget set you are describing a convex set
  - $\Sigma\, p_i x_i \leq y, \quad x_i \geq 0.$
- When you "open up" the model to trade you change
  - from a world where $\Phi(\cdot)$ determines the constraint
  - to a world where a budget set determines the constraint
- In the new situation you can apply the separation theorem.

THE ROBINSON CRUSOE ECONOMY

- The global maximum is simple.
- But can be split up into two separate parts.
  - Profit (national income) maximisation.
  - Utility maximisation.
- All this relies on the fundamental *decentralisation* result for the price system.
- Follows from the *separating hyperplane* result.
- "You can always separate two eggs with a single sheet of paper"