MICROECONOMICS
Principles and Analysis

GENERAL EQUILIBRIUM: PRICE TAKING

PUZZLES IN COMPETITIVE EQUILIBRIUM ANALYSIS

- So far we have focused on competitive equilibrium analysis.
  - But why?
  - Why concentrate on equilibrium?
  - Why assume competitive behaviour?

- Here we re-examine the basics of market interaction by agents.

- Let's start by having another look at the exchange economy.

- We'll redraw the Edgeworth box.

OVERVIEW...

The offer curve as a tool of analysis

General Equilibrium: price taking

An exchange economy

The solution concept

Prices and the Core

THE EDGEWORTH BOX

- ✖ Remember that the Edgeworth Box is a 2×2 representation of an exchange economy:
  - ✚ Two goods.
  - ✚ Two persons Alf and Bill.

- ✖ Represent the equilibrium for each person given:
  - ✚ Price-taking behaviour.
  - ✚ Ownership of the resources.

- ✖ Introduce the materials balance condition...

- ✖ ...achieved by inverting one diagram to complete the "box."

BEHAVIOUR OUT OF EQUILIBRIUM

* First let's see why the CE is of such significance.
* To do so consider a simple question:
  + If Alf and Bill are price takers, what will they do in situations other than equilibrium?

* To answer this use a familiar tool.
  + The *offer curve*.
  + Introduced in consumer demand.
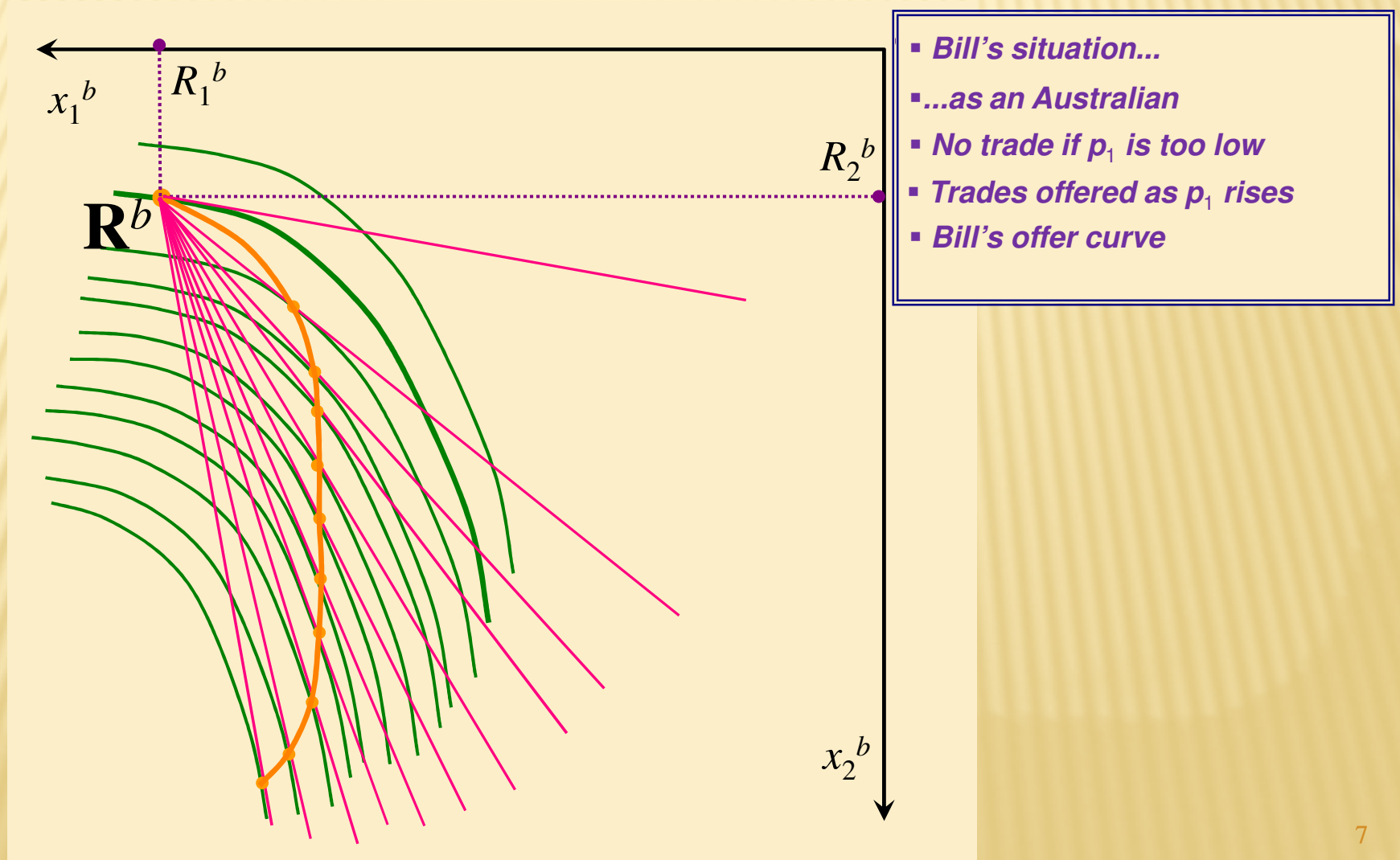* To get this re-examine the optimisation problems
  + First Alf
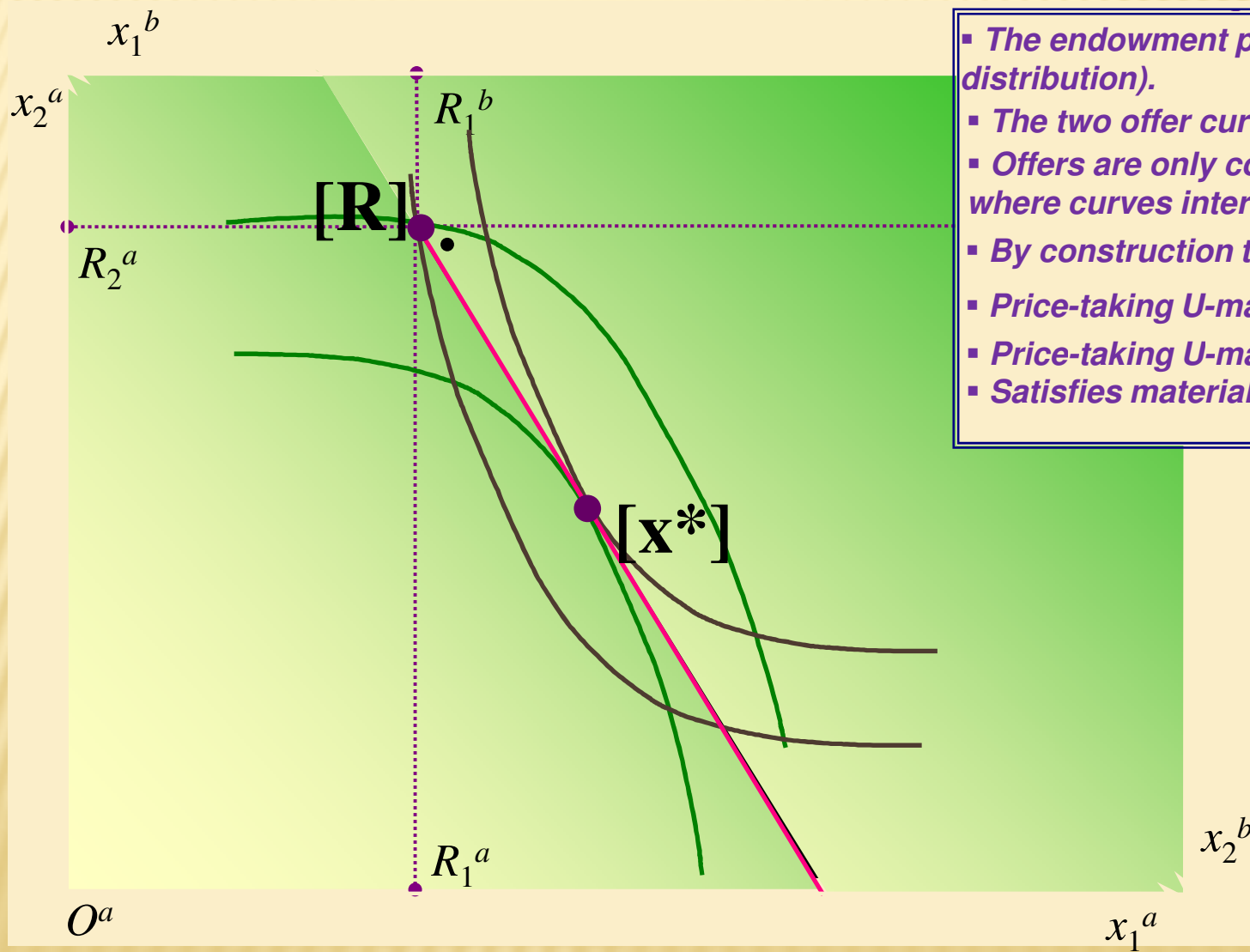 + Then Bill

ALF'S RESPONSES TO CHANGES IN P_1/P_2
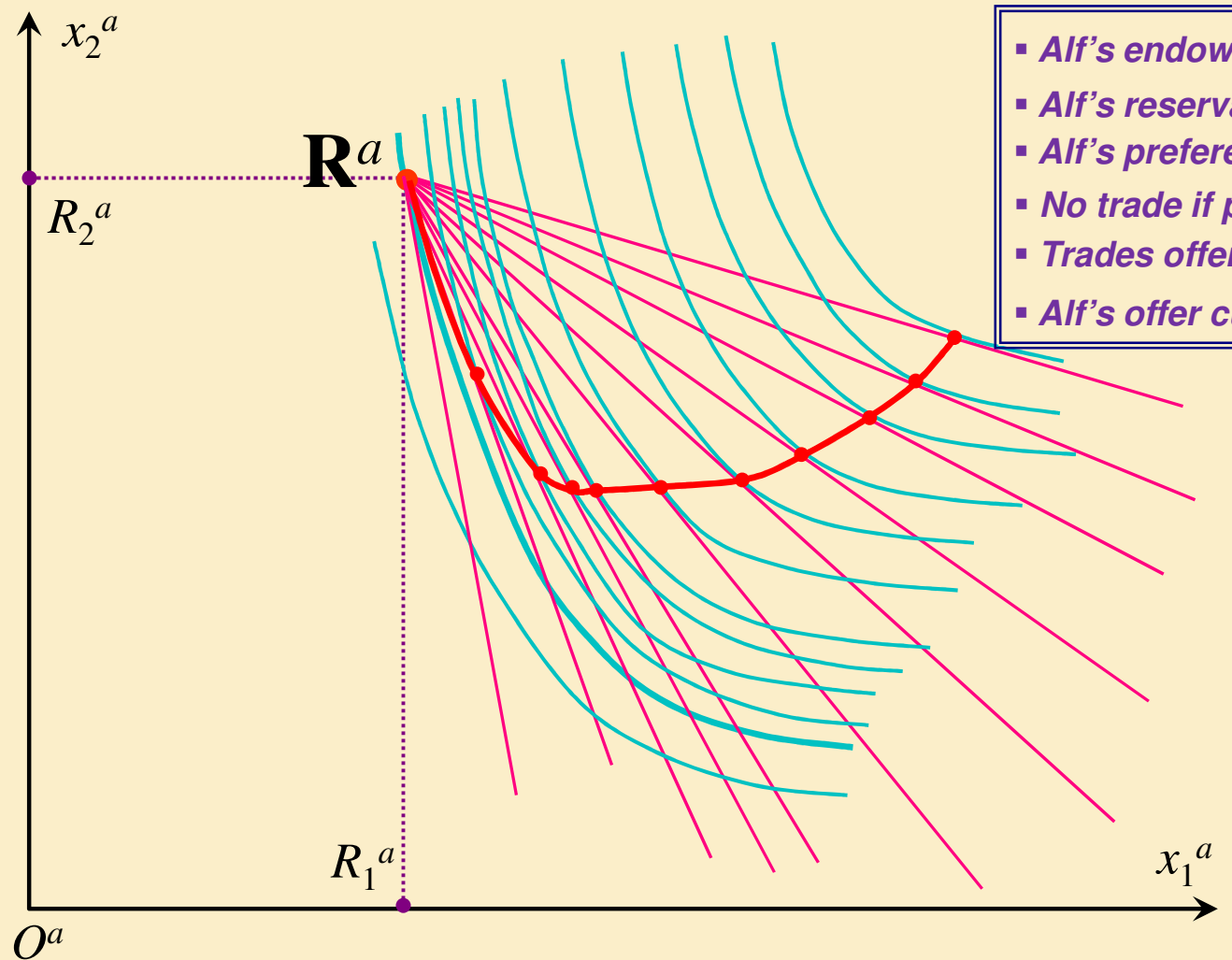
- Alf's endowment
- Alf's reservation utility
- Alf's preference map
- No trade if p_1 is too high
- Trades offered as p_1 falls
- Alf's offer curve

- *Bill's situation...*
- *...as an Australian*
- *No trade if p_1 is too low*
- *Trades offered as p_1 rises*
- *Bill's offer curve*

x_1^b

R_1^b

R_2^b

$\mathbf{R}^b$

x_2^b

EDGEWORTH BOX AND CE

x_1^b

x_2^a

R_1^b

R_2^a

[R]

R_1^a

O^a

x_2^b

x_1^a

[x*]

- **The endowment point (property distribution).**
- **The two offer curves**
- **Offers are only consistent where curves intersect**
- **By construction this is CE**
- **Price-taking U-maximising Alf**
- **Price-taking U-maximising Bill**
- **Satisfies materials balance**
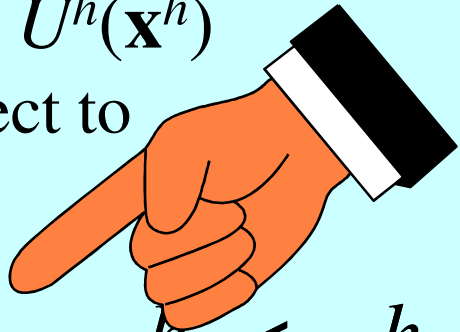
THE NATURE OF CE

- ✖ Given competitive behaviour, the CE is the only "consistent" allocation.
- ✖ Clearly the location of the CE depends upon the initial resource endowment [R].
- ✖ But why assume competitive behaviour?
- ✖ Why should Alf and Bill behave as price-takers?

WHERE DO THE PRICES COME FROM?

- The "rules of the game" assume that people act as price takers and that prices are "given"

- Then people can solve the standard optimisation problem.

- But where do the prices come from?

- We can't appeal to invented "shadow" prices

- Nor to "world markets"

- Nor to some external agency…

$$\max\ U^h(\mathbf{x}^h)$$
subject to

$$\sum_{i=1}^{n} p_i x_i^h \leq y^h$$

HOW TO MAKE PROGRESS

- It would be convenient to assume there is a big hand....
  - ...given the prices the system almost solves itself
  - But we have to manage without the artificial construct.
  - How?
- We need a more general solution concept.
- Base this on a broader concept of trading behaviour.
- We will describe the type of equilibrium associated with this concept.
- Then we examine how price-taking equilibrium relates to this.

OVERVIEW...

Blocking and the core

General Equilibrium: price taking

An exchange economy

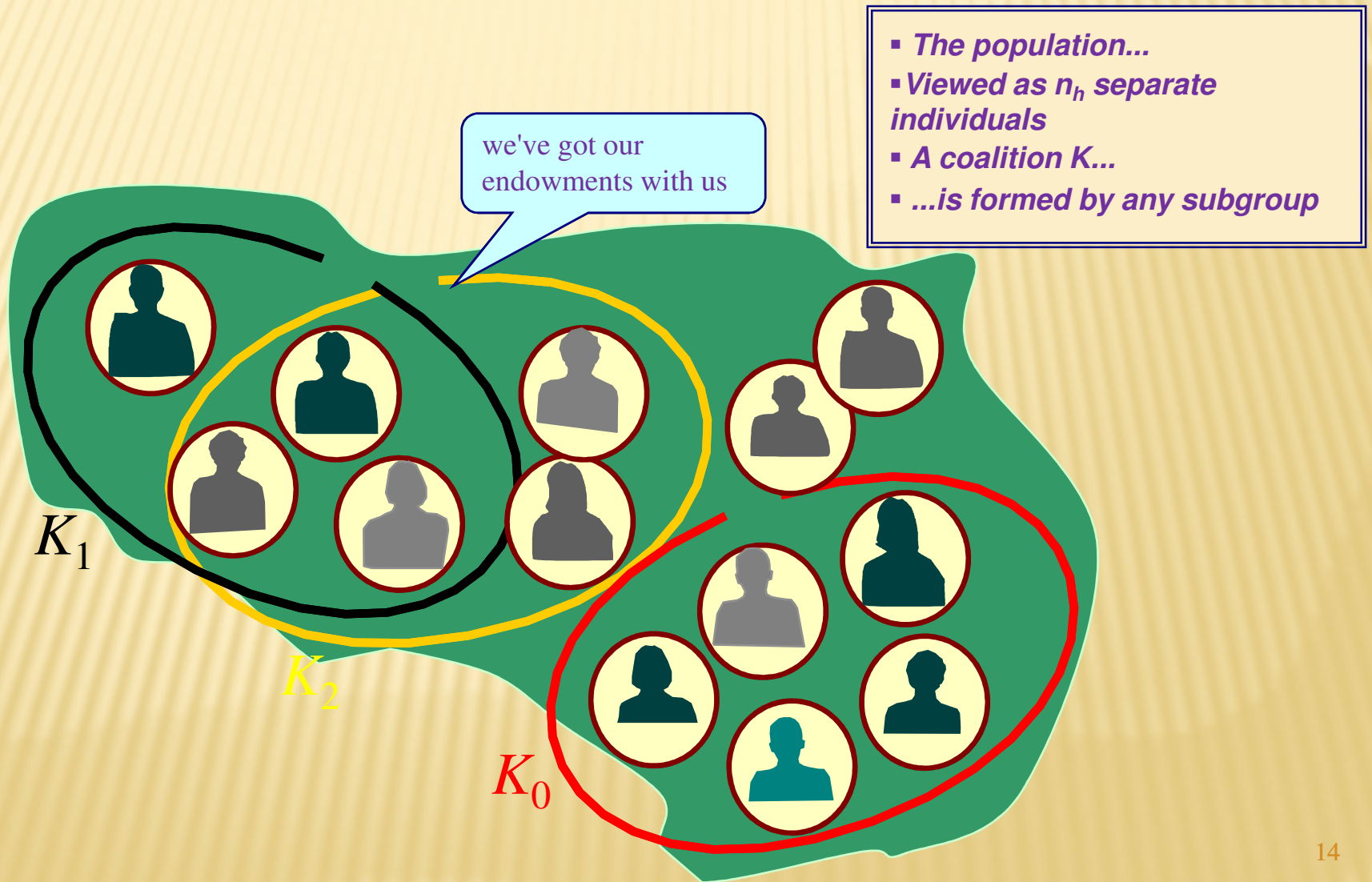The solution concept

Prices and the Core

A FRESH APPROACH

- ✖ Develop the approach for an exchange economy.
- ✖ But it could apply to more interesting economies.
  - ✛ To do it for production usually involves some strong assumptions.
- ✖ Imagine this as the economics of a PoW camp.
- ✖ The rules of the game are very simple:

  $\mathbf{R}^h$

  - ✛ Each person is endowed with a given bundle of goods
  - ✛ Each person has absolute right of disposal over this bundle.

  No-one is forced to trade/exchange

  - ✛ Everyone is free to associate with others to form coalitions.

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COALITIONS

we've got our endowments with us

- **The population...**
- **Viewed as n_h separate individuals**
- **A coalition K...**
- **...is formed by any subgroup**

K_1

K_2

K_0

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THE IDEA OF *BLOCKING*: A STORY

- One day you take your bundle to the "swap shop."
- Some bossy person there proposes (insists on?) a particular feasible allocation.
- You and some others don't like the bundle you all get under this allocation.
- Your group finds that, just by using its own resources, you could all get as much or more utility as that offered under the proposed allocation.
- You guys therefore refuse to accept the proposal.
- Your coalition has *blocked* the proposed allocation

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A FORMAL APPROACH

- ✖ Consider a proposed allocation for the economy
  $[\hat{\mathbf{x}}]$
- ✖ A coalition
  $K \subseteq \{1,2,...,n_h\}$
- ✖ An allocation [x] preferred by the coalition K:
  $\forall h \in K: U^h(\mathbf{x}^h) \geq U^h(\hat{\mathbf{x}}^h),$ for some $h \in K: U^h(\mathbf{x}^h) > U^h(\hat{\mathbf{x}}^h)$
- ✖ The allocation [x] of bundles is feasible for K if:
  $\Sigma_{h \in K} \mathbf{x}^h \leq \Sigma_{h \in K} \mathbf{R}^h$
- ✖ If there is a feasible, preferred bundle for K then
  ... $[\hat{\mathbf{x}}]$ is *blocked* by K

An allocation is blocked by a coalition if the coalition members can do better for themselves

EQUILIBRIUM CONCEPT

- Use the idea of blocking to introduce a basic solution concept.

- Surely no blocked allocation could be a solution to the trading game?

- So we use the following definition of a solution:

- The *Core* is the set of unblocked, feasible allocations.

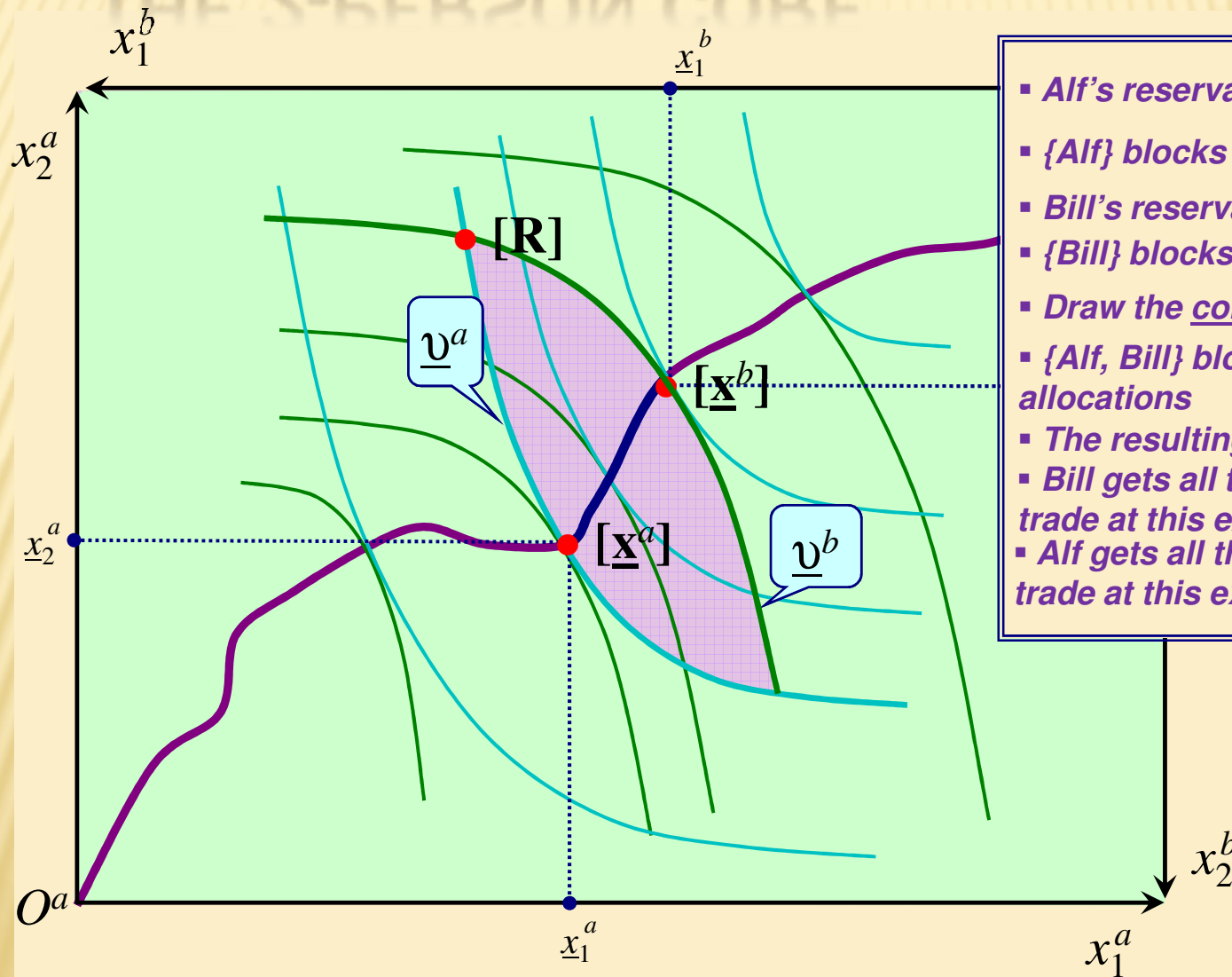- Let's apply it in the two-trader case.

COALITIONS

  ✖ In a 2-person world there are few coalitions:

   {Alf }

   {Bill}

   {Alf & Bill}

THE 2-PERSON CORE

- **Alf's reservation utility**
- **{Alf} blocks these allocations**
- **Bill's reservation utility**
- **{Bill} blocks these allocations**
- **Draw the <u>contract curve</u>**
- **{Alf, Bill} blocks these allocations**
- **The resulting core**
- **Bill gets all the advantage from trade at this extreme point.**
- **Alf gets all the advantage from trade at this extreme point.**

▪ *The contract curve is the locus of common tangencies*

THE CORE: SUMMARY

- Definition of the core follows immediately from:
  + The definition of an allocation.
  + The definition of blocking.
- It is a general concept.
- To find the core you need just:
  + A complete description of the property distribution.
  + An enumeration of the possible coalitions.
  + A certain amount of patience.
- The major insight from the core comes when we examine the relation to CE.

OVERVIEW...

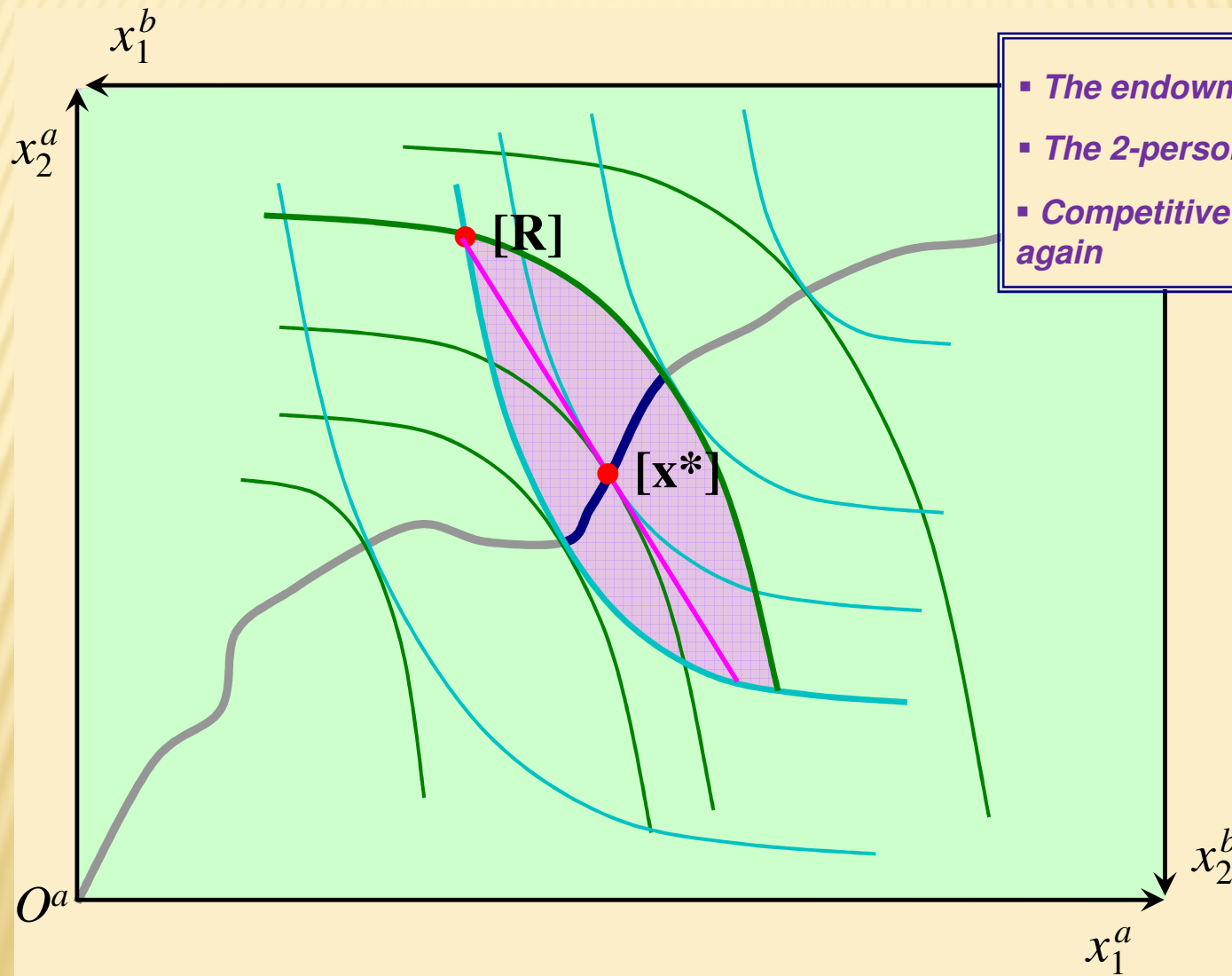**Competitive equilibrium, large numbers and a limit theorem**

General Equilibrium: price taking

An exchange economy

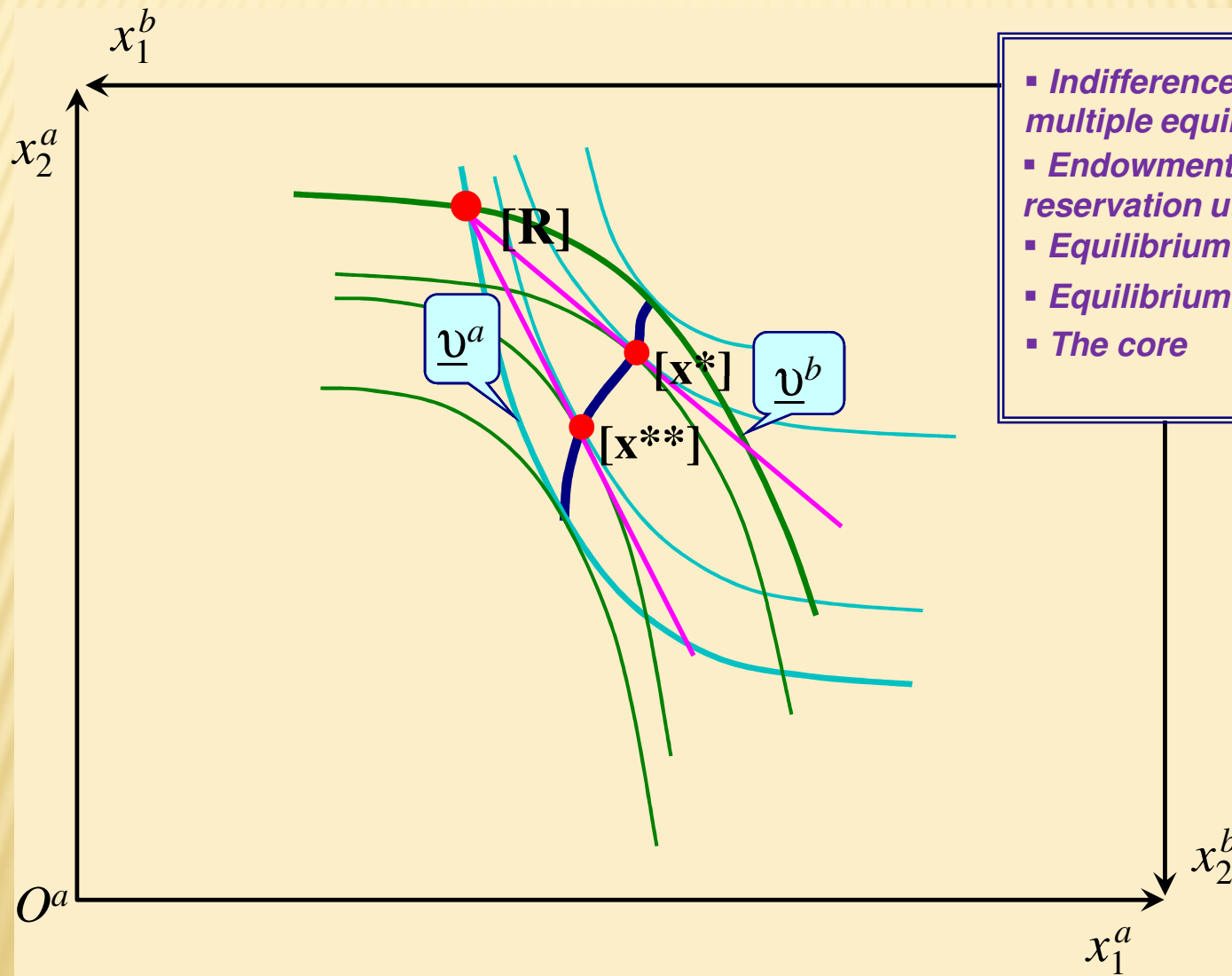The solution concept

Prices and the Core

- *The endowment point*
- *The 2-person core again*
- *Competitive equilibrium again*

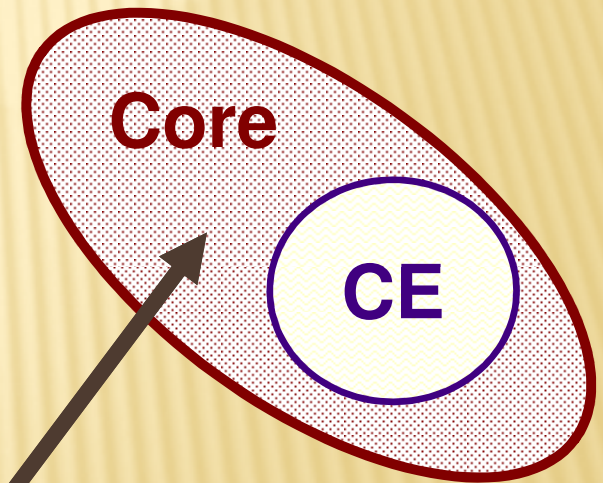- *A competitive equilibrium must always be a core allocation*

x_1^b

x_2^a

$[R]$

$[x^*]$

O^a

x_2^b

x_1^a

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A SIMPLE RESULT

- Every CE allocation must belong to the core.
- It is possible that no CE exists.
- But what of other core allocations which are not CE?
  - Remember we are dealing with a 2-person model.
  - But will there always be non-CE points in the core?
- Let's take a closer look...

Core

CE

SO LET'S CLONE THE ECONOMY

- Assume that the economy is *replicated* by a factor N, so that there are $2N$ persons.
- Start with $N=2$:
  - We move from a 2-person economy to a 4-person economy.
  - Alf and his twin brother Arthur have the same preferences and endowments.
  - Likewise the twins Bill and Ben.
- Now of course there are more possibilities of forming coalitions.

✖ All old coalitions are still possible…

▪ … plus some new ones

{Alf}   {Arthur}

{Bill}   {Ben}

{Alf & Bill}   {Arthur & Ben}

{Alf & Arthur}   {Bill&Ben}

{Alf, Arthur &Bill}   {etc, etc}

EFFECT OF CLONING ON THE CORE

[R], Ben, Alf, Arthur, Bill, $[\mathbf{x}^b]$, $[\mathbf{x}^a]$
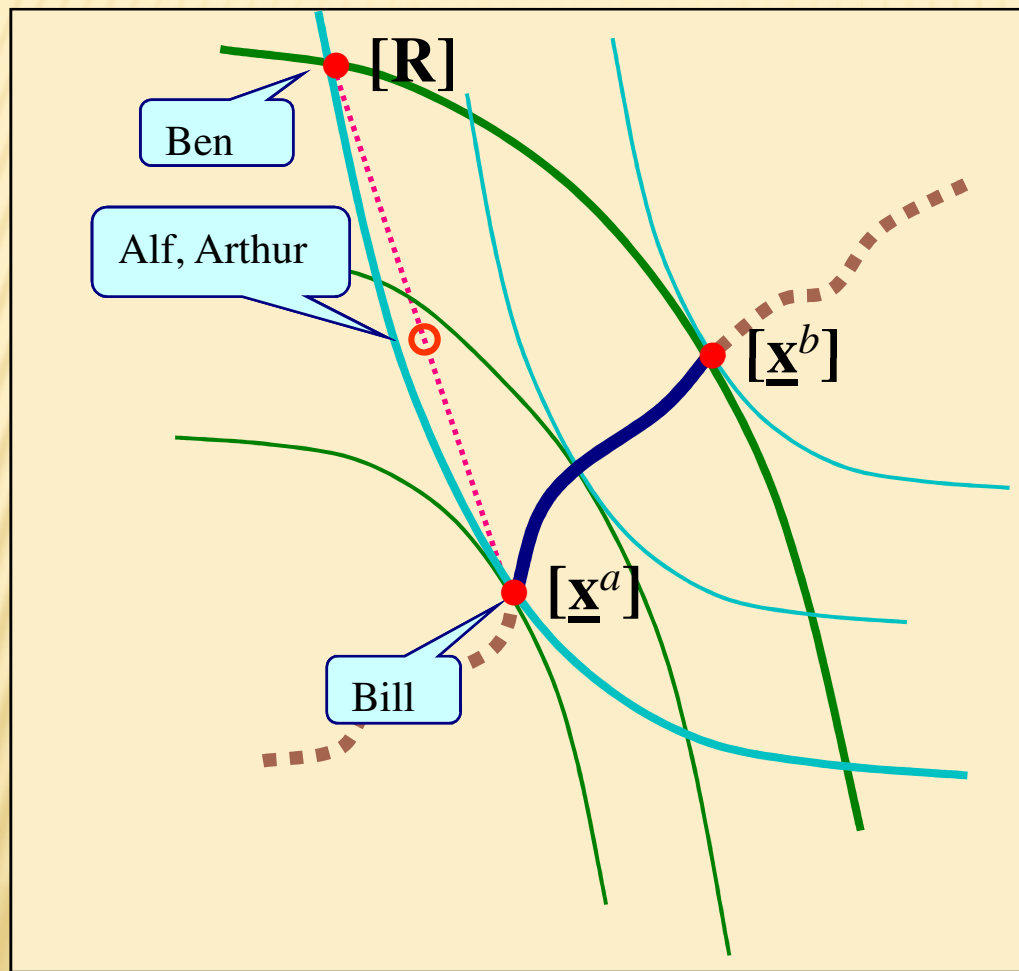
- *The core in the 2-person case*
- *The extremes of the two-person core*
- *{Alf,Arthur,Bill} can block* $[\underline{\mathbf{x}}^a]$...
- *...leaving the Ben twin* *outside the coalition*

- *Are the extremes still core allocations in the 4-person economy?*
- *This new allocation is not a solution...*
- *But it shows that the core must have become smaller*

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- *Consumption in the coalition*
- *Sum to get resource requirement*

- *Consumption out of coalition*

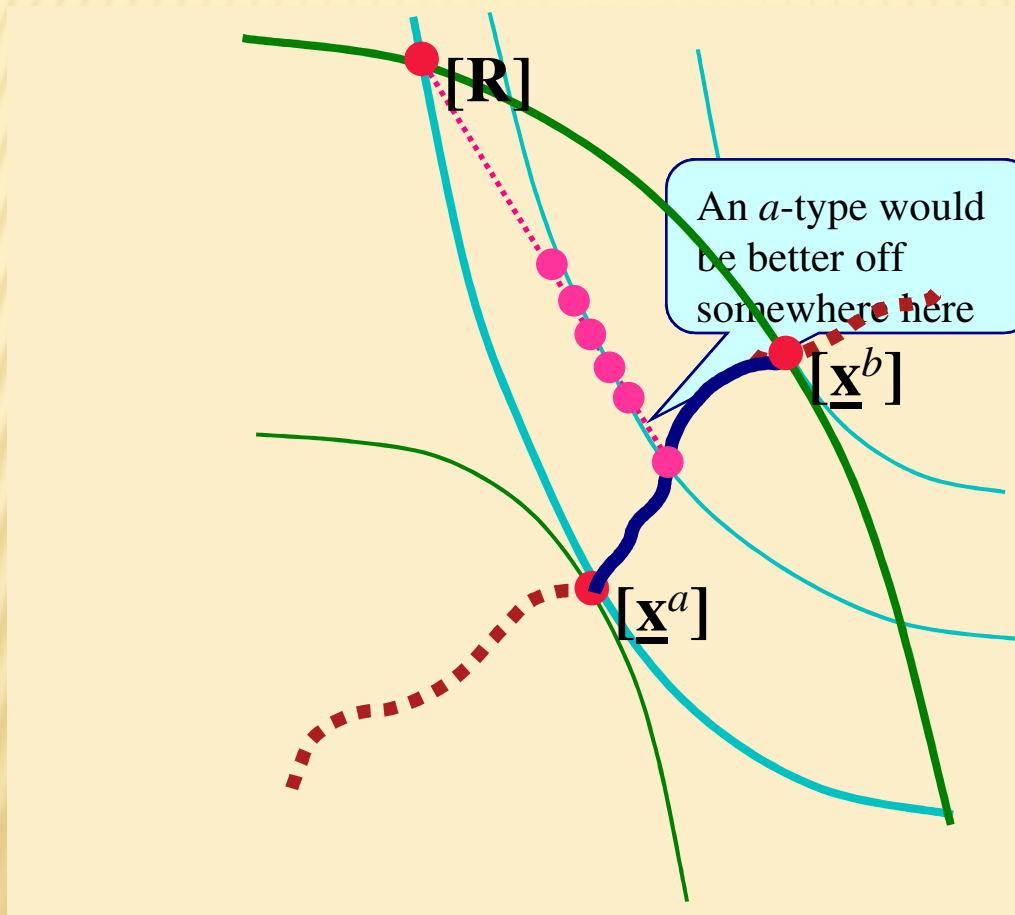|	
Alf	$\mathbf{x}^a = \tfrac{1}{2}[\underline{\mathbf{x}}^a + \mathbf{R}^a]$
Arthur	$\mathbf{x}^a = \tfrac{1}{2}[\underline{\mathbf{x}}^a + \mathbf{R}^a]$
Bill	$\dfrac{[2\mathbf{R}^a + \mathbf{R}^b - 2\mathbf{x}^a]}{2\mathbf{R}^a + \mathbf{R}^b}$
Ben	$\mathbf{R}^b$

- *The consumption within the coalition equals the coalition's resources.*

- *So the allocation is feasible.*

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IF *N* IS BIGGER: MORE BLOCKING COALITIONS?

[R]

An *a*-type would be better off somewhere here
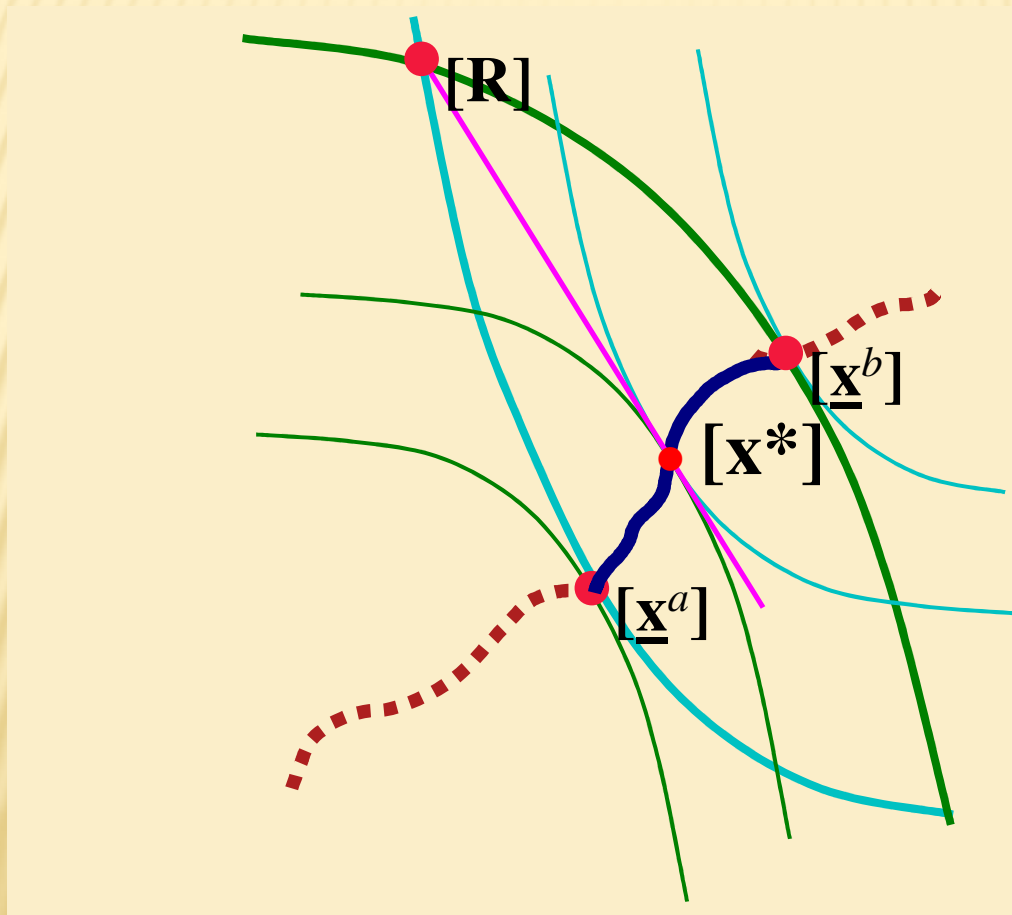
[$\underline{\mathbf{x}}^b$]

[$\underline{\mathbf{x}}^a$]

- *The 2-person core*
- *An arbitrary allocation - can it be blocked?*
- *Draw a line to the endowment*
- *Take N=500 of each tribe.*
- *Divide the line for different coalition numbers.*

- *We've found the blocking coalition.*

- *If line is not a tangent this can always be done.*

numbers of…	
a-tribe	*b-tribe*
500	450

IN THE LIMIT

- *If N→∞ a coalition can be found that divides the line to [R] in any proportion you want.*

- *Only if the line is like this will the allocation be impossible to block.*

- *With the large N the core has "shrunk" to the set of CE*

A POWERFUL RESULT: THE SHRINKING CORE

- As you clone the economy the core becomes smaller.
- If you make N large enough you will find some coalition that blocks any non-CE allocation.
- So in the limit the core contains only CE allocations.
- In a suitably large economy the core exactly equals the set of competitive equilibria.

THE SHRINKING CORE: DISCUSSION

- ✖ The result rules out non-price-taking behaviour as a solution. But:
- ✖ There are some weasel words: "suitably large".
  - ✛ In principle N should be infinite
- ✖ Process requires balanced replication of the Alf and Bill tribes.
  - ✛ Problems arise if there is one large b-trader and many a-traders
- ✖ All possible coalitions are assumed relevant to negotiations about blocking.
  - ✛ Only valid if communication and other coalition costs are negligible. The Internet?
- ✖ We have argued only using an exchange economy.
  - ✛ Can be extended to production economies with CRTS and (with some difficulty) others too.

REVIEW

- ✖ Basic components of trading equilibrium:
    - ✚ Coalitions
    - Blocking
    - Core as an equilibrium concept
- Relation to CE
    - ✚ Every CE must lie in the core
    - ✚ In the limit of a replication economy the core consists *only* of CE
-
- ✖ Answer to question: why price-taking?
    - ✚ In a large economy with suitably small agents...
    - ✚ ....it's the only thing to do.