MICROECONOMICS
Principles and Analysis

GENERAL EQUILIBRIUM: PRICE TAKING

PUZZLES IN COMPETITIVE EQUILIBRIUM ANALYSIS

- So far we have focused on competitive equilibrium analysis.
  - But why?
  - Why concentrate on equilibrium?
  - Why assume competitive behaviour?
- Here we re-examine the basics of market interaction by agents.
- Let's start by having another look at the exchange economy.
- We'll redraw the Edgeworth box.

OVERVIEW...

- General Equilibrium: price taking
  - An exchange economy
  - The solution concept
  - Prices and the Core

The offer curve as a tool of analysis

THE EDGEWORTH BOX

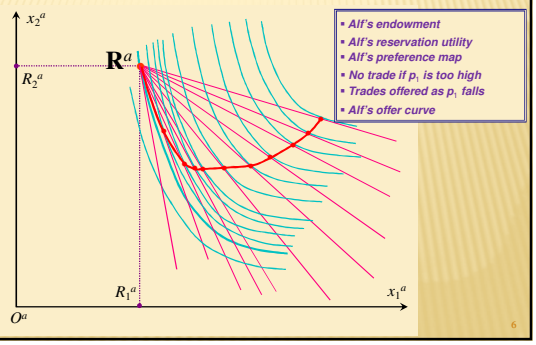
- Remember that the Edgeworth Box is a 2×2 representation of an exchange economy:
  - Two goods.
  - Two persons Alf and Bill.
- Represent the equilibrium for each person given:
  - Price-taking behaviour.
  - Ownership of the resources.
- Introduce the materials balance condition...
- ...achieved by inverting one diagram to complete the "box."

BEHAVIOUR OUT OF EQUILIBRIUM

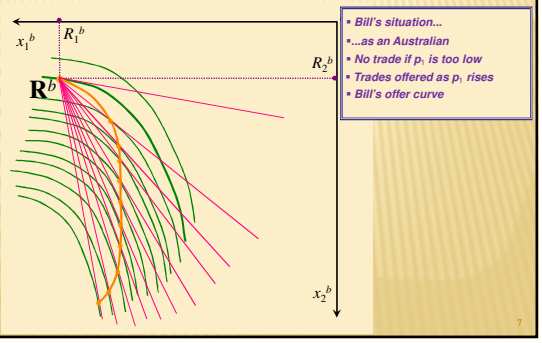
- First let's see why the CE is of such significance.
- To do so consider a simple question:
  - If Alf and Bill are price takers, what will they do in situations other than equilibrium?
- To answer this use a familiar tool.
  - The *offer curve*.
  - Introduced in consumer demand.
- To get this re-examine the optimisation problems
  - First Alf
  - Then Bill

Jump to consumer demand

ALF'S RESPONSES TO CHANGES IN P_1/P_2

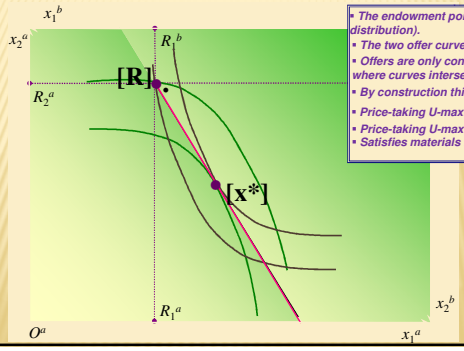


BILL'S RESPONSES TO CHANGES



- Bill's situation...
- ...as an Australian
- No trade if p_1 is too low
- Trades offered as p_1 rises
- Bill's offer curve

EDGEWORTH BOX AND CE



- The endowment point (property distribution).
- The two offer curves
- Offers are only consistent where curves intersect
- By construction this is CE
- Price-taking U-maximising Alf
- Price-taking U-maximising Bill
- Satisfies materials balance

THE NATURE OF CE

- Given competitive behaviour, the CE is the only "consistent" allocation.
- Clearly the location of the CE depends upon the initial resource endowment [**R**].
- But why assume competitive behaviour?
- Why should Alf and Bill behave as price-takers?

WHERE DO THE PRICES COME FROM?

- The "rules of the game" assume that people act as price takers and that prices are "given"
- Then people can solve the standard optimisation problem.
- But where do the prices come from?
- We can't appeal to invented "shadow" prices
- Nor to "world markets"
- Nor to some external agency…

$$\max U^h(\mathbf{x}^h) \text{ subject to } \sum_{i=1}^{n} p_i x_i^h \leq y^h$$

HOW TO MAKE PROGRESS

- It would be convenient to assume there is a big hand....
  - ...given the prices the system almost solves itself
  - But we have to manage without the artificial construct.
  - How?
- We need a more general solution concept.
- Base this on a broader concept of trading behaviour.
- We will describe the type of equilibrium associated with this concept.
- Then we examine how price-taking equilibrium relates to this.

OVERVIEW...

General Equilibrium: price taking

An exchange economy

Blocking and the core

The solution concept

Prices and the Core

A FRESH APPROACH

- Develop the approach for an exchange economy.
- But it could apply to more interesting economies.
  - To do it for production usually involves some strong assumptions.
- Imagine this as the economics of a PoW camp.
- The rules of the game are very simple:
  - Each person is endowed with a given bundle of goods $\mathbf{R}^h$
  - Each person has absolute right of disposal over this bundle.
  - Everyone is free to associate with others to form coalitions.

No-one is forced to trade/exchange

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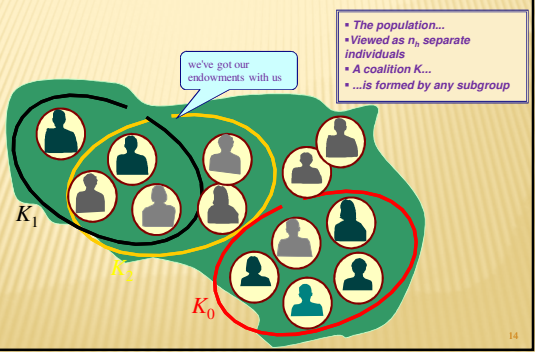
COALITIONS



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THE IDEA OF *BLOCKING*: A STORY

- One day you take your bundle to the "swap shop."
- Some bossy person there proposes (insists on?) a particular feasible allocation.
- You and some others don't like the bundle you all get under this allocation.
- Your group finds that, just by using its own resources, you could all get as much or more utility as that offered under the proposed allocation.
- You guys therefore refuse to accept the proposal.
- Your coalition has *blocked* the proposed allocation

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A FORMAL APPROACH

- Consider a proposed allocation for the economy $[\hat{\mathbf{x}}]$
- A coalition $K \subseteq \{1,2,\ldots,n_h\}$
- An allocation $[\mathbf{x}]$ preferred by the coalition K: $\forall h \in K: U^h(\mathbf{x}^h) \geq U^h(\hat{\mathbf{x}}^h)$, for some $h \in K: U^h(\mathbf{x}^h) > U^h(\hat{\mathbf{x}}^h)$
- The allocation $[\mathbf{x}]$ of bundles is feasible for K if: $\Sigma_{h\in K}\mathbf{x}^h \leq \Sigma_{h\in K}\mathbf{R}^h$
- If there is a feasible, preferred bundle for K then ... $[\hat{\mathbf{x}}]$ is *blocked* by K

An allocation is blocked by a coalition if the coalition members can do better for themselves

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EQUILIBRIUM CONCEPT

- Use the idea of blocking to introduce a basic solution concept.
- Surely no blocked allocation could be a solution to the trading game?
- So we use the following definition of a solution:
- The *Core* is the set of unblocked, feasible allocations.
- Let's apply it in the two-trader case.

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COALITIONS

- In a 2-person world there are few coalitions:

{Alf }

{Bill}

{Alf & Bill}

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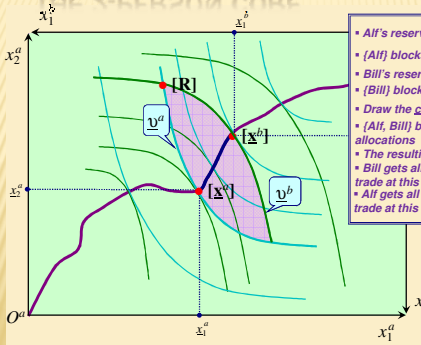
THE 2-PERSON CORE



- *Alf's reservation utility*
- *{Alf} blocks these allocations*
- *Bill's reservation utility*
- *{Bill} blocks these allocations*
- *Draw the contract curve*
- *{Alf, Bill} blocks these allocations*
- *The resulting core*
- *Bill gets all the advantage from trade at this extreme point.*
- *Alf gets all the advantage from trade at this extreme point.*

▪ *The contract curve is the locus of common tangencies*

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THE CORE: SUMMARY

- Definition of the core follows immediately from:
  - The definition of an allocation.
  - The definition of blocking.
- It is a general concept.
- To find the core you need just:
  - A complete description of the property distribution.
  - An enumeration of the possible coalitions.
  - A certain amount of patience.
- The major insight from the core comes when we examine the relation to CE.

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OVERVIEW...

Competitive equilibrium, large numbers and a limit theorem

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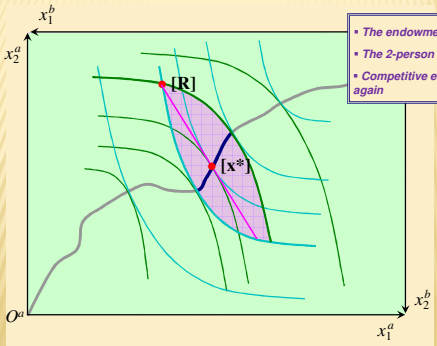
THE CORE AND CE



- *The endowment point*
- *The 2-person core again*
- *Competitive equilibrium again*

▪ ***A competitive equilibrium must always be a core allocation***

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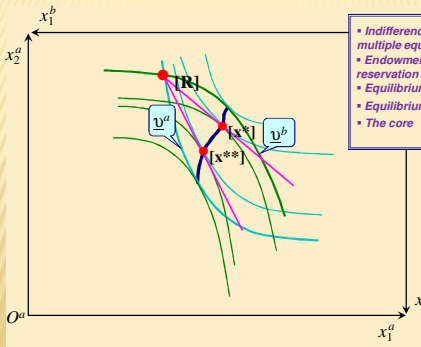
THE CORE AND CE (2)



- *Indifference curves that yield multiple equilibria*
- *Endowment point and reservation utility*
- *Equilibrium: low p_1/p_2*
- *Equilibrium: high p_1/p_2*
- *The core*

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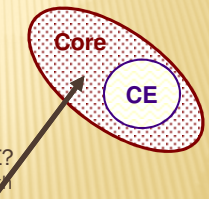
A SIMPLE RESULT

- Every CE allocation must belong to the core.
- It is possible that no CE exists.
- But what of other core allocations which are not CE?
  - Remember we are dealing with a 2-person model.
  - But will there always be non-CE points in the core?
- Let's take a closer look...



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SO LET'S CLONE THE ECONOMY

- Assume that the economy is *replicated* by a factor *N*, so that there are 2*N* persons.
- Start with *N*=2:
  - We move from a 2-person economy to a 4-person economy.
  - Alf and his twin brother Arthur have the same preferences and endowments.
  - Likewise the twins Bill and Ben.
- Now of course there are more possibilities of forming coalitions.

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COALITIONS IN THE *N*=2 ECONOMY

- All old coalitions are still possible...
- ... plus some new ones

{Alf}	{Arthur}
{Bill}	{Ben}
{Alf & Bill}	{Arthur & Ben}
{Alf & Arthur}	{Bill&Ben}
{Alf, Arthur &Bill}	{etc, etc}

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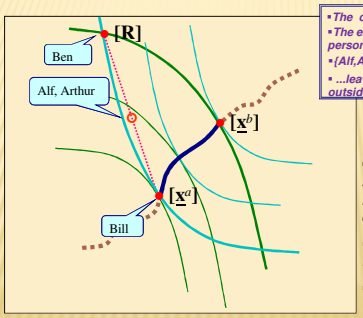
EFFECT OF CLONING ON THE CORE



- *The core in the 2-person case*
- *The extremes of the two-person core*
- *{Alf,Arthur,Bill} can block [x^a]...*
- *...leaving the Ben twin outside the coalition*

- ***Are the extremes still core allocations in the 4-person economy?***
- ***This new allocation is not a solution...***
- ***But it shows that the core must have become smaller***

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HOW THE BLOCKING COALITION WORKS

- *Consumption in the coalition*
- *Sum to get resource requirement*
- *Consumption out of coalition*

Alf	$\mathbf{x}^a = \frac{1}{2}[\underline{\mathbf{x}}^a + \mathbf{R}^a]$
Arthur	$\mathbf{x}^a = \frac{1}{2}[\underline{\mathbf{x}}^a + \mathbf{R}^a]$
Bill	$[2\mathbf{R}^a + \mathbf{R}^b - 2\mathbf{x}^a]$
	$2\mathbf{R}^a + \mathbf{R}^b$
Ben	$\mathbf{R}^b$

- ***The consumption within the coalition equals the coalition's resources.***
- ***So the allocation is feasible.***

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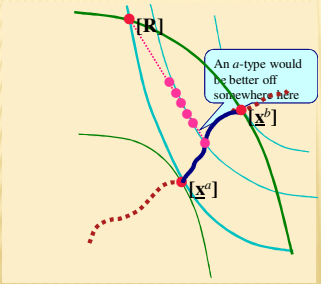
IF *N* IS BIGGER: MORE BLOCKING COALITIONS?



- *The 2-person core*
- *An arbitrary allocation - can it be blocked?*
- *Draw a line to the endowment*
- *Take N=500 of each tribe.*
- *Divide the line for different coalition numbers.*

- *We've found the blocking coalition.*
- *If line is not a tangent this can always be done.*

numbers of...	
a-tribe	*b-tribe*
500	450

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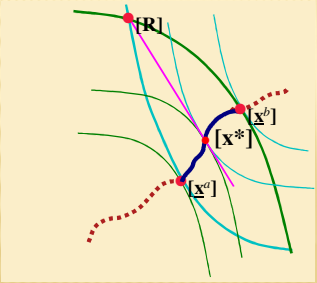
IN THE LIMIT



- *If N→∞ a coalition can be found that divides the line to [**R**] in any proportion you want.*
- *Only if the line is like this will the allocation be impossible to block.*
- *With the large N the core has "shrunk" to the set of CE*

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A POWERFUL RESULT: THE SHRINKING CORE

- As you clone the economy the core becomes smaller.
- If you make *N* large enough you will find some coalition that blocks any non-CE allocation.
- So in the limit the core contains only CE allocations.
- In a suitably large economy the core exactly equals the set of competitive equilibria.

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THE SHRINKING CORE: DISCUSSION

- The result rules out non-price-taking behaviour as a solution. But:
- There are some weasel words: "suitably large".
  - In principle *N* should be infinite
- Process requires balanced replication of the Alf and Bill tribes.
  - Problems arise if there is one large *b*-trader and many *a*-traders
- All possible coalitions are assumed relevant to negotiations about blocking.
  - Only valid if communication and other coalition costs are negligible. The Internet?
- We have argued only using an exchange economy.
  - Can be extended to production economies with CRTS and (with some difficulty) others too.

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REVIEW

- Basic components of trading equilibrium:
  - Coalitions
  - Blocking
  - Core as an equilibrium concept
  - Relation to CE
  - Every CE must lie in the core
  - In the limit of a replication economy the core consists *only* of CE
- Answer to question: why price-taking?
  - In a large economy with suitably small agents…
  - ….it's the only thing to do.

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