

## WHAT IS MONOPOLY?

* Consider a simple model of market power

One seller, multiple buyers
Buyers act as price-takers
Seller determines price

* An artificial construct?

What prevents there being other firms in the industry?
Or other firms that could potentially replace this firm?
Or firms producing very close substitutes?
Assume monopoly position is guaranteed by an exogenous factor (the law?)
$\times$ Here we will examine:
...monopoly with different types of market power
.. the relationship with competitive market equilibrium
A useful baseline case for more interesting models of the market
Begin with an elementary model...


## A SIMPLE PRICE-SETTING FIRM

* Contrast with the price-taking firm:
$\times$ Output price is no longer exogenous
* We assume a determinate demand curve
* No other firm's actions are relevant
x Profit maximisation is still the objective


## MONOPOLY - MODEL STRUCTURE

* We are given the inverse demand function:
$p=p(q)$
Gives the (uniform) price that would rule if the monopolist chose to deliver $q$ to the market.
For obvious reasons, consider it as the average revenue curve (AR).
x Total revenue is:
$p(q) q$.
x Differentiate to get monopolist's marginal revenue (MR): $p(q)+p_{q}(q) q$
$p_{q}(\bullet)$ means $d p(\bullet) / d q$
$\times$ Clearly, if $p_{q}(q)$ is negative (demand curve is downward sloping), then MR < AR.

AVERAGE AND MARGINAL REVENUE


## MONOPOLY - OPTIMISATION PROBLEM

$\times$ Introduce the firm's cost function C(q).

+ Same basic properties as for the competitive firm.
$\times$ From $C$ we derive marginal and average cost: + MC: $C_{q}(q)$.
AC: $C(q) / q$.
* Given $C(q)$ and total revenue $p(q) q$ profits are: $\Pi(q)=p(q) q-C(q)$
$\times$ The shape of $\Pi$ is important:
We assume it to be differentiable
Whether it is concave depends on both $C(\bullet)$ and $p(\bullet)$. Of course $\Pi(0)=0$.
$\times$ Firm maximises $\Pi(q)$ subject to $q \geq 0$.


## MONOPOLY - SOLVING THE PROBLEM

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* Problem is " max \Pi(q) s.t. q\geq0, where:
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    \(\Pi(q)=p(q) q-C(q)\).
    * First- and second-order conditions for interior
maximum:
$\Pi_{q}(q)=0$.
$\Pi_{q q}(q)<0$.
$\times$ Evaluating the FOC:
$p(q)+p_{q}(q) q-C_{q}(q)=0$.
* Rearrange this:
$p(q)+p_{q}(q) q=C_{q}(q)$
"Marginal Revenue $=$ Marginal cost"
- This condition gives the solution.
From above get optimal output $q$
Put $q^{*}$ in $p(\bullet)$ to get monopolist's price
$p^{*}=p\left(q^{*}\right)$.


## MONOPOLY - PRICING RULE

$\times$ Introduce the elasticity of demand $\eta$ :
$+\eta:=\mathrm{d}(\log q) / \mathrm{d}(\log p)$
$=q p_{q}(q) / p$
$\eta<0$
$\times$ First-order condition for an interior maximum $p(q)+p_{q}(q) q=C_{q}(q)$
x ...can be rewritten as

$$
p(q)[1+1 / \eta]=C_{q}(q)
$$

* This gives the monopolist's pricing rule:

$$
p(q)=\frac{C_{q}(q)}{1+1 / \eta}
$$

## MONOPOLY - THE ROLE OF DEMAND

x Suppose demand were changed to
$a+b p(q)$
$a$ and $b$ are constants.

* Marginal revenue and demand elasticity are now: $\operatorname{MR}(q)=b p_{q}(q) q+[a+b p(q)]$
$\eta=[a / b+b p(q)] / p_{q}(q)$
* Rotate the demand curve around ( $p^{*}, q^{*}$ ).
$\mathrm{db}>0$ and $\mathrm{d} a=-p\left(q^{*}\right) \mathrm{db}<0$
Price at $q^{*}$ remains the same.
Marginal revenue at $q^{*}$ increases - $\mathrm{dMR}\left(q^{\star}\right)>0$.
Abs value of elasticity at $q^{*}$ decreases - $d|\eta|<0$
But what happens to optimal output?
Differentiate FOC in the neighbourhood of $q^{*}$ :
$\mathrm{dMR}\left(q^{*}\right) \mathrm{db}+\Pi_{q q} \mathrm{~d} q^{*}=0$
So $\mathrm{d} q^{*}>0$ if $\mathrm{d} b>0$.


## MONOPOLY - ANALYSING THE OPTIMUM

$\times$ Take the basic pricing rule
$+p(q)=\frac{C_{q}(q)}{1+1 / \eta}$

- Use the definition of demand elasticity
- $p(q) \geq C_{q}(q)$
- $p(q)>C_{q}(q)$ if $|\eta|<\infty$.
- "price > marginal cost"
- Clearly as $|\eta|$ decreases :
- output decreases
- gap between price and marginal cost increases.
- What happens if $\eta \geq-1$ ?


## WHAT IS GOING ON?

## PROFIT IN THE TWO EXAMPLES

* To understand why there may be no solution consider two examples
* A firm in a competitive market: $\eta=-\infty$ $+p(q)=\bar{p}$
* A monopoly with inelastic demand: $\eta=-1 / 2$ $+p(q)=a q^{-2}$
* Same quadratic cost structure for both:

$$
C(q)=c_{0}+c_{1} q+c_{2} q^{2}
$$

$\times$ Examine the behaviour of $\Pi(q)$

## THE RESULT OF SIMPLE MARKET POWER

* There's no supply curve:

For competitive firm market price is sufficient to determine output.

- Here output depends on shape of market demand curve.
* Price is artificially high:

Price is above marginal cost
Price/MC gap is larger if demand is inelastic

* There may be no solution:

What if demand is very inelastic?


OVERVIEW... Monopoly

Simple model
increased power for the monopolist?

## Exploitation

## Discriminating

monopolist

Product diversity

## COULD THE FIRM HAVE MORE POWER?

$\times$ Consider how the simple monopolist acts:

- Chooses a level of output $q$
+ Market determines the price that can be borne $p=p(q)$
- Monopolist sells all units of output at this price $p$
* Consumer still makes some gain from the deal Consider the total amount bought as separate units Perhaps would pay more than $p$ for previous units (for $x<q$ )
* What is total gain made by the consumer? This is given by area under the demand curve and above price $p$ Conventionally known as consumer's surplus $\int_{0} p(x) d x-p q$
* Use this to modify the model of monopoly power...


## THE FIRM WITH MORE POWER

$\times$ Suppose monopolist can charge for the right to purchase

Charges a fixed "entry fee" F for customers Only works if it is impossible to resell the good
x This changes the maximisation problem
Profits are now
$F+p q-C(q)$
where $F=\int_{0} p(x) d x-p q$
which can be simplified to
$\int_{0}^{q} p(x) d x-C(q)$
$\times$ Maximising this with respect to $q$ we get the FOC $p(q)=C(q)$
× This yields the optimum output...

MONOPOLIST WITH ENTRY FEE


## MULTIPLE MARKETS

* Monopolist sells same product in more than one market An alternative model of increased power Perhaps can discriminate between the markets
- Can the monopolist separate the markets? Charge different prices to customers in different markets In the limit can see this as similar to previous case... ...if each "market" consists of just one customer
$\times$ Essentials emerge in two-market case
$\times$ For convenience use a simplified linear model: Begin by reviewing equilibrium in each market in isolation Then combine model....
...how is output determined...?
...and allocated between the markets



## MONOPOLY WITH SEPARATED MARKETS

$\times$ Problem is now " $\max \Pi\left(q^{1}, q^{2}\right)$ s.t. $q^{1}, q^{2} \geq 0$, where: $\Pi\left(q^{1}, q^{2}\right)=p^{1}\left(q^{1}\right) q^{1}+p^{2}\left(q^{2}\right) q^{2}-C\left(q^{1}+q^{2}\right)$.

* First-order conditions for interior maximum:
$\Pi_{i}\left(q^{1}, q^{2}\right)=0, i=1,2$
$p^{1}\left(q^{1}\right) q^{1}+p^{1}{ }_{q}\left(q^{1}\right)=C_{q}\left(q^{1}+q^{2}\right)$
$p^{2}\left(q^{2}\right) q^{2}+p_{q}^{2}\left(q^{2}\right)=C_{q}\left(q^{1}+q^{2}\right)$
$\times$ Interpretation:
"Market 1 MR =MC overall"
"Market 2 MR $=$ MC overall"
So output in each market adjusted to equate MR
* Implication

Set price in each market according to what it will bear
Price higher in low-elasticity market

OPTIMUM WITH SEPARATED MARKETS


OPTIMUM WITH SEPARATED MARKETS


TWO MARKETS: NO SEPARATION



## MARKET POWER AND PRODUCT DIVERSITY

* Nature of product is a major issue in classic monopoly + No close substitutes?
+ Otherwise erode monopoly position
* Now suppose potentially many firms making substitutes
+ Firms' products differ one from another
Each firm is a local monopoly - downward-sloping demand curve
New firms can enter with new products
Diversity may depend on size of market
Like corner shops dotted around the neighbourhood
* Use standard analysis
+ Start with a single firm - use monopoly paradigm Then consider entry of others, attracted by profit... ...process similar to competitive industry



## WHAT NEXT?

* All variants reviewed here have a common element...
* Firm does not have to condition its behaviour on what other firms do...
$x$ Does not attempt to influence behaviour of other firms

Not even of potential entrants
$\times$ Need to introduce strategic interdependence

