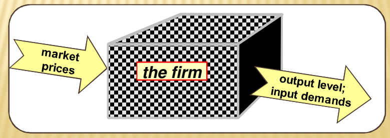
MICROECONOMICS

THE FIRM: DEMAND AND SUPPLY

1

MOVING ON FROM THE OPTIMUM…

- We derive the firm's reactions to changes in its environment.
- These are the *response functions*.
  - We will examine three types of them
  - Responses to different types of market events.
- In effect we treat the firm as a Black Box.



2

THE FIRM AS A "BLACK BOX"

- Behaviour can be predicted by necessary and sufficient conditions for optimum.
- The FOC can be solved to yield behavioural response functions.
- Their properties derive from the solution function.
- We need the solution function's properties…
- …again and again.

3

OVERVIEW…

Response function for stage 1 optimisation

Firm: Comparative Statics

- Conditional Input Demand
- Output Supply
- Ordinary Input Demand
- Short-run problem

4

THE FIRST RESPONSE FUNCTION

- Review the cost-minimisation problem and its solution
- Choose **z** to minimise

$$\sum_{i=1}^{m} w_i z_i \text{ subject to } q \le \phi(\mathbf{z}),\ \mathbf{z}\ge\mathbf{0}$$

- The "stage 1" problem

- The firm's cost function:

$$C(\mathbf{w}, q) := \min_{\{\phi(\mathbf{z}) \ge q\}} \sum w_i z_i$$

- The solution function

- Cost-minimising value for each input:

$$z_i^* = H^i(\mathbf{w}, q),\ i=1,2,\dots,m$$

may be a well-defined function or may be a correspondence

vector of input prices

Specified output level

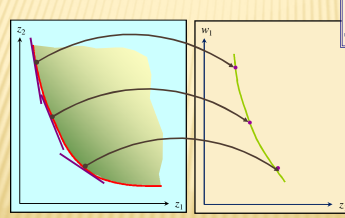
- H^i is the conditional input demand function.
- Demand for input *i*, conditional on given output level *q*

A graphical approach

5

MAPPING INTO (Z_1,W_1)-SPACE

- *Conventional case of Z.*
- *Start with any value of w_1 (the slope of the tangent to Z).*
- *Repeat for a lower value of w_1.*
- *…and again to get…*
- *…the conditional demand curve*



- *Constraint set is convex, with smooth boundary*
- *Response function is a continuous map:*

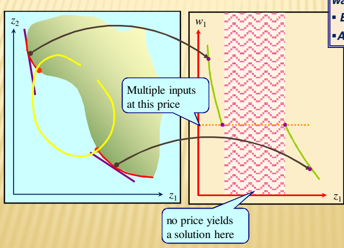
$$H^1(\mathbf{w},q)$$

Now try a different case

6

ANOTHER MAP INTO (Z_1, W_1)-SPACE

- Now take case of nonconvex Z.
- Start with a high value of w_1.
- Repeat for a very low value of w_1.
- Points "nearby" work the same way.
- But what happens in between?
- A demand correspondence



- Constraint set is nonconvex.
- Response is a discontinuous map: jumps in z*
- Map is multivalued at the discontinuity

7

CONDITIONAL INPUT DEMAND FUNCTION

- Assume that single-valued input-demand functions exist.
- How are they related to the cost function?
- What are their properties?
- How are they related to properties of the cost function?

Do you remember these...?

8

USE THE COST FUNCTION

The slope: $\frac{\partial C(\mathbf{w}, q)}{\partial w_i}$

Optimal demand for input i

- Recall this relationship?

$C_i(\mathbf{w}, q) = z_i^*$

- ...yes, it's Shephard's lemma

conditional input demand function

- So we have:

$C_i(\mathbf{w}, q) = H^i(\mathbf{w}, q)$

- Link between conditional input demand and cost functions

Second derivative

- Differentiate with respect to w_j

$C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$

- Slope of input demand function

Two simple results:

9

SIMPLE RESULT 1

- Use a standard property

$$\frac{\partial^2(\bullet)}{\partial w_i \partial w_j} = \frac{\partial^2(\bullet)}{\partial w_j \partial w_i}$$

- second derivatives of a function "commute"

- So in this case

$C_{ij}(\mathbf{w}, q) = C_{ji}(\mathbf{w}, q)$

- The order of differentiation is irrelevant

- Therefore we have:

$H_j^i(\mathbf{w}, q) = H_i^j(\mathbf{w}, q)$

- The effect of the price of input i on conditional demand for input j equals the effect of the price of input j on conditional demand for input i.

10

SIMPLE RESULT 2

- Use the standard relationship:

$C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$

- Slope of conditional input demand function derived from second derivative of cost function

- We can get the special case:

$C_{ii}(\mathbf{w}, q) = H_i^i(\mathbf{w}, q)$

- We've just put j=i

- Because cost function is concave:

$C_{ii}(\mathbf{w}, q) \leq 0$

- A general property

- Therefore:

$H_i^i(\mathbf{w}, q) \leq 0$

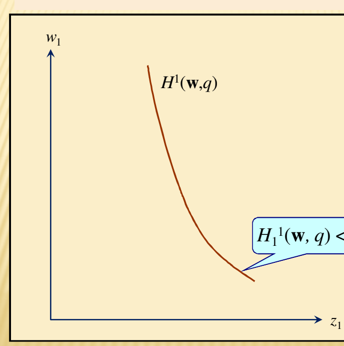
- The relationship of conditional demand for an input with its own price cannot be positive.

and so...

11

CONDITIONAL INPUT DEMAND CURVE

- Consider the demand for input 1
- Consequence of result 2?



- "Downward-sloping" conditional demand
- In some cases it is also possible that $H_i^i=0$
- Corresponds to the case where isoquant is kinked: multiple **w** values consistent with same ***z****.

12

FOR THE CONDITIONAL DEMAND FUNCTION…

- Nonconvex *Z* yields discontinuous *H*
- Cross-price effects are symmetric
- Own-price demand slopes downward.
- (exceptional case: own-price demand could be constant)

13

OVERVIEW…

Response function for stage 2 optimisation

- Firm: Comparative Statics
  - Conditional Input Demand
  - Output Supply
  - Ordinary Input Demand
  - Short-run problem

14

THE SECOND RESPONSE FUNCTION

- Review the profit-maximisation problem and its solution
- Choose q to maximise:

$$pq - C(\mathbf{w}, q)$$

- *The "stage 2" problem*
- From the FOC:

$$p \leq C_q(\mathbf{w}, q^*)$$

$$pq^* \geq C(\mathbf{w}, q^*)$$

- *"Price equals marginal cost"*
- *"Price covers average cost"*
- profit-maximising value for output:

$$q^* = S(\mathbf{w}, p)$$

input prices

output price

- *S is the supply function*
- *(again it may actually be a correspondence)*

15

SUPPLY OF OUTPUT AND OUTPUT PRICE

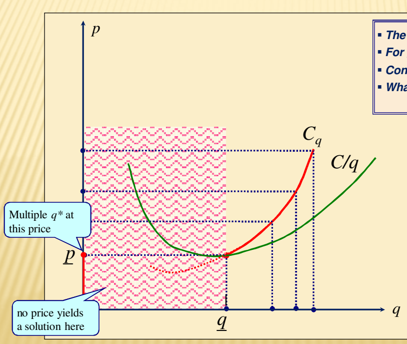
- Use the FOC:
  $C_q(\mathbf{w}, q) = p$
- *"marginal cost equals price"*
- Use the supply function for q:
  $C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$
- *Gives an equation in w and p*
- Differentiate with respect to p
  $C_{qq}(\mathbf{w}, S(\mathbf{w}, p))\, S_p(\mathbf{w}, p) = 1$
- Differential of S with respect to p
- *Use the "function of a function" rule*
- Rearrange:

$$S_p(\mathbf{w}, p) = \frac{1}{C_{qq}(\mathbf{w}, q)}.$$

- Positive if MC is increasing.
- *…e of the supply …nction.*

16

THE FIRM'S SUPPLY CURVE



- *The firm's AC and MC curves.*
- *For given p read off optimal q**
- *Continue down to p*
- *What happens below p*

- *Supply response is given by $q=S(\mathbf{w},p)$*
- *Case illustrated is for ϕ with first IRTS, then DRTS. Response is a discontinuous map: jumps in q**
- *Map is multivalued at the discontinuity*

17

SUPPLY OF OUTPUT AND PRICE OF INPUT J

- Use the FOC:
  $C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$
- *Same as before: "price equals marginal cost"*
- Differentiate with respect to w_j
  $C_{qj}(\mathbf{w}, q^*) + C_{qq}(\mathbf{w}, q^*)\, S_j(\mathbf{w}, p) = 0$
- *Use the "function of a function" rule again*
- Rearrange:

$$S_j(\mathbf{w}, p) = -\frac{C_{qj}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}$$

- Remember, this is positive
- *Supply of output must fall with w_j if marginal cost increases with w_j.*

18

FOR THE SUPPLY FUNCTION…

- Supply curve slopes upward.
- Supply decreases with the price of an input, if MC increases with the price of that input.
- Nonconcave ϕ yields discontinuous S.
- IRTS means ϕ is nonconcave and so S is discontinuous.

19

OVERVIEW…

Response function for combined optimisation problem

- Firm: Comparative Statics
  - Conditional Input Demand
  - Output Supply
  - Ordinary Input Demand
  - Short-run problem

20

THE THIRD RESPONSE FUNCTION

- Recall the first two response functions:

$$z_i^* = H^i(\mathbf{w}, q)$$

Demand for input i, conditional on output q

$$q^* = S(\mathbf{w}, p)$$

Supply of output

- Now substitute for q^*:

$$z_i^* = H^i(\mathbf{w}, S(\mathbf{w}, p))$$

Stages 1 & 2 combined…

- Use this to define a new function:

$$D^i(\mathbf{w}, p) := H^i(\mathbf{w}, S(\mathbf{w}, p))$$

input prices

output price

Demand for input i (unconditional)

Use this relationship to analyse further the firm's response to price changes

21

DEMAND FOR *I* AND THE PRICE OF OUTPUT

- Take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w}, \ldots$$

"function of a function" rule again

- Differentiate with respect to p:

$$D_p^i(\mathbf{w}, p) = H_q^i(\mathbf{w}, q^*)\, S_p(\mathbf{w}, p)$$

D^i increases with p iff H^i increases with q. Reason? Supply increases with price (S_p>0).

- But we also have, for any q:

$$H^i(\mathbf{w}, q) = C_i(\mathbf{w}, q)$$
$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q)$$

Shephard's Lemma again

- Substitute in the above:

$$D_p^i(\mathbf{w}, p) = C_{qi}(\mathbf{w}, q^*) S_p(\mathbf{w}, p)$$

Demand for input i (D^i) increases with p iff marginal cost (C_q) increases with w_i.

22

DEMAND FOR *I* AND THE PRICE OF *J*

- Again take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w}, S(\mathbf{w}, p)).$$

"function of a function" rule yet again

- Differentiate with respect to w_j:

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) + H_q^i(\mathbf{w}, q^*) S_j(\mathbf{w}, p)$$

- Use Shephard's Lemma again:

$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q) = C_{qi}(\mathbf{w}, q)$$

- Use this and the previous r… p) to give a decomposition into a "substitu… effect" and an "output effect":

"substitution effect"

"output effect"

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*) C_{jq}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}$$

23

RESULTS FROM DECOMPOSITION FORMULA

- Take the general relationship:

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*) C_{jq}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}.$$

The effect w_i on demand for input j equals the effect of w_j on demand for input i.

We already know this is symmetric in i and j.

Obviously symmetric in i and j.

- Now take the special case where $j = i$:

$$D_i^i(\mathbf{w}, p) = H_i^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*)^2}{C_{qq}(\mathbf{w}, q^*)}$$

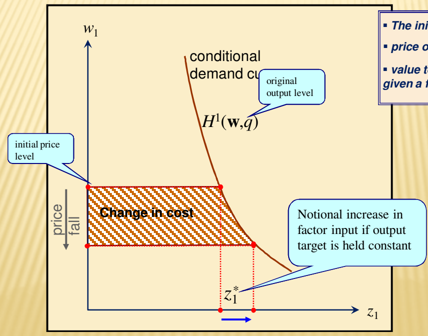
If w_i increases, the demand for input i cannot rise.

We already know this is negative or zero.

cannot be positive.

24

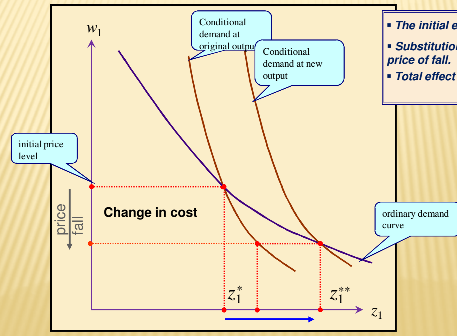
INPUT-PRICE FALL: SUBSTITUTION EFFECT



- The initial equilibrium
- price of input falls
- value to firm of price fall, given a fixed output level

25

INPUT-PRICE FALL: TOTAL EFFECT



- The initial equilibrium
- Substitution effect of input-price of fall.
- Total effect of input-price fall

26

THE ORDINARY DEMAND FUNCTION…

- Nonconvex Z may yield a discontinuous D
- Cross-price effects are symmetric
- Own-price demand slopes downward
- Same basic properties as for H function

27

OVERVIEW…

Optimisation subject to side-constraint

Firm: Comparative Statics

- Conditional Input Demand
- Output Supply
- Ordinary Input Demand
- Short-run problem

28

THE SHORT RUN…

- This is not a moment in time but…
- … is defined by additional constraints within the model
- Counterparts in other economic applications where we sometimes need to introduce side constraints

29

THE SHORT-RUN PROBLEM

- We build on the firm's standard optimisation problem
- Choose q and $\mathbf{z}$ to maximise

$$\Pi := pq - \sum_{i=1}^{m} w_i z_i$$

- subject to the standard constraints:

$$q \le \phi(\mathbf{z})$$

$$q \ge 0, \mathbf{z} \ge \mathbf{0}$$

- But we add a *side condition* to this problem:

$$z_m = \bar{z}_m$$

- Let $\bar{q}$ be the value of q for which $z_m = \bar{z}_m$ would have been freely chosen in the unrestricted cost-min problem…

30

THE SHORT-RUN COST FUNCTION

$$\tilde{C}(\mathbf{w}, q, \bar{z}_m) := \min_{\{z_m = \bar{z}_m\}} \Sigma\, w_i z_i$$

- The solution function with the side constraint.

- Short-run demand for input i:

$$\tilde{H}^i(\mathbf{w}, q, \bar{z}_m) = \tilde{C}_i(\mathbf{w}, q, \bar{z}_m)$$

- Follows from Shephard's Lemma

- Compare with the ordinary cost function

$$C(\mathbf{w}, q) \leq \tilde{C}(\mathbf{w}, q, \bar{z}_m)$$

- By definition of the cost function. We have "=" if $q = \bar{q}$.

- So, dividing by q:

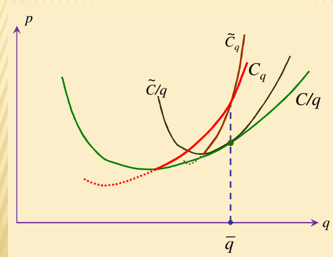
$$\frac{C(\mathbf{w}, q)}{q} \leq \frac{\tilde{C}(\mathbf{w}, q, \bar{z}_m)}{q}$$

- Short-run AC ≥ long-run AC. SRAC = LRAC at $q = \bar{q}$

Supply curves

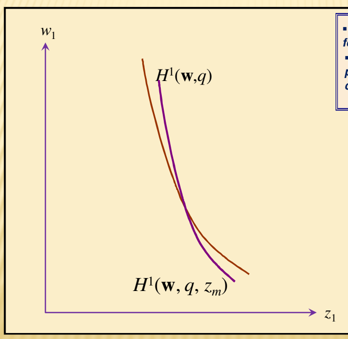
MC, AC AND SUPPLY IN THE SHORT AND LONG RUN

- AC if all inputs are variable
- MC if all inputs are variable
- Fix an output level.
- AC if input m is now kept fixed
- MC if input m is now kept fixed
- Supply curve in long run
- Supply curve in short run



- SRAC touches LRAC at the given output
- SRMC cuts LRMC at the given output
- The supply curve is steeper in the short run

CONDITIONAL INPUT DEMAND



- The original demand curve for input 1
- The demand curve from the problem with the side constraint.

- "Downward-sloping" conditional demand
- Conditional demand curve is steeper in the short run.

KEY CONCEPTS

- Basic functional relations
- price signals → firm → input/output responses

- $H^i(\mathbf{w},q)$ demand for input i, conditional on output
- $S(\mathbf{w},p)$ supply of output
- $D^i(\mathbf{w},p)$ demand for input i (unconditional)

And they all hook together like this:

- $H^i(\mathbf{w}, S(\mathbf{w},p)) = D^i(\mathbf{w},p)$

WHAT NEXT?

- Analyse the firm under a variety of market conditions.
- Apply the analysis to the consumer's optimisation problem.