MICROECONOMICS

Principles and Analysis

HOUSEHOLD DEMAND AND SUPPLY

WORKING OUT CONSUMER RESPONSES

- The analysis of consumer optimisation gives us some powerful tools:
  - The primal problem of the consumer is what we are really interested in.
  - Related dual problem can help us understand it.
  - The analogy with the firm helps solve the dual.
- The work we have done can map out the consumer's responses
  - to changes in prices
  - to changes in income

OVERVIEW...

The basics of the consumer demand system.

- Household Demand & Supply
  - **Response functions**
  - Slutsky equation
  - Supply of factors
  - Examples

SOLVING THE MAX-UTILITY PROBLEM

- The primal problem and its solution

$$\max U(\mathbf{x}) + \mu\left[\, y - \sum_{i=1}^{n} p_i x_i \,\right]$$

The Lagrangean for the max U problem

$$\left.\begin{array}{l} U_1(\mathbf{x}^*) = \mu p_1 \\ U_2(\mathbf{x}^*) = \mu p_2 \\ \dots\ \dots\ \dots \\ U_n(\mathbf{x}^*) = \mu p_n \end{array}\right\} \qquad \sum_{i=1}^{n} p_i x_i^* = y$$

The n+1 first-order conditions, assuming all goods purchased.

- Solve this set of equations:

$$\left.\begin{array}{l} x_1^* = D^1(\mathbf{p}, y) \\ x_2^* = D^2(\mathbf{p}, y) \\ \dots\ \dots\ \dots \\ x_n^* = D^n(\mathbf{p}, y) \end{array}\right\}$$

Gives a set of demand functions, one for each good. Functions of prices and incomes.

$$\sum_{i=1}^{n} p_i D^i(\mathbf{p}, y) = y$$

A restriction on the n equations. Follows from the budget constraint

THE RESPONSE FUNCTION

- The response function for the primal problem is demand for good i:

$$x_i^* = D^i(\mathbf{p}, y)$$

Should be treated as just one of a set of n equations.

- The system of equations must have an "adding-up" property:

$$\sum_{i=1}^{n} p_i\, D^i(\mathbf{p}, y) = y$$

Reason? This follows immediately from the budget constraint: left-hand side is total expenditure.

- Each equation in the system must be homogeneous of degree 0 in prices and income. For any $t > 0$:

$$x_i^* = D^i(\mathbf{p}, y) = D^i(t\mathbf{p}, ty)$$

Reason? Again follows immediately from the budget constraint.

To make more progress we need to exploit the relationship between primal and dual approaches again...

HOW YOU WOULD USE THIS IN PRACTICE...

- Consumer surveys give data on expenditure for each household over a number of categories...
- ...and perhaps income, hours worked etc as well.
- Market data are available on prices.
- Given some assumptions about the structure of preferences...
- ...we can estimate household demand functions for commodities.
- From this we can recover information about utility functions.

OVERVIEW…

Household Demand & Supply
- Response functions
- Slutsky equation
- Supply of factors
- Examples

A fundamental decomposition of the effects of a price change.

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CONSUMER'S DEMAND RESPONSES

- What's the effect of a budget change on demand?
- Depends on the type of budget constraint.
  - Fixed income?
  - Income endogenously determined?
- And on the type of budget change.
  - Income alone?
  - Price in primal type problem?
  - Price in dual type problem?
- So let's tackle the question in stages.
- Begin with a type 1 (exogenous income) budget constraint.

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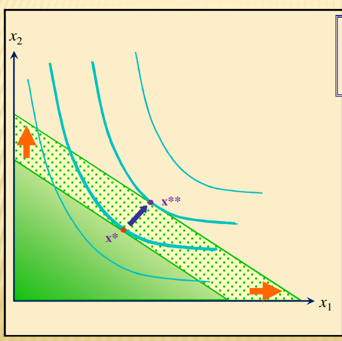
EFFECT OF A CHANGE IN INCOME



- *Take the basic equilibrium*
- *Suppose income rises*
- *The effect of the income increase.*

- *Demand for each good does not fall if it is "normal"*
- *But could the opposite happen?*

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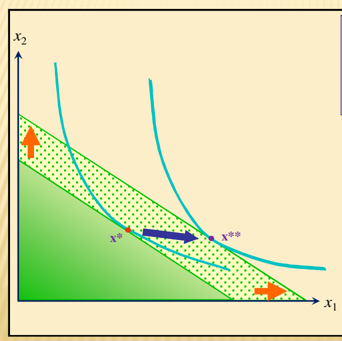
AN "INFERIOR" GOOD



- *Take same original prices, but different preferences*
- *Again suppose income rises*
- *The effect of the income increase.*

- *Demand for good 1 rises, but…*
- *Demand for "inferior" good 2 falls a little*
- *Can you think of any goods like this?*
- *How might it depend on the categorisation of goods?*

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A GLIMPSE AHEAD…

- We can use the idea of an "income effect" in many applications.
- Basic to an understanding of the effects of prices on the consumer.
- Because a price cut makes a person better off, as would an income increase…

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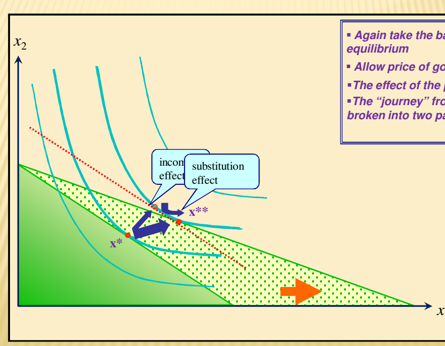
EFFECT OF A CHANGE IN PRICE



- *Again take the basic equilibrium*
- *Allow price of good 1 to fall*
- *The effect of the price fall.*
- *The "journey" from x* to x** broken into two parts*

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AND NOW LET'S LOOK AT IT IN MATHS

- We want to take both primal and dual aspects of the problem…
- …and work out the relationship between the response functions…
- … using properties of the solution functions.
- (Yes, it's time for Shephard's lemma again…)

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A FUNDAMENTAL DECOMPOSITION

compensated demand

ordinary demand

- …e the two metho… …riting x_i^*:
  $H^i(\mathbf{p},\upsilon) = D^i(\mathbf{p},y)$
- Use cost function to substitute for y:
  $H^i(\mathbf{p},\upsilon) = D^i(\mathbf{p}, C(\mathbf{p},\upsilon))$
- Differentiate with respect to p_j :
  $H_j^i(\mathbf{p},\upsilon) = D_j^i(\mathbf{p},y) + D_y^i(\mathbf{p},y)C_j(\mathbf{p},\upsilon)$
- Simplify :
  $H_j^i(\mathbf{p},\upsilon) = D_j^i(\mathbf{p},y) + D_y^i(\mathbf{p},y)\, H^j(\mathbf{p},\upsilon)$
  $= D_j^i(\mathbf{p},y) + D_y^i(\mathbf{p},y)\, x_j^*$
- And so we get:
  $D_j^i(\mathbf{p},y) = H_j^i(\mathbf{p},\upsilon) - x_j^* D_y^i(\mathbf{p},y)$

- *Remember: they are two ways of representing the same thing*
- *Gives us an implicit relation in prices and utility.*
- *Uses function-of-a-function rule again. Remember $y=C(\mathbf{p},u)$*
- *Using cost function and Shephard's Lemma again*
- *From the comp. demand function*
- *This is the Slutsky equation*

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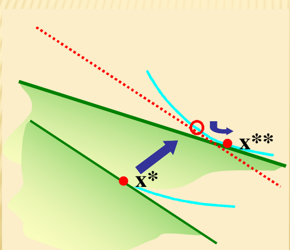
THE SLUTSKY EQUATION

$$D_j^i(\mathbf{p},y) = H_j^i(\mathbf{p},\upsilon) - x_j^* D_y^i(\mathbf{p},y)$$



- *Gives fundamental breakdown of effects of a price change*
- *Income effect: "I'm better off if the price of elly falls, so I buy more things, including cecream"*
- *"Substitution effect: When the price of elly falls and I'm kept on the same utility level, I prefer to switch from cecream for dessert"*

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SLUTSKY: POINTS TO WATCH

- Income effects for some goods may be negative
  - inferior goods.
- For $n > 2$ the substitution effect for some pairs of goods could be positive…
  - net substitutes
  - Apples and bananas?
- … while that for others could be negative
  - net complements
  - Gin and tonic?
- A neat result is available if we look at the special case where $j = i$.

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THE SLUTSKY EQUATION: OWN-PRICE

- Set $j = i$ to get the effect of the price of icecream on the demand for icecream

$$D_i^i(\mathbf{p},y) = H_i^i(\mathbf{p},\upsilon) - x_i^* D_y^i(\mathbf{p},y)$$

- Own-price substitution effect must be negative
- Income effect of price increase is non-positive for normal goods
- So, if the demand for i does not decrease when y rises, then it must decrease when p_i rises.

- *Follows from the results on the firm*
- *Price increase means less disposable income*

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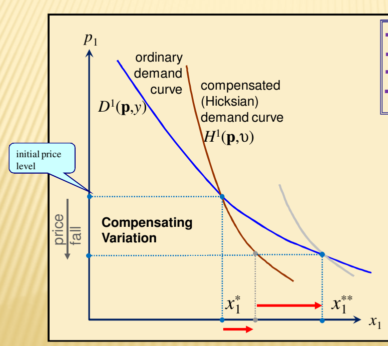
PRICE FALL: NORMAL GOOD



- ***The initial equilibrium***
- ***price fall: substitution effect***
- ***total effect: normal good***
- ***income effect: normal good***

- *For normal good income effect must be positive or zero*

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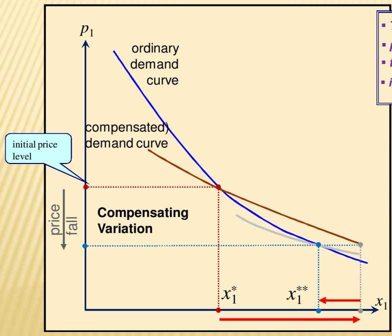
PRICE FALL: INFERIOR GOOD



- *The initial equilibrium*
- *price fall: substitution effect*
- *total effect: inferior good*
- *income effect: inferior good*

- *Note relative slopes of these curves in inferior-good case.*
- *For inferior good income effect must be negative*

FEATURES OF DEMAND FUNCTIONS

- Homogeneous of degree zero.
- Satisfy the “adding-up” constraint.
- Symmetric substitution effects.
- Negative own-price substitution effects.
- Income effects could be positive or negative:
  - in fact they are nearly always a pain.

OVERVIEW...

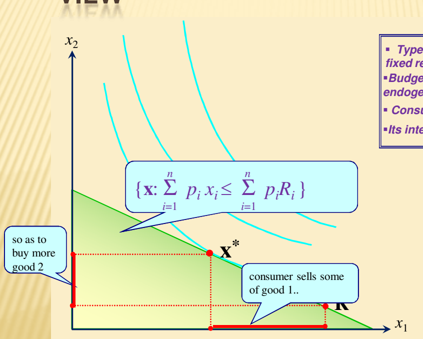
Extending the Slutsky analysis.

- Household Demand & Supply
  - Response functions
  - Slutsky equation
  - **Supply of factors**
  - Examples

CONSUMER DEMAND: ALTERNATIVE APPROACH

- Now for an alternative way of modelling consumer responses.
- Take a type-2 budget constraint (endogenous income).
- Analyse the effect of price changes...
- ...allowing for the impact of price on the valuation of income

CONSUMER EQUILIBRIUM: ANOTHER VIEW



- *Type 2 budget constraint: fixed resource endowment*
- *Budget constraint with endogenous income*
- *Consumer's equilibrium*
- *Its interpretation*

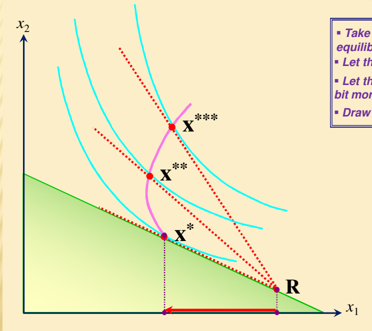
- *Equilibrium is familiar: same FOCs as before.*

TWO USEFUL CONCEPTS

- From the analysis of the endogenous-income case derive two other tools:

1. The offer curve:
   - The path of equilibrium bundles mapped out by price variation
   - Depends on “pivot point” - the endowment vector **R**
2. The household’s supply curve:
   - The “mirror image” of household demand.
   - Again the role of **R** is crucial.

THE OFFER CURVE



- Take the consumer's equilibrium
- Let the price of good 1 rise
- Let the price of good 1 rise a bit more
- Draw the locus of points

- This path is the <u>offer curve</u>.
- Amount of good 1 that household supplies to the market

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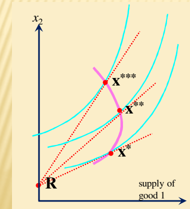
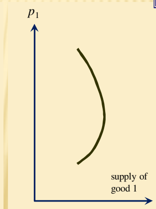
HOUSEHOLD SUPPLY

- Flip horizontally, to make supply clearer
- Rescale the vertical axis to measure price of good 1.
- Plot p_1 against x_1.





- This path is the <u>household's supply curve of good 1</u>.
- Note that the curve "bends back" on itself.
- Why?

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DECOMPOSITION – ANOTHER LOOK

- Take ordinary demand for good i:
  $x_i^* = D^i(\mathbf{p},y)$
- Substitute in for y :
  $x_i^* = D^i(\mathbf{p}, \Sigma_j p_j R_j)$
- Differentiate with respect to p_j:
  $$\frac{dx_i^*}{dp_j} = D_j^i(\mathbf{p}, y) + D_y^i(\mathbf{p}, y)\frac{dy}{dp_j}$$
  $$= D_j^i(\mathbf{p}, y) + D_y^i(\mathbf{p}, y) R_j$$
- Now recall the Slutsky relation:
  $D_j^i(\mathbf{p},y) = H_j^i(\mathbf{p},\upsilon) - x_j^* D_y^i(\mathbf{p},y)$
- Use this to substitute for D_j^i in the above:
  $$\frac{dx_i^*}{dp_j} = H_j^i(\mathbf{p},\upsilon) + [R_j - x_j^*]\, D_y^i(\mathbf{p},y)$$

direct effect of p_j on demand

indirect effect of p_j on demand via the impact on income

- Function of prices and income
- Income itself now depends on prices
- The indirect effect uses function-of-a-function rule again
- Just the same as on earlier slide
- This is the <u>modified Slutsky equation</u>

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THE MODIFIED SLUTSKY EQUATION:

$$\frac{dx_i^*}{dp_j} = H_j^i(\mathbf{p},\upsilon) + [R_j - x_j^*]\, D_y^i(\mathbf{p},y)$$

- Substitution effect has same interpretation as before.
- Income effect has two terms.
- This term is just the same as before.
- This term makes all the difference:
  - Negative if the person is a net demander.
  - Positive if he is a net supplier.

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OVERVIEW...

Household Demand & Supply

- Response functions
- Slutsky equation
- Supply of factors
- **Examples**

Labour supply, savings...

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SOME EXAMPLES

- Many important economic issues fit this type of model :
  - Subsistence farming.
  - Saving.
  - Labour supply.
- It's important to identify the components of the model.
  - How are the goods to be interpreted?
  - How are prices to be interpreted?
  - What fixes the resource endowment?
- To see how key questions can be addressed.
  - How does the agent respond to a price change?
  - Does this depend on the type of resource endowment?

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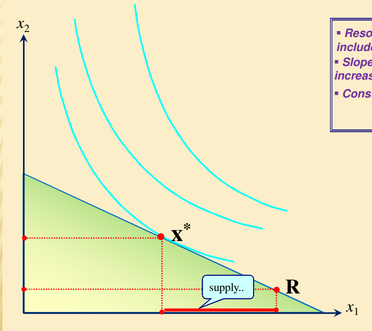
SUBSISTENCE AGRICULTURE...



- Resource endowment includes a lot of rice
- Slope of budget constraint increases with price of rice
- Consumer's equilibrium

- x_1,x_2 are "rice" and "other goods"
- Will the supply of rice to export rise with the world price...?.

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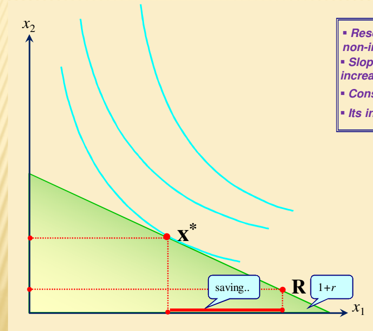
THE SAVINGS PROBLEM...



- Resource endowment is non-interest income profile
- Slope of budget constraint increases with interest rate, r
- Consumer's equilibrium
- Its interpretation

- x_1,x_2 are consumption "today" and "tomorrow"
- Determines time-profile of consumption
- What happens to saving when the interest rate changes...?.

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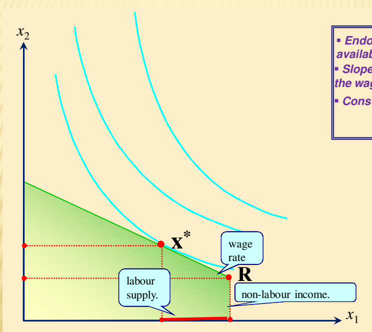
LABOUR SUPPLY...



- Endowment is total time available & non-labour income.
- Slope of budget constraint is the wage rate
- Consumer's equilibrium

- x_1,x_2 are leisure and "consumption"
- Determines labour supply
- Will people work harder if their wage rate goes up?.

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MODIFIED SLUTSKY: LABOUR SUPPLY

- Take the modified Slutsky:

$$\frac{dx_i^*}{dp_j} = H^i_j(\mathbf{p},\upsilon) + [R_j - x_j^*]\, D^i_y(\mathbf{p},y)$$

- Assume that supply of good i is the *only* source of income (so $y = p_i[R_i - x_i]$). Then, for the effect of p_i on x_i^* we get:

$$\frac{dx_i^*}{dp_i} = H^i_i(\mathbf{p},\upsilon) + \frac{y}{p_i}\, D^i_y(\mathbf{p},y)$$

- Rearranging :

$$-\frac{p_i}{R_i - x_i^*}\,\frac{dx_i^*}{\ldots} \ldots \frac{y}{R_i - x_i^*}\, D^i_y(\mathbf{p},y)$$

- Write ...

$$\varepsilon_{\text{total}} = \varepsilon_{\text{subst}} + \varepsilon_{\text{income}}$$

Total labour supply elasticity: could ... (backward-bendi...

must be positive

negative if leisure is a normal good

- *The general form. We are going to make a further simplifying assumption*
- *Suppose good i is labour time; then $R_i - x_i$ is the labour you sell in the market (I.e. leisure time not consumed); p_i is the wage rate*
- *Divide by labour supply; multiply by (-) wage rate*
- *The Modified Slutsky equation in a simple form*

Estimate the whole demand system from family expenditure data...

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SIMPLE FACTS ABOUT LABOUR SUPPLY

Source: Blundell and Walker (*Economic Journal*, 1982)

	total	*subst*	*income*
Men:	−0.23	+0.13	−0.36
Women:			
No children	+0.43	+0.65	−0.22
One child	+0.10	+0.32	−0.22
Two children	−0.19	+0.03	−0.22

- The estimated elasticities...
- Men's labour supply is backward bending!
- Leisure is a "normal good" for everyone
- Children tie down women's substitution effect...

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SUMMARY

- How it all fits together:
- Compensated (H) and ordinary (D) demand functions can be hooked together.
- Slutsky equation breaks down effect of price i on demand for j.
- Endogenous income introduces a new twist when prices change.

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WHAT NEXT?

- The welfare of the consumer.
- How to aggregate consumer behaviour in the market.