MICROECONOMICS

Principles and Analysis

CONSUMPTION BASICS

OVERVIEW...

The environment for the basic consumer optimisation problem.

- Consumption: Basics
  - The setting
  - Budget sets
  - Revealed Preference
  - Axiomatic Approach

A METHOD OF ANALYSIS

- Some treatments of micro-economics handle consumer analysis first.
- But we have gone through the theory of the firm first for a good reason:
- We can learn a lot from the ideas and techniques in the theory of the firm...
- ...and reuse them.

REUSING RESULTS FROM THE FIRM

- What could we learn from the way we analysed the firm....?
- How to set up the description of the environment.
- How to model optimization problems.
- How solutions may be carried over from one problem to the other
- ...and more .

NOTATION

- **Quantities**
  - x_i •amount of commodity i
  - $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ •commodity vector (a "basket of goods")
  - X •consumption set ($\mathbf{x} \in X$ denotes feasibility)
- **Prices**
  - p_i •price of commodity i
  - $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ •price vector
  - y •income

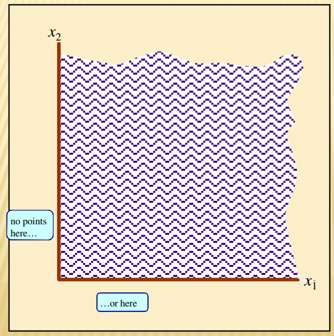
THINGS THAT SHAPE THE CONSUMER'S PROBLEM

- The set *X* and the number *y* are both important.
- But they are associated with two distinct types of constraint.
- We'll save *y* for later and handle *X* now.
- (And we haven't said anything yet about objectives...)

THE CONSUMPTION SET

- The set *X* describes the basic entities of the consumption problem.
- Not a description of the consumer's opportunities.
  - That comes later.
- Use it to make clear the type of choice problem we are dealing with; for example:
  - Discrete versus continuous choice (refrigerators vs. contents of refrigerators)
  - Is negative consumption ruled out?
- "$\mathbf{x} \in X$" means "**x** belongs the set of logically feasible baskets."

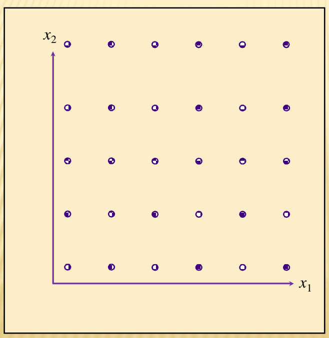
THE SET *X*: STANDARD ASSUMPTIONS



- Axes indicate quantities of the two goods x_1 and x_2.
- Usually assume that *X* consists of the whole non-negative orthant.
- Zero consumptions make good economic sense
- But negative consumptions ruled out by definition

- Consumption goods are (theoretically) divisible…
- …and indefinitely extendable…
- But only in the ++ direction

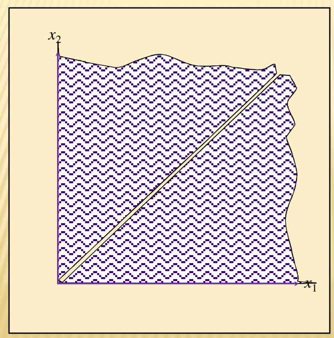
RULES OUT THIS CASE...



- Consumption set *X* consists of a countable number of points

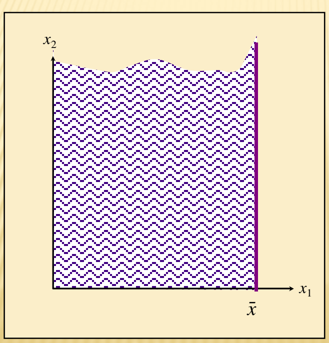
- Conventional assumption does not allow for indivisible objects.
- But suitably modified assumptions may be appropriate

... AND THIS



- Consumption set *X* has holes in it

... AND THIS



- Consumption set *X* has the restriction $x_1 < \bar{x}$

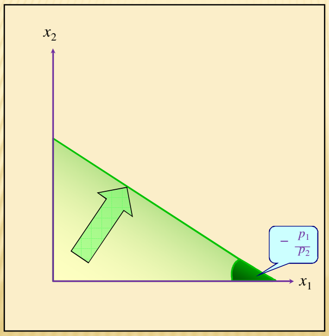
- Conventional assumption does not allow for physical upper bounds
- But there are several economic applications where this is relevant

OVERVIEW...

Budget constraints: prices, incomes and resources

- Consumption: Basics
  - The setting
  - **Budget sets**
  - Revealed Preference
  - Axiomatic Approach

THE BUDGET CONSTRAINT

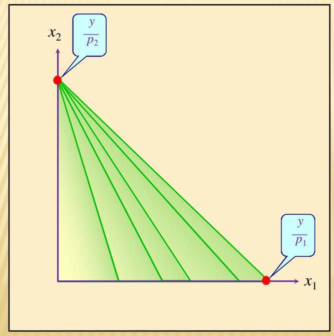


- *The budget constraint typically looks like this*
- *Slope is determined by price ratio.*
- *"Distance out" of budget line fixed by income or resources*

Two important subcases determined by

1. *… amount of money income y.*
2. *…vector of resources **R***

CASE 1: FIXED NOMINAL INCOME

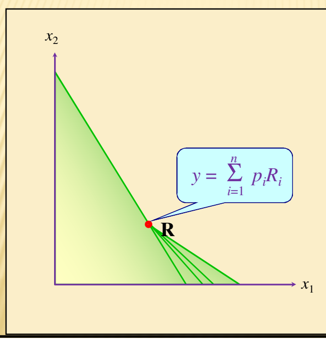


- *Budget constraint determined by the two end-points*
- *Examine the effect of changing p_1 by "swinging" the boundary thus…*

- *Budget constraint is*

$$\sum_{i=1}^{n} p_i x_i \leq y$$

CASE 2: FIXED RESOURCE ENDOWMENT



- *Budget constraint determined by location of "resources" endowment R.*
- *Examine the effect of changing p_1 by "swinging" the boundary thus…*

- *Budget constraint is*

$$\sum_{i=1}^{n} p_i x_i \leq \sum_{i=1}^{n} p_i R_i$$

BUDGET CONSTRAINT: KEY POINTS

- Slope of the budget constraint given by price ratio.
- There is more than one way of specifying "income":
  - Determined exogenously as an amount *y*.
  - Determined endogenously from resources.
- The exact specification can affect behaviour when prices change.
  - Take care when income is endogenous.
  - Value of income is determined by prices.

OVERVIEW…

Deducing preference from market behaviour?

- Consumption: Basics
  - The setting
  - Budget sets
  - **Revealed Preference**
  - Axiomatic Approach

A BASIC PROBLEM

- In the case of the firm we have an observable constraint set (input requirement set)…
- …and we can reasonably assume an obvious objective function (profits)
- But, for the consumer it is more difficult.
- We have an observable constraint set (budget set)…
- But what objective function?

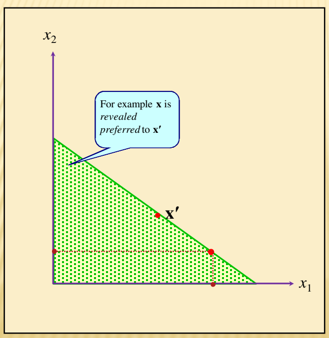
THE AXIOMATIC APPROACH

- We could "invent" an objective function.
- This is more reasonable than it may sound:
  - It is the standard approach.
  - See later in this presentation.
- But some argue that we should only use what we can observe:
  - Test from market data?
  - The "revealed preference" approach.
  - Deal with this now.
- Could we develop a coherent theory on this basis alone?

USING OBSERVABLES ONLY

- Model the opportunities faced by a consumer.
- Observe the choices made.
- Introduce some minimal "consistency" axioms.
- Use them to derive testable predictions about consumer behaviour

"REVEALED PREFERENCE"



- *Let market prices determine a person's budget constraint..*
- *Suppose the person chooses bundle x...*
- *Use this to introduce Revealed Preference*

AXIOMS OF REVEALED PREFERENCE

- **Axiom of Rational Choice**

  the consumer always makes a choice, and selects the most preferred bundle that is available.

  Essential if observations are to have meaning

- **Weak Axiom of Revealed Preference (WARP)**

  If **x** RP **x'** then **x'** not-RP **x**.

  If **x** was chosen when **x'** was available then **x'** can never be chosen whenever **x** is available

WARP is more powerful than might be thought

WARP IN THE MARKET

- Suppose that **x** is chosen when prices are **p**.
- If **x'** is also affordable at **p** then:

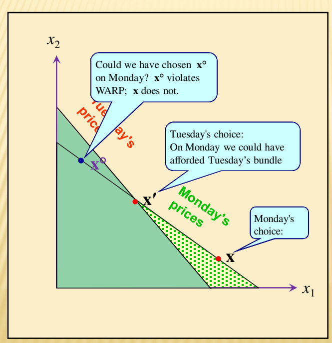
$$\sum_{i=1}^{n} p_i x_i \geq \sum_{i=1}^{n} p_i x'_i$$

- Now suppose **x'** is chosen at prices **p'**
- This must mean that **x** is not affordable at **p'**:

$$\sum_{i=1}^{n} p'_i x_i > \sum_{i=1}^{n} p'_i x'_i$$

Otherwise it would violate WARP

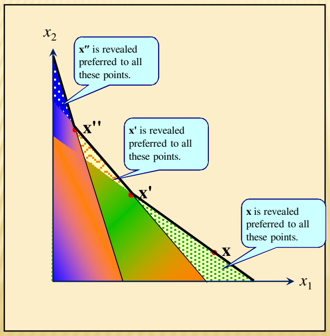
WARP IN ACTION



- *Take the original equilibrium*
- *Now let the prices change...*
- *WARP rules out some points as possible solutions*

- *Clearly WARP induces a kind of negative substitution effect*
- *But could we extend this idea...?*

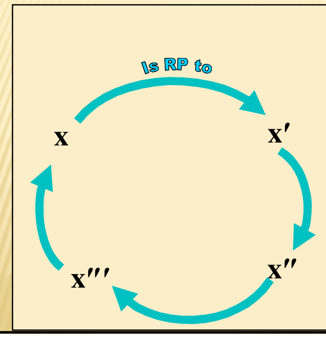
TRYING TO EXTEND WARP



- Take the basic idea of revealed preference
- Invoke revealed preference again
- Invoke revealed preference yet again
- Draw the "envelope"

- *Is this an "indifference curve"...?*
- *No. Why?*

LIMITATIONS OF WARP



- *WARP rules out this pattern*
- *...but not this*

- *WARP does not rule out cycles of preference*
- *You need an extra axiom to progress further on this:*
- *the strong axiom of revealed preference.*

REVEALED PREFERENCE: IS IT USEFUL?

- You can get a lot from just a little:
  - You can even work out substitution effects.
- WARP provides a simple consistency test:
  - Useful when considering consumers en masse.
  - WARP will be used in this way later on.
- You do not need any special assumptions about consumer's motives:
  - But that's what we're going to try right now.
  - It's time to look at the mainstream modelling of preferences.

OVERVIEW...

Consumption: Basics

- The setting
- Budget sets
- Revealed Preference
- Axiomatic Approach

Standard approach to modelling preferences

THE AXIOMATIC APPROACH

- Useful for setting out a priori what we mean by consumer preferences.
- But, be careful...
- ...axioms can't be "right" or "wrong,"...
- ... although they could be inappropriate or over-restrictive.
- That depends on what you want to model.
- Let's start with the basic relation...

THE (WEAK) PREFERENCE RELATION

- The basic weak-preference relation:

  x < x'

  "Basket ***x*** *is regarded as at least as good as basket* ***x'*** *..."*

- From this we can derive the indifference relation.

  x ∨ x'

  "x < x'" and "x' < x."

- ...and the strict preference relation...

  x Â x'

  "x < x'" and not "x' < x."

FUNDAMENTAL PREFERENCE AXIOMS

- Completeness
- Transitivity
- Continuity
- Greed
- (Strict) Quasi-concavity
- Smoothness

For every **x, x'** $\in X$ *either* **x**<**x**' *is true, or* **x**'<**x** *is true, or both statements are true*

FUNDAMENTAL PREFERENCE AXIOMS

- Completeness
- Transitivity
- Continuity
- Greed
- (Strict) Quasi-concavity
- Smoothness

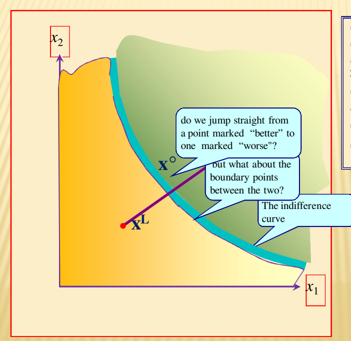
For all **x, x' , x"** $\in X$ *if* **x**<**x**' *and* **x**'<**x"** *then* **x**<**x"**.

FUNDAMENTAL PREFERENCE AXIOMS

- Completeness
- Transitivity
- Continuity
- Greed
- (Strict) Quasi-concavity
- Smoothness

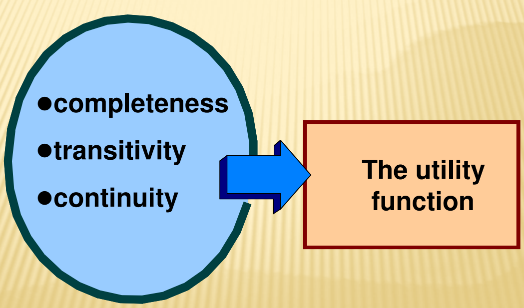
For all **x'** $\in X$ *the not-better-than-***x'** *set and the not-worse-than-***x'** *set are closed in* X

CONTINUITY: AN EXAMPLE

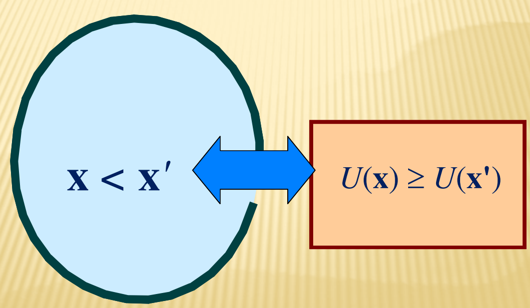


- *Take consumption bundle* **x°**.
- *Construct two other bundles,* $\mathbf{x}^L$ *with Less than* **x°**, $\mathbf{x}^M$ *with More*
- *There is a set of points like* $\mathbf{x}^L$, *and a set like* $\mathbf{x}^M$
- *Draw a path joining* $\mathbf{x}^L$, $\mathbf{x}^M$.
- *If there's no "jump"…*

AXIOMS 1 TO 3 ARE CRUCIAL …



A CONTINUOUS UTILITY FUNCTION THEN REPRESENTS PREFERENCES…



TRICKS WITH UTILITY FUNCTIONS

- *U*-functions represent preference *orderings*.
- So the utility scales don't matter.
- And you can transform the *U*-function in any (monotonic) way you want...

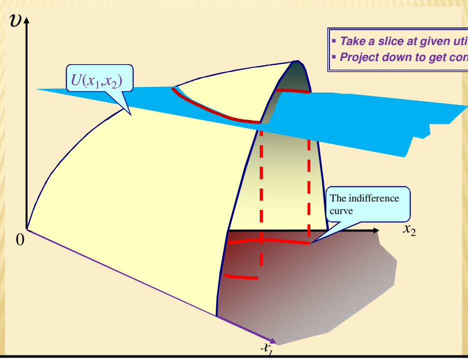
IRRELEVANCE OF CARDINALISATION

- $U(x_1, x_2,..., x_n)$
- $\log(\ U(x_1, x_2,..., x_n)\ )$
- $\exp(\ U(x_1, x_2,..., x_n)\ )$
- $\sqrt{(\ U(x_1, x_2,..., x_n)\ )}$
- $\varphi(\ U(x_1, x_2,..., x_n)\ )$

▪ *So take any utility function...*
▪ *This transformation represents the same preferences...*
▪ *...and so do both of these*
▪ *And, for any monotone increasing φ, this represents the same preferences.*

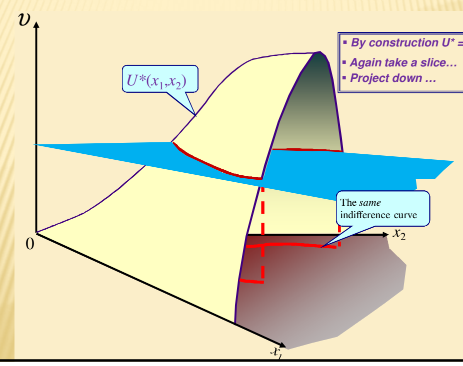
▪ *U is defined up to a monotonic transformation*
▪ *Each of these forms will generate the same contours.*
▪ *Let's view this graphically.*

A UTILITY FUNCTION



▪ *Take a slice at given utility level*
▪ *Project down to get contours*

ANOTHER UTILITY FUNCTION

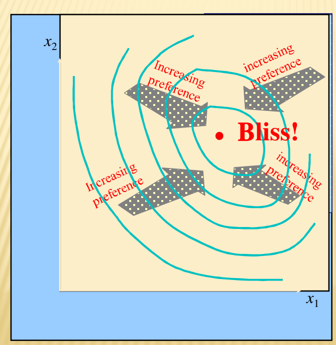


▪ *By construction U* = φ(U)*
▪ *Again take a slice…*
▪ *Project down …*

ASSUMPTIONS TO GIVE THE *U*-FUNCTION SHAPE

- Completeness
- Transitivity
- Continuity
- Greed
- (Strict) Quasi-concavity
- Smoothness

THE GREED AXIOM



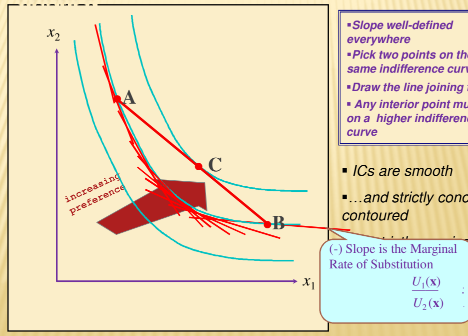
▪ *Pick any consumption bundle in X.*
▪ *Greed implies that these bundles are preferred to x'.*
▪ *Gives a clear "North-East" direction of preference.*
▪ *What can happen if consumers are not greedy*

▪ *Greed: utility function is monotonic*

A KEY MATHEMATICAL CONCEPT

- We've previously used the concept of concavity:
  - Shape of the production function.
- But here simple concavity is inappropriate:
  - The *U*-function is defined only up to a monotonic transformation.
  - *U* may be concave and U^2 non-concave even though they represent the same preferences.
- So we use the concept of "quasi-concavity":
  - "Quasi-concave" is equivalently known as "concave contoured".
  - A concave-contoured function has the same contours as a concave function (the above example).
  - Somewhat confusingly, when you draw the IC in (x_1,x_2)-space, common parlance describes these as "convex to the origin."
- It's important to get your head round this:
  - Some examples of ICs coming up...

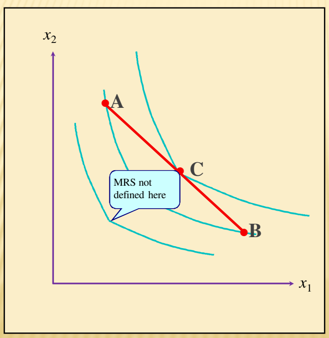
CONVENTIONALLY SHAPED INDIFFERENCE CURVES



- *Slope well-defined everywhere*
- *Pick two points on the same indifference curve.*
- *Draw the line joining them.*
- *Any interior point must line on a higher indifference curve*

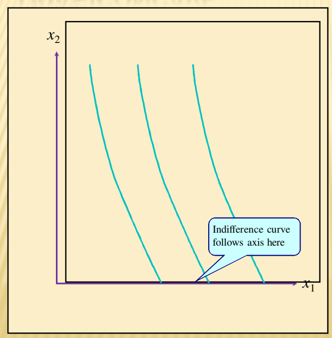
- *ICs are smooth*
- *…and strictly concaved-contoured*

OTHER TYPES OF IC: KINKS



- *Strictly quasiconcave*
- *But not everywhere smooth*

OTHER TYPES OF IC: NOT STRICTLY QUASICONCAVE



- *Slope well-defined everywhere*
- *Not quasiconcave*
- *Quasiconcave but not strictly quasiconcave*

- *Indifference curves with flat sections make sense*
- *But may be a little harder to work with...*

SUMMARY: WHY PREFERENCES CAN BE A PROBLEM

- Unlike firms there is no "obvious" objective function.
- Unlike firms there is no observable objective function.
- And who is to say what constitutes a "good" assumption about preferences...?

REVIEW: BASIC CONCEPTS

- Consumer's environment
- How budget sets work
- WARP and its meaning
- Axioms that give you a utility function
- Axioms that determine its shape

WHAT NEXT?

- Setting up consumer's optimisation problem
- Comparison with that of the firm
- Solution concepts.