UNIVERSITY OF ATHENS
Department of Economics
Course: Advanced Microeconomics
Instructor: Vassilis T. Rapanos

Assignment no. 1 *The answers are due on November 20*

Exercise 1.

A firm uses two inputs in the production of a single good. The input requirements per unit of output for a number of alternative techniques are given by the following table:

Process	1	2	3	4	5	6
Input 1	9	15	7	1	3	4
Input 2	4	2	6	10	9	7

The firm has exactly 140 units of input 1 and 410 units of input 2 at its disposal.

1. Discuss the concepts of technological and economic efficiency with reference to this example.

2. Describe the optimal production plan for the firm.

3. Would the firm prefer 10 extra units of input 1 or 20 extra units of input2?

Exercise 2.

Suppose a .rm.s production function has the Cobb-Douglas form

$$q = z_1^{\alpha_1} z_2^{\alpha_2}$$

where z_1 and z_2 are inputs, q is output and α_1, α_2 are positive parameters.

1. Draw the isoquants. Do they touch the axes?

2. What is the elasticity of substitution in this case?

3. Using the Lagrangean method and the cost-minimising values of the inputs and the cost function.

4. Under what circumstances will the production function exhibit (a) decreasing (b) constant (c) increasing returns to scale? Explain this using first the production function and then the cost function.

5. Find the conditional demand curve for input 1.

Exercise 3.

Suppose a firm's production function has the Leontief *form*

$$q = \min\left\{\frac{z_1}{\alpha_1}, \frac{z_2}{\alpha_2}\right\}$$

where the notation is the same as in Exercise 2.4.

1. *Draw the isoquants.*
2. *For a given level of output identify the cost-minimising input combination(s) on the diagram.*
3. *Hence write down the cost function in this case. Why would the Lagrangean method of Exercise* 2 *be inappropriate here?*
4. *What is the conditional input demand curve for input 1?*
5. *Repeat parts 1-4 for each of the two production functions*

$$\begin{aligned} q &= \alpha_1 z_1 + \alpha_2 z_2 \\ q &= \alpha_1 z_1^2 + \alpha_2 z_2^2 \end{aligned}$$

   Explain carefully how the solution to the cost-minimisation problem differs in these two cases.

Exercise 4.

Assume the production function

$$\phi(\mathbf{z}) = \left[\alpha_1 z_1^\beta + \alpha_2 z_2^\beta\right]^{\frac{1}{\beta}}$$

where z_i is the quantity of input i and $\alpha_i \geq 0$, $-\infty < \beta \leq 1$ are parameters. This is an example of the CES (Constant Elasticity of Substitution) production function.

1. *Show that the elasticity of substitution is $\frac{1}{1-\beta}$.*
2. *Explain what happens to the form of the production function and the elasticity of substitution in each of the following three cases: $\beta \to -\infty$, $\beta \to 0$, $\beta \to 1$.*