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INTERSUBJECTIVITY IN MATHEMATICS LEARNING: A CHALLENGE TO THE RADICAL CONSTRUCTIVIST PARADIGM?

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Radical constructivism is currently a major, if not the dominant, theoretical orientation in the mathematics education community, in relation to children’s learning. There are, however, aspects of children’s learning that are challenges to this perspective, and what appears to be “at least temporary states of intersubjectivity” (Cobb, Wood, & Yackel, 1991, p. 162) in the classroom is one such challenge. In this paper I discuss intersubjectivity and through it offer an examination of the limitations of the radical constructivist perspective. I suggest that the extension of radical constructivism toward a social constructivism, in an attempt to incorporate intersubjectivity, leads to an incoherent theory of learning. A comparison of Piaget’s positioning of the individual in relation to social life with that of Vygotsky and his followers is offered, in support of the claim that radical constructivism does not offer enough as an explanation of children’s learning of mathematics.

Constructivists, whether radical, weak, or social (Cobb, 1994), draw their inspiration from Piaget, for whom the individual is the central element in meaning-making. In this paper I intend to register serious objections to this central thesis. Throughout the paper I will draw on Vygotsky’s psychological theories as contrast to Piaget’s. Vygotsky attempted to develop a fully cultural psychology (Kozulin, 1990, p. 1), by which I mean placing communication and social life at the center of meaning-making, which is a challenge to Piaget’s ideas. Vygotsky died in 1934 at an early age, without the opportunity to fully develop his program. His collaborators and followers described and describe Vygotsky as the inspiration for their work; consequently, I will draw on the ideas of the Vygotskian school as extensions and developments of his work, recognizing the inevitable contradictions and limitations of a theorist whose output became more prolific as he approached his death. Many modern writers on cultural psychology (e.g., Harré & Gillett, 1994) also acknowledge the ideas of Vygotsky and his followers as forerunners of their own. I will argue that Vygotsky’s and Piaget’s programs have fundamentally different orientations, the former placing the social life as primary and the latter placing the individual as primary. Through an examination of the issue of intersubjectivity in the mathematics classroom I will argue that the integration of notions of the social construction of knowledge into a radical constructivist view of learning is, at the very least, problematic. I will suggest that it does not do justice to the implications of cultural psychology, indeed that it cannot do so; that the assumption of complementarity leads to incoherence; and as a consequence, that mathematics education would benefit from abandoning constructivism as a view of how people learn.

The paper commences with a discussion of the challenge that intersubjectivity poses for radical constructivism and an argument for a fully cultural view as an alternative.
Of course, there are different versions of constructivism, and their different orientations, together with issues of complementarity and paradigms forms the third part of that discussion. In the fourth part, I offer a number of examples of research programs that, I argue, cannot easily be accommodated by an individualistic psychology.

INTERSUBJECTIVITY AND SOCIAL CONSTRUCTIVISM

A major difficulty for radical constructivism is an adequate explanation of intersubjectivity. The problem is stated clearly by Cobb, Wood, and Yackel (1991):

Constructivism, at least as it has been applied to mathematics education, has focused almost exclusively on the processes by which individual students actively construct their own mathematical realities .... However, far less attention has been given to the interpersonal or social aspects of mathematics learning and teaching .... [H]ow ... does mathematics as cultural knowledge become "interwoven" with individual children’s cognitive achievements? In other words, how is it that the teacher and the children manage to achieve at least temporary states of intersubjectivity when they talk about mathematics? (p. 162)

Although social interactions, together with sensorimotor material and graphic representations (von Glasersfeld, 1992), are seen as leading to the development of an individual’s knowledge and are seen as providing the perturbations that lead to the adaptation of concepts (Steffe, 1993), this does not constitute, nor does it explain, intersubjectivity. Indeed, for radical constructivists it is difficult to see how there could be such a thing as the intersubjective construction of knowledge, because if there were, the problem that Piaget attempted to answer through genetic epistemology would reappear, namely, how an individual might gain access to such knowledge.

Nevertheless, some radical constructivists see the need to face the issue. Social constructivism, in its various forms, has grown out of the attempt to incorporate an explanation for intersubjectivity into an overall constructivist position. It appears, though, that the only way those writers who have taken that direction can achieve this is in a complementary fashion. Consider the following quotations:

We can observe that when we talk of students’ constructive activities we are emphasising the cognitive aspect of mathematical learning. It then becomes apparent that we need to complement the discussion by noting that learning is also a process of acculturation. (Cobb, Yackel, & Wood, 1992, p. 28)

Although the primacy of focus of each of conventionalism and radical constructivism is sacrificed in social constructivism, their conjunction in it serves to compensate for their individual weaknesses.... (Ernest, 1991, p. 86)

The fundamental orientation of the work in our own classroom springs from the radical constructivist principle and an integrated and compatible elaboration of the role of the social dimension in these individual processes of constructing as well as the processes of social interaction in the classroom. (Bauersfeld, 1992, p. 2)

These extracts demonstrate attempts to elaborate on the way in which social interactions, conceived as part of the active subject’s efforts to achieve viability for her
or his concepts, function in the mathematics classroom while placing a much stronger emphasis on those social interactions. They suggest a desire on the part of the authors to situate themselves within the radical constructivist position but at the same time to recognize and accept that intersubjective construction of shared knowledge appears to take place in the classroom. The authors’ theoretical move at this point seems to be to add to the two well known hypotheses (von Glasersfeld, 1990, pp. 22–23, quoted below) a third hypothesis, namely, that sometimes the actors in the classroom share knowledge. It is as if, in interpreting the learning processes taking place in the classroom, one sometimes sets aside the notion that individuals are constructing their own knowledge, and one observes instead that learning appears to be involuntary, through acculturation. Interestingly, here Cobb et al. specify the constructive activities as cognitive, acculturation presumably being something else. Indeed, Cobb (1994) is more specific elsewhere, describing his concern with students’ mathematical activities in the classroom as arising from a need to go beyond “the purely cognitive by locating students’ activity in social and cultural circumstances” (p. 9).

In these quotations, “acculturation” and “at least temporary states of intersubjectivity” go beyond the interpretation of social interactions as setting up perturbations for the individual, which is the Piagetian view. There is a strong sense of preexisting cultural tools, or concepts, outside of the individual’s mind, in the notion of acculturation, perhaps implying a positioning of the individual within a culture or cultures, rather than the individual’s autonomous construction of her or his own subjectivity. Similarly, the phrase “states of intersubjectivity” suggests something more than “the role of the social dimension in these individual processes of constructing,” something that takes place between people that the individual internalizes only secondarily. Bauersfeld’s interactionism appears to accept that meanings are cultural and thus the core of what is learned is “when to do what and how to do it ... The core part of school mathematics enculturation comes into effect on the meta-level and is ‘learned’ indirectly” (quoted in Cobb, 1994, p. 10). There is still a problem with the process of that indirect learning, however, if the fundamental notion is that all learning is the construction of the individual. Cobb’s recent work (e.g., 1994), which draws together Bauersfeld’s interactionism and constructivism into what he calls an “emergent perspective,” still relies on unexplained phrases such as “profoundly influenced by” (p. 9) in order to explain how the sociocultural setting leads to learning. There is an undeniable conflict here precisely because, as Goldin (1990) argues, “radical constructivism does not in principle ever permit us to conclude that two individuals have ‘the same’ knowledge.” (p. 39).

Before elaborating further on the various strategies adopted by the constructivists to deal with intersubjectivity within a radical constructivist framework, I will offer an alternative interpretation of intersubjectivity that, I argue, demonstrates a world view quite distinct from that offered by radical constructivism.

**INTERSUBJECTIVITY AS THE CONSTRUCTION OF MIND**

Any talk of acculturation or the social construction of knowledge or intersubjectivity...
appears to lead to the problem, for constructivists of all sorts, of how that knowledge, or knowing, can be internalized by the individual. Cobb’s use of the phrase “profoundly influence” and Bauersfeld’s “indirect learning” are examples of the struggle. Steffe (1993) is consistent and quite specific when he recognizes that, in the radical constructivist view, the process of internalization makes no sense:

Vygotsky’s notion of internalization is an observer’s concept in that what the observer regards as external to the child eventually becomes in some way part of the child’s knowledge. But Bickhard (in press) has pointed out that there is no explanatory model of the process. (p. 30)

In contrast, I wish to argue, with Leont’ev (1981), that the process of internalization is not the transferal of an external to a pre-existing, internal “plane of consciousness”; it is the process in which this plane is formed. (p. 57)

It is not that we need to take greater account of social interactions and language, because it is through these that the individual constructs thoughts and concepts, which is the shift that Bauersfeld, Cobb, and others make. It is in discourses, subjectivities, significations, and positionings that psychological phenomena actually exist (Evans & Tsatsaroni, 1994). A fully cultural psychology is a different world view and a challenge to the mentalism that lies at the heart of Piagetian psychology and therefore constructivism in all its forms (Harré & Gillett, 1994). In order to engage with Vygotsky’s argument that “the true direction of the development of thinking is not from the individual to the socialised, but from the social to the individual” (Vygotsky, 1986, p. 32), it is necessary to recognize the shift from a view of the autonomous cognizing subject constructing her or his subjectivity and knowing to one of the construction of human consciousness in and through communication. Thus the individual is, in a primary sense, a product of her or his time and place.

Acts of remembering, for instance, are not manifestations of hidden subjective psychological phenomena—they are the psychological phenomena (Harré & Gillett, 1994). So, too, for emotions, attitudes, and so on. When an action gains significance for a child, becoming bound up with goals, aims, and needs and associated with a purpose, it is a social event.

Voluntary attention is not biological in its origin but a social act, and that it can be interpreted as the introduction of factors which are the product, not of the biological maturing of the organism but of forms of activity created in the child during his relations with adults, into the complex regulation of this selective mental activity [sic]. (Luria, 1973, p. 262)

Consciousness is constructed in communication, in discursive practices, and through acculturation. It is in this sense that Vygotsky writes of the social plane as primary both in time and in fact. This is not to see individuality and difference as non-existent, nor is it to argue that the individual mind is reducible to the social plane. It is to argue that the gaze of the psychologist must be on the social and cultural practices in which meanings and purposes function so that people act in the world. “The study of the mind is a way of understanding the phenomena that arise when different sociocultural discourses are integrated within an identifiable human individual situated in relation to those discourses” (Harré & Gillett, 1994, p.22). Social settings,
such as the mathematics classroom, are determined by all the actors, both present and absent, and so the intersubjectivity is a function of the time and place and the goals of the activity and the actors. Cultural psychology is sometimes described as a transmission view of teaching and learning (Murray, Olivier, & Human, 1993; Cobb, 1994) because writers assume that from this perspective the mind is open to the input of the parent or the teacher, but this ignores the specificity of the intersubjectivity of every teaching/learning situation as a function of the setting, the activity, the actors, the texts, and so on. It assumes that a cultural psychology ignores the differences that the individuals, as unique “collections” of subjectivities and positionings, bring to the situation. Finally, it ignores the claim, common to Piaget and Vygotsky, that objects or tools become part of an individual’s life when they are acted upon; they cannot become the individual’s in a “transmitted” sense. For Luria, following Vygotsky, those actions, including attending, distinguishing, and so on, are themselves social acts, learned in interaction with others.

Language plays the central role in the development of consciousness, as Luria’s work on the organizational structure of the brain demonstrates (Luria, 1973). This is not a language of thought, nor is it, as Piaget (1969, p. 126) argues, “playing a central role in the formation of thinking only in so far as language is one of the manifestations of the symbolic function.” Language provides the tools of thought, and carries the cultural inheritance of the communities (ethnic, gender, class, etc.) in which the individual grows up. “It is the world of words which creates the world of things” (Lacan, 1966, p. 155). Language is not seen as giving structure to the already conscious cognizing mind; rather, the mind is constituted in discursive practices. Thus the semiotic function becomes the focus of study, rather than the reversible mental structures that, for Piaget (1964), “constitute the basis of knowledge, the natural psychological reality, in terms of which we must understand the development of knowledge” (p. 9).

It is incorrect to consider language as correlative of thought, language is a correlative of consciousness. The mode of language correlative to consciousness is meanings. The work of consciousness with meanings leads to the generation of sense, and in the process consciousness acquires a sensible (meaningful) structure. To study human consciousness means to study this sensible structure, and verbal meaning is the methodological unit of this study. Such a study can be carried out at the abstractive as well as the concrete level. At the level of abstract psychology we can study general rules of signification; at the concrete level we should be concerned with specific “sense-generating” activity that changes the consciousness of a person. (Kozulin, 1990, p. 190)

More poetically, Vygotsky says (1986),

Consciousness is reflected in a word as the sun in a drop of water. A word relates to consciousness as a living cell relates to an organism, as an atom relates to the universe. A word is the microcosm of human consciousness. (p. 256)

Rejecting a picture theory of mind, that mental representations of reality are exact replicas of the real world, leads, for the radical constructivist, to the conclusion that one can only argue that all representations are constructed by the individual, and hence meanings are ultimately those in the individual’s mental plane (Confrey, 1992).
In his rejection of his own picture theory, Wittgenstein (1974) did not make that move. On the contrary, he argued that private languages make no sense and that all experiences gain their meaning and significance for the individual from their use. In his discussion of pain as a social phenomenon (op cit.), he demonstrates how consciousness is socially constituted. To constructivists the two alternatives are absolutism of a platonic or empirical kind or the autonomous cognizing individual who constructs her or his own world. The notion of the mind as constituted in social and cultural experience is only seen as another form of absolutism where there is no mechanism for internalization.

Although Vygotsky died in 1934, his focus on meanings being socially and culturally determined and consciousness being constituted in social practices created a psychology that could engage with the insights offered by poststructuralism in recent decades (Harré & Gillett, 1994; Lerman, 1994a). Thus, although Vygotsky and his collaborators and followers would not have talked of regulation of subjectivities within discourses, at the heart of their psychology was the Marxist notion that human behavior and social relationships are products of economic and other relations. I would suggest that Vygotsky’s own upbringing as an orthodox Jew in a segregated region of Russia would have given him a clear sense of the role that culture played in the construction of the individual (although, of course, Marx denied the significance of nationalities).

Finally, where the individual is seen as being constructed socially and culturally, teaching and learning are integrated. Processes of human thinking, such as attending, are social acts, and cultural tools, most particularly language, are appropriated by the individual. “The child does not create his own speech, his own verbal meanings … he masters the speech of adults” (Davydov, 1990, p. 179). Learning takes place as theoretical/scientific concepts ascend from the abstract to the concrete in the zone of proximal development (z. p. d.), in interaction with more knowledgeable others. From this world view, teaching and learning cannot be discussed separately.

**CONSTRUCTIVISMS, PARADIGMS, AND COMPLEMENTARITY**

There is a substantial body of literature that argues that there can be no resolution of different world views. There is no way of stepping outside of paradigms and meanings in order to evaluate and compare alternatives, and there is no way of stepping outside of language in order to discuss language (Derrida, 1973). Neither the mentalistic, individualistic psychology nor the cultural, discursive psychology can be discounted or disproved by the other. I want to argue, however, that a merger of these two views is incoherent and can only be attempted by not engaging fully with their distinct interpretations of the individual in her or his actions in the world. In this section I will further examine the moves made in the attempts and hope to demonstrate that incoherence. I would argue that this examination is a very important task for mathematics education, because we otherwise might lose the strength of the insights of cultural studies over the last few decades by subsuming them into mentalistic psychology.
Bauersfeld, Ernest, Cobb, and others appear to wish to draw on different theories of learning at different times, one that places the individual at the center of meaning-making and denies the possibility of internalizing something that occurs first outside the individual, and the other that asserts that all learning is a process of internalizing what is first on the social plane and that the mind is constituted in intersubjectivities. The notion of complementarity (e.g., Bartolini-Bussi, 1991) is often used to suggest that in the literature one can find support for taking apparently contradictory theories and incorporating them in some fashion. Reference is made to Steiner’s work (e.g., 1985) in particular. However, a careful reading of Steiner’s arguments for complementarity in theorizing in mathematics education shows that he refers to a complementarity between what he calls two levels, theory and activity:

The concept of complementarity also turns out to be an adequate tool for better understanding the relations between different types and levels of knowledge and activity as the [sic] appear in contrapositions like “scientific theory vs. everyday knowledge.” (p. 15, emphasis in original)

Using the notion to slip from theory to theory, ignoring the contradictions and disagreements, on the grounds that each offers a richer explanation than the other at different times is, I would argue, to do an injustice to theory and also to be in danger of losing the coherence of each and the insights that each theory, when taken in full, can offer. I argue here that social constructivists, for the most part, are guilty of such moves. Confrey (1991, 1992) makes, in my view, a more worthy attempt to identify the strengths and weaknesses of Piaget and Vygotsky and create a new approach. I discuss her work below.

Incoherence arises also in the partial use of a fundamental aspect of the Piagetian view of achieving or at least challenging viability, namely, the filter of the individual’s conceptual system. “We come to see knowledge and competence as products of the individual’s conceptual organization of the individual’s experience.” (von Glasersfeld, 1983, p. 66).

Individuals construct their own knowledge from experiences; in the school setting, those experiences often result from activities arranged by the teacher. Whether the individual modifies her or his ideas or not is dependent on the challenge to the viability of her or his existing conceptual structures when faced with the new experiences. However, whether a new experience constitutes a challenge to a person’s existing schemata or not is also dependent on the individual’s schemata. To be consistent, one needs to retain that view. As Confrey (1990, p. 108) writes: “We construct our understanding through our experiences and the character of our experience is influenced profoundly by our cognitive lenses.” Thus what happens in the classroom may or may not generate a cognitive conflict for an individual; social interactions have no “force,” the individual construes (the sense of “force” as used here will be elaborated below). Jaworski (1994) describes an incident where a researcher who was observing two 11-year-old children looking at two triangles, one with angles of 45, 45, and 90 degrees and the other of 30, 60, and 90, hears one child identify the first as a triangle but not the second. Jaworski goes on to discuss the idea that one cannot say that the boy is wrong and the researcher who would see both as triangles
is right without “taking into account the circumstances from which the statement arises” (p. 14) and the experiences that would have led the boy to make the statement. She draws on this example to support Noddings’s (1990) claim that constructivism is postepistemological and that one can only talk of constructivism as a theory of knowing, not of knowledge. Negotiation, through the intervention of the researcher, the teacher, or the other child, may have led to disequilibrium for the boy, or it may not. Jaworski is, in my view, drawing in a coherent and consistent manner on Piaget’s notion of social interaction and seems to find it adequate. Nevertheless, later in that paper she argues for a social constructivist position, despite expressing some concern about theories of learning that strengthen the social role and talk of learning as integration into a community of practice. She claims that linking constructivism and sociocultural theory offers “the potential to explain children’s development of mathematical knowledge in terms of its individual and social construction under the influence of social and cultural practices” (p. 17). Taking constructivism’s view of the autonomy of the individual in the construction of her or his knowing, given her or his particular conceptual system and its particular filter, leads to a consistent, albeit very restricted, view. To argue for an integrated social view is to argue that sometimes the filter has very large holes and what is occurring beyond the individual can somehow enter without constraint. I argue, then, that it makes no sense to strengthen the functioning of the “social” into a social constructivism. Either there is no force to cultural knowledge and discursive practices, or there is. If the former, then the Piagetian view of social interaction as one of the processes that may or may not challenge the viability of the organism’s cognitive structure is inadequate to avoid solipsism, but as a postepistemological theory this is not a problem. If it is a problem then the answer is not found in constructivism.

In my reading of Steffe’s work it seems that he does not fall into the trap of arguing for the complementarity of social and individual meaning-making. He finds Piaget’s approach adequate for the interpretation of the function of social interaction:

In this sense it is legitimate to interpret Piaget’s work as a social-cultural approach in which he explained the mathematical development of children as self-regulating, autonomous organisms interacting in their environments. He seemed to take the social-cultural milieu of the children as a given without attempting to alter their most general experiences. (Steffe, 1993, pp. 3–4)

For Piaget there was no doubt that the real world constrains the possibilities of the individual’s schemata. As Goldin (1990, p. 39) points out: “Piaget not only recognized ‘logical necessity’ but also accorded an important role to ‘structure’ apart from idiosyncratic construction by individuals.” The radical constructivist position, as described by von Glasersfeld (1990), draws away from Piaget’s commonalities and is radical precisely because it argues that reality is unknowable, in that one can never know that what one perceives as reality is anything other than one’s own construction.

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.
2. a. The function of cognition is adaptive in the biological sense of the term, tending towards fit or viability;

In terms of an outside and an inside, the external real world and the internal mental world(s), the postepistemological radical constructivist position has coherence in that it denies the value of talk of the real world at all. It does not have, but does not need, an explanatory model of how knowledge becomes internal, because any sense of knowledge or knowing, other than the internal, is excluded. This is not the case with the Cobb position, or with other versions of constructivism that emphasize a stronger role for social interactions. Some writers, then, appear to consider that Piaget’s formulation of what Bauersfeld calls here “the social dimension” is adequate for an explanation of what appears to be processes of acculturation and intersubjectivity. Others consider the social dimension to be more substantial and seek to complement radical constructivism with a sociocultural view of learning.

The former draw on notions of “fit, not match” and “taken as shared.” As long as the gaze is on the individual mind, with meaning-making being interpreted as the sense an individual makes of her or his experiences, one has to continue to reject the possibility of a match between the conceptualizations of one person with another or between the individual’s representations and reality. The biological metaphor of viability is completely appropriate for such a characterization of representation. If the gaze shifts, however, to human consciousness, to how culture and social life are embedded in language (verbal and other), and the focus moves to how people are regulated and constituted through communication, the need for the distinction between “fit” and “match” falls away. As discussed above, each individual is a unique collection of subjectivities and positionings (Harré & Gillett, 1994). Those collections are not ultimately private, however, but are each shared with others of common culture; they are communicable even though they may not be communicated. I would argue that no sense can be made of their being private (Wittgenstein, 1974), because they are internalized from cultural experiences and carried in language and ritual and other forms of essentially human communication. It is inconceivable that a “wolf-child” might choose or construct Judaism for her or himself. “Fit” and “match,” or rather fit but not match, are part of the language of a learning theory and an epistemology that centers meaning-making on an autonomous individual. Although I would argue that such a position is very limited in that it does not account for so much of social and cultural life, it is at least consistent and coherent as a view of learning with implications for epistemology and ontology. The complementary strategy tries to bring together two different world views but does not achieve coherence.

Confrey (1992) also criticizes many social constructivists for putting together the theories of Vygotsky and Piaget in a manner that addresses neither the insights nor the drawbacks of each. Her paper focuses on Vygotsky and radical constructivism as alternative positions, and she examines both theories and argues for a new approach that incorporates Maturana and Varela’s (1986) theory of autopoiesis.

Confrey’s major criticism of Vygotsky’s work is the different ways he characterizes
everyday, spontaneous, or natural concepts, and scientific concepts\(^1\), and she draws on the work of Wertsch (1991) in which he addresses the “emergent interactionism between natural and social strands” (Confrey, 1992, p.9). Although she rightly argues that the simple insertion of Piaget into that deficiency is inadequate, she maintains that there is a significant inadequacy in Vygotsky’s ideas here. She (with Wertsch) argues that Vygotsky seems to suggest that cultural tools and learning are evident in the development of scientific concepts, but that everyday concepts appear to be closer to Piaget’s vision of the autonomous cognizing subject’s constructions from her or his experiences. Although she refers to Leont’ev’s interpretation of internalization as the process whereby the individual plane is constituted, it is again related just to the development of scientific concepts. In this paper I have indicated that consciousness is constituted in and through communication and this relates to the whole development of the child. The distinction between scientific concepts and everyday concepts can certainly be seen as the interaction of intuitive notions with the formalized theoretical notions of the community in the zone of proximal development, but that does not imply that those intuitive spontaneous concepts are not formed in communication, nor does it imply that they only occur later in the child’s life (Newman & Holzman, 1993). On the contrary, taking Luria’s and Leont’ev’s elaborations of Vygotsky’s interpretation of the development of human consciousness indicates that a nonsocial origin to spontaneous concepts makes no sense. Newman and Holzman (1993) suggest that Vygotsky never defined the terms spontaneous and scientific concepts or their difference, but that the major difference is that the learning of the former is never made conscious (p. 61). Davydov’s elaboration of Vygotsky’s description of the leading activities at different stages in children’s lives (Davydov, 1988) suggests that children are engaged in learning in the z. p. d. at all stages, although play is the leading activity before school and school learning after entering school. The difference is in the intentioned learning, concomitant with the new goals of the child, in school. However, in the stage led by play such intentioned learning, usually by the parent, is always present. Confrey’s description of natural learning resembles Piaget’s notion of prelinguistic thought but not, in my view, the interpretation of cultural, discursive psychology, nor that of the developments of Vygotsky’s theory.

In the following section I discuss the issue of intersubjectivity in three sections: intersubjectivity as manifested in the social construction of subjectivity, situated cognition, and mathematics as cultural knowledge.

THREE ASPECTS OF INTERSUBJECTIVITY

Subjectivity Constituted Through Social Practices

From a sociocultural position (Walkerdine, 1988; Evans & Tsatsaroni, 1994), subjectivity is constituted through social practices, not independently arrived at by a

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\(^1\) Given the times in which he was writing, Vygotsky would have used the term “scientific” in a sense that pre-dates sociological critiques of the reification of science and positivism.
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decontextualised individual. Development takes place in social settings in which meanings are carried, rather than meanings being freely and “rationally” constructed by the individual. (For Piaget, development is a natural human process, which is primary to learning; for Vygotsky, learning and social interactions are what form consciousness, and learning leads development.) As a consequence, the individual is more appropriately thought of as constituted through multiple subjectivities, a fragmented self rather than an autonomous unitary subject. Below are two studies of the forms these subjectivities take, and how they are manifested in the classroom.

Study 1. Taken from Walkerdine and Girls and Math Unit (1989)

Annie takes a piece of Lego to add to a construction she is building. Terry tries to take it away from her to use himself .... The teacher tells him to stop and Sean tries to mess up another child’s construction. The teacher tells him to stop. Then Sean says: ‘Get out of it, Miss Baxter paxter.’

Terry: Get out of it, knickers Miss Baxter.
Sean: Get out of it, Miss Baxter paxter.
Terry: Get out of it, Miss Baxter the knickers paxter knickers, bum.
Sean: Knickers, shit, bum.
Miss Baxter: Sean, that’s enough. You’re being silly.
Sean: Miss Baxter, knickers, show your knickers.
Terry: Miss Baxter, show off your bum (they giggle).
Miss Baxter: I think you’re being very silly.
Terry: Shit, Miss Baxter, shit Miss Baxter.
Sean: Miss Baxter, show your knickers your bum off.
Sean: Take all your clothes off, your bra off... (pp. 65–66)

This transcript is quite shocking in its content and is a very vivid illustration of how power is situated within linguistic practices and is not simply a characteristic of a particular person or of the relative statuses endowed by a particular formal relationship. These boys did not choose to “take control”; there was some inadvertent, unconscious slippage of associations and meanings, perhaps through words, perhaps through rhyming. The actors found themselves in a different discourse, a sexist discourse in which women are abused and dominated, even when the woman is an experienced teacher and the boys are very young. Only when that discourse shifted again could the teacher regain control and change the power relationships. Power relations and the particular ways in which people are positioned are carried in, and regulated through, discursive practices. This is the sense of the “force” of language and social practices mentioned above.

Study 2. Redmond (1992)

Discursive practices are not clearly bounded, they are continually changing, and one switches from one discursive practice to another. A researcher (Redmond, 1992) asked 5–6-year-old children, in pairs, questions about “bigger” in two settings, which
she termed objective and subjective. The former focused on objects for comparison that the researcher produced, whereas the latter focused on comparisons of objects, such as towers made of building blocks, with which the children were playing.

These two were happy to compare two objects put in front of them and tell me why they had chosen the one they had. However when I allocated the multilinks to them (the girl had 8 the boy had 5) to make a tower ... and I asked them who had the taller one, the girl answered correctly but the boy insisted that he did. Up to this point the boy had been putting the objects together and comparing them. He would not do so on this occasion and when I asked him how we could find out whose tower was the taller he became very angry. I asked him why he thought that his tower was taller and he just replied “Because IT IS.” He would go no further than this and seemed to be almost on the verge of tears. (p. 24)

Again, one can see the action of multiple subjectivities in this short extract. The interpretation of the “objective” questions by researcher and children are shared, by virtue of a school mathematics setting being called up by the tasks for the participants (Evans, 1993), but the “subjective” questions call up a different practice for this boy. One can conjecture what that practice might be: the inner need for the boy to assert his superiority over a girl in all things (this explanation is supported by another incident involving these two children in which they were asked to compare their heights. The boy, who was the smaller, pushed the girl on the floor and exclaimed “I am”); rivalry with a sibling; an unconscious response open to a psychoanalytic interpretation; and so on. Thus, it is not appropriate to talk of a child “understanding size,” nor indeed lacking an understanding; such judgments are far too simplistic and rely on a single fixed meaning for an activity such as “comparison of size.” “Understanding” size takes different forms in different settings.

Clearly these are texts, with all the hermeneutic implications that reading texts conveys (Brown, 1991, 1993). A radical constructivist reading might argue that in these examples individuals construe their own meaning. I want to argue that subjectivities are regulated within and through the discourses, or practices, carrying the “force” mentioned above.

Cognition as Situated in Practices

In Piagetian theory the individual achieves the highest level of intellectual thought when engagement with the real world becomes completely formal, when actions become abstract entities to be manipulated mentally. “For cognitive progress is not only assimilation of information; it entails a systematic decenteration process which is a necessary condition of objectivity itself” (Piaget, 1970, p. 710).

Given our academic pursuits, it is difficult to conceive of another order of values regarding thinking, and this is what I take Foucault (1977) to mean when he talks of a “regime of truth.” When the work of Lave (1988), Carraher (1988), and Scribner (1986) is examined, some may tend to think in terms of how this research can help teachers to remediate children’s learning of mathematics. Perhaps it is more appropriate, however, to examine our assumptions and values and their origins, which lead to the privileging of the abstract, particularly in what constitutes mathematics
(although I do not wish to deny that access to esoteric mathematics [Dowling, 1992] is also access to power) (Brown, Collins, & Duguid, 1989; Resnick, 1987). Within practices, thought is contextualised and is valued through criteria of relevance, applicability, innovation, and so on. An academic activity is another specific practice; however, the “pure” abstract cognitive functioning that appears to emanate from that practice is reified as the exemplar of the highest level of intellectual activity. Lave (1988) challenges that reification process:

It is not at the level of cognitive processes that the unique, the non-routine, the crisis, the exception, the creative novelty, the scientific discovery, major contributions to knowledge, ideal modes of thought, the expert and the powerful, are brought into being and given significance and experienced as such. These are all matters of constitutive order in the broadest and most complex sense, and they are constructed in dialectical relations between the experienced lived-in world and its constitutive order—in practice. (p. 190)

The situating of cognition within practices is given a Marxist connection with the view that thinking is a dialectic process, goal-oriented and never without mediation, be it material tools, sign systems, or other people.

The most central claim I wish to pursue is that human action typically employs “mediational means” such as tools and language, and that these mediational means shape the actions in essential ways ... the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of “individual(s)-acting with mediational means” than to speak simply of “individual(s)” (Wertsch, 1991, p. 12)

I would also argue that the valuing of thought divorced from action, and of decontextualized knowledge, mystifies learning and knowledge and leads to oppression rather than empowerment (Freire, 1985; Mellin-Olsen, 1987; Lerman, 1992a). This manifests itself both in terms of individual voice (Ellsworth, 1989) and in a collective sense. Cultural and social settings as well as goals, needs, and purposes are seen as integral to cognition rather than the interference of distinct and subordinate issues labeled “affective,” when meanings are viewed as socially constructed. The overt expression of individual subjectivities in the classroom setting forms part of the constitution of the specific intersubjective activity. Similarly, at the collective level, academic mathematics can be seen as another practice, albeit with considerable cultural capital (Bourdieu, 1977). It draws on processes of thought that turn actions into objects that can themselves be acted on, in ever increasing layers of abstraction (Restivo, 1992). However, the reification, both of the thought processes and of academic mathematics as the highest goal of intellectual development, is expressive of power relations of social and discursive practices and therefore oppressive. (For similar arguments in the context of science, see O’Loughlin, 1992.)

In substituting thought for action, mental transformations for real-world transformations, cognitivism veils the objective sources and bases of social life and relegates individual potency to the inner world of gymnastics. (Sampson, 1981, p. 735)

**Mathematics as Cultural Knowledge**

In mathematics education we are concerned with students acquiring the language
and concepts of the community of mathematicians. We draw on images of ownership and autonomy in an attempt to support the notion of constructions of the individual students, but the meanings that we wish students to construct are quite specific. As teachers of mathematics we aim to structure the classroom activities and the style of discourse in order to facilitate this appropriation and participation. Pupils may well offer ideas that we work on in class but that are either incomplete or differ from accepted meanings. An instance that Bishop (1981, personal communication) has offered will illustrate the argument here. Children are asked for examples of even numbers. After a number of acceptable ones, a child offers “1.” When asked why is it even, the child answers “Because there are two halves.” Similarly, she answers that “a half” is even because there are two quarters, but quarters are odd. This idiosyncratic definition may result from a clock face having been used frequently to illustrate fractions. The children’s responses and ideas are delightful, and we may well report such an anecdote to colleagues. It is entirely appropriate that children offer ideas and that teacher and the rest of the class take these on board and examine them, but there must come a stage when those ideas are extended and compared with other interpretations and meanings from other discourses. There are many meanings to the word “half,” some of which (“your half is bigger than mine”) are in the domain of everyday discourses and only one of which (two halves are only “halves” if they are identical and together make a whole) is in the mathematical discourse. It is not wrong to hold both; it is important to learn which is appropriate in which discourse. The aim of the mathematics teacher might be seen as assisting pupils to appropriate the culture of the community of mathematicians as a further social practice. Objects in mathematics are objective in an intersubjective sense, agreed, useful, long-lasting but potentially changeable. Concepts are socially determined and thus socially acquired. Because concepts gain their meaning in their use, the acquisition of a concept, or “understanding,” can be interpreted as that of an individual coming to share in that meaning through negotiation and discussion. The language creates reality and then describes that creation, and it can appear as though the description is that of a timeless, fixed reality (Eco, 1976; Rotman, 1988). There is a sense in which the “grammar” of the created reality takes on a life of its own, just as the rules of chess create almost endless possibilities. Yet there are always new situations. In tennis, were it possible for a person to throw the ball into the air to a height of 100 metres when about to serve, doubtless a new rule would have to be invented (Shankar, 1987).

In the literature, a pedagogy that is based on constructivism is problem-solving centered and is often unsure of the kinds of interventions that the teacher should make, to the extent that some almost refrain from any interventions. Cobb, et al. (1992) and others rightly criticize this tendency as an abdication of the responsibility of the teacher and at the same time a dependence on students “seeing” the mathematics embodied in the materials. However, the issue of pedagogic practices that follow from constructivist principles can only be couched in a discussion of ways of encouraging rich constructions (or correct constructions mathematically?) on the part of students. Constructivism, after all, just attempts to describe how people learn, and
that process will take place even in a formal lecture hall of 300 students. There is
an independence of the two discussions, radical constructivism and teaching. However, for Vygotsky there is no separation; mediation by materials, tools,
peers, and teacher are constitutive of learning.

In summary, I want to suggest there is only a problem concerning the enculturation of children into mathematical signifiers and into the discourse of mathematicians if we insist that somehow people construct their own private languages and meanings. A fully developed sociocultural theory treats discourse as being at the center of human intellectual development.

CONCLUSION

For Piaget, the individual is a self-regulating autonomous organism, making sense and meaning from sensorimotor, social, and textual experiences. Through social interaction individuals attempt to reach a fit of their images and theories. There are transcultural commonalities in the mental structures individuals develop, for example, those of unit, plurality, and number (Steffe, von Glasersfeld, Richards, & Cobb, 1983; von Glasersfeld, 1992). The social setting provides the possibility for negotiation of meaning through which the fit may be achieved. For Vygotsky, consciousness comes about through communication, through mediation, and through language in particular. Individuals are always situated in time and place. The individual is acculturated from infancy, if not before, and concepts are absorbed from the specific cultures within which the individual develops. In different contexts, with different sets of social relationships2, individuals occupy different “positionings,” a term used by Evans (1993) and Evans and Tsatsaroni (1994) to capture the interplay of sociocultural origins and individuality. Thus both views place the individual and the social life at the center of their theories, and it is unhelpful to claim that either view ignores or downplays the individual or the social life; the terms carry different significations, however. I have argued above and elsewhere (Lerman, 1992[b], 1994b) that the difference is encapsulated in their identification of the source of meaning, the one identifying the cognizing individual and the other cultural and discursive practices.

I have attempted to elaborate the differences between radical constructivism and a sociocultural view of learning and to highlight the unsatisfactory outcome of adding on the latter to the former, because that latter is more than a greater emphasis on social interactions. Indeed, I have argued that these two positions are fundamentally different, the former seeing the individual as meaning-maker and the latter seeing meaning as first sociocultural, to be internalized by the subject’s regulation within discursive practices. I have drawn on the issue of intersubjectivity as a focus through which to offer a critique of the whole constructivist paradigm. Intersubjectivity is a problem for radical constructivism, which is internally consistent and coherent without it, albeit

2As is well known, Vygotsky’s work was an attempt to take Marx’s social theory: “The mode of production of material life conditions the general process of social, political and intellectual life” (Marx 1859/1970, p. 21) as inspiration for the development of a Marxist psychology.
trapped in the Cartesian separation of subject and object. Attempts to incorporate intersubjectivity into radical constructivism make it an incoherent theory: on the one hand, seeing the subject as separate from object, and on the other, drawing on the notion of the object as extension of the subject; on the one hand, seeing language as individual and secondary to intelligence, and on the other, allowing that ideas are carried in language, which is first on the social plane, in context-specific ways.

Rejecting constructivism, the individual is integrally part of the social world, and thinking is in a dialectic relationship with that world. Individual mental structures are not the fundamental unit of cognition; meanings, which are first on the social plane, perform this function. Inevitable biological development is not seen to lead to human functioning; the development of consciousness, which only takes place in social life, is the essence. Intersubjectivity is no longer a problem; on the contrary:

Intersubjectivity and interpersonal communication are the driving belts of the sensible structure of consciousness. (Kozulin, 1990, p. 190)

REFERENCES


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