Chapter 5

Shared Memory Programming with OpenMP
Roadmap

- Writing programs that use OpenMP.
- Using OpenMP to parallelize many serial for loops with only small changes to the source code.
- Task parallelism.
- Explicit thread synchronization.
- Standard problems in shared-memory programming.
OpenMP

- An API for shared-memory parallel programming.
- MP = multiprocessing
- Designed for systems in which each thread or process can potentially have access to all available memory.
- System is viewed as a collection of cores or CPU’s, all of which have access to main memory.
A shared memory system
Pragmas

- Special preprocessor instructions.
- Typically added to a system to allow behaviors that aren’t part of the basic C specification.
- Compilers that don’t support the pragmas ignore them.

`#pragma`
#include <stdio.h>
#include <stdlib.h>
#include <omp.h>

void Hello(void); /* Thread function */

int main(int argc, char* argv[]) {
    /* Get number of threads from command line */
    int thread_count = strtol(argv[1], NULL, 10);

    # pragma omp parallel num_threads(thread_count)
    Hello();

    return 0;
} /* main */

void Hello(void) {
    int my_rank = omp_get_thread_num();
    int thread_count = omp_get_num_threads();

    printf("Hello from thread %d of %d\n", my_rank, thread_count);
} /* Hello */
gcc -g -Wall -fopenmp -o omp_hello omp_hello.c

./omp_hello 4

compiling
running with 4 threads

Hello from thread 0 of 4
Hello from thread 1 of 4
Hello from thread 2 of 4
Hello from thread 3 of 4

possible outcomes
Hello from thread 1 of 4
Hello from thread 2 of 4
Hello from thread 0 of 4
Hello from thread 3 of 4
Hello from thread 3 of 4
Hello from thread 1 of 4
Hello from thread 2 of 4
Hello from thread 0 of 4
OpenMp pragmas

- # pragma omp parallel
  - Most basic parallel directive.
  - The number of threads that run the following structured block of code is determined by the run-time system.
A process forking and joining two threads
clause

- Text that modifies a directive.
- The num_threads clause can be added to a parallel directive.
- It allows the programmer to specify the number of threads that should execute the following block.

```
# pragma omp parallel num_threads ( thread_count )
```
Of note…

- There may be system-defined limitations on the number of threads that a program can start.
- The OpenMP standard doesn’t guarantee that this will actually start thread_count threads.
- Most current systems can start hundreds or even thousands of threads.
- Unless we’re trying to start a lot of threads, we will almost always get the desired number of threads.
Some terminology

- In OpenMP parlance the collection of threads executing the parallel block — the original thread and the new threads — is called a **team**, the original thread is called the **master**, and the additional threads are called **slaves**.
In case the compiler doesn’t support OpenMP

```c
#include <omp.h>
#ifdef _OPENMP
#include <omp.h>
#endif
```
In case the compiler doesn’t support OpenMP

```c
#ifdef _OPENMP
    int my_rank = omp_get_thread_num ();
    int thread_count = omp_get_num_threads ();
#else
    int my_rank = 0;
    int thread_count = 1;
#endif
```
THE TRAPEZOIDAL RULE
The trapezoidal rule
Serial algorithm

/* Input:  a, b, n */
h = (b–a)/n;
approx = (f(a) + f(b))/2.0;
for (i = 1; i <= n–1; i++) {
    x_i = a + i*h;
    approx += f(x_i);
}
approx = h*approx;
1) We identified two types of tasks:
   a) computation of the areas of individual trapezoids, and
   b) adding the areas of trapezoids.

2) There is no communication among the tasks in the first collection, but each task in the first collection communicates with task 1b.
3) We assumed that there would be many more trapezoids than cores.

- So we aggregated tasks by assigning a contiguous block of trapezoids to each thread (and a single thread to each core).
Assignment of trapezoids to threads
Unpredictable results when two (or more) threads attempt to simultaneously execute:

```
global_result += my_result ;
```
Mutual exclusion

```
#pragma omp critical
    global_result += my_result;
```

only one thread can execute the following structured block at a time
#include <stdio.h>
#include <stdlib.h>
#include <omp.h>

void Trap(double a, double b, int n, double* global_result_p);

int main(int argc, char* argv[]) {
    double global_result = 0.0;  /* Store result in global_result */
    double a, b;                /* Left and right endpoints */
    int n;                      /* Total number of trapezoids */
    int thread_count;

    thread_count = strtol(argv[1], NULL, 10);
    printf("Enter a, b, and n\n");
    scanf("%lf %lf %d", &a, &b, &n);

    #pragma omp parallel num_threads(thread_count)
    Trap(a, b, n, &global_result);

    printf("With n = %d trapezoids, our estimate\n", n);
    printf("of the integral from %f to %f = %.14e\n", a, b, global_result);
    return 0;
} /* main */
void Trap(double a, double b, int n, double* global_result_p) {
    double h, x, my_result;
    double local_a, local_b;
    int i, local_n;
    int my_rank = omp_get_thread_num();
    int thread_count = omp_get_num_threads();

    h = (b—a)/n;
    local_n = n/thread_count;
    local_a = a + my_rank*local_n*h;
    local_b = local_a + local_n*h;
    my_result = (f(local_a) + f(local_b))/2.0;
    for (i = 1; i <= local_n—1; i++) {
        x = local_a + i*h;
        my_result += f(x);
    }
    my_result = my_result*h;

    # pragma omp critical
    *global_result_p += my_result;
}
} /* Trap */
SCOPE OF VARIABLES
Scope

- In serial programming, the scope of a variable consists of those parts of a program in which the variable can be used.

- In OpenMP, the scope of a variable refers to the set of threads that can access the variable in a parallel block.
Scope in OpenMP

- A variable that can be accessed by all the threads in the team has **shared** scope.

- A variable that can only be accessed by a single thread has **private** scope.

- The default scope for variables declared before a parallel block is **shared**.
THE REDUCTION CLAUSE
We need this more complex version to add each thread’s local calculation to get global_result.

```c
void Trap(double a, double b, int n, double* global_result_p);
```

Although we’d prefer this.

```c
double Trap(double a, double b, int n);
```

```c
global_result = Trap(a, b, n);
```
If we use this, there’s no critical section!

```c
double Local_trap(double a, double b, int n);
```

If we fix it like this…

```c
global_result = 0.0;
#pragma omp parallel num_threads(thread_count)
{
#pragma omp critical
    global_result += Local_trap(double a, double b, int n);
}
```

… we force the threads to execute sequentially.
We can avoid this problem by declaring a private variable inside the parallel block and moving the critical section after the function call.

```c
global_result = 0.0;
#pragma omp parallel num_threads(thread_count)
{
    double my_result = 0.0; /* private */
    my_result += Local_trap(double a, double b, int n);
#pragma omp critical
    global_result += my_result;
}
```
I don’t like it.

Neither do I.

I think we can do better.
Reduction operators

- A reduction operator is a binary operation (such as addition or multiplication).
- A reduction is a computation that repeatedly applies the same reduction operator to a sequence of operands in order to get a single result.
- All of the intermediate results of the operation should be stored in the same variable: the reduction variable.
A reduction clause can be added to a parallel directive.

\[
\text{reduction}(<\text{operator}>: <\text{variable list}>)
\]

\[
+ , *, -, &, |, ^, &&, ||
\]

\[
\text{global_result} = 0.0;
\]

\[
\text{# pragma omp parallel num_threads(thread_count) \}
\]
\[
\text{reduction}(+: \text{global_result})
\]

\[
\text{global_result} += \text{Local_trap}(\text{double } a, \text{double } b, \text{int } n);
\]
THE “PARALLEL FOR” DIRECTIVE
Parallel for

- Forks a team of threads to execute the following structured block.
- However, the structured block following the parallel for directive must be a for loop.
- Furthermore, with the parallel for directive the system parallelizes the for loop by dividing the iterations of the loop among the threads.
h = (b−a)/n;
approx = (f(a) + f(b))/2.0;
for (i = 1; i <= n−1; i++)
    approx += f(a + i*h);
approx = h*approx;

# pragma omp parallel for num_threads(thread_count) \
    reduction(+: approx)
for (i = 1; i <= n−1; i++)
    approx += f(a + i*h);
approx = h*approx;
Legal forms for parallelizable for statements

```
for

index = start ; index >= end ; index > end

{  
    index++
    ++index
    index--
    --index
    index += incr
    index -= incr
    index = index + incr
    index = incr + index
    index = index - incr
}
```
Caveats

- The variable `index` must have integer or pointer type (e.g., it can’t be a float).

- The expressions `start`, `end`, and `incr` must have a compatible type. For example, if `index` is a pointer, then `incr` must have integer type.
Caveats

- The expressions `start`, `end`, and `incr` must not change during execution of the loop.

- During execution of the loop, the variable `index` can only be modified by the “increment expression” in the `for` statement.
Data dependencies

fibo[0] = fibo[1] = 1;
for (i = 2; i < n; i++)
fibo[i] = fibo[i - 1] + fibo[i - 2];

fibo[0] = fibo[1] = 1;
#pragma omp parallel for num_threads(2)
for (i = 2; i < n; i++)
fibo[i] = fibo[i - 1] + fibo[i - 2];

1 1 2 3 5 8 13 21 34 55
this is correct

1 1 2 3 5 8 0 0 0 0
but sometimes we get this

note 2 threads
What happened?

1. OpenMP compilers don’t check for dependences among iterations in a loop that’s being parallelized with a parallel for directive.

2. A loop in which the results of one or more iterations depend on other iterations cannot, in general, be correctly parallelized by OpenMP.
Estimating $\pi$

\[
\pi = 4 \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right] = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}
\]

```java
double factor = 1.0;
double sum = 0.0;
for (k = 0; k < n; k++) {
    sum += factor / (2*k+1);
    factor = -factor;
}
pi_approx = 4.0*sum;
```
OpenMP solution #1

```c
# double factor = 1.0;
double sum = 0.0;
#pragma omp parallel for num_threads(thread_count) \ 
    reduction(+:sum)
for (k = 0; k < n; k++) {
    sum += factor/(2*k+1);
    factor = -factor;
}
pi_approx = 4.0*sum;
```

loop dependency
OpenMP solution #2

```c
double sum = 0.0;

#pragma omp parallel for num_threads(thread_count) \ reduction(+:sum) private(factor)
for (k = 0; k < n; k++) {
    if (k % 2 == 0)
        factor = 1.0;
    else
        factor = -1.0;
    sum += factor/(2*k+1);
}
```

Insures factor has private scope.
The default clause

- Lets the programmer specify the scope of each variable in a block.

  \texttt{default\ (none)}

- With this clause the compiler will require that we specify the scope of each variable we use in the block and that has been declared outside the block.
double sum = 0.0;
#pragma omp parallel for num_threads(thread_count) \
  default(none) reduction(+:sum) private(k, factor) \
  shared(n)
for (k = 0; k < n; k++) {
  if (k % 2 == 0)
    factor = 1.0;
  else
    factor = -1.0;
  sum += factor/(2*k+1);
}
MORE ABOUT LOOPS IN OPENMP: SORTING
Bubble Sort

```c
for (list_length = n; list_length >= 2; list_length--)
    for (i = 0; i < list_length - 1; i++)
        if (a[i] > a[i+1]) {
            tmp = a[i];
            a[i] = a[i+1];
            a[i+1] = tmp;
        }
```
Serial Odd-Even Transposition Sort

```c
for (phase = 0; phase < n; phase++)
    if (phase % 2 == 0)
        for (i = 1; i < n; i += 2)
            if (a[i-1] > a[i]) Swap(&a[i-1], &a[i]);
    else
        for (i = 1; i < n-1; i += 2)
            if (a[i] > a[i+1]) Swap(&a[i], &a[i+1]);
```
### Serial Odd-Even Transposition Sort

<table>
<thead>
<tr>
<th>Phase</th>
<th>Subscript in Array</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>
for (phase = 0; phase < n; phase++) {
    if (phase % 2 == 0)
        # pragma omp parallel for num_threads(thread_count) \
            default(none) shared(a, n) private(i, tmp)
        for (i = 1; i < n; i += 2) {
            if (a[i-1] > a[i]) {
                tmp = a[i-1];
                a[i-1] = a[i];
                a[i] = tmp;
            }
        }
    else
        # pragma omp parallel for num_threads(thread_count) \
            default(none) shared(a, n) private(i, tmp)
        for (i = 1; i < n-1; i += 2) {
            if (a[i] > a[i+1]) {
                tmp = a[i+1];
                a[i+1] = a[i];
                a[i] = tmp;
            }
        }
}
Second OpenMP Odd-Even Sort

```c
#pragma omp parallel num_threads(thread_count) \
  default(none) shared(a, n) private(i, tmp, phase)
for (phase = 0; phase < n; phase++) {
  if (phase % 2 == 0)
    # pragma omp for
    for (i = 1; i < n; i += 2) {
      if (a[i-1] > a[i]) {
        tmp = a[i-1];
        a[i-1] = a[i];
        a[i] = tmp;
      }
    }
  else
    # pragma omp for
    for (i = 1; i < n-1; i += 2) {
      if (a[i] > a[i+1]) {
        tmp = a[i+1];
        a[i+1] = a[i];
        a[i] = tmp;
      }
    }
}
```
Odd-even sort with two parallel for directives and two for directives. (Times are in seconds.)

<table>
<thead>
<tr>
<th>thread_count</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two parallel <strong>for</strong> directives</td>
<td>0.770</td>
<td>0.453</td>
<td>0.358</td>
<td>0.305</td>
</tr>
<tr>
<td>Two <strong>for</strong> directives</td>
<td>0.732</td>
<td>0.376</td>
<td>0.294</td>
<td>0.239</td>
</tr>
</tbody>
</table>
SCHEDULING LOOPS
We want to parallelize this loop.

\[
\begin{align*}
\text{sum} &= 0.0; \\
\text{for } (i = 0; i <= n; i++) \\
&\quad \text{sum } += f(i);
\end{align*}
\]

<table>
<thead>
<tr>
<th>Thread</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0, (n/t), 2n/t, \ldots</td>
</tr>
<tr>
<td>1</td>
<td>1, (n/t + 1), 2n/t + 1, \ldots</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>(t-1)</td>
<td>(t - 1), (n/t + t - 1), 2n/t + t - 1, \ldots</td>
</tr>
</tbody>
</table>

Assignment of work using cyclic partitioning.
double f(int i) {
    int j, start = i*(i+1)/2, finish = start + i;
    double return_val = 0.0;

    for (j = start; j <= finish; j++) {
        return_val += sin(j);
    }
    return return_val;
} /* f */

Our definition of function $f$. 
Results

- $f(i)$ calls the sin function $i$ times.
- Assume the time to execute $f(2i)$ requires approximately twice as much time as the time to execute $f(i)$.

- $n = 10,000$
  - one thread
  - run-time = 3.67 seconds.
Results

- n = 10,000
  - two threads
  - default assignment
  - run-time = 2.76 seconds
  - speedup = 1.33

- n = 10,000
  - two threads
  - cyclic assignment
  - run-time = 1.84 seconds
  - speedup = 1.99
The Schedule Clause

- Default schedule:

```c
sum = 0.0;
#pragma omp parallel for num_threads(thread_count) \ 
  reduction(+:sum)
  for (i = 0; i <= n; i++)
    sum += f(i);
```

- Cyclic schedule:

```c
sum = 0.0;
#pragma omp parallel for num_threads(thread_count) \ 
  reduction(+:sum) schedule(static,1)
  for (i = 0; i <= n; i++)
    sum += f(i);
```
schedule ( type , chunksize )

- Type can be:
  - static: the iterations can be assigned to the threads before the loop is executed.
  - dynamic or guided: the iterations are assigned to the threads while the loop is executing.
  - auto: the compiler and/or the run-time system determine the schedule.
  - runtime: the schedule is determined at run-time.
- The chunksize is a positive integer.
The Static Schedule Type

twelve iterations, 0, 1, \ldots, 11, and three threads

\texttt{schedule(static,1)}

Thread 0 : 0, 3, 6, 9
Thread 1 : 1, 4, 7, 10
Thread 2 : 2, 5, 8, 11
The Static Schedule Type

twelve iterations, 0, 1, . . . , 11, and three threads

\texttt{schedule(static,2)}

Thread 0: 0, 1, 6, 7
Thread 1: 2, 3, 8, 9
Thread 2: 4, 5, 10, 11
The Static Schedule Type

twelve iterations, 0, 1, \ldots, 11, and three threads

\texttt{schedule(static, 4)}

\begin{align*}
\text{Thread 0 :} & \quad 0, 1, 2, 3 \\
\text{Thread 1 :} & \quad 4, 5, 6, 7 \\
\text{Thread 2 :} & \quad 8, 9, 10, 11
\end{align*}
The Dynamic Schedule Type

- The iterations are also broken up into chunks of \textit{chunksize} consecutive iterations.
- Each thread executes a chunk, and when a thread finishes a chunk, it requests another one from the run-time system.
- This continues until all the iterations are completed.
- The \textit{chunksize} can be omitted. When it is omitted, a \textit{chunksize} of 1 is used.
The Guided Schedule Type

- Each thread also executes a chunk, and when a thread finishes a chunk, it requests another one.
- However, in a guided schedule, as chunks are completed the size of the new chunks decreases.
- If no \textit{chunksize} is specified, the size of the chunks decreases down to 1.
- If \textit{chunksize} is specified, it decreases down to \textit{chunksize}, with the exception that the very last chunk can be smaller than \textit{chunksize}.
Assignment of trapezoidal rule iterations 1–9999 using a guided schedule with two threads.

<table>
<thead>
<tr>
<th>Thread</th>
<th>Chunk</th>
<th>Size of Chunk</th>
<th>Remaining Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 – 5000</td>
<td>5000</td>
<td>4999</td>
</tr>
<tr>
<td>1</td>
<td>5001 – 7500</td>
<td>2500</td>
<td>2499</td>
</tr>
<tr>
<td>1</td>
<td>7501 – 8750</td>
<td>1250</td>
<td>1249</td>
</tr>
<tr>
<td>1</td>
<td>8751 – 9375</td>
<td>625</td>
<td>624</td>
</tr>
<tr>
<td>0</td>
<td>9376 – 9687</td>
<td>312</td>
<td>312</td>
</tr>
<tr>
<td>1</td>
<td>9688 – 9843</td>
<td>156</td>
<td>156</td>
</tr>
<tr>
<td>0</td>
<td>9844 – 9921</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>1</td>
<td>9922 – 9960</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>9961 – 9980</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>1</td>
<td>9981 – 9990</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>9991 – 9995</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>9996 – 9997</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>9998 – 9998</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>9999 – 9999</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
The Runtime Schedule Type

- The system uses the environment variable \texttt{OMP\_SCHEDULE} to determine at run-time how to schedule the loop.

- The \texttt{OMP\_SCHEDULE} environment variable can take on any of the values that can be used for a static, dynamic, or guided schedule.
PRODUCERS AND CONSUMERS
Queues

- Can be viewed as an abstraction of a line of customers waiting to pay for their groceries in a supermarket.
- A natural data structure to use in many multithreaded applications.
- For example, suppose we have several “producer” threads and several “consumer” threads.
  - Producer threads might “produce” requests for data.
  - Consumer threads might “consume” the request by finding or generating the requested data.
Message-Passing

- Each thread could have a shared message queue, and when one thread wants to "send a message" to another thread, it could enqueue the message in the destination thread’s queue.

- A thread could receive a message by dequeuing the message at the head of its message queue.
Message-Passing

```c
for (sentmsgs = 0; sentmsgs < send_max; sentmsgs++) {
    Send_msg();
    Try_receive();
}

while (!Done())
    Try_receive();
```
mesg = random();
dest = random() % thread_count;

#pragma omp critical
Enqueue(queue, dest, my_rank, mesg);
if (queue_size == 0) return;
else if (queue_size == 1)
    # pragma omp critical
    Dequeue(queue, &src, &mesg);
else
    Dequeue(queue, &src, &mesg);
    Print_message(src, mesg);
queue_size = enqueued - dequeued;
if (queue_size == 0 && done_sending == thread_count)
    return TRUE;
else
    return FALSE;

each thread increments this after completing its for loop
When the program begins execution, a single thread, the master thread, will get command line arguments and allocate an array of message queues: one for each thread.

This array needs to be shared among the threads, since any thread can send to any other thread, and hence any thread can enqueue a message in any of the queues.
One or more threads may finish allocating their queues before some other threads.

We need an explicit barrier so that when a thread encounters the barrier, it blocks until all the threads in the team have reached the barrier.

After all the threads have reached the barrier all the threads in the team can proceed.

```
#pragma omp barrier
```
The Atomic Directive (1)

- Unlike the critical directive, it can only protect critical sections that consist of a single C assignment statement.

  # pragma omp atomic

- Further, the statement must have one of the following forms:

  \[
  x \ <op> = \ <expression> ; \\
  x++ ; \\
  ++x ; \\
  x--; \\
  --x ; 
  \]
The Atomic Directive (2)

- Here <op> can be one of the binary operators +, *, −, /, &, ^, |, <<, or >>

- Many processors provide a special load-modify-store instruction.

- A critical section that only does a load-modify-store can be protected much more efficiently by using this special instruction rather than the constructs that are used to protect more general critical sections.
Critical Sections

- OpenMP provides the option of adding a name to a critical directive:

  ```
  # pragma omp critical(name)
  ```

- When we do this, two blocks protected with critical directives with different names can be executed simultaneously.

- However, the names are set during compilation, and we want a different critical section for each thread’s queue.
Locks

- A lock consists of a data structure and functions that allow the programmer to explicitly enforce mutual exclusion in a critical section.
/* Executed by one thread */
Initialize the lock data structure;

/* Executed by multiple threads */
Attempt to lock or set the lock data structure;
Critical section;
Unlock or unset the lock data structure;

/* Executed by one thread */
Destroy the lock data structure;
Using Locks in the Message-Passing Program

```c
#pragma omp critical
/* q_p = msg_queues[dest] */
Enqueue(q_p, my_rank, msg);

/* q_p = msg_queues[dest] */
omp_set_lock(&q_p->lock);
Enqueue(q_p, my_rank, msg);
omp_unset_lock(&q_p->lock);
```
Using Locks in the Message-Passing Program

```c
#pragma omp critical
/* q_p = msg_queues[my_rank] */
Dequeue(q_p, &src, &msg);

/* q_p = msg_queues[my_rank] */
omp_set_lock(&q_p->lock);
Dequeue(q_p, &src, &msg);
omp_unset_lock(&q_p->lock);
```
Some Caveats

1. You shouldn’t mix the different types of mutual exclusion for a single critical section.
2. There is no guarantee of fairness in mutual exclusion constructs.
3. It can be dangerous to “nest” mutual exclusion constructs.
Matrix-vector multiplication

\[ y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots + a_{i,n-1}x_{n-1} \]

\[
\begin{array}{cccc}
  a_{00} & a_{01} & \cdots & a_{0,n-1} \\
  a_{10} & a_{11} & \cdots & a_{1,n-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{i0} & a_{i1} & \cdots & a_{i,n-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m-1,0} & a_{m-1,1} & \cdots & a_{m-1,n-1} \\
\end{array}
\]

\[
\begin{array}{c}
  x_0 \\
  x_1 \\
  \vdots \\
  x_{n-1} \\
\end{array}
\]

\[
\begin{array}{c}
  y_0 \\
  y_1 \\
  \vdots \\
  y_{m-1} \\
\end{array}
\]

\[
\begin{align*}
  y_i &= a_{i0}x_0 + a_{i1}x_1 + \cdots + a_{i,n-1}x_{n-1} \\
  &\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad }
Matrix-vector multiplication

```c
#pragma omp parallel for num_threads(thread_count) \
    default(none) private(i, j) shared(A, x, y, m, n)
for (i = 0; i < m; i++) {
    y[i] = 0.0;
    for (j = 0; j < n; j++)
        y[i] += A[i][j]*x[j];
}
```

Run-times and efficiencies of matrix-vector multiplication (times are in seconds)

<table>
<thead>
<tr>
<th>Threads</th>
<th>Matrix Dimension</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8,000,000 × 8</td>
<td>8000 × 8000</td>
<td>8 × 8,000,000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.322</td>
<td>1.000</td>
<td>0.264</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.219</td>
<td>0.735</td>
<td>0.189</td>
<td>0.698</td>
</tr>
<tr>
<td>4</td>
<td>0.141</td>
<td>0.571</td>
<td>0.119</td>
<td>0.555</td>
</tr>
</tbody>
</table>
void Tokenize(
    char* lines[] /* in/out */,
    int line_count /* in */,
    int thread_count /* in */) {
    int my_rank, i, j;
    char *my_token;

    #pragma omp parallel num_threads(thread_count) \
    default(none) private(my_rank, i, j, my_token) \ 
    shared(lines, line_count)
{
    my_rank = omp_get_thread_num();
    #pragma omp for schedule(static, 1)
    for (i = 0; i < line_count; i++) {
        printf("Thread %d > line %d = %s", my_rank, i, lines[i]);
        j = 0;
        my_token = strtok(lines[i], " \t\n");
        while (my_token != NULL) {
            printf("Thread %d > token %d = %s\n", my_rank, j, my_token);
            my_token = strtok(NULL, " \t\n");
            j++;
        }
    } /* for i */
} /* omp parallel */

} /* Tokenize */
Concluding Remarks (1)

- OpenMP is a standard for programming shared-memory systems.
- OpenMP uses both special functions and preprocessor directives called pragmas.
- OpenMP programs start multiple threads rather than multiple processes.
- Many OpenMP directives can be modified by clauses.
A major problem in the development of shared memory programs is the possibility of race conditions.

OpenMP provides several mechanisms for insuring mutual exclusion in critical sections.

- Critical directives
- Named critical directives
- Atomic directives
- Simple locks
Concluding Remarks (3)

- By default most systems use a block-partitioning of the iterations in a parallelized for loop.
- OpenMP offers a variety of scheduling options.
- In OpenMP the scope of a variable is the collection of threads to which the variable is accessible.
Concluding Remarks (4)

- A reduction is a computation that repeatedly applies the same reduction operator to a sequence of operands in order to get a single result.