M105 – Ανάλυση και Μοντελοποίηση Δικτύων

Some Review Questions

Winter 2023

Probability and Stochastic Processes overview – Poisson Process

1. Understand the relation between events (sets of ω) and individual outcomes ω. When does an event occur in relation an the outcome ω that is observed/occurs? When do multiple events occur? When does the occurrence of an event imply the occurrence of another event?
2. Derive in detail, step-by-step and with rigor the binomial distribution (prob for k successes in n trials). You are supposed to understand the justification of all steps. Understand how to calculate the probability of joint events (composed of individual events that are independent) in terms of the marginal probabilities of the associated events. Also, how to calculate the probability of the union of mutually exclusive events in terms of the probability of the associated events.
3. Understand the conditions under which the Poisson RV is a good model for the superposition of multiple event processes.
4. Formulate the question and the related condition that will show that the length of a phone call has no memory. Study the proof of the memoryless property of the exponential distribution. Understand the implications of the lack of memory.
5. Understand the various definitions / interpretations of a stochastic process and their relation to random variables.
6. Through aspects of the presented construction, justify the key properties of the Wiener process and its applicability. Derivative of the Wiener process and the integral of a white process and their autocorrelation
7. Ergodicity: definition (qualitative and quantitative) and why it is important.
8. Formal definition of a Poisson process and the 3 basic properties required by a counting process to be Poisson.
9. Poisson process behavior over very small time intervals in terms of number of events producing.
10. Alternative definitions of a Poisson process and ways to check if data points conform to a Poisson process.
11. Understand the uniformity of the distribution of the time of arrivals of a number of arrivals that are known to have occurred over an interval.

12. Conditions under which a Poisson could be approximated by a Normal distributions and why.
13. Burstiness of the Poisson process – exponential interarrival times.
14. Memorylessness of the Poisson interarrival times.
15. Forward recurrence times of a Poisson process and its implication.
16. Merging and splitting Poisson processes (no prrofs): characterize the resulting processes.
17. Basic difference between a compound Poisson and a Poisson.

Markov Processes and Applications

18. Key property and definition of a MC.
19. Chapman Kolmogorov equations.
20. Clarify the difference between $p^n(i,j)$ and $[p(I,j)]^2$.
21. Examples for checking the Markov property and setting up the stochastic matrix. The Markov property can be claimed based on the “nature” or “physics” of the system based on which the key Markovian property can be claimed. It can also be justified by writing up a simple equation for the evolution of the process and by showing that the state in the next timestep depends only on the state in the current step and maybe some other independent processes/RVs. For example, example slide 13-15, etc.
22. Conditional independence of a function of a MC from past visits given the current
23. Visits to a state: familiarize yourselves with the quantities $N_j(\omega)$, $F_k(i,j)$ and its recursive computation, and the potential R(i,j) and its relation to $F_k(i,j)$. Also understand for which states $F_k(i,j)$ will be 0 or 1.
24. Classification of states - definitions of states
25. What are the limiting probabilities for visiting a transient or a recurrent positive (non-null) state?
26. What is an irreducible set of states or an irreducible MC?
27. Learn how to Identify closed sets of states in a MC – examples
28. Understand how to organize recurrent states of a MC in irreducible sets
29. Characterization of states of an irreducible MC. Can there be a mix of recurrent null and recurrent positive (non-null)?
30. What type of states exist in an irreducible MC with a finite number of states?
31. Learn how to determine the type of states in a MC with a finite number of states –example

32. What type of states exist in an irreducible MC with an infinite number of states?
33. What is the condition (involving the transition matrix) in order for all states to be recurrent non-null, in an irreducible MC with an infinite number of states?
34. Understand how to derive the matrices for S and F shown on slides 56 and 61.
35. What is the invariant distribution of an irreducible MC with recurrent positive (non-null) states and how is it related to the limiting probabilities of that MC?
36. Understand how we can use the limiting probabilities of an irreducible MC with recurrent non-null states to calculate average value of a bounded function on the state space.
37. Consider a discrete-time buffer occupancy system and investigate if the occupancy process is a MC, under various assumptions on the arrival and service processes.
38. Embedded MCs: Understand how by embedding a process on a specific sequence of times can yield a MC, when this is not generally the case for a random time.
39. M/G/1 and G/M/1: What is the environment they can model? How is the Markovian property ensured or proven? How is the stochastic matrix derived? What is the basic parameter whose value shapes the type of states (transient, recurrent null, non-null) of the resulting MC?
40. Examples of MCs – Understand how to set up a MC and its stochastic matrix from a system description and answer some performance question. For instance, the study of the slotted Aloha protocol.