Renewal-Theoretical Dynamic Spectrum Access in Cognitive Radio Network with Unknown Primary Behavior

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Abstract—Dynamic spectrum access in cognitive radio networks can greatly improve the spectrum utilization efficiency. Nevertheless, interference may be introduced to the Primary User (PU) when the Secondary Users (SUs) dynamically utilize the PU's licensed channels. If the SUs can be synchronous with the PU's time slots, the interference is mainly due to their imperfect spectrum sensing of the primary channel. However, if the SUs have no knowledge about the PU's exact communication mechanism, additional interference may occur. In this paper, we propose a dynamic spectrum access protocol for the SUs confronting with unknown primary behavior and study the interference caused by their dynamic access. Through analyzing the SUs' dynamic behavior in the primary channel which is modeled as an ON-OFF process, we prove that the SUs' communication behavior is a renewal process. Based on the Renewal Theory, we quantify the interference caused by the SUs and derive the corresponding closed-form expressions. With the interference analysis, we study how to optimize the SUs' performance under the constraints of the PU's communication quality of service (QoS) and the secondary network's stability. Finally, simulation results are shown to verify the effectiveness of our analysis.

Index Terms—Cognitive radio, dynamic spectrum access, interference analysis, renewal theory.

I. INTRODUCTION

COGNITIVE radio is considered as an effective approach to mitigate the problem of crowded electromagnetic radio spectrums. Compared with static spectrum allocation, dynamic spectrum access (DSA) technology can greatly enhance the utilization efficiency of the existing spectrum resources [1]. In DSA, devices with cognitive capability can dynamically access the licensed spectrum in an opportunistic way, under the condition that the interference to the communication activities in the licensed spectrum is minimized [2]. Such cognitive devices are called as Secondary Users (SUs), while the licensed users as Primary Users (PUs) and the available spectrum resource for the SUs is referred to as "spectrum hole".

One of the most important issues in the DSA technology is to control the SUs' adverse interference to the normal communication activities of the PU in licensed bands [3]. One way is to strictly prevent the SUs from interfering the PU in both time domain and frequency domain [4], and the other approach is to allow interference but minimizing the interference effect to the PU [5]. For the latter approach, the key problem is to model and analyze the interference caused by the SUs to reveal the quantitative impacts on the PU. Most of the existing works on interference modeling can be categorized into two classes: spatial interference model and accumulated interference model. The spatial interference model is to study how the interference caused by the SUs varies with their spatial positions [6], [7], [8], while the accumulated interference model focuses on analyzing the accumulated interference power of the SUs at primary receiver through adopting different channel fading models such as what discussed in [9], [10] with exponential path loss, and in [11], [12], [13] with both exponential path loss and log-normal shadowing. Moreover, in [14], [15], [16], [17], the SUs are modeled as separate queuing systems, where the interference and interactions among these queues are analyzed to satisfy the stability region.

However, most traditional interference analysis approaches are based on aggregating the SUs' transmission power with different path fading coefficients, regardless the communication behaviors of the PU and the SUs. In this paper, we will study the interference through analyzing the relationship between the SUs' dynamic access and the states of the primary channel in the MAC layer. Especially, we will focus on the situation when the SUs are confronted with unknown primary behavior. If the SUs have the perfect knowledge of the PU's communication mechanism, the interference is mainly from the imperfect sensing which has been well studied [18]. To simplify the analysis and give more insights into the interference analysis, we assumed perfect spectrum sensing in this paper. We show that the SUs' dynamic behavior in the primary channel is a renewal process and quantify the corresponding interference caused by the SUs' behavior based on the Renewal Theory [19]. There are some works using renewal theory for cognitive radio networks. In [20], the primary channel was modeled as an ON-OFF renewal process to study how to efficiently discover spectrum holes through dynamically adjusting the SUs' sensing period. As the extension works of [20], Xue et. al. designed a periodical

MAC protocol for the SUs in [21], while Tang and Chew analyzed the periodical sensing errors in [22]. In [23], [24], the authors discussed how to efficiently perform channel access and switch according to the residual time of the ON-OFF process in the primary channel. Based on the assumption that the primary channel is an ON-OFF renewal process, the delay performance of the SUs were analyzed in [25], [26]. However, all these related works have only modeled the PU's behavior in the primary channel as an ON-OFF process. In this paper, we further show and study the renewal characteristic of the SUs' communication behavior and analyze the interference to the PU when they dynamically access the primary channel.

The main contributions of this paper are summarized as follows.

1) We propose a dynamic spectrum access protocol for the SUs under the scenario that they have no knowledge about the PU's exact communication mechanism. By treating the SUs as customers and the primary channel as the server, our system can be regarded as a queuing system.

2) Different from the traditional interference analysis which calculates the SUs' aggregated signal power at the primary receiver in the physical layer, we introduce a new way to quantify the interference caused by the SUs in the MAC layer. This interference quantity represents the proportion of the periods when the PUs' communication are interfered by the SUs' dynamic access.

3) We prove that the SUs' communication behavior in the primary channel is a renewal process and derive the closed-form expressions for the interference quantity using the Renewal Theory.

4) To guarantee the PUs' communication quality of service (QoS) and maintain the stability of the secondary network, we formulate the problem of controlling the SUs' dynamic access as an optimization problem, where the objective function is to maximize the SUs' average data rate with two constraints: the PU's average data rate should not be lower than a pre-determined threshold and the SUs' arrival interval and transmission time should satisfy the stability condition.

The rest of this paper is organized as follows. Firstly, our system model is described in Section II. Then, we present the proposed dynamic spectrum access protocol for the SUs in Section III. We derive the closed-form expressions for the interference quantity of two different scenarios in Section IV and V, respectively. In Section VI, we discuss how to optimize the SUs' communication performance according to the interference analysis. Finally, simulation results are shown in Section VII and conclusion is drawn in Section VIII.

II. SYSTEM MODEL

A. Network Entity

As shown in Fig. 1, we consider a cognitive radio network with one PU and M SUs operating on one primary channel. The PU has priority to occupy the channel at any time, while the SUs are allowed to temporarily access the channel under the condition that the PU's communication QoS is guaranteed. An important feature of our system is that the communication
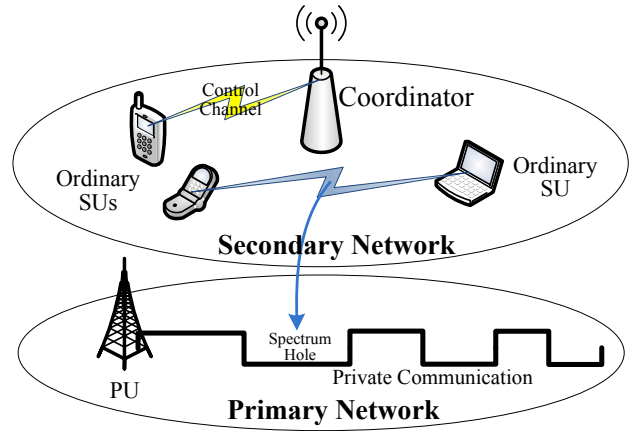

Fig. 1. Network entity.

mechanism in the primary network is private, i.e., the SUs have no knowledge when the PU's communication will arrive.

For the secondary network, M SUs form a group under the management of one coordinator. The coordinator is in charge of observing the PU's behavior, deciding the availability of the primary channel, coordinating and controlling the SUs' dynamic access. There is a control channel for command exchange between the ordinary SUs and the coordinator [27]. The SUs need to opportunistically access the primary channel to acquire more bandwidth for high data rate transmission, e.g., multimedia transmission. Considering that the ordinary SUs are usually small-size and power-limit mobile terminals, spectrum sensing is only performed by the coordinator. Meanwhile, we assume that all the ordinary SUs are half-duplex, which means that they cannot simultaneously transmit and receive data packet [28].

B. Primary Channel State Model

Since the SUs have no idea about the exact communication mechanism of the primary network and hence cannot be synchronous with the PU, there is no concept of "time slot" in the primary channel from the SUs' points of view. Instead, the primary channel just alternatively switches between ON state and OFF state. The ON state means the channel is being occupied by the PU, while the OFF state is the "spectrum hole" which can be freely occupied by the SUs.

We model the length of the ON state and OFF state by two random variables T_{ON} and T_{OFF}, respectively. According to different types of the primary services (e.g., digital TV broadcasting or cellular communication), T_{ON} and T_{OFF} statistically satisfy different distributions. In this paper, we assume that T_{ON} and T_{OFF} are independent and satisfy exponential distributions with parameter λ_1 and λ_0, denoted by $f_{\text{ON}}(t) = \frac{1}{\lambda_1} e^{-t/\lambda_1}$ and $f_{\text{OFF}}(t) = \frac{1}{\lambda_0} e^{-t/\lambda_0}$. In such a case, the expected lengths of the ON state and OFF state are λ_1 and λ_0 accordingly. These two important parameters λ_1 and λ_0 can be effectively estimated by a maximum likelihood estimator [20]. Such an ON-OFF behavior of the PU is a combination of two Poisson process, which is a renewal process [19]. The renewal interval is $T_p = T_{\text{ON}} + T_{\text{OFF}}$ and the distribution of T_p, denoted by $f_p(t)$,
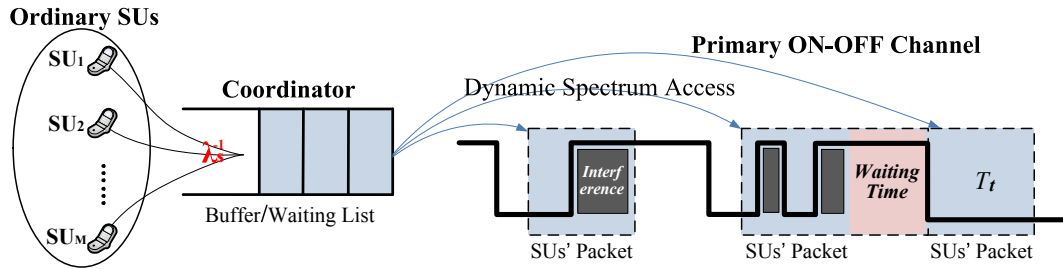
Fig. 2.   Illustration of the queuing system, the SUs' dynamic spectrum access and interference to the PU.

is

$$f_p(t) = f_{\text{ON}}(t) * f_{\text{OFF}}(t), \tag{1}$$

where the symbol "$*$" represents the convolution operation.

III. SECONDARY USERS' DYNAMIC SPECTRUM ACCESS PROTOCOL

In this section, we will design and analyze the SUs' communication behavior including how the SUs dynamically access the primary channel and how the coordinator manages the group of SUs. Based on the behavior analysis, we can further study the interference caused by the SUs' access.

A. Dynamic Spectrum Access Protocol

In our protocol, the SUs who want to transmit data must first inform the coordinator with a *request command*, which can also be listened by the corresponding receiver. The coordinator sequentially responds to the requesting SUs by a *confirmation command* according to the First-In-First-Out (FIFO) rule. The SU who has received the confirmation should immediately transmit data with time T_t over the primary channel. During the whole process, all the spectrum sensing and channel estimation works are done by the coordinator simultaneously. The proposed dynamic access protocol for both ordinary SUs and the coordinator is summarized in Algorithm 1.

B. Queuing Model

According to the proposed access protocol, the secondary network can be modeled as a queueing system as shown in Fig. 2. We assume that the requests from all SUs arrive by a Poisson process at the coordinator with rate λ_s^{-1}. In such a case, the arrival intervals of SUs' requests at the coordinator, denoted by T_s, satisfies the exponential distribution with expectation λ_s, i.e., $T_s \sim f_s(t) = \frac{1}{\lambda_s} e^{-t/\lambda_s}$.

In this queuing system, the coordinator's buffer only records the sequence of the SUs' request, instead of the specific data packets. The packets are stored in each SU's own data memory, which is considered as infinite length. In such a case, we can also regard the buffer in the coordinator as infinite length. Note that the coordinator removes the SU's request after it responses to the corresponding SU with a confirmation command regardless of whether that SU's transmission is successful or not. Thus, for the service time of each SU, it is the sum of the transmission time T_t and the waiting time if T_t ends in the ON state as show in Fig. 2. The reason of introducing such a waiting time is that although the SUs may

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Algorithm 1 SUs' Dynamic Spectrum Access Protocol.

I. For the ordinary SU
1: **if** A SU has one packet to transmit **then**
2:     • Send a *request command* to the coordinator through the secondary control channel
3:     **while** No confirmation from the coordinator **do**
4:        • Store the packet in its memory and wait
5:     **end while**
6:     • Transmit its packet after confirmation
7: **end if**

II. For the coordinator
1: • Estimate the primary channel's parameters λ_0 and λ_1
2: **if** A *request command* is received **then**
3:     • Register the SU's request on the waiting list according to the FIFO rule
4: **end if**
5: **while** The primary channel is in the OFF state **do**
6:     • Response to the requesting SU on the top of the waiting list with a *confirmation command*
7:     • Wait for transmission time T_t
8: **end while**
9: • Keep sensing the primary channel

---

be beneficial from transmission during the ON state, it can cause sever interference to the PU and thus is forbidden in our system. In our model, the time consumed by command exchange between ordinary SUs and the coordinator is not taken into account, since it is negligible compared to T_t, λ_0 and λ_1. Based on this queuing model, we can analyze the interference caused by the SUs' dynamic access.

C. Interference Quantity

If the SUs have the perfect knowledge of communication scheme in the primary network, e.g. the primary channel is slotted and all SUs can be synchronous with the PU, then the SUs can immediately vacate the occupied channel by the end of the slot. In such a case, the potential interference only comes from the SUs' imperfect spectrum sensing. However, when an SU is confronted with unknown primary behavior, additional interference will appear since the SU may fail to discover the PU's recurrence when it is transmitting packet in the primary channel, as shown by the shaded regions in Fig. 2. The essential reason is that the half-duplex SUs cannot receive any command from the coordinator during data transmission or receiving. Therefore, the interference under such a scenario is mainly due to the SUs' failure of discovering the PU's recurrence during their access time.

In most of the existing works [5]-[17], interference to the PU was usually measured as the quantity of SUs' signal power at primary receiver in the physical layer. In this paper, we will measure the interference quantity based on communication behaviors of the PU and SUs in the MAC layer. The shaded regions in Fig. 2 indicate the interference periods in the ON state of the primary channel. In order to illustrate the impacts of these interference periods on the PU, we define the interference quantity Q_I as follows.

Definition 1: The interference quantity Q_I is the proportion of accumulated interference periods to the length of all ON states in the primary channel within a long period time, which can be written by

$$Q_I = \lim_{T \to +\infty} \frac{\sum_T \text{Interference periods}}{\sum_T T_{\text{ON}}}. \tag{2}$$

In the following sections, we will derive the closed-form of Q_I in two different scenarios listed below.

- Q_{I_1}: SUs with arrival interval $\lambda_s = 0$.
- Q_{I_2}: SUs with constant average arrival interval $\lambda_s \neq 0$.

IV. INTERFERENCE CAUSED BY SUS WITH ZERO ARRIVAL INTERVAL

In this section, we will discuss the interference to the PU when the average arrival interval of all SUs' requests $\lambda_s = 0$, i.e., the arrival rate $\lambda_s^{-1} = +\infty$. In the practical scenario, $\lambda_s^{-1} = +\infty$ is corresponding to the situation when each SU has plenty of packets to transmit, resulting in an extremely high arrival rate of all SUs' requests at the coordinator, i.e., $\lambda_s^{-1} \to +\infty$. In such a case, the coordinator's buffer is non-empty all the time, which means the SUs always want to transmit packets in the primary channel. Such a scenario is the worst case for the PU since the maximum interference from the SUs is considered.

A. SUs' Communication Behavior Analysis

Since $\lambda_s = 0$ means the coordinator always has requests in its buffer, the SUs are either transmitting one packet or waiting for the OFF state. The waiting time, denoted by T_w, will appear if the previous transmission ends in the ON state, and the value of T_w is determined by the length of the remaining time in the primary channel's ON state. As we discussed in Section III-C, the interference to the PU only occurs during the SUs' transmission time T_t. Therefore, the interference quantity is determined by the occurrence probability of T_t. In the following, we will analyze the SUs' communication behavior based on Renewal Theory.

Theorem 1: When the SUs' transmission requests arrive by Poisson process with average arrival interval $\lambda_s = 0$, the SUs' communication behavior is a renewal process in the primary channel.

Proof: As shown in Fig. 3, the SUs' behavior dynamically switches between transmitting one packet and waiting for the OFF state. The property of SUs's behavior is that their transmissions always start in an OFF state and the following waiting time is only dependent with the last transmission. Since the primary channel is memoryless, if we take one
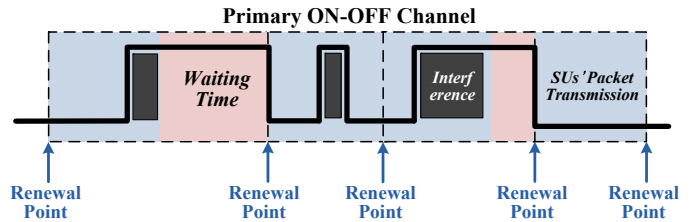

Fig. 3. SUs' renewal process.

transmission and waiting time as a renewal period, the SUs' communication behavior is a renewal process. ∎

B. Interference Quantity Analysis

In order to analyze the interference during the SUs' one packet transmission time T_t, we first introduce a new function, $I(t)$, defined as follows.

Definition 2: $I(t)$ is the expected accumulated interference to the PU within a period of time t, where t has two special characteristics listed as follows

- period t always begins at the OFF state of the primary channel,
- during t, the SUs keep transmitting packets in the primary channel.

According to *Definition 1*, *Definition 2* and *Theorem 1*, the interference quantity Q_{I_1} (when $\lambda_s = 0$) can be calculated by

$$Q_{I_1} = \frac{I(T_t)}{I(T_t) + \mathbb{E}(T_w)}, \tag{3}$$

where $I(T_t)$ is the expected interference generated during the SUs' transmission time T_t, $\mathbb{E}(T_w)$ is the expectation of SUs' waiting time T_w, during which the primary channel is always in the ON state and no interference from the SUs occurs. In the following, we will derive the closed-form expressions for $I(T_t)$ and $\mathbb{E}(T_w)$, respectively.

1) Expected interference $I(T_t)$: According to *Definition 2*, $I(t)$ is the expected length of all ON states within a period of time t, given that t begins at the OFF state. According to the Renewal Theory [19], the PU's ON-OFF behavior is a renewal process. Therefore, we can derive $I(t)$ through solving the renewal equation (4) according to the following *Theorem 2*.

*Theorem 2:* $I(t)$ satisfies the renewal equation as follows

$$I(t) = \lambda_1 F_p(t) + \int_0^t I(t-w) f_p(w) dw, \tag{4}$$

where $f_p(t)$ is the *p.d.f* of the PU's renewal interval given in (1) and $F_p(t)$ is the corresponding cumulative distribution function (*c.d.f*).

Proof: Let X denote the first OFF state and Y denote the first ON state, as shown in Fig. 4. Thus, we can write the recursive expression of function $I(t)$ as follows

$$I(t|X, Y) = \begin{cases} 0 & t \leq X, \\ t - X & X \leq t \leq X + Y, \\ Y + I(t - X - Y) & X + Y \leq t, \end{cases} \tag{5}$$

where $X \sim f_{\text{OFF}}(x) = \frac{1}{\lambda_0} e^{-x/\lambda_0}$ and $Y \sim f_{\text{ON}}(y) = \frac{1}{\lambda_1} e^{-y/\lambda_1}$.
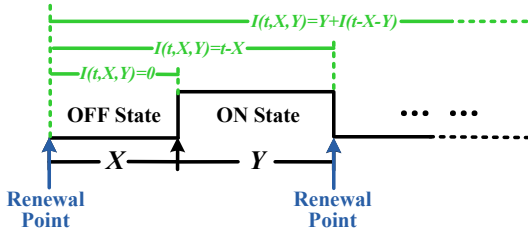
Fig. 4.   Illustration of function $I(t)$.

Fig. 5.   SUs' waiting time T_w.

Since X and Y are independent, their joint distribution $f_{XY}(x,y) = f_{\text{OFF}}(x)f_{\text{ON}}(y)$. In such a case, $I(t)$ can be re-written as follows

$$
\begin{aligned}
I(t) &= \iint_{xy} I(t|x,y)f_{XY}(x,y)dxdy, \\
&= \iint_{x \le t \le x+y} (t-x)f_{XY}(x,y)dxdy + \\
&\quad \iint_{x+y \le t} [y + I(t-x-y)]f_{XY}(x,y)dxdy, \\
&= \int_0^t (t-x)f_{\text{OFF}}(x)dx + \\
&\quad \iint_{x+y \le t} I(t-x-y)f_{\text{OFF}}(x)f_{\text{ON}}(y)dxdy - \\
&\quad \iint_{x+y \le t} (t-x-y)f_{\text{OFF}}(x)f_{\text{ON}}(y)dxdy, \\
&= I_1(t) + I_2(t) - I_3(t), \quad (6)
\end{aligned}
$$

where $I_1(t)$, $I_2(t)$ and $I_3(t)$ represent those three terms in the second equality, respectively. By taking Laplace transforms on the both sides of (6), we have

$$
\mathbb{I}(s) = \mathbb{I}_1(s) + \mathbb{I}_2(s) - \mathbb{I}_3(s), \quad (7)
$$

where $\mathbb{I}_1(s)$, $\mathbb{I}_2(s)$, $\mathbb{I}_3(s)$ are the Laplace transforms of $I_1(t)$, $I_2(t)$, $I_3(t)$, respectively.

According to the expression of $I_1(t)$ in (6), we have

$$
I_1(t) = \int_0^t (t-x)f_{\text{OFF}}(x)dx = t * f_{\text{OFF}}(t). \quad (8)
$$

Thus, the Laplace transform of $I_1(t)$, $\mathbb{I}_1(s)$ is

$$
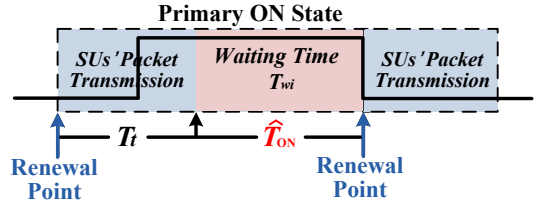\mathbb{I}_1(s) = \frac{1}{s^2}\mathbb{F}_{\text{OFF}}(s), \quad (9)
$$

where $\mathbb{F}_{\text{OFF}}(s) = \frac{1}{\lambda_0 s + 1}$ is the Laplace transform of $f_{\text{OFF}}(t)$.

With the expression of $I_2(t)$ in (6), we have

$$
\begin{aligned}
I_2(t) &= \iint_{x+y \le t} I(t-x-y)f_{\text{OFF}}(x)f_{\text{ON}}(y)dxdy \\
&= I(t) * f_{\text{ON}}(t) * f_{\text{OFF}}(t) \\
&= I(t) * f_p(t), \quad (10)
\end{aligned}
$$

where the last step is according to (1). Thus, the Laplace transform of $I_2(t)$, $\mathbb{I}_2(s)$ is

$$
\mathbb{I}_2(s) = \mathbb{I}(s)\mathbb{F}_p(s), \quad (11)
$$

where $\mathbb{I}(s)$ and $\mathbb{F}_p(s) = \frac{1}{(\lambda_1 s+1)(\lambda_0 s+1)}$ are Laplace transforms of $I(t)$ and $f_p(t)$, respectively.

Similar to (10), we can re-written $I_3(t)$ as $I_3(t) = t * f_p(t)$. Thus, the Laplace transform of $I_3(t)$, $\mathbb{I}_3(s)$ is

$$
\mathbb{I}_3(s) = \frac{1}{s^2}\mathbb{F}_p(s). \quad (12)
$$

By substituting (9), (11) and (12) into (7), we have

$$
\begin{aligned}
\mathbb{I}(s) &= \frac{1}{s^2}\mathbb{F}_{\text{OFF}}(s) + \mathbb{I}(s)\mathbb{F}_p(s) - \frac{1}{s^2}\mathbb{F}_p(s) \\
&= \lambda_1 \frac{\mathbb{F}_p(s)}{s} + \mathbb{I}(s)\mathbb{F}_p(s). \quad (13)
\end{aligned}
$$

Then by taking the inverse Laplace transform on the both sides of (13), we have

$$
\begin{aligned}
I(t) &= \lambda_1 \int_0^t f_p(w)dw + \int_0^t I(t-w)f_p(w)dw \\
&= \lambda_1 F_p(t) + \int_0^t I(t-w)f_p(w)dw. \quad (14)
\end{aligned}
$$

This completes the proof of the theorem.   ∎

Theorem 2 illustrates the renewal characteristic of $I(t)$. By substituting $\mathbb{F}_p(s) = \frac{1}{(\lambda_1 s+1)(\lambda_0 s+1)}$ into (13), the Laplace transform of $I(t)$ can be calculated by

$$
\mathbb{I}(s) = \frac{\lambda_1 \mathbb{F}_p(s)}{s(1 - \mathbb{F}_p(s))} = \frac{\lambda_1}{s^2(\lambda_0\lambda_1 s + \lambda_0 + \lambda_1)}. \quad (15)
$$

Then, by taking inverse Laplace transform on (15), we can obtain the closed-form expression for $I(t)$ as

$$
I(t) = \frac{\lambda_1}{\lambda_0 + \lambda_1}t - \frac{\lambda_0\lambda_1^2}{(\lambda_0+\lambda_1)^2}\left(1 - e^{-\frac{\lambda_0+\lambda_1}{\lambda_0\lambda_1}t}\right). \quad (16)
$$

2) Expected waiting time $\mathbb{E}(T_w)$: As shown in Fig. 3, on one hand, if the transmission time T_t ends in the OFF state, the following waiting time T_w will be 0; on the other hand, if T_t ends in the ON state, the length of T_w will depend on when this ON state terminates, which can be specifically illustrated in Fig. 5. In the second case, according to the Renewal Theory [19], T_w is equivalent to the *forward recurrence time* of the ON state, $\widehat{T}_{\text{ON}}$, the distribution of which is only related to that of the ON state. Thus, we can summarize T_w as follows

$$
T_w = \begin{cases} 0 & T_t \text{ ends in the OFF state,} \\ \widehat{T}_{\text{ON}} & T_t \text{ ends in the ON state.} \end{cases} \quad (17)
$$

To compute the closed-form expression for T_w, we introduce a new function defined as follows.

Definition 3: $P_{\text{ON}}(t)$ is the average probability that a period of time t begins at the OFF state and ends at the ON state.
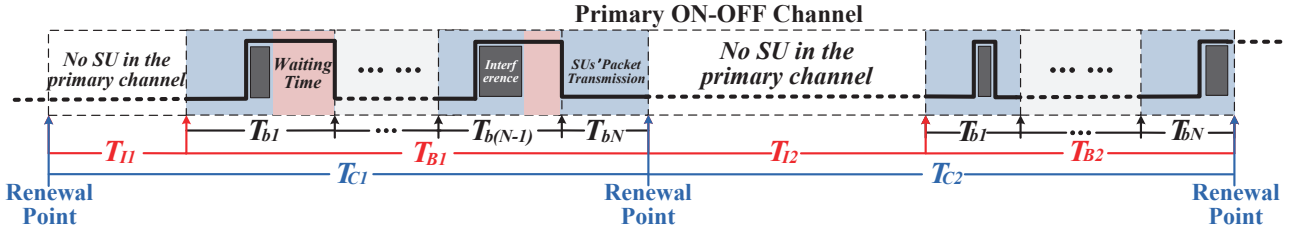
Fig. 6.   Illustration of the SUs' *idle-busy* behavior in the primary channel when $\lambda_s \neq 0$.

According to *Definition 3* and (17), the SUs' average waiting time $\mathbb{E}(T_w)$ can be written by

$$\mathbb{E}(T_w) = P_{\text{ON}}(T_t)\mathbb{E}(\widehat{T}_{\text{ON}}). \tag{18}$$

Similar to the analysis of $I(t)$ in Section IV-B1, $P_{\text{ON}}(t)$ can also be obtained through solving the following renewal equation

$$P_{\text{ON}}(t) = \lambda_1 f_p(t) + \int_0^t P_{\text{ON}}(t - w)f_p(w)dw. \tag{19}$$

By solving (19), we can obtain the closed-form expression of $P_{\text{ON}}(t)$ as

$$P_{\text{ON}}(t) = \frac{\lambda_1}{\lambda_0 + \lambda_1}\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1}t}\right). \tag{20}$$

Note that (20) can also be derived by the theory of continuous Markov chains [29].

The $\widehat{T}_{\text{ON}}$ is the *forward recurrence time* of the primary channel's ON state. Since all ON sates follow a Poisson process. According to Renewal Theory [19], we have

$$\widehat{T}_{\text{ON}} \sim \frac{1}{\lambda_1}e^{-t/\lambda_1}, \quad \mathbb{E}(\widehat{T}_{\text{ON}}) = \lambda_1. \tag{21}$$

By combining (20) and (21), the SUs' average waiting time $\mathbb{E}(T_w)$ can be obtained as follows

$$\mathbb{E}(T_w) = \frac{\lambda_1^2}{\lambda_0 + \lambda_1}\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1}T_t}\right). \tag{22}$$

Finally, by substituting (16) and (22) into (3), we can obtain the quantity of interference Q_{I_1} as follows

$$Q_{I_1} = \frac{(\lambda_0 + \lambda_1)T_t - \lambda_0\lambda_1\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1}T_t}\right)}{(\lambda_0 + \lambda_1)T_t + \lambda_1^2\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1}T_t}\right)}. \tag{23}$$

V. INTERFERENCE CAUSED BY SUs WITH NON-ZERO ARRIVAL INTERVAL

In this section, we will discuss the case when the SUs' requests arrive by a Poisson process with average arrival interval $\lambda_s \neq 0$. Under such a scenario, the buffer at the coordinator may be empty during some periods of time. Similar to the analysis in Section IV, we will start with analyzing the SUs' communication behavior, and then quantify the interference to the PU.
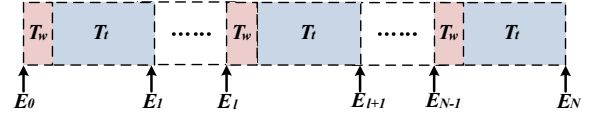

Fig. 7.   Illustration of buffer status E_l when $n = N$.

A. SUs' Communication Behavior Analysis

Compared with the SUs' behavior when $\lambda_s = 0$, another state that may occur when $\lambda_s \neq 0$ is there is no SUs' request in the coordinator's buffer. We call this new state as an *idle state* of the SUs' behavior, while the opposite *busy state* refers to the scenario when the coordinator's buffer is not empty. The length of the *idle state* and *busy state* are denoted by T_I and T_B, respectively. As shown in Fig. 6, the SUs' behavior switches between the *idle state* and *busy state*, which is similar to the PU's ON-OFF model. In the following, we prove that the SUs' such *idle-busy* switching is also a renewal process.

Theorem 3: When the SUs' transmission requests arrive by Poisson process with constant rate λ_s^{-1}, the SUs' communication behavior is a renewal process in the primary channel.

Proof: In Fig. 6, we use T_c to denote one cycle of the SUs' *idle* and *busy* state, i.e., $T_c = T_I + T_B$. For the idle state, since the SUs' requests arrive by Poisson process, $T_I \sim \frac{1}{\lambda_s}e^{-t/\lambda_s}$ and hence the lengths of all *idle states* are *i.i.d.* For the *busy state*, $T_B = \sum_{i=1}^{n} T_{bi}$ as shown in Fig. 6, where n is the number of SUs' transmitting-waiting times during one *busy state*. Since all T_{bi} are *i.i.d* as proved in *Theorem 1*, T_{B1}, T_{B2}, ... will also be *i.i.d* if we can prove that the n of all *busy states* are *i.i.d*. It is obvious that the n of all *busy states* are independent since the SUs' requests arrive by a Poisson process. In the following, we will focus on proving its property of identical distribution.

In Fig. 7, we illustrate the case when there are N transmitting-waiting times during one *busy state*, i.e., $n = N$. E_l represents the number of requests waiting in the coordinator's buffer at the end of the lth T_t, i.e., the time right after the transmission of the SUs' lth packet. We can see that E_l $(0 \leq l \leq N)$ should satisfy the following condition

$$\begin{cases} E_0 = 1, & 1 \leq E_1 \leq N - 1, \\ \cdots, & \\ 1 \leq E_l \leq N - l, & \\ \cdots, & \\ E_{N-1} = 1, & E_n = 0. \end{cases} \tag{24}$$

According to the queuing theory [30], the sequence E_1, E_2,

$\ldots$, E_N is an embedded Markov process. Thus, the probability $P(n = N)$ can be written as

$$P(n = N) = P(E_1|E_0)P(E_2|E_1)\ldots P(E_N|E_{N-1}), \quad (25)$$

where $P(E_{l+1}|E_l)$ denotes the probability that the last T_t ends with E_l requests in the coordinator's buffer and current T_t ends with E_{l+1} requests. Suppose $E_{l+1} = j$ and $E_l = i$, we can denote $P(E_{l+1}|E_l)$ simply as P_{ij}, which represents the probability that there are $j - i + 1$ requests arriving at the coordinator during the period $T_w + T_t$. Since the SUs' requests arrive by a Poisson process with arrival interval T_s, P_{ij} can be calculated by

$$P_{ij} = P\left(\sum_{k=1}^{j-i+1} T_{s_k} \le (T_w + T_t) \le \sum_{k=1}^{j-i+2} T_{s_k}\right)$$
$$= \int_{T_t}^{+\infty} \frac{(t/\lambda_s)^{j-i+1}}{(j-i+1)!}e^{-t/\lambda_s}P(T_w + T_t = t)dt$$
$$= \int_0^{+\infty} \frac{\left((t+T_t)/\lambda_s\right)^{j-i+1}}{(j-i+1)!}e^{\frac{-(t+T_t)}{\lambda_s}}P(T_w = t)dt, (26)$$

where T_{s_k} is the SUs' kth arrival interval satisfying the exponential distribution with parameter λ_s, the first equality is because $\sum_{k=1}^{j-i+1} T_{s_k}$ and $\sum_{k=1}^{j-i+2} T_{s_k}$ satisfy Erlang distribution.

According to (17) and (21), the probability distribution of T_w, $P(T_w = t)$, can be written as follows

$$P(T_w = t) = \begin{cases} P_{\text{OFF}}(T_t) & t = 0, \\ \frac{P_{\text{ON}}(T_t)}{\lambda_1}e^{-t/\lambda_1} & t > 0, \end{cases} \quad (27)$$

where $P_{\text{OFF}}(T_t) = 1 - P_{\text{ON}}(T_t) = \frac{\lambda_0}{\lambda_0+\lambda_1}\left(1 - e^{-\frac{\lambda_0+\lambda_1}{\lambda_0\lambda_1}T_t}\right)$.

By substituting (27) into (26), we can re-write P_{ij} as

$$P_{ij} = P_{\text{OFF}}(T_t)\frac{(T_t/\lambda_s)^{j-i+1}}{(j-i+1)!}e^{-T_t/\lambda_s} +$$
$$\int_{0+}^{+\infty} \frac{\left((t+T_t)/\lambda_s\right)^{j-i+1}}{(j-i+1)!}\frac{P_{\text{ON}}(T_t)}{\lambda_1}e^{-\left(\frac{\lambda_1+\lambda_s}{\lambda_1\lambda_s}t+\frac{T_t}{\lambda_s}\right)}dt. \quad (28)$$

According to (28), we can see that there are $(N-1)!$ possible combinations of $(E_0, \ldots, E_l, \ldots, E_N)$. We denote each case as $C(a)$, where $1 \le a \le (N-1)!$. For each case, the probability is the product of N terms $P_{ij}\big(C(a), b\big)$, where $1 \le b \le N$. Thus, $P(n = N)$ can be expressed as follows

$$P(n = N) = \sum_{a=1}^{(N-1)!} \prod_{b=1}^{N} P_{ij}\big(C(a), b\big). \quad (29)$$

From (29), we can see that n of all *busy states* are identical distributed, and hence *i.i.d*.

Up to now, we have come to the conclusion that T_I of all *idle states* are *i.i.d*, as well as T_B of all *busy states*. Since T_I and T_B are independent with each other, the sequence of all cycles' lengths T_{c1}, T_{c2}, $\ldots$ are *i.i.d*. Therefore, the SUs' communication behavior is a renewal process. ∎

B. Interference Quantity Analysis

According to *Definition 1* and *Theorem 3*, the interference quantity Q_{I_2} can be calculated by

$$Q_{I_2} = \mu_B Q_{I_1}, \quad (30)$$

where $\mu_B = \frac{\mathbb{E}(T_B)}{\mathbb{E}(T_I)+\mathbb{E}(T_B)}$ is the occurrence probability of the SUs' *busy state*.

Our system can be treated as an $M/G/1$ queuing system, where the customers are the SUs' data packets and the server is the primary channel. The service time S of one SU is the sum of its transmission time T_t and the waiting time of the next SU T_w. In such a case, the expected service time is $\mathbb{E}(S) = T_t + \mathbb{E}(T_w)$. According to the queuing theory [30], the load of the server is $\rho = \mathbb{E}(S)/\lambda$, where λ is the average arrival interval of the customers. By Little's law [30], ρ is equivalent to the expected number of customers in the server. In our system, there can be at most one customer (SUs' one packet) in the server, which means the expected number of customers is equal to the probability that there is a customer in the server. Therefore, ρ is equal to the proportion of time that the coordinator is busy, i.e.,

$$\rho = \frac{T_t + \mathbb{E}(T_w)}{\lambda_s} = \mu_B = \frac{\mathbb{E}(T_B)}{\mathbb{E}(T_I) + \mathbb{E}(T_B)}. \quad (31)$$

Thus, combining (23), (30) and (31), the closed-form expression of Q_{I_2} can be obtained as follows

$$Q_{I_2} = \frac{(\lambda_0 + \lambda_1)T_t - \lambda_0\lambda_1\left(1 - e^{-\frac{\lambda_0+\lambda_1}{\lambda_0\lambda_1}T_t}\right)}{\lambda_s(\lambda_0 + \lambda_1)}. \quad (32)$$

VI. OPTIMIZING SECONDARY USERS' COMMUNICATION PERFORMANCE

In this section, we will discuss how to optimize the SUs' communication performance while maintaining the PU's communication QoS and the stability of the secondary network. In our system, the SUs' communication performance is directly dependent on the expected arrival interval of their packets λ_s[1] and the length of the transmission time T_t. These two important parameters should be appropriately chosen so as to minimize the interference caused by the SUs' dynamic access and also to maintain a stable secondary network.

We consider two constraints for optimizing the SUs' λ_s and T_t as follows

- the PU's average data rate should be at least $R_p^\downarrow$, which is the PU's lowest data rate,
- the stability condition of the secondary network should be satisfied.

In the following, we will first derive the expressions for these two constraints based on the analysis in Section IV and V. Then we formulate the problem of finding the optimal λ_s^* and T_t^* as an optimization problem to maximize the SUs' average data rate.

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[1]To evaluate the stability condition, we only consider the scenario when $\lambda_s \neq 0$.

A. The Constraints

1) PU's Average Data Rate: If there is no interference from the SUs, the PU's instantaneous rate is $\log(1 + \mathrm{SNR}_p)$, where SNR_p denotes the Signal-to-Noise Ratio of primary signal at the PU's receiver. On the other hand, if the interference occurs, the PU's instantaneous rate will be $\log\left(1 + \frac{\mathrm{SNR}_p}{\mathrm{INR}_p+1}\right)$, where INR_p is the Interference-to-Noise Ratio of secondary signal received by the PU. According to *Definition 1*, Q_{I_2} represents the ratio of the interference periods to the PU's overall communication time. Thus, the PU's average data rate R_p can be calculated by

$$R_p = \left(1 - Q_{I_2}\right)\log\left(1 + \mathrm{SNR}_p\right) + Q_{I_2}\log\left(1 + \frac{\mathrm{SNR}_p}{\mathrm{INR}_p + 1}\right). \quad (33)$$

2) SUs' Stability Condition: In our system, the secondary network and the primary channel can be modeled as a single-server queuing system. According to the queuing theory [30], the stability condition for a single-server queue with Poisson arrivals is that the load of the server should satisfy $\rho < 1$ [31]. In our system, we have

$$\rho = \frac{T_t + \mathbb{E}(T_w)}{\lambda_s} < 1. \quad (34)$$

In such a case, the SUs' stability condition function, $S(T_t, \lambda_s)$, can be written as follows

$$S(T_t, \lambda_s) = \lambda_s - T_t - \frac{\lambda_1^2}{\lambda_0 + \lambda_1}\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t}\right) > 0. \quad (35)$$

B. Objective Function: SUs' Average Data Rate

If a SU encounters the PU's recurrence, i.e., the ON state of the primary channel, during its transmission time T_t, its communication is also interfered by the PU's signal. In such a case, the SU's instantaneous rate is $\log\left(1 + \frac{\mathrm{SNR}_s}{\mathrm{INR}_s+1}\right)$, where SNR_s is the SU's Signal-to-Noise Ratio and INR_s is the Interference-to-Noise Ratio of primary signal received by the SU. According to *Theorem 1* and *Theorem 3*, the occurrence probability of such a phenomenon is $\mu_B \frac{I(T_t)}{T_t + \mathbb{E}(T_w)} = \frac{I(T_t)}{\lambda_s}$. On the other hand, if no PU appears during the SU's transmission, its instantaneous rate will be $\log(1 + \mathrm{SNR}_s)$ and the corresponding occurrence probability is $\mu_B \frac{T_t - I(T_t)}{T_t + \mathbb{E}(T_w)} = \frac{T_t - I(T_t)}{\lambda_s}$. Thus, the SU's average data rate R_s is

$$R_s = \frac{T_t - I(T_t)}{\lambda_s}\log\left(1 + \mathrm{SNR}_s\right) + \frac{I(T_t)}{\lambda_s}\log\left(1 + \frac{\mathrm{SNR}_s}{\mathrm{INR}_s + 1}\right). \quad (36)$$

C. Optimizing SUs' Communication Performance

Based on the analysis of constraints and objective function, the problem of finding optimal T_t^* and λ_s^* for the SUs can be formulated by (37) below.

Proposition 1: The SUs' average data rate $R_s(T_t, \lambda_s)$ is a strictly increasing function in terms of the their transmission time T_t and a strictly decreasing function in terms of their average arrival interval λ_s, i.e.,

$$\frac{\partial R_s}{\partial T_t} > 0, \quad \frac{\partial R_s}{\partial \lambda_s} < 0. \quad (38)$$

The PU's average data rate $R_p(T_t, \lambda_s)$ is a strictly decreasing function in terms of T_t and a strictly increasing function in terms of λ_s, i.e.,

$$\frac{\partial R_p}{\partial T_t} < 0, \quad \frac{\partial R_p}{\partial \lambda_s} > 0. \quad (39)$$

The stability condition function $S(T_t, \lambda_s)$ is a strictly decreasing function in terms of T_t and a strictly increasing function in terms of λ_s, i.e.,

$$\frac{\partial S}{\partial T_t} < 0, \quad \frac{\partial S}{\partial \lambda_s} > 0. \quad (40)$$

Proof: For simplification, we use R_{s0} to express $\log\left(1 + \mathrm{SNR}_s\right)$ and R_{s1} to express $\log\left(1 + \frac{\mathrm{SNR}_s}{\mathrm{INR}_s+1}\right)$. According to (36) and (16), $\frac{\partial R_s}{\partial T_t}$ and $\frac{\partial R_s}{\partial \lambda_s}$ can be calculated as follows

$$\begin{aligned}\frac{\partial R_s}{\partial T_t} &= \frac{R_{s0}}{\lambda_s} - \frac{R_{s0} - R_{s1}}{\lambda_s} \cdot \frac{\partial I(T_t)}{\partial T_t} \\ &= \frac{1}{\lambda_s(\lambda_0 + \lambda_1)}\Big(\lambda_0 R_{s0} + \lambda_1 R_{s1} + \\ &\quad \lambda_1(R_{s0} - R_{s1})\Big(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t}\Big)\Big),\end{aligned} \quad (41)$$

$$\frac{\partial R_s}{\partial \lambda_s} = -\frac{1}{\lambda_s^2}\Big(\big(T_t - I(T_t)\big)R_{s0} + I(T_t)R_{s1}\Big). \quad (42)$$

Since $R_{s0} > R_{s1}$, $e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t} < 1$, and $T_t \geq I(T_t)$, we have

$$\frac{\partial R_s}{\partial T_t} > 0, \quad \frac{\partial R_s}{\partial \lambda_s} < 0. \quad (43)$$

Similarly, we use R_{p0} to express $\log\left(1 + \mathrm{SNR}_p\right)$ and R_{p1} to express $\log\left(1 + \frac{\mathrm{SNR}_p}{\mathrm{INR}_p+1}\right)$. According to (33), $\frac{\partial R_p}{\partial T_t}$ and $\frac{\partial R_p}{\partial \lambda_s}$ can be calculated as follows
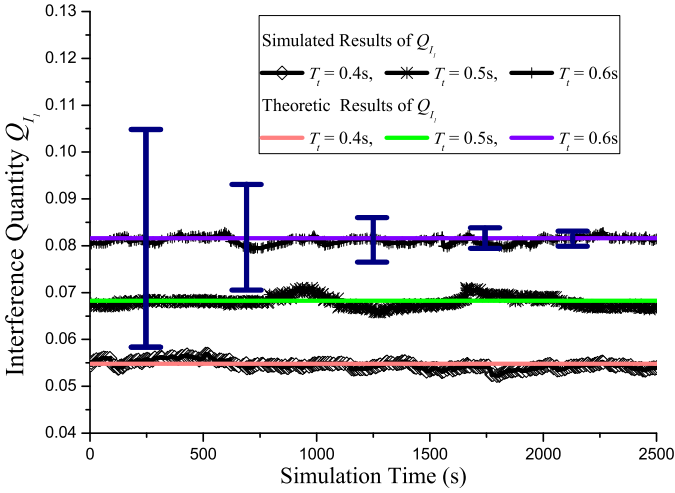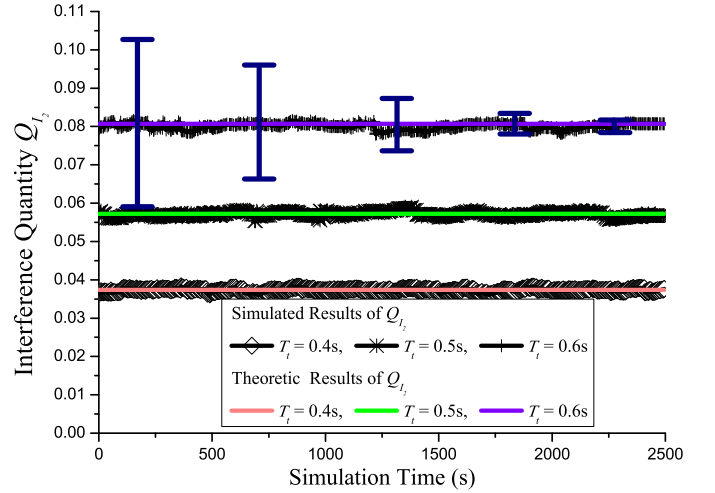
$$\frac{\partial R_p}{\partial T_t} = -\frac{\partial Q_{I_2}}{\partial T_t}(R_{s0} - R_{s1}), \quad (44)$$

$$\frac{\partial R_p}{\partial \lambda_s} = -\frac{\partial Q_{I_2}}{\partial \lambda_s}(R_{s0} - R_{s1}). \quad (45)$$

$$\begin{aligned}\max_{(T_t, \lambda_s)} \quad & R_s(T_t, \lambda_s) = \frac{T_t - I(T_t)}{\lambda_s}\log\left(1 + \mathrm{SNR}_s\right) + \frac{I(T_t)}{\lambda_s}\log\left(1 + \frac{\mathrm{SNR}_s}{\mathrm{INR}_s + 1}\right), \\ \text{s.t.} \quad & R_p(T_t, \lambda_s) = \left(1 - Q_{I_2}\right)\log\left(1 + \mathrm{SNR}_p\right) + Q_{I_2}\log\left(1 + \frac{\mathrm{SNR}_p}{\mathrm{INR}_p + 1}\right) \geq R_p^{\downarrow}, \\ & S(T_t, \lambda_s) = \lambda_s - T_t - \frac{\lambda_1^2}{\lambda_0 + \lambda_1}\left(1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t}\right) > 0.\end{aligned} \quad (37)$$

Fig. 8. Interference quantity Q_{I_1} with $\lambda_s = 0$ s.

Fig. 9. Interference quantity Q_{I_2} with $\lambda_s = 1.3$ s.

According to (32), we have

$$\frac{\partial Q_{I_2}}{\partial T_t} = \frac{1 - e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t}}{\lambda_s} > 0, \quad \frac{\partial Q_{I_2}}{\partial \lambda_s} < 0. \quad (46)$$

Thus, combining (44), (45) and (46), we have

$$\frac{\partial R_p}{\partial T_t} < 0, \quad \frac{\partial R_p}{\partial \lambda_s} > 0. \quad (47)$$

According to (35), $\frac{\partial S}{\partial T_t}$ and $\frac{\partial S}{\partial \lambda_s}$ can be calculated as follows

$$\frac{\partial S}{\partial T_t} = -\left(1 + \frac{\lambda_1}{\lambda_0} e^{-\frac{\lambda_0 + \lambda_1}{\lambda_0 \lambda_1} T_t}\right) < 0, \quad \frac{\partial S}{\partial \lambda_s} = 1 > 0. \quad (48)$$

This completes the proof of the theorem. ∎

From *Proposition 1*, we can see that the objective function and the constraints are all monotonous functions in terms of T_t and λ_s. Thus, the solution to the optimization problem (37) can be found using gradient descent method [32].

VII. SIMULATION RESULTS

In this section, we conduct simulations to verify the effectiveness of our analysis. The parameters of primary ON-OFF channel are set to be $\lambda_0 = 2.6$ s and $\lambda_1 = 3.6$ s. According to Fig. 2, we build a queuing system using Matlab to simulate the PU's and SUs' behaviors.

A. Interference Quantity Q_I

In Fig. 8 and Fig. 9, we illustrate the theoretic and simulated results of Q_{I_1} and Q_{I_2}, respectively. The theoretic Q_{I_1} and Q_{I_2} are computed according to (23) and (32) with different values of the SUs' transmission time T_t. The average arrival interval of the SUs' packets λ_s is set to be 1.3 s when calculating theoretic Q_{I_2}. For the simulated results, once the interference occurs, we calculate and record the ratio of the accumulated interference periods to the accumulated periods of the ON states. We perform 2000 times simulation runs and average all of them to obtain the final simulation results.

From Fig. 8 and Fig. 9, we can see that all the simulated results of Q_{I_1} and Q_{I_2} are accord with the corresponding theoretic results, which means that the closed-form expressions in (23) and (32) are correct and can be used to calculate

the interference caused by the SUs in the practical cognitive radio system. We also denote the standard deviation of Q_I at several simulation time points when $T_t = 0.6$ s and the results show that the standard deviation converges to 0 along with the increasing of the simulation time, i.e., the system gradually tends to steady state. Moreover, we can also see that the interference increases as the SUs' transmission time T_t increases. Such a phenomenon is because the interference to the PU can only occur during T_t and the increase of T_t enlarges the occurrence probability of T_t. Finally, we find that due to the existence of the *idle state* when $\lambda_s \neq 0$, Q_{I_2} is less than Q_{I_1} under the same condition.

B. Stability of The Secondary Network

Since we have modeled the secondary network as a queuing system shown in Fig. 2, the stability of the network is reflected by the status of the coordinator's buffer. A stable network means that the requests waiting in the coordinator's buffer do not explode as time goes to infinite, while the requests in the buffer of an unstable network will eventually go to infinite. In Section VI-A2, we have shown the stability condition of the secondary network in (35). On one hand, if the SUs' access time T_t is given in advance, the SUs' minimal average arrival interval λ_s can be computed by (35). On the other hand, if λ_s is given, the maximal T_t can be obtained to restrict the SUs' transmission time.

In this simulation, we set $T_t = 0.6$ s, and thus λ_s should be larger than 1.25 s to ensure the SUs' stability according to (35). In Fig. 9, we show the queuing length, i.e., the number of requests in the coordinator's buffer, versus the time. The black lines shows the queuing length of a stable network, in which $\lambda_s = 1.3$ s is larger than the threshold 1.25 s. It can be seen that the requests dynamically vary between 0 and 60. However, if we set $\lambda_s = 1.2$ s, which is smaller than the lower limit, from Fig. 9, we can see that the queuing length will finally go to infinite, which represents an unstable network. Therefore, the stability condition in (35) should be satisfied to maintain a stable secondary network.
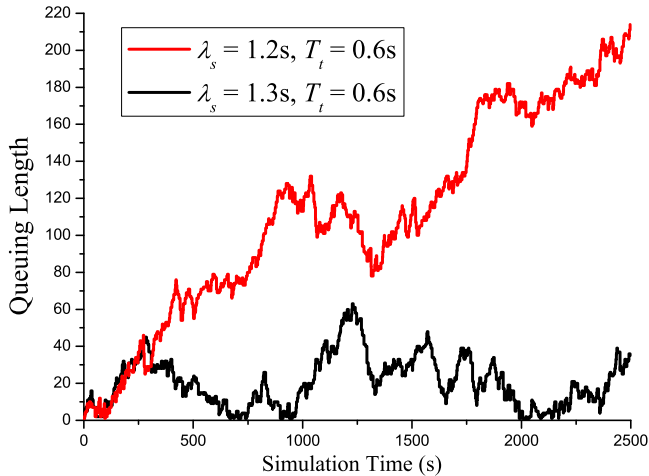
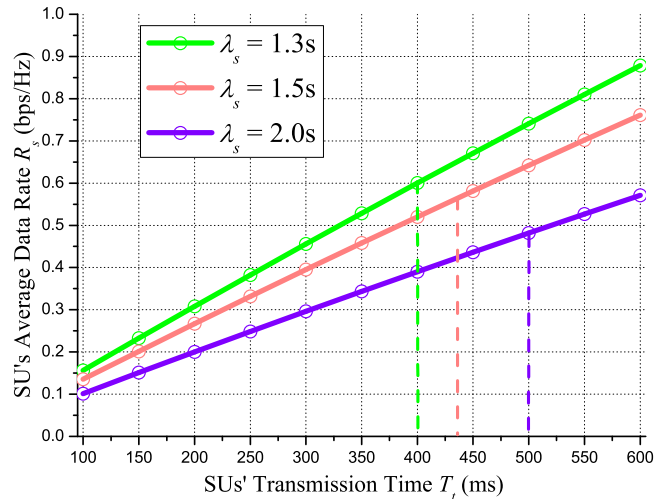Fig. 10. Queuing length under stable and unstable conditions.
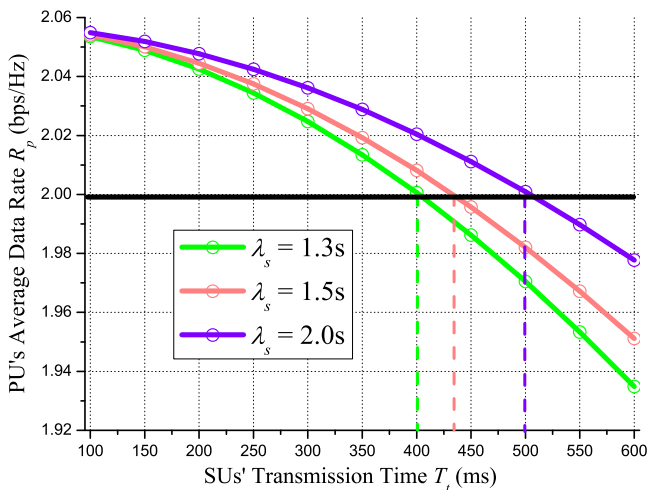

Fig. 11. PU's average data rate.

Fig. 12. SUs' average data rate.

C. PU's and SUs' Average Data Rate

The simulation results of the PU's average data rate R_p versus the SUs' transmission time T_t and arrival interval λ_s are shown in Fig. 11, where we set $\text{SNR}_p = \text{SNR}_s = 5\,\text{dB}$ and $\text{INR}_p = \text{INR}_s = 3\,\text{dB}$. We can see that R_p is a decreasing function in terms of T_t given a certain λ_s, and an increasing function in terms of λ_s for any fixed T_t, which is in accordance with *Proposition 1*. Such a phenomenon is because an increase of T_t or a decrease of λ_s will cause more interference to the PU and thus degrade its average data rate. In Fig. 12, we illustrate the simulation results of the SUs' average data rate R_s versus T_t and λ_s. Different from R_p, R_s is an increasing function in terms of T_t given a certain λ_s, and a decreasing function in terms of λ_s for any fixed T_t, which also verifies the correctness of *Proposition 1*.

Suppose that the PU's data rate should be at least $2.0\,\text{bps/Hz}$, i.e., $R_p^{\downarrow} = 2.0\,\text{bps/Hz}$. Then, according to the constraints in (37), T_t should be no larger than the location of those three colored vertical lines in Fig. 11 corresponding to $\lambda_s = 1.3\,\text{s}$, $1.5\,\text{s}$, $2.0\,\text{s}$, respectively. For example, when $\lambda_s = 1.3\,\text{s}$, the optimal T_t^* should be around $400\,\text{ms}$ to satisfy both the $R_p^{\downarrow}$ and stability condition constraints. In such a case,

the SUs' average data rate can achieve around $0.6\,\text{bps/Hz}$ according to Fig. 12. For any fixed $R_p^{\downarrow}$, the optimal values of T_t^* and λ_s^* are determined by the channel parameters λ_0 and λ_1. Therefore, the SUs should dynamically adjust their communication behaviors according to the channel parameters.

VIII. CONCLUSION

In this paper, we analyzed the interference caused by the SUs confronted with unknown primary behavior. Based on the Renewal Theory, we showed that the SUs' communication behaviors in the ON-OFF primary channel is a renewal process and derived the closed-form for the interference quantity. We further discussed how to optimize the SUs' arrival rate and transmission time to control the level of interference to the PU and maintain the stability of the secondary network. Simulation results are shown to validate our closed-form expressions for the interference quantity. In the practical cognitive radio networks, these expressions can be used to evaluate the interference from the SUs when configuring the secondary network. In the future work, we will study how to concretely coordinate the primary spectrum sharing among multiple SUs.

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