Leveraging Information in Parking Assistance Systems

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***Abstract*—This paper seeks to systematically explore the efficiency of uncoordinated parking space allocation in urban environments with two types of parking facilities. Drivers decide whether to go for inexpensive but limited on-street parking spots or expensive yet over-dimensioned parking lots, incurring an additional cruising cost when they decide for on-street parking spots but fail to actually acquire one. Their decisions are made under perfect knowledge of the total parking supply and costs and different levels of information about the parking demand, *i.e.*, *complete/probabilistic* information and *uncertainty*. We take a game-theoretic approach and analyze the parking space allocation process in each case as *resource selection game* instances. We derive their equilibria, compute the related *Price of Anarchy* values and study the impact of pricing on them.**

It is shown that, under typical pricing policies on the two types of parking facilities, drivers tend to over-compete for the on-street parking space, giving rise to redundant cruising cost. Yet this inefficiency can be alleviated through the systematic manipulation of the information that is announced to the drivers. In particular, counterintuitive *less-is-more effects* emerge regarding the way information availability modulates the resulting efficiency of the process, which underpin general competitive service provision settings.

I. Introduction

The tremendous increase of urbanization necessitates the efficient and environmentally sustainable management of various urban processes and operations. Recent advances in wireless networking and sensing technologies can address this need by enabling efficient monitoring mechanisms for these processes and higher flexibility to control them, thus paving the way for the so-called *smart cities*. Intelligent networked sensor nodes, placed on buildings' surfaces or mounted on vehicles, constitute pervasive monitoring platforms that can measure environmental parameters such as pollution concentration, radiation level, road traffic congestion or public transport utilization. The data generated by this monitoring infrastructure can then be exploited by municipal authorities to more efficiently manage both the environmental and man-made city resources. In the case of parking space resources, in particular, the challenge is to efficiently manage the available parking space and reduce the vehicle volumes that cruise in search of it, in order to alleviate not only traffic congestion but also the related environmental burden.

To this end, academic research but also public and/or private initiatives in the past years have been primarily directed towards the design and deployment of *parking assistance systems*. Common to these systems is the exploitation of wireless communications and sensing technologies to collect and broadcast (in centralized systems, *i.e.*, [1], [2]) or share (in distributed systems, *i.e.*, [3], [4]) information about the supply of (and demand for) parking resources. This information ideally saves drivers from redundant cruising trips in search of a parking spot and assists in the management of parking resources, with centralized systems even implementing parking spot reservation. Parking assistance systems may also enable smart demand-responsive pricing schemes on the parking facilities, resulting in higher parking availability in overused parking zones and preventing double-parking and excessive cruising phenomena (*i.e.*, in [5]).

This paper seeks to systematically explore the impact of information that these systems make available, on the efficiency of the parking search process and resource utilization, when the parking resource allocation is not controlled by a centralized entity, *e.g.*, through a reservation mechanism. The drivers choose independently to either compete for the inexpensive but scarce on-street parking spots or head for the more expensive parking lot(s). In the first case, they run the risk of failing to get a spot and having to *a posteriori* take the more expensive alternative, this time suffering the additional *cruising* cost in terms of time, fuel consumption (and stress) of the failured attempt. Drivers make their decisions drawing on various levels of information about the parking demand (number of drivers) and perfect knowledge of the parking supply (capacity) and the applied fees on the parking facilities. The questions that arise in this respect are: How do different amounts of information on the parking demand modulate drivers' parking choices? Could this be controlled by the parking service operator to minimize the cost that drivers incur and the redundant cruising cost? How do prices charged for the two types of parking facilities modulate the information impact?

We take a game-theoretic approach and view the drivers as rational strategic selfish agents that try to minimize the cost they pay for the acquired parking space. More precisely, we assume that the decisions are made by automatic software agent implementations on-board the vehicles rather than humans and the drivers' actions fully comply with the agents' suggestions. We formulate the uncoordinated parking spot selection problem as an instance of *resource selection games* in

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Section II. The accuracy of the information that drivers possess about the parking demand is abstracted at three levels giving rise to an equal number of game variants. In Section III, we describe the game variant under complete information, which establishes a first comparison reference for the efficiency of the uncoordinated parking spot selection process. In Section IV we analyze more realistic game variants, where decisions are made under probabilistic information and full uncertainty, respectively. We first derive the equilibria of the three game variants. Then we quantify their efficiency through their Price of Anarchy values with respect to an optimal centralized system implementing parking space reservation and explore how they, hence the efficiency of the parking allocation process, depend on various factors.

The numerical results in Section V suggest that, for typical prices of the two types of parking resources, drivers tend to over-compete for the on-street parking spots giving rise to redundant cruising cost. However, there are important degrees of freedom in alleviating this inefficiency. One possibility is through the choice of charging costs for the two facilities. A second, much less intuitive, possibility is through the systematic manipulation of information that becomes available to the drivers. In this respect, counterintuitive less-is-more effects emerge, implying that the possession of less accurate information on the parking demand alleviates competition and induces equilibrium behaviors that outperform those under complete knowledge. We outline related research and position our work against it in Section VI and conclude in Section VII.

II. Modeling the parking spot selection process

In our model, drivers are faced with a decision whether to compete for the low-cost but scarce on-street parking space or directly head for the over-dimensioned but more expensive parking lots. Those who manage to park in curbside pay $c_{osp,s}$ per-time cost units, whereas those heading directly for the safer parking lot option pay $c_{pl} = \beta \cdot c_{osp,s}, \beta > 1$, units. However, drivers that decide first to search for low-cost parking spots but fail to acquire one and finally resort to a parking lot, pay $c_{osp,f} = \gamma \cdot c_{osp,s}, \gamma > \beta$, units. The excess cost $\delta \cdot c_{osp,s}$, with $\delta = \gamma - \beta > 0$, reflects the actual cost of cruising and the "virtual" cost of wasted time till eventually reaching the more expensive parking facility. Notice that the drivers' decisions are essentially made on the two *sets* of parking facilities, *i.e.*, on-street parking space *vs.* parking lots, rather than individual set items, *i.e.*, parking spots.

Under an optimal centralized parking spot allocation scheme, with R on-street parking spots and $N > R$ drivers, exactly R $(N-R)$ drivers would be directed to the low-cost (resp. more expensive) facilities and no one would pay the excess cruising cost. In the absence of central coordination, each driver acts selfishly aiming at minimizing her parking cost. However, the intuitive tendency to head for the low-cost on-street parking space, combined with its scarcity in urban center areas, give rise to *tragedy of commons* effects and highlight the game-theoretic dynamics behind the parking spot selection task. Thus, the collective decision-making on parking space selection can be formulated as an instance of *resource selection games*, whereby N players (*i.e.*, drivers/software agents) compete against each other for a finite number R of common resources (*i.e.*, curbside parking) [6]. In this game-theoretic view of the parking spot selection process, the agents are assumed to be rational strategic players. They explicitly consider the presence of identical counter-actors that also make rational decisions, weight the costs related to every possible action profile, and act as cost-minimizers. In doing so, they may or may not hold precise information about the actual competition for parking resources.

In the following sections, we analyze the parking spot selection game under different levels of uncertainty (or amounts of information) for the overall parking demand, ranging from exact knowledge to simply an upper bound on the potential competitors. In all cases, we derive the stable operational conditions and the associated costs incurred by the players, and compare them with those under optimal centralized parking spot allocation.

III. Parking spot selection under complete knowledge of parking demand

Besides the number of parking spots and the parking fees, which are assumed to be known throughout the paper, drivers are assumed to also possess perfect information about the level of parking demand, *i.e.*, the number of drivers searching for parking space. Then the one-shot parking spot selection game under complete information is defined as follows:

Definition III.1. *A* strategic Parking Spot Selection Game *is a tuple* $\Gamma(N) = (\mathcal{N}, \mathcal{R}, (A_i)_{i\in\mathcal{N}}, (w_j)_{j\in(osp,pl)})$*, where:*

- $\mathcal{N} = \{1, ..., N\}$*,* $N > 1$ *is the set of drivers who seek for parking space,*
- $\mathcal{R} = \mathcal{R}_{osp} \cup \mathcal{R}_{pl}$ *is the set of parking spots;* $\mathcal{R}_{osp}$ *is the set of on-street spots, with* $R = |\mathcal{R}_{osp}| \geq 1$*;* $\mathcal{R}_{pl}$ *is the set of spots in parking lot, with* $|\mathcal{R}_{pl}| \geq N$*,*
- $A_i = \{osp, pl\}$ *is the action set for each driver* $i \in \mathcal{N}$*, comprising of the actions "on-street" (osp) and "parking lot" (pl),*
- $w_{osp}(\cdot)$ *and* $w_{pl}(\cdot)$ *are the cost functions of the two actions, respectively*[1].

The parking spot selection game falls under the broader family of *congestion games*. The players' payoffs (here: costs) are non-decreasing functions of the *number* of players competing for the parking capacity, rather than their identities, and common to all players. More specifically, drivers who decide to compete for the curbside parking space undergo the risk of not being among the R winner-drivers to get a spot. In this case, they have to eventually resort to a parking lot, only after wasting extra time and fuel (plus patience supply) on the failed attempt. The expected cost for a driver that plays the action osp, $w_{osp} : A_1 \times ... \times A_N \rightarrow \mathbb{R}$, is therefore a function of the number of drivers k taking it, and is given by

$$w_{osp}(k) = min(1, R/k)c_{osp,s} + (1 - min(1, R/k))c_{osp,f} \quad (1)$$

[1] Note that the cost functions are defined over the action set of each user; in the original definition of resource selection games in [6], cost functions are defined over the resources but the resource set coincides with the action set.

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On the other hand, the cost for those that head directly to the parking lot facilities is fixed

$$w_{pl}(k) = c_{pl} = \beta \cdot c_{osp,s} \tag{2}$$

We denote an action *profile* by the vector $a = (a_i, a_{-i}) \in \times_{k=1}^{N} A_k$, where a_{-i} denotes the actions of all other drivers but player i in the profile a. Besides the two *pure* actions reflecting the pursuit of parking spots in curbside and in parking lots, the drivers may also randomize over them. In particular, if $\Delta(A_i)$ is the set of probability distributions over the action set of player i, a player's *mixed action* corresponds to a vector $p = (p_{osp}, p_{pl}) \in \Delta(A_i)$, where p_{osp} and p_{pl} are the probabilities of the two pure actions, with $p_{osp}+p_{pl} = 1$, while its cost is a weighted sum of the cost functions $w_{osp}(\cdot)$ and $w_{pl}(\cdot)$ of the pure actions. We draw on concepts from [7] and theoretical results from [6], [8] to derive the equilibrium strategies for the game $\Gamma(N)$ and assess their (in)efficiency.

A. Pure equilibrium strategies

Existence: The parking spot selection game constitutes a symmetric game, where the action set is common to all players and consists of two possible actions, *osp* and *pl*. Cheng *et al.* have shown in ([8], Theorem 1) that every symmetric congestion game with two strategies has an equilibrium in pure strategies.

Derivation: Due to the game's symmetry, the full set of 2^N different action profiles maps into $N+1$ different action *meta-profiles*. Each meta-profile $a(m), m \in [0, N]$ encompasses all $\binom{N}{m}$ different action profiles that correspond to the same number of drivers competing for on-street parking space. The expected costs for these m drivers and for the $N - m$ ones choosing directly the parking lot alternative are functions of $a(m)$ rather than the exact action profile. In general, the cost for driver i under the action profile $a = (a_i, a_{-i})$ is

$$c_i^N(a_i, a_{-i}) = \begin{cases} w_{osp}(N_{osp}(a)), & \text{for } a_i = osp \\ w_{pl}(N - N_{osp}(a)), & \text{for } a_i = pl \end{cases} \tag{3}$$

where $N_{osp}(a)$ is the number of competing drivers for on-street parking under action profile a. Equilibrium action profiles combine the players' *best-responses* to their opponents' actions. Formally, an action profile $a = (a_i, a_{-i})$ is a pure Nash equilibrium if for all $i \in \mathcal{N}$, it holds that $a_i \in \arg\min_{a'_i \in A_i}(c_i^N(a'_i, a_{-i}))$, so that no player has anything to gain by changing her decision unilaterally. Therefore, to derive the equilibrium states, we determine the conditions on N_{osp} that break the equilibrium definition and reverse them. More specifically, given an action profile a with $N_{osp}(a)$ competing drivers, a player gains by changing her decision to play action a_i in two circumstances:

$$when\ a_i = pl \text{ and } w_{osp}(N_{osp}(a) + 1) < c_{pl} \tag{4}$$

$$when\ a_i = osp \text{ and } w_{osp}(N_{osp}(a)) > c_{pl} \tag{5}$$

Lemma III.1. *In $\Gamma(N)$, a driver is motivated to change her action a_i in the following circumstances:*

- $a_i = pl$ *and (a)* $N_{osp}(a) < R \leq N$ *or*

  (b) $R \leq N_{osp}(a) < N_0 - 1 \leq N$ *or*

  (c) $N_{osp}(a) < N \leq R$ (6)

- $a_i = osp$ *and* $R < N_0 < N_{osp}(a) \leq N$ (7)

where $N_0 = \frac{R(\gamma-1)}{\delta} \in \mathbb{R}$.

Proof. Conditions (6a) and (6c) are trivial. Since the current number of competing vehicles is less than the on-street parking capacity, every driver having originally chosen the parking lot option has the incentive to change her decision due to the price differential between $c_{osp,s}$ and c_{pl}. When $N_{osp}(a)$ exceeds the curbside parking supply, a driver who has decided to avoid competition, profits from switching her action when (4) holds, which combined with (1) yields (6b). Similarly, a driver that first decides to compete, profits by switching her action if (5) holds, which combined with (1) yields (7). □

Theorem III.1. *The game $\Gamma(N)$ has:*

(a) *for $N \leq N_0$, a unique Nash equilibrium profile a^* with $N_{osp}(a^*) = N_{osp}^{NE,1} = N$*

(b.1) *for $N > N_0$ and $N_0 \in (R, N)\backslash\mathbb{N}^*$, $\binom{N}{\lfloor N_0 \rfloor}$ Nash equilibrium profiles a' with $N_{osp}(a') = N_{osp}^{NE,2} = \lfloor N_0 \rfloor$*

(b.2) *for $N > N_0$ and $N_0 \in [R+1, N] \cap \mathbb{N}^*$, $\binom{N}{N_0}$ Nash equilibrium profiles a' with $N_{osp}(a') = N_{osp}^{NE,2} = N_0$ and $\binom{N}{N_0-1}$ Nash equilibrium profiles $a^\star$ with $N_{osp}(a^\star) = N_{osp}^{NE,3} = N_0 - 1$.*

Proof. Theorem III.1 follows directly from Lemma III.1. The equilibrium states satisfy both the conditions $N_{osp} \geq N_0 - 1$ and $N_{osp} \leq N_0$. Thus, the game has two equilibrium states on N_{osp} for $N > N_0$ with integer N_0 (case b.2), or a unique state, otherwise (cases a, b.1). □

In [9], we describe an alternative way to derive the equilibria of $\Gamma(N)$ via potential functions.

Efficiency: The efficiency of the equilibria is assessed through the broadly used metric of the Price of Anarchy (PoA) [7]. It expresses the ratio of the social cost in the worst-case equilibria over the optimal social cost under ideal coordination of the drivers' strategies.

Proposition III.1. *In $\Gamma(N)$, the pure PoA equals:*

$$\mathtt{PoA} = \begin{cases} \frac{\gamma N - (\gamma-1)\min(N,R)}{\min(N,R)+\beta\max(0,N-R)}, & if\ N_0 \geq N \\ \\ \frac{\lfloor N_0 \rfloor \delta - R(\gamma-1)+\beta N}{R+\beta(N-R)}, & if\ N_0 < N \end{cases}$$

Proof. The social cost under action profile a equals:

$$C(N_{osp}(a)) = \sum_{i=1}^{N} c_i^N(a) = c_{osp,s}(N\beta - N_{osp}(a)(\beta - 1)) \tag{8}$$

if $N_{osp}(a) \leq R$ and

$$C(N_{osp}(a)) = c_{osp,s}(N_{osp}(a)\delta - R(\gamma - 1) + \beta N) \tag{9}$$

if $R < N_{osp}(a) \leq N$. The numerators of the two ratios are obtained directly by replacing the first two N_{osp}^{NE} values (a) and (b1) (worst-cases) computed in Theorem III.1. On the other hand, under the socially optimal action profile a_{opt}, exactly R drivers pursue on-street parking space, and, hence, no drivers have to pay the additional cruising cost. The optimal social cost, C_{opt} is given by:

$$C_{opt} = \sum_{i=1}^{N} c_i^N(a_{opt}) = c_{osp,s}[min(N, R) + \beta \cdot max(0, N - R)]$$

□

Proposition III.2. *In $\Gamma(N)$, the pure PoA is upper-bounded by $\frac{1}{1-R/N}$ with $N > R$.*

Proof. The condition is obtained directly from Proposition III.1, when $N > R$. □

B. Mixed-action equilibrium strategies

We consider *symmetric* mixed-action equilibria since these can be more helpful in dictating practical strategies in real systems (asymmetric mixed-action equilibria are discussed at the end of the section).

Existence: In ([6], Theorem 1) it is proved that a unique symmetric mixed equilibrium exists for the broader family of resource selection games with more than two players and increasing cost functions. This is easily shown to hold for the game $\Gamma(N)$, with $N > R$ and cost functions $w_{osp}(\cdot)$ and $w_{pl}(\cdot)$ that are non-decreasing functions of the number of players.

Derivation: The expected costs of choosing parking spots in curbside and in parking lot, when all other drivers play the mixed-action $p = (p_{osp}, p_{pl})$, are given by $c_i^N(pl, p) = c_{pl}$ and

$$c_i^N(osp, p) = \sum_{N_{osp}=0}^{N-1} w_{osp}(N_{osp}+1)B(N_{osp}; N-1, p_{osp})$$

where $B(N_{osp}; N-1, p_{osp})$ is the Binomial probability distribution with parameters $N-1$ and p_{osp}, for N_{osp} drivers choosing curbside parking. The cost of the symmetric profile where everyone plays the mixed-action p is given by

$$c_i^N(p, p) = p_{osp} \cdot c_i^N(osp, p) + p_{pl} \cdot c_i^N(pl, p) \qquad (10)$$

We can now postulate the following Theorem, whose proof is given in [9].

Theorem III.2. *The game $\Gamma(N)$ has a unique symmetric mixed-action Nash equilibrium $p^{NE} = (p_{osp}^{NE}, p_{pl}^{NE})$, where $p_{osp}^{NE} = 1$, if $N \leq N_0$ and $p_{osp}^{NE} = \frac{N_0}{N}$, if $N > N_0$, with $p_{osp}^{NE} + p_{pl}^{NE} = 1$ and $N_0 \in \mathbb{R}$.*

Asymmetric mixed-action equilibria: In Section III-A, we showed that there may exist multiple asymmetric pure equilibria, when the number of drivers exceeds N_0. In general, the derivation of results for asymmetric mixed-action equilibria is much harder than for either their pure or their symmetric counterparts since the search space is much larger. Moreover, asymmetric mixed-action equilibria have two more undesirable properties: a) they do not treat all players equally, *i.e.*, different players end up with *a-priori* worse chances to come up with an inexpensive parking spot; b) their realization in practical situations is much more difficult than that of their symmetric counterparts. Therefore, we focus our analysis and discussion on symmetric equilibria and their (in)efficiency.

IV. Parking spot selection under incomplete knowledge of parking demand

The availability of complete information about the drivers' (*i.e.*, players') population is a fairly strong and unrealistic assumption. In this section we relax it and study two game variants under incomplete demand information, where the players either share common probabilistic information about the overall demand or are totally uncertain about it. Note that the parking service operator, depending on the network and sensing infrastructure at her disposal, may provide the competing drivers with different amounts of information about the parking demand (*e.g.*, based on historical statistical data). However, again, drivers are assumed to have perfect knowledge of the parking supply; this is fairly realistic and feasible since the sensing of the parking space and the broadcasting of the collected data can be viewed as less complex tasks.

A. Probabilistic knowledge of parking demand

In the *Bayesian* model of the game, the drivers determine their actions on the basis of private information, their *types*. The type in this game is a binary variable indicating whether a driver is in search of parking space (*active* player) or not. Every driver knows her own type along with the strategy space and the cost functions, and draws on common *prior* probabilistic information about the types of other drivers to estimate the expected cost of her actions. Formally, the Bayesian parking spot selection game is defined as follows:

Definition IV.1. *A* Bayesian Parking Spot Selection Game *is a tuple $\Gamma_B(N) = (\mathcal{N}, \mathcal{R}, (A_i)_{i\in\mathcal{N}}, (w_j)_{j\in(osp,pl)}, (\Theta_i)_{i\in\mathcal{N}}, f_\Theta)$, where $\mathcal{N}$ and $\mathcal{R}$ are as defined for $\Gamma(N)$, and*

- *$A_i = \{osp, pl, \oslash\}$, the set of potential actions for each driver $i \in \mathcal{N}$,*
- *$\Theta_i = \{0, 1\}$, the set of types for each driver $i \in \mathcal{N}$, where 1 (0) stands for active (inactive) drivers,*
- *$S_i : \Theta_i \to A_i$, the set of possible strategies for each driver $i \in \mathcal{N}$,*
- *$c_i^{N_B}(s(\vartheta), \vartheta)$, the cost functions for each driver $i \in \mathcal{N}$, for every type profile $\vartheta \in \times_{k=1}^N \Theta_k$ and strategy profile $s(\vartheta) \in \times_{k=1}^N S_k$, that are functions of $w_{osp}(\cdot)$ and $w_{pl}(\cdot)$ as defined for $\Gamma(N)$, and also written as $c_i^{N_B}(s(\vartheta), \vartheta) = c_i^{N_B}(s_i(\vartheta_i), s_{-i}(\vartheta_{-i}), \vartheta_i, \vartheta_{-i})$,*
- *p_{act} is the probability for a driver to be active.*

In $\Gamma_B(N)$, for all inactive drivers i, $s_i(\vartheta_i = 0) = \oslash$. For active players i, $s_i(\vartheta_i = 1) \in \{osp, pl\}$, under pure-action strategy, or $s_i(\vartheta_i = 1) \in \Delta(\{osp, pl\})$, when they randomize over this subset of A_i under mixed-action strategy. The game is symmetric when, besides the action set, drivers share the same activity probability, p_{act} and hence, the same prior joint probability distribution of the drivers' activity (types), f_Θ. The number of active players upon each time depends on their types and is given by $n_{act} = \sum_k \vartheta_k$. The action profile is the effect of players' strategies on their types and is noted as $a = (s(\vartheta), \vartheta) \in \times_{k=1}^N A_k$.

Equilibria: For the game $\Gamma_B(N)$, the strategy profile $s' \in \times_{k=1}^N S_k(\vartheta_k = 1)$ is a Bayesian Nash equilibrium if for all $i \in \mathcal{N}$ with $\vartheta_i = 1$:

$$s_i(\vartheta_i) \in \arg\min_{s_i' \in S_i} c_i^{N_B}(s_i(\vartheta_i), s_{-i}(\vartheta_{-i}), \vartheta_i, \vartheta_{-i}) \qquad \text{or,}$$

$$s_i(\vartheta_i) \in \arg\min_{s_i' \in S_i} \sum_{\vartheta_{-i}} f_\Theta(\vartheta_{-i}/\vartheta_i) c_i^{\sum_k \vartheta_k}(s_i', s_{-i}(\vartheta_{-i}), \vartheta_i, \vartheta_{-i})$$

where $c_i^{\sum_k \vartheta_k}(s_i', s_{-i}(\vartheta_{-i}), \vartheta_i, \vartheta_{-i})$, with $s_l(\vartheta_l = 0) = pl$, $\forall l \neq i$, is the cost $c_i^m(s_i', s_{-i})$ of driver i under profile s in the game $\Gamma(m)$ with $m = \sum_k \vartheta_k$ drivers, and $f_\Theta(\vartheta_{-i}/\vartheta_i)$

the posterior conditional probability of the active drivers *given that* user i is active, as derived from the application of the Bayesian rule. Therefore, s' minimizes the expected cost over all possible combinations of the other drivers' types and strategies so that no active player can further lower its expected cost by unilaterally changing her strategy.

Theorem IV.1. *The game $\Gamma_B(N)$ has unique symmetric equilibrium profiles $p^{NE_B} = (p_{osp}^{NE_B}, p_{pl}^{NE_B})$, with $p_{osp}^{NE_B} + p_{pl}^{NE_B} = 1$. More specifically, with $N_0 \in \mathbb{R}$:*

- *a unique pure (Bayesian Nash) equilibrium with $p_{osp}^{NE_B} = 1$, if $p_{act} < \frac{N_0}{N}$,*
- *a unique symmetric mixed-action Bayesian Nash equilibrium with $p_{osp}^{NE_B} = \frac{N_0}{N p_{act}}$, if $p_{act} \geq \min(\frac{N_0}{N}, 1)$.*

Proof. We present the proof in [9]. □

B. Strictly incomplete information about parking demand

The worst-case scenario with respect to the information drivers possess is represented by the *pre-Bayesian* game variant, under which the drivers are aware of only the upper limit of the vehicles that are *potential* competitors for parking resources.

Pre-Bayesian games do not necessarily have *ex-post* Nash equilibria, even in mixed actions. The ex-post Nash equilibrium consists of strategies that, for every joint type profile, result in actions which are in Nash equilibrium in the corresponding strategic game. On the other hand, all quasi-concave pre-Bayesian games *do* have at least one mixed-strategy *safety-level equilibrium* [6]. In the safety-level equilibrium, every player minimizes over her strategy set S_i the worst-case (maximum) cost she may suffer over all possible types and actions of her competitors (S_{-i}, Θ_{-i}). The result of interest for our pre-Bayesian variant of the parking spot selection model $\Gamma_{pB}(N)$ is the following Proposition, due to [6], whose implications for the efficiency of the equilibrium behaviors of the drivers are discussed in Section V.

Proposition IV.1. *An action profile a is the unique symmetric mixed-action safety-level equilibrium of the pre-Bayesian parking spot selection game, $\Gamma_{pB}(N)$, with non-decreasing resource cost functions, iff a is the unique symmetric mixed-action equilibrium of the respective strategic game with deterministic knowledge of the number of players, $\Gamma(N)$.*

V. Numerical results

In this section, we first systematically study the efficiency of the parking search process when the parking assistance systems provide information of perfect accuracy about the demand. Then, we comparatively discuss how this efficiency is affected when the process is executed under probabilistic information and uncertainty. For the numerical results we adopt per-time unit normalized cost values used in typical municipal parking systems in big European cities [10]. The parking fee for on-street space is set to $c_{osp,s} = 1$ unit whereas the cost of parking lots β ranges in $(1, 16]$ units and the excess cruising cost parameter δ is let vary within $[1, 5]$ units.

A. Parking search under complete information

1) Impact of parking demand and on-street parking supply: An optimal (centralized) mechanism would assign exactly $\min(N, R)$ on-street parking spots to $\min(N, R)$ drivers. If $N \leq R$, in the absence of (central) coordination, all drivers go for the on-street parking space and, trivially, $\texttt{PoA} = 1$. Hereafter, we focus on the more interesting case $N > R$, where a number of drivers end up paying the extra cruising cost $\delta c_{osp,s}$ (see Lemma III.1, Theorem III.1). Under a fixed pricing scheme, this inefficiency depends on N and R. In Figure 1, we plot the $\texttt{PoA}$ against N and R ranging in $[55, 195]$ and $[10, 50]$, respectively. The following remarks suggest joint conditions on N and R that result in more efficient parking search, namely:

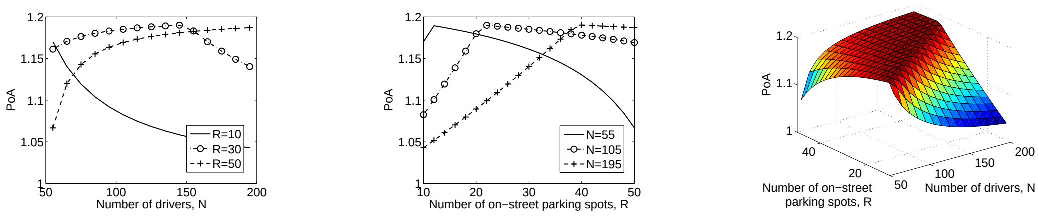
Varying N or R: For $N \leq N_0$ or, equivalently, for $R \geq \frac{N\delta}{\gamma-1}$, it holds that $\frac{\vartheta PoA}{\vartheta N} > 0$ and $\frac{\vartheta PoA}{\vartheta R} < 0$. Therefore, the $\texttt{PoA}$ is strictly increasing in N and decreasing in R. On the contrary, for $N > N_0$ or $R < \frac{N\delta}{\gamma-1}$, the $\texttt{PoA}$ is strictly decreasing in N and increasing in R, since $\frac{\vartheta PoA}{\vartheta N} < 0$ and $\frac{\vartheta PoA}{\vartheta R} > 0$.

When all drivers choose to compete, that is, if $N \leq N_0$ or $R \geq \frac{N\delta}{\gamma-1}$, exactly R drivers pay $c_{osp,s}$ while the rest of them, *i.e.*, $N - R$ drivers, pay $\gamma c_{osp,s}$. Thus, under a fixed pricing scheme, the social cost is optimized when the maximum charging cost ($\gamma c_{osp,s}$) is incurred by the minimum possible set of drivers, namely when the parking demand is the lowest possible one ($N = R + 1$) or, equivalently, as R is increased so that only one driver fails the competition for the low-cost parking spots. On the other hand, when $N > N_0$ or $R < \frac{N\delta}{\gamma-1}$, R drivers pay $c_{osp,s}$, $N - N_0$ drivers pay $\beta c_{osp,s}$ and $N_0 - R$ drivers pay $\gamma c_{osp,s}$. Under the optimal operation of the service, the latter two sets of drivers head directly for space in parking lot. Thus, the efficiency of the uncoordinated parking search is improved as the parking demand increases, making the total cost paid by the $N - N_0$ drivers the most critical factor for the overall social cost and hence, minimizing the impact of the total cost paid by the $N_0 - R$ drivers due to the lack of coordination. Equivalently, the set of $N_0 - R = \frac{R(\beta-1)}{\delta}$ drivers that fail the competition, is minimized when the on-street parking capacity becomes the lowest possible one, *i.e.*, $R = 1$.

On the other hand, the extra cruising cost δ may change as the result of *e.g.*, an addition of a parking lot closer to the search area or a change in driving conditions. Figure 2 displays the $\texttt{PoA}$ against δ and suggests the following trends:

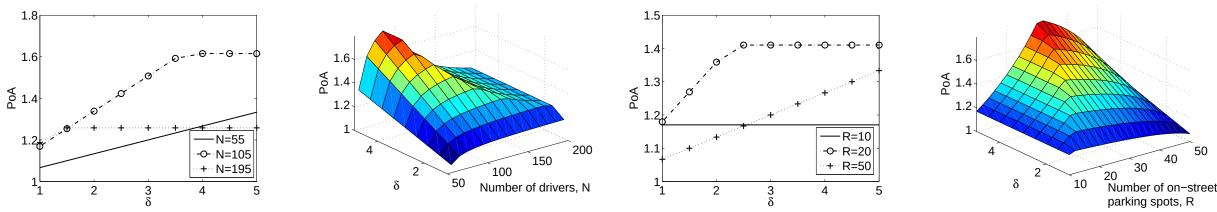
Varying δ: For $N \leq N_0$ or, equivalently, for $\delta \leq \frac{R(\beta-1)}{N-R}$, it holds that $\frac{\vartheta PoA}{\vartheta \delta} > 0$. Therefore, the $\texttt{PoA}$ is strictly increasing in δ. For $\delta > \frac{R(\beta-1)}{N-R}$, we get $\frac{\vartheta PoA}{\vartheta \delta} = 0$. Hence, if δ exceeds $\frac{R(\beta-1)}{N-R}$, $\texttt{PoA}$ is insensitive to changes of the excess cost δ.

For given charging costs and on-street parking capacity, the construction of expensive parking lots in the proximity of the on-street parking area does not work effectively, when the competition is high (see Fig. 2a, 2b for high N values and Fig. 2c, 2d for low R values). Otherwise, under medium or low competition, there is a monotonic trend that suggests, if possible, to decrease the distance between the two options in order to increase the efficiency of the parking search process. Overall, changes in this distance and hence, the cruising cost,



a. $2D$ PoA(N) b. $2D$ PoA(R) c. $3D$ PoA(N, R)

Fig. 1. Price of Anarchy as a function of the parking demand and supply, under fixed pricing scheme $\beta = 5, \delta = 1$.



a. $2D$ PoA(N, δ), $R = 50$ b. $3D$ PoA(N, δ), $R = 50$ c. $2D$ PoA(R, δ), $N = 55$ d. $3D$ PoA(R, δ), $N = 55$

Fig. 2. Price of Anarchy as a function of the parking demand and supply, under variable cruising cost δ and fixed parking fee $\beta = 5$.

are meaningless for high δ values, over $\frac{R(\beta-1)}{N-R}$. In this case, when $N > N_0$, $N_0 - R = \frac{R(\beta-1)}{\delta}$ drivers pay the extra cruising cost and end up in a parking lot together with the $N - N_0$ drivers that head directly for this kind of parking space. Thus, the increase of cruising cost has a double-edge effect. On the one hand, drivers are discouraged from competing so that fewer end up paying the cruising overhead. On the other hand, failing the competition for on-street parking costs more. In addition, the total number of drivers that incur the more (less) expensive parking fee is $N - R$ (R), irrespective of the exact δ value. As a result, changes in δ do not affect either the total cost spent for space in the on-street or parking lot facilities, or the aggregate cruising overhead. Thus, the social cost can be decreased by locating a parking lot in the proximity of the on-street parking area so that the additional travel distance is reduced to the point of bringing the excess cost δ below $\frac{R(\beta-1)}{N-R}$.

Although low PoA values denote high efficiency in the parking search process, they are not always coupled with low *absolute* social costs. For instance, this may happen under very intense competition, namely, under high parking demand for very low curbside capacity (*i.e.*, see Fig. 1 at $N = 195$, $R = 10$). In the following section, we study the sensitivity of the social cost to the parking demand and supply as well as the prices charged for the two types of parking facilities.

2) Impact of pricing scheme: Figure 3 plots the social costs $C(N_{osp})$ under pure (Eq. 8, 9) and $C(p_{osp})$ under mixed-action strategies as a function of the number of competing drivers N_{osp} and competition probability p_{osp}, respectively, where

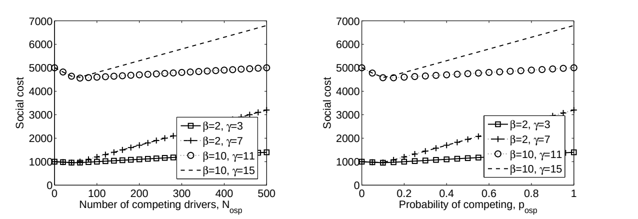
$$C(p) = c_{osp,s} \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} \cdot [min(n, R) + max(0, n-R)\gamma + (N-n)\beta] \quad (11)$$

Figure 3 motivates two remarks. First, the social cost curves for pure and mixed-action profiles have the same shape. This comes as no surprise since for given N, any value for the expected number of competing players $0 \leq N_{osp} \leq N$ can be realized through an appropriate choice of the symmetric mixed-action profile p. Second, the cost is minimized when the number of competing drivers is equal to the number of on-street parking spots. The cost rises when either the competition exceeds the available on-street parking capacity or the drivers are overconservative in (and refrain more than they should from) competing for on-street parking. In both cases, the drivers pay the penalty for the lack of coordination in their decisions. The deviation from optimal grows faster with increasing price differential between the on-street spots and the space in parking lot (*i.e.*, β) or the distance between the on-street and parking lot facilities (*i.e.*, δ).

If $N > R$, in the worst-case equilibrium (*i.e.*, the equilibrium state with the maximum number of competing drivers and hence, the maximum social cost, among all equilibria) the number of drivers that actually compete for on-street parking spots exceeds the real curbside parking capacity by a factor which is a function of β and γ (equivalently, δ) (see Lemma III.1, Theorem III.1). This inefficiency is captured in the PoA plots in Figures 4a, 4b for β and δ ranging in $[1.1, 16]$ and $[1, 5]$, respectively. The plots illustrate the following trends:

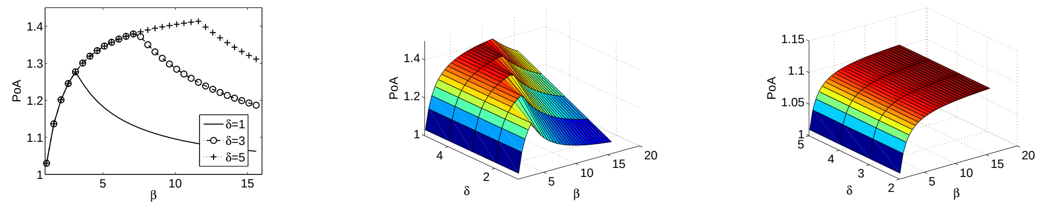
Varying β**:** For $N \leq N_0$ or, equivalently, for $\beta \geq \frac{\delta(N-R)+R}{R}$, it holds that $\frac{\vartheta PoA}{\vartheta \beta} < 0$ and therefore, the PoA is strictly decreasing in β. On the contrary, for $\beta < \frac{\delta(N-R)+R}{R}$, the PoA is strictly increasing in β, since $\frac{\vartheta PoA}{\vartheta \beta} > 0$.

Practically, the equilibrium strategy emerging from this kind of assisted parking search behavior can approximate the optimal coordinated mechanism, provided that the operation of parking lots properly accounts for the drivers' preferences as well as estimates of the typical parking demand and supply. More specifically, if, as part of the pricing policy, the fee of parking lot is less than $\frac{\delta(N-R)+R}{R}$ times the cost of on-street



a. Pure-action profiles b. Symmetric mixed-action profiles

Fig. 3. Social cost for $N = 500$ drivers when N_{osp} drivers compete (a) or when all drivers decide to compete with probability p_{osp} (b), for $R = 50$.



a. $2D$ $\texttt{PoA}(\beta)$, $R = 160$ b. $3D$ $\texttt{PoA}(\beta, \delta)$, $R = 160$ c. $3D$ $\texttt{PoA}(\beta, \delta)$, $R = 50$

Fig. 4. Price of Anarchy for $N = 500$ and varying R, under different pricing schemes.

parking, then the social cost in the equilibrium profile approximates the optimal social cost as the price differential between on-street and parking lot decreases. This result is inline with the statement in [11], arguing that "*price differentials between on-street and off-street parking should be reduced in order to reduce traffic congestion*".

Note that the $\texttt{PoA}$ metric also decreases monotonically for high values of the parking lot fee, specifically when the parking operator desires to gain more than $\frac{\delta(N-R)+R}{R}$ times the cost of on-street parking, towards a bound that depends on the excess cost δ. Nevertheless, these operating points correspond to high absolute social cost, *i.e.*, the minimum achievable social cost is already unfavorable due to the high fee paid by the $N - R$ drivers that use space in parking lots (see Fig. 3, large β). However, there are instances, as in the case of $R = 50$ (see Fig. 4c), where the value $\frac{\delta(N-R)+R}{R}$ corresponds to a non-realistic (too large) option for the cost of the space in parking lots, already for $\delta > 1$. Thus, contrary to the previous case, the $\texttt{PoA}$ only improves as the cost for parking lot decreases.

B. Parking search under incomplete information

Looking at the mixed-action equilibria, Theorem III.2 indicates that drivers' intention to compete for on-street parking resources is shaped by the pricing schemes, the number of players and the curbside parking capacity. Indeed, players start to withdraw from competition as competition intensity rises over the threshold $N_0 = \frac{R(\gamma-1)}{\delta}$. For the Bayesian implementation, the rationale behind the active players' behavior is almost the same. The only difference is that the players adjust their strategies, based on estimations for the demand level as expressed in the commonly known probabilistic information of competition. Therefore, the probability to compete decreases with the expected number of competitors Np_{act}, if this number exceeds the threshold N_0 of the strategic games (see Theorem IV.1). Furthermore, for both game formulations, players start to renege from competition as the distance between on-street and parking lot facilities (*i.e.*, δ) is increased or the number of opportunities for curbside parking (*i.e.*, R) decreases or the price for space reservation in parking lot (*i.e.*, β) drops. Figure 5 depicts the effect of these parameters on the equilibrium mixed-action, for strategic ($p_{act} = 1$) and Bayesian ($p_{act} \in \{0.5, 0.7\}$) games.

Less-is-more phenomena under uncertainty: Less intuitive are the game dynamics in its pre-Bayesian variant, when users only possess an estimate of the maximum number of drivers that are *potentially* interested in parking space. From Proposition IV.1, the mixed-action safety-level equilibrium corresponds to the mixed-action equilibrium of the strategic game $\Gamma(N)$. However, we have seen that, when the players outnumber the on-street parking capacity: a) the mixed-action equilibrium in the strategic game generates higher expected number of competitors than the optimal value R (see Theorem III.2); b) the social cost conditionally increases with the probability of competing (see Fig. 3b, for $p_{osp} > \frac{R}{N}$); c) the probability of competition decreases with N (see Fig. 5, for $N > N_0$). Therefore, at the safety-level equilibrium of the game, the drivers end up randomizing the pure action "on-street" with a lower probability than that corresponding to the game they actually play, with $k \leq N$ players. Hence, the resulting number of competing vehicles is smaller and, cumulatively, they may end up paying less than they would if they knew deterministically the competition they face.

One question that becomes relevant is for which (real) number K of competing players do the drivers end up paying the *optimal* cost. Practically, if $p_N^{NE} = (p_{osp,N}^{NE}, p_{pl,N}^{NE})$ denotes the symmetric mixed-action equilibrium for $\Gamma(N)$, we are looking for the value of K satisfying:

$$Kp_{osp,N} = R \Rightarrow K = \frac{RN}{N_0} = \frac{\delta}{\gamma - 1}N$$

namely, when $\frac{\delta}{\gamma-1}N$ (rounded to the nearest integer) drivers are seeking for parking space under uncertainty conditions, in the induced equilibrium they end up paying the minimum possible cost, which is better than what they would pay under complete information about the parking demand.

VI. Related work

Most work on parking assistance systems initially focused on *centralized* parking (reservation) mechanisms [12] [1] [13] [14] [5] [2]. For a comparative description of these systems and a broader summary on work on the parking problem the interested reader is referred to [15]. Work on opportunistic parking search assistance, on the other hand, where information about the location and vacancy of parking spots is opportunistically disseminated among vehicles, is rarer and more recent. In [3], vehicles are allowed to exchange aggregate availability information of variable accuracy about clusters of parking places covering large regions, in an effort to limit the volume of the disseminated information for the sake of

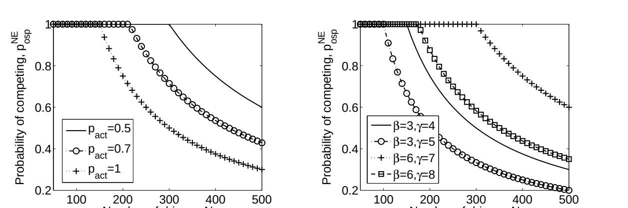


Fig. 5. Probability of competing in equilibrium, for $R = 50$. Left: Strategic and Bayesian games under fixed pricing scheme $\beta = 5, \gamma = 7$. Right: Strategic games under various pricing schemes $\beta \in \{3, 6\}, \gamma \in \{4, 5, 7, 8\}$.

scalability. The way the opportunistic exchange of information among vehicles may sharpen competition for parking space is treated in [16] and [4]. In [16], Kokolaki *et al.* simulate a fully cooperative opportunistic parking space assistance scheme and show that the full exchange of information may give rise to synchronization effects (vehicles are steered towards similar locations), sharpen competition, and eventually render the search process inefficient. Motivated by similar findings, Delot *et al.* propose in [4] a distributed virtual parking space reservation mechanism, whereby vehicles vacating a parking spot selectively distribute this information to their proximity. Hence, they mitigate the competition for the scarce parking spots by opportunistically controlling the diffusion of the parking information. Interestingly, the systems in [17] and [18] realize almost the same idea for parking management in the cities of Athens (Greece) and New York, respectively. Both applications leverage the social network element: users can offer their parking spot to the rest of the users or find a parking spot for themselves by claiming a spot another user is offering. A rating mechanism on drivers' sharing and reserving habits, shapes parking spot seekers' likelihood to be chosen by a parking spot sharer (defender) in [17] or get informed about a vacancy prior to other seekers in [18].

Pricing and the more general economic dimensions of the parking allocation problem are analyzed from a microeconomical point of view in [19]. Anderson and de Palma view the parking spots as common property resource and question whether free access or some pricing structure result in more efficient use of the parking capacity. Working on a simple model of city and parking spot distribution, they show that this use is more efficient (in fact, optimal) when the spots are charged with the fee chosen in the monopolistically competitive equilibrium under private ownership; whereas drivers are better off when access to the parking spots is free of charge. Subsequent research contributions have explicitly catered for strategic behavior and the related game-theoretic dimensions of general parking applications. In [20], the games are played among parking facility providers and concern the location and capacity of their parking facility as well as which pricing structure to adopt. Whereas, in the two other works, the strategic players are the drivers. In [21], which seeks to provide cues for optimal parking lot size dimensioning, the drivers decide on the arriving time at the lot, accounting for their preferred time as well as their desire to secure a space. In a work more relevant to ours, Ayala *et al.* in [22] define a game setting where drivers exploit (or not) information on the location of others to occupy an available parking spot at the minimum possible travelled distance, irrespective of the distance between the spot and driver's actual travel destination. The authors present distributed parking spot assignment algorithms to realize or approximate the Nash equilibrium states.

Our work also draws on game theoretic analysis. Similarly to [21] and [22], the decision-maker (player) is the driver; and as in [16] and [4], we are particularly concerned with a broader phenomenon, evidenced in several instances of information provision within non-cooperative environments: the double-edged impact of information dissemination on the overall process efficiency, *i.e.*, its assistance with resource/service discovery against the sharpening of competition for its usage. On the other hand, contrary to [19], [21] and [22], we explicitly discriminate between on-street parking spots and parking lots as two types of resources drivers choose among. We argue that this dilemma between searching for cheaper yet non-guaranteed on-street parking space and heading directly for the granted yet costlier parking lot(s) is a frequently recurring situation in real urban every day life; and we set our focus on the impact of information availability and accuracy on the efficiency of these decisions. Moreover, rather than considering a particular system or algorithm, as [1] – [5], [12] – [14], [17] and [18] do, we leverage game-theoretic abstractions to capture the different levels of information accuracy. This way, we can derive *closed-form expressions* for the stable operational points in the different settings and insightful counterintuitive results about the impact of information on the efficiency of the parking search process.

VII. Conclusions

In this paper, we seek to assess the ultimate impact that different types of parking assistance systems, collecting and sharing information of variable accuracy on parking demand, can have on the parking space selection process. To this end, we formulate the information-assisted parking search process as an instance of resource selection games with three game variants (strategic, Bayesian, and pre-Bayesian) providing normative prescriptions for the impact of the information factor on drivers's decisions. Essentially, this work derives some bounds on what may be achievable by *fully rational*, strategic agents that aim at minimizing the cost of their decisions. Our results describe how different amounts of information for the parking demand steer the equilibrium strategies, reduce the inefficiency of the parking search process, and favor the social welfare. Actually, the dissemination of parking information constitutes an instance of service provision within competitive networking environments, where more information does not necessarily improve the efficiency of service delivery but, even worse, may hamstring users' efforts to maximize their benefit. This result, has direct practical implications since it challenges the need for more elaborate information mechanisms and promotes certain policies for information dissemination on the service provider side.

In the remainder of this section, we iterate on two implicit assumptions behind the game models we introduced in Sections III and IV, which can motivate further research work.

Drivers' indifference among individual parking spots: The formulation of the parking spot selection game assumes that drivers do not have any preference order over the R on-street parking spots. This could be the case when these R spots are quite close to each other, resulting in practically similar driving times to them and walking times from them to the drivers' ultimate destinations. When drivers express preferences over different parking spots, we come up with an instance of the *stable marriage problem*, potentially with indifference [23], whereby the option of parking lot would commonly rank as the last one for all drivers. At a theoretical level, the search is for mechanisms that treat all drivers fairly, are strategy-proof, *i.e.*, the drivers are motivated to advertise their true preference orders because they cannot gain by lying about them, and efficient in some Pareto-optimality sense.

Drivers' rationality: Full (or global) rationality demands an exhaustively analysis of the possible strategies available to decision-makers and realization of the best-response actions. This may be feasible when decisions are made by well-programmed automated software agents on-board the vehicles. However, when humans are actively involved in the decision-making process the assumption of full rationality becomes much more problematic. Indeed, Simon, already more than half a century ago [24], challenged both the *normative* and *descriptive* capacity of the fully rational decision-maker, arguing that human decisions, are most often made under knowledge, time and computational constraints. One way to accommodate the first constraints is through *(pre-)Bayesian* games of incomplete information; whereas the latter ones are well expressed in alternative solution concepts to the Nash Equilibrium [25] [26], arguing that "individuals are more likely to select better choices than worse choices, but do not necessarily succeed in selecting the very best choice". However, models that completely depart from the utility-maximization norm and draw on fairly simple *cognitive heuristics*, *e.g.*, [27], reflect better Simon's argument that humans are satisficers rather than maximizers. For example, the authors in [28] explore the impact of the *fixed-distance* heuristic on a simpler version of the unassisted parking search problem. The comparison of normative and more descriptive decision-making modeling approaches both in the context of the parking spot selection problem and more general decision-making contexts, is an interesting area worth of further exploration.

References

[1] S. Mathur, T. Jin, N. Kasturirangan, J. Chandrasekaran, W. Xue, M. Gruteser, and W. Trappe, "Parknet: Drive-by sensing of road-side parking statistics," in *Proc. 8th ACM MobiSys*, USA, 2010.

[2] H. Wang and W. He, "A reservation-based smart parking system," in *Proc. 5th International Workshop on Cyber-Physical Networking Systems (colocated with IEEE INFOCOM)*, April 2011.

[3] M. Caliskan, D. Graupner, and M. Mauve, "Decentralized discovery of free parking places," in *Proc. 3rd VANET (in conjunction with ACM MobiCom)*, Los Angeles, CA, USA, 2006.

[4] T. Delot, N. Cenerario, S. Ilarri, and S. Lecomte, "A cooperative reservation protocol for parking spaces in vehicular ad hoc networks," in *Proc. 6th Inter. Conference on Mobile Technology, Application and Systems*, ser. MOBILITY, 2009.

[5] SFpark: parking system for San Francisco, available online in http://sfpark.org/.

[6] I. Ashlagi, D. Monderer, and M. Tennenholtz, "Resource selection games with unknown number of players," in *Proc. AAMAS*, Japan, 2006.

[7] E. Koutsoupias and C. H. Papadimitriou, "Worst-case equilibria," *Computer Science Review*, vol. 3, no. 2, pp. 65–69, 2009.

[8] S.-G. Cheng, D. M. Reeves, Y. Vorobeychik, and M. P. Wellman, "Notes on the equilibria in symmetric games," in *Proc. 6th Workshop On Game Theoretic And Decision Theoretic Agents (colocated with IEEE AAMAS)*, New York, USA, August 2004.

[9] E. Kokolaki, M. Karaliopoulos, and I. Stavrakakis, "Leveraging information in parking assistance systems," Dept. of Informatics and Telecom., Univ. of Athens, http://cgi.di.uoa.gr/~evako/park_info_TR.pdf, Tech. Rep., 2013.

[10] http://www.city-parking-in-europe.eu/.

[11] R. C. Larson and K. Sasanuma, "Congestion pricing: A parking queue model," *Journal of Industrial and Systems Engineering*, vol. 4, 2010.

[12] R. Lu, X. Lin, H. Zhu, and X. Shen, "SPARK: A new VANET-based smart parking scheme for large parking lots," in *Proc. IEEE INFOCOM 2009*, Rio de Janeiro, Brazil, April 2009.

[13] Parkomotivo: parking system for the city of Lugano, available online in http://www.bmob-park.com.

[14] J. Boehle, L. Rothkrantz, and M. V. Wezel, "CBPRS: A city based parking and routing system," Erasmus Research Institute of Management (ERIM), University Rotterdam, Tech. Rep., 2008.

[15] E. Kokolaki, M. Karaliopoulos, and I. Stavrakakis, "Leveraging information in vehicular parking games," Dept. of Informatics and Telecommunications, Univ. of Athens, http://cgi.di.uoa.gr/~evako/tr1.pdf, Tech. Rep., 2012.

[16] ——, "Opportunistically assisted parking service discovery: Now it helps, now it does not," *Pervasive and Mobile Computing*, vol. 8, no. 2, pp. 210 – 227, 2012.

[17] Parking Defenders mobile application, available online in http://www.parkingd.com.

[18] ParkShark: parking system for New York City, available online in http://www.parkshark.mobi/www/.

[19] S. P. Anderson and A. de Palma, "The economics of pricing parking," *Journal of Urban Economics*, vol. 55, no. 1, pp. 1 – 20, 2004.

[20] R. Arnott, "Spatial competition between parking garages and downtown parking policy," *Transport Policy (Elsevier)*, pp. 458–469, 2006.

[21] M. Arbatskaya, K. Mukhopadhaya, and E. Rasmusen, "The parking lot problem," Dept of Economics, Emory Univ., Atlanta, Tech. Rep., 2007.

[22] D. Ayala, O. Wolfson, B. Xu, B. Dasgupta, and J. Lin, "Parking slot assignment games," in *Proc. 19th ACM SIGSPATIAL GIS*, 2011.

[23] R. W. Irving, "Stable marriage and indifference," *Discrete Applied Mathematics*, vol. 48, no. 3, pp. 261 – 272, 1994.

[24] H. A. Simon, "A behavioral model of rational choice," *The Quarterly Journal of Economics*, vol. 69, no. 1, pp. 99–118, February 1955.

[25] R. Rosenthal, "A bounded-rationality approach to the study of noncooperative games," *Int. J. Game Theory*, pp. 273–292, 1989.

[26] R. McKelvey and T. Palfrey, "Quantal response equilibria for normal form games," *Games and Economic Behavior*, pp. 6–38, 1995.

[27] D. G. Goldstein and G. Gigerenzer, "Models of ecological rationality: The recognition heuristic," *Psychological Review*, vol. 109, no. 1, 2002.

[28] J. M. C. Hutchinson, C. Fanselow, and P. M. Todd, *Ecological rationality: intelligence in the world*. New York: Oxford Univ. Press, 2012.

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