

ON THE BREAKDOWN OF PERTURBATION THEORY

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The production of $\mathcal{O}(\frac{1}{g^2})$ particles in a weakly-coupled theory is believed to be non-perturbatively suppressed. I comment on the prospects of (a) establishing this rigorously, and (b) estimating the effect to exponential accuracy semiclassically, by discussing two closely-related problems: the large-order behavior of few-point Green functions, and induced excitation in quantum mechanics. Induced tunneling in the latter case is exponentially enhanced for frequencies of the order of the barrier height.

1. Introduction. The issue of baryon- and lepton-number violation in the standard electroweak model [1–3] has brought back to the limelight the limitations of perturbation theory. The problem, in a nutshell, is that naive weak-coupling expansions cannot be used to estimate processes in which a large number of particles is involved. If we denote for instance by $N \equiv \frac{\nu}{g^2}$ the number of W bosons produced in a high-energy collision, then an expansion of the inclusive cross-section in terms of Feynman diagrams corresponds to expanding simultaneously in the gauge coupling g and in ν . This is of no use if one is interested in the region $g \rightarrow 0$ with ν held fixed and not small. Indeed the leading-order result for this process, whether accompanied [2] or not [4] by vacuum tunneling, violates for large ν the unitarity bounds, and is hence manifestly unreliable. A breakdown of perturbation theory of course also occurs in other contexts, such as at high temperature, high densities or large external fields. However, in all these cases it is obvious that the properties of the perturbative vacuum are being modified drastically, so that the theory is no more weakly-coupled. In high-energy two-particle collisions on the other hand, our difficulty in calculating multiparticle production looks more like a technical nuisance, rather than a signal that these processes are unsuppressed. This sounds intuitively obvious if one thinks of the time-reversed process: how could we send 50 W bosons in an interaction region, and expect only an energetic e^+e^- pair to emerge?

There has been much learned debate on this issue, which I could not possibly review in this short talk. Here I will focus briefly on two closely-related problems where definitive answers are available: the problem of induced excitation in quantum mechanics, and the large-order behavior of typical Euclidean Green functions. Based on these I will then make a few comments on the prospects (i) of proving that multiparticle production in a weakly-coupled theory is non-perturbatively small, and (ii) of actually calculating it to exponential accuracy in a semiclassical approximation. Given the intuitive evidence for exponential suppression, and the ridiculously small low-energy tunneling probability in the electroweak model, $e^{-\frac{4\pi}{\alpha_w}} \sim 10^{-156}$, one may wonder whether calculating zero to exponential accuracy is really worth all this effort. I think the answer is yes for two reasons: first, the issue is sufficiently important to deserve that we close all loopholes in our intuitive arguments. Second, learning how to calculate such non-perturbative phenomena may turn out to be academic in the electroweak model, but very interesting in other contexts. In this respect simpler models could be more valuable for their own sake, rather than as paradigms of multi- W and $-Z$ production.

Large-Order Behavior. We are all well-accustomed to the fact that in field theory perturbative expansions are usually divergent. Indeed, a finite radius of convergence would be in contradiction with the vacuum instability that typically develops for negative square coupling [5]. Consider for definiteness

a scalar field in d dimensions, whose action in appropriate mass units is

$$S = \int d^d x \left[\frac{1}{2} (\partial \phi)^2 + \frac{1}{2} \phi^2 + \frac{g^2}{4} \phi^4 \right], \quad (1)$$

and let

$$G^{(N)}(g^2 | p_1, \dots, p_N) \asymp \sum_{n=0}^{\infty} g^{2n} G_n^{(N)}(p_1, \dots, p_N) \quad (2)$$

be the expansion of a Euclidean N -point function. For $n \gg 1$ we can compute the coefficients of the series semiclassically [6,7]. The idea is to relate them by a dispersion integral to the discontinuity of the function on the negative- g^2 axis,

$$G_n^{(N)}(p_1, \dots, p_N) = \frac{1}{2\pi i} \int_{-\infty}^0 \frac{dg^2}{g^{2n+2}} \text{disc } G^{(N)}(g^2 | p_1, \dots, p_N). \quad (3)$$

The integral is then evaluated at the saddle point $\phi_{cl}(x) \equiv \frac{1}{\sqrt{-g^2}} f(x)$, which describes the decay of the vacuum for negative square coupling, with the result

$$G_n^{(N)}(p_1, \dots, p_N) \underset{n \rightarrow \infty}{\sim} (2\pi)^{\frac{d}{2}-1} \Gamma\left(n + \frac{N+d}{2}\right) (-)^n a^{-n-\frac{N}{2}} \prod_{j=1}^N f(p_j). \quad (4)$$

Here $S(\phi_{cl}) \equiv \frac{a}{-g^2}$ is the saddle-point action, and we have dropped a momentum-conserving δ -function. Note that a weak-coupling expansion of the discontinuity is justified in the large- n limit, provided N and p_j are all kept finite and fixed.

The rapid factorial growth of the coefficients, typical of bosonic field theory, makes the series (2) diverge for any g . A closer look at Feynman diagrams in fact reveals that these large contributions at high orders arise when $n \sim o(\frac{1}{g^2})$ vertices are concentrated in a real-space region of size $\sim o(1)^1$. From the functional-integral point of view these contributions probe the region of *large* fields, or of large virtual *non-linear* waves. It is because such waves are not described adequately, when one expands around the usual vacuum, that perturbation theory breaks down. Intuitively we of course expect that large fluctuations, though perhaps hard to calculate precisely, are nevertheless *exponentially suppressed* and thus often negligible. This expectation is in practice confirmed by the enormous success of QED: keeping for instance three terms in the expansion of the anomalous magnetic moment of the electron, we find agreement with experiment to better than seven significant digits [9]. The more precise mathematical statement one would like to make is that the perturbative series is asymptotic to some rigorously defined function, and that the semiclassical estimate, eq.(4), can be converted into a *uniform bound* for the series remainder

$$\text{Rem}_n(G) \equiv \left| G^{(N)}(g^2) - \sum_{l=0}^{n-1} G_l^{(N)} g^{2l} \right| < n! g^{2n} a_n, \quad (5)$$

where $\lim_{n \rightarrow \infty} (a_n)^{-1/n} = \tilde{a}$ is a finite constant, and g^2 lies in some finite interval $(0, \tilde{g}^2)$ on the positive real axis. From the above bound and from Stirling's formula, $n! \simeq \sqrt{2\pi}(n+1)^{n+\frac{1}{2}} e^{-n-1}$, we could conclude

1) Fermionic theories fare in this respect better thanks to the Pauli exclusion principle. In closed-string theory, on the other hand, the divergence of perturbative expansions appears to be even more severe [8].

that in the $g \rightarrow 0$ limit

$$\min_n \text{Rem}_n(G) \preceq \exp\left(-\frac{\tilde{a}}{g^2}\right), \quad (6)$$

so that the divergent high-order contributions indeed sum up into a controllable non-perturbatively small ambiguity²⁾.

Proving this statement has been one of the aims of formal (constructive) field theory [10]. For the Euclidean scalar theory, eq.(1), in the superrenormalizable ($d < 4$) domain, the stronger result of Borel summability [11] ensures in particular the existence of the above bound³⁾. Furthermore, the large-order semiclassical estimate can be shown to correctly predict the relevant singularity of the Borel transform [13], so that the bound is optimized for $\tilde{a} = a$. The extension of these results to (i) the Minkowski region in field theory, (ii) the double-well potential in quantum mechanics, or (iii) the $d = 4$ renormalizable theory is not straightforward. Controlling non-perturbative effects in the first case is hard because bounds do not continue analytically. Borel summability has nevertheless been established for the on-shell four-point function, but only below the particle-production threshold [14]. In the case of the double-well potential of quantum mechanics, obtained by flipping the sign of the quadratic term in (1), there is a hindrance to Borel summability due to the presence of real instantons. Indeed the semiclassical large-order analysis [15] predicts a singularity of the Borel transform at the same distance, $a = \frac{4}{3}$, from the origin as in the single-well case, but lying on the positive real axis. This singularity corresponds to the action of an infinitely-separated instanton-antiinstanton pair, and is expected to control the non-perturbative ambiguities of the asymptotic expansions in the trivial and one-instanton sectors [7], but rigorous bounds of the type (5) have not to my knowledge been established [16]. In the third case of the ϕ_4^4 model, Borel summability is obstructed by the so-called renormalons [17], which are a warning that non-perturbative ambiguities could *a priori* be affected by renormalization. Since, however, the theory most probably does not exist, there is little point in worrying about bounds (5). Needless to say, finally, that non-perturbative effects in four-dimensional gauge theories are still beyond rigorous technical control [10], since in addition to renormalization and the presence of real instantons one must also face the problems of gauge fixing and scale invariance.

There are three lessons to retain from this blitz review of large-order behaviour: first that naive perturbation theory around the vacuum breaks down when one tries to estimate the contribution of large fields, or of (real or virtual) non-linear waves; second that such contributions to few-particle processes are expected to stay exponentially small, and can be estimated to exponential accuracy semi-classically; and third that a real proof of exponential suppression requires a non-perturbative control of the vacuum and takes us into the rough territory of constructive field theory. These comments should be kept in mind when moving on to the problem of multi-particle production, which in a vague sense is the *square root* of the large-order problem: indeed, large-field fluctuations are in this case created from, but do not have to disappear back to a few-particle state.

3. Multi-Leg Functions in Quantum Mechanics. The quantum mechanical analog of multiparticle production is the induced excitation of an anharmonic oscillator under the action of a weak but very energetic external force [18–20]. We concentrate first on the single-well potential with action given by eq.(1). The amplitude of interest is proportional to the matrix element $\langle 0|\phi|N\rangle$, where $N \equiv \frac{\nu}{g^2} \gg 1$ is the level of the excited final state and $E \equiv \frac{\epsilon}{g^2}$ is its energy. To motivate this scaling of parameters consider

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- 2) The simple statement of asymptoticity of the series says nothing about the size of this ambiguity, which could for instance have been as large as $\exp(-1/g^{1/100})$.
 - 3) Borel summability in fact allows a reconstruction of the function from its series with arbitrary precision, by using appropriate conformal mappings [12,7] .

the weak-coupling expansion of the energy,

$$E = (N + \frac{1}{2}) + \frac{3g^2}{16}(2N^2 + 2N + 1) - \frac{g^4}{128}(34N^3 + 51N^2 + 59N + 21) + \dots \quad (7a)$$

which can be clearly reorganized as follows:

$$\epsilon = \epsilon(\nu) + g^2 \epsilon_1(\nu) + g^4 \epsilon_2(\nu) + \dots \quad (7b)$$

The effective expansion parameter in (7b) is Planck's constant, as would be made obvious if we were to restore all units in our equations. In particular, the function $\epsilon(\nu) = \nu + \frac{3}{8}\nu^2 + \frac{34}{128}\nu^3 + \dots$ is the inverse of the integrated classical density of states,

$$\nu(\epsilon) = \frac{2}{\pi} \int_0^{x(\epsilon)} dx \sqrt{2\left(\epsilon - \frac{x^2}{2} - \frac{x^4}{4}\right)} \quad (8)$$

where $\phi \equiv x/g$ defines the rescaled position variable, and $x(\epsilon) = \sqrt{\sqrt{1+4\epsilon}-1}$ is the rescaled classical turning point. This relation between energy and level illustrates that, for coherent states of many quanta, it is the *classical* but not the naive *weak-coupling* approximation that is adequate.

Let us then consider an analogous semiclassical expansion of the matrix-element $\langle N' | x^M | N \rangle$, when all quantum numbers $N \equiv \frac{\nu}{g^2}$, $N' \equiv \frac{\nu'}{g^2}$, and $M \equiv \frac{\mu}{g^2}$ are large⁴⁾,

$$\langle N' | x^M | N \rangle = \exp\left(\frac{1}{g^2} F(\nu, \nu', \mu) + F_1(\nu, \nu', \mu) + \dots\right) \quad (9)$$

The form of this expansion is based on our expectation that in some range of parameters the overlap integral should be non-perturbatively suppressed. We are of course ultimately interested in the matrix element $\langle 0 | \phi | N \rangle$, since we want to mimic the effect of a few-particle initial state. We must therefore hope that this matrix element can be obtained, at least to exponential accuracy, by taking the $\mu, \nu' \rightarrow 0$ limit, i.e. in formulae:

$$\lim_{g \rightarrow 0} g^2 \log \langle 0 | \phi | N \rangle = F(\nu) \quad (10)$$

where

$$F(\nu) \equiv \lim_{\mu, \nu' \rightarrow 0} F(\nu, \nu', \mu) \quad (11)$$

The simple unitarity relation

$$\langle 0 | \phi^2 | 0 \rangle = \sum_{m=0}^{\infty} |\langle 0 | \phi | m \rangle|^2 \quad (12)$$

implies in particular that $|\langle 0 | \phi | N \rangle|^2 < \langle 0 | \phi^2 | 0 \rangle \simeq \frac{1}{2} + o(g^2)$, so that the function $F(\nu)$ must clearly stay *non-positive*. The first term of a naive weak-coupling expansion violates this unitarity bound, because

4) Because of reflection symmetry $N + N' + M$ must be even.

of the same factorial growth of graphs which is responsible for the large-order divergence. A precise calculation in fact gives [21]⁵⁾

$$\langle 0|\phi|N\rangle = \sqrt{\frac{8}{g^2}} \sqrt{N!} \left(\frac{g^2}{16}\right)^{N/2} [1 + o(g^2)] , \quad (13a)$$

from which we find

$$F(\nu) = -\frac{\nu}{2} \left(1 + \log \frac{16}{\nu}\right) + o(\nu^2 \log \nu) . \quad (13b)$$

The Born approximation thus hits the unitarity bound at $\nu = 16e$, at which point it is manifestly unreliable. In reality, however, the matrix element stays exponentially small for all finite ν . This can be established explicitly both (a) by a semiclassical calculation [22,18,23], and (b) by deriving rigorous upper bounds [20], as I will now briefly explain:

a). The semiclassical calculation of the overlap integral $\int x^M \Psi_{N'}^\dagger \Psi_N$ for $N \gg N' \gg 1$ was suggested a long time ago by Landau [22]. The idea is to deform the integration contour in the complex- x plane away from the classical turning points, use a WKB approximation for the wavefunctions Ψ_N and $\Psi_{N'}$, and evaluate the resulting integral at its saddle point. The full details of a careful calculation have only recently been completed [23], and confirm Landau's result [18, 22] in the $\mu \rightarrow 0$ limit

$$F(\epsilon, \epsilon') = - \int_{\epsilon'}^{\epsilon} du \int_{x(u)}^{\infty} \frac{dx}{\sqrt{2(\frac{x^2}{2} + \frac{x^4}{4} - u)}} = - \int_{\epsilon'}^{\epsilon} \frac{du}{(1+4u)^{\frac{1}{4}}} \mathbf{K} \left(\sqrt{\frac{1 + \sqrt{1+4u}}{2\sqrt{1+4u}}} \right) \quad (14)$$

where \mathbf{K} is the complete elliptic integral. The result is here given as a function of energies, but can be also expressed in terms of the levels ν and ν' via the classical relation (8). As was already anticipated, F has a smooth $\nu' \rightarrow 0$ limit, which can be used to estimate $\langle 0|\phi|N\rangle$ to exponential accuracy. Furthermore $F(\nu)$ is a monotone decreasing function of energy or level, so that the matrix element for finite ν is indeed exponentially small. Notice also that by using the asymptotic expansion $\mathbf{K}(1-\delta) \sim \log(\sqrt{\frac{8}{\delta}}) + o(\delta \log \delta)$, one can recover the Born result, eq.(13b).

b). The more rigorous proof of exponential suppression [20] makes use of an exact recursion relation between matrix elements of powers of ϕ . Defining the recursion coefficients $\langle N'|\phi^M|N\rangle \equiv \mathcal{R}_M^{(N,N')} \times \langle N'|\phi^{M+2}|N\rangle$ one finds⁶⁾

$$\mathcal{R}_M = \frac{g^2 \binom{M+1}{2}}{6 \binom{M}{4} \mathcal{R}_{M-2} \mathcal{R}_{M-4} + 2 \binom{M}{2} (E + E') \mathcal{R}_{M-2} + ((E - E')^2 - M^2)} , \quad (15)$$

where $\binom{a}{b}$ are the binomial coefficients, and E and E' are the energies at the N and N' levels. This is the analog of Schwinger-Dyson equations, which in field theory are instrumental for proving the uniform

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- 5) The calculation is identical to the Born approximation for the production of N scalar particles at threshold, up to a normalization $\frac{1}{\sqrt{N!}}$, and an extra factor of $\sqrt{2}$ per external leg appearing in the LSZ reduction of quantum mechanics [20].
- 6) This equation allows a fast numerical evaluation of the low-lying energy levels and wavefunctions with very high precision [24]. Notice also that in the semiclassical limit it reduces to

$$\frac{\mu^4}{6} \eta^3 + \mu^2 (\epsilon + \epsilon') \eta^2 + [(\epsilon - \epsilon')^2 - \mu^2] \eta - \frac{\mu^2}{2} = 0$$

where $-\frac{1}{2} \log \eta \equiv \frac{\partial F}{\partial \mu}(\epsilon, \epsilon', \mu)$. It can therefore be used to determine the μ -dependence of the function F completely.

remainder bounds (5). The problem we are here facing is very similar: indeed, since the first N terms in the asymptotic power series for $\langle 0|\phi|N\rangle$ are zero, we have for all $n < N$

$$\text{Rem}_n\left(\langle 0|\phi|N\rangle\right) = \langle 0|\phi|N\rangle, \quad (16)$$

so that bounding the series remainder up to this order would automatically put bounds on the matrix element itself. Skipping further details on the derivation of these bounds, let me simply point out that they finally take the form

$$F(\epsilon) \leq \min_{0 \leq \xi \leq \frac{\epsilon}{2}} B(\epsilon, \xi), \quad (17)$$

where ξ/g^2 plays the role of the order of the expansion. This should be chosen appropriately so as to optimize the bound B , making sure in particular that the coupling-constant suppression is not overwhelmed by factorial growth. The optimal bound has been shown to decrease monotonically with energy [20], like the semiclassical estimate eq.(14). Proving that the two actually coincide would require some more work.

The double-well potential, $V(\phi) = \frac{g^2}{16} \left(\phi^2 - \frac{2}{g^2} \right)^2$, can be treated with similar techniques. Induced excitation, whether accompanied or not by quantum tunneling, can be again bounded by an exponential envelope whose decay with energy is monotonic [20]. The leading-order instanton calculation [19], on the other hand, predicts that induced tunneling grows fast with energy in the low-energy region. To be more precise, the transition amplitude from the ground state in one well to the N th excited state in the other is given to exponential accuracy, and in the limit of small ν , by

$$F(\nu) = -\frac{4}{3} + \frac{\nu}{2} \left(1 + \log \frac{16}{\nu} \right) + \text{subleading}. \quad (18)$$

This rapid initial growth with energy is the result of two competing effects: the enhancement of spontaneous tunneling, starting from some optimal jumping state, is partially compensated by the fast-falling probability of exciting this latter from the vacuum [25]. The presence of extra real turning points complicates in this case the WKB analysis, and the calculation of the full function $F(\nu)$ has not yet been completed [23]. The exponential envelope tells us, however, that it has to reach a maximum at some finite distance below the zero-axis, before dropping indefinitely in the $\nu \rightarrow \infty$ limit. This means that induced tunneling has a *resonance of exponential proportions*, which could, for instance, be observable in semiconductor physics [26].

4. Comments on Field Theory. I will conclude with a few telegraphic comments on the prospects of extending the results reviewed in the previous two sections to multi-particle production in field theory. We saw that the large-order behavior shows no qualitative difference as one passes from ($d = 1$) quantum mechanics, to ($d = 2, 3$) superrenormalizable Euclidean theories. It should therefore be possible to prove that, like induced excitation in the former, multiparticle production in the latter is exponentially suppressed. Repeated use of the Schwinger-Dyson equations could indeed lead to bounds for Euclidean multi-leg Green functions, but there is at present no obvious strategy on how to extend such bounds to the Minkowski region. Closely related are efforts to establish bounds by exploiting the relation between the forward elastic amplitude and the total inclusive cross-section [27]

$$\sigma_{incl}(\sqrt{s}) = \frac{1}{\sqrt{(s-4)s}} \text{Im} \mathcal{A}_{el}(\sqrt{s}). \quad (19)$$

This is the analog of the quantum-mechanical unitarity relation, eq.(12). Taking the remainder at order N , we can express the inclusive cross section for producing at least N particles in the following form:

$$\sigma_{n \geq N}(\sqrt{s}) = \frac{1}{\sqrt{(s-4)s}} \text{Im} \text{Rem}_N(\mathcal{A}_{el}) + \text{Rem}_N(\sigma_{n < N}). \quad (20)$$

Rigorous bounds on the large-order behaviour of the right-hand side could thus be used to establish exponential suppression of the left-hand side. Though very plausible, this strategy stumbles again on the

fact that little can be said rigorously about the large-order behaviour of perturbation theory deep in the Minkowskian region.

In view of these difficulties, attempting to calculate the process semiclassically⁷⁾ remains, I believe, the most promising prospect. This prospect has received a boost recently thanks to a clever suggestion by Rubakov and Tinyakov [29], who proposed to first calculate a quantity which appears to have a manifest semi-classical expansion. This quantity is the inclusive cross-section for an ensemble of $N' \equiv \frac{\nu'}{g^2}$ incoming particles, distributed randomly in phase space, but with fixed total energy in the center-of-mass frame. Assuming we can calculate it, we may then hope to take the $\nu' \rightarrow 0$ limit, so as to recover the leading exponential estimate for processes with only few particles in the initial state. This has been illustrated explicitly in our discussion of the single-well potential in quantum mechanics. There is, furthermore, some preliminary evidence for the smoothness of this limit in the one-instanton sector of the standard electroweak model [30]. Whether a useful solution to this complicated saddle-point problem can, however, be found remains to be seen.

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References

- [1] 't Hooft G. // Phys. Rev. 1976. V. D14. P. 3432.
- [2] Ringwald A.// Nucl. Phys. 1990. V. B330. P. 1; Espinosa O.// Nucl. Phys. 1990. V. B334. P. 310; McLerran L., Vainshtein A., Voloshin M.B.// Phys. Rev. 1990. V. D42 1990. P. 171.
- [3] Mattis M.// Phys. Rep. 1992. V. 214. P. 159.
- [4] Cornwall J.M.// Phys. Lett. 1990. V. 243B. P. 27; Goldberg H.// Phys. Lett. 1990. V. 246B. P. 445.
- [5] F.J.Dyson // Phys. Rev. 1951. V. 85. P. 631.
- [6] Bender C.M., Wu T.T.// Phys. Rev. 1973. V. D7. P. 1620; Lipatov L.N.// Sov. Phys. JETP 1976. V. 44. P. 1055; 1977. V. 45 P. 361; Brézin E., Le Guillou J.C., Zinn-Justin J.// Phys. Rev. 1977. V. D15. P. 1544, 1558; Parisi G.// Phys. Lett. 1977. V. 66B. P. 167.
- [7] Le Guillou J.C., Zinn-Justin J. editors. Large-Order Behaviour of Perturbation Theory. Amsterdam: North Holland, 1989; Zinn-Justin J. Quantum Field Theory and Critical Phenomena, Oxford U. Press, 1989.
- [8] Gross D., Periwal V.// Phys. Rev. Lett. 1988. V. 60 P. 2105; Shenker S.// Random Surfaces and Quantum Gravity / O. Alvarez et al., eds. New York: Plenum Press, 1991.
- [9] Kinoshita T., Lindquist W.B.// Phys. Rev. Lett. 1981. V. 47. P. 1573.
- [10] Glimm J., Jaffe A. Quantum Physics, a functional integral point of view. New York: McGraw-Hill, 1981; Rivasseau V. From Perturbative to Constructive Renormalization, Princeton: Princeton U. Press, 1991.
- [11] Graffi S., Grecchi V., Simon B.// Phys. Lett. 1970. V. 32B; Eckmann J.P., Magnen J., Sénéor R.// Comm. Math. Phys. 1975. V. 39. P. 251; Magnen J., Sénéor R.// Comm. Math. Phys. 1977. V. 56. P. 237.
- [12] Hirsbrunner B.// Helv. Phys. Acta. 1982. V. 55. P. 295.
- [13] Magnen J., Rivasseau V.// Comm. Math. Phys. 1985. V. 102. P. 59; Feldman J., Rivasseau V.// Ann. Inst. H. Poincaré, 1986. V. 44. P. 427.
- [14] Eckmann J.-P., Epstein H. // Comm. Math. Phys. 1979. V. 68. P. 245.
- [15] Brézin E., Parisi G., Zinn-Justin J.// Phys. Rev. 1977. V. D16. P. 408.
- [16] Simon B.// Int. J. Quant. Chem. 1982. V. XXI. P. 3; Caliceti E., Grecchi V., Maioli M.// Comm. Math. Phys. 1988. V. 113. P. 625.

7) That a part of this problem, i.e. the calculation of final-state corrections, admits a semiclassical expansion has been known for a while [28,3].

- [17] *Parisi G.*// Phys. Rep. 1979. V. 49. P. 215; *'t Hooft G.*// The whys of subnuclear physics / Ed. by Zichichi A. New York: Plenum Press, 1979.
- [18] *Voloshin M.B.*// Phys. Rev 1991. V. D43. P. 1726; *Porrati M., Girardello L.*// Phys. Lett. 1991. V. B271. P. 148; *Cornwall J.M., Tiktopoulos G.*// Phys. Lett. 1992. V. B282. P. 195.
- [19] *Bachas C., Lazarides G., Shafi Q., Tiktopoulos G.* // Phys. Lett. 1991. V. 268B. P. 401; *Kiselev V.G.*// Phys. Lett. 1992. V. B278. P. 454; *Diamandis G., Georgalas B., Lahanas A., Papantonopoulos E.* UA/NPPS-8-92. August 1992.
- [20] *Bachas C.*// Nucl. Phys. 1992. V. B377. P. 622.
- [21] *Voloshin M.B.* TPI-MINN-92/38-T. August 1992; *Argyres E.N., Kleiss R., Papadopoulos C.* CERN-TH. 6496. May 1992.
- [22] *Landau L.D., Lifshitz E.M.* Quantum Mechanics, Non-Relativistic Theory. 3rd edition, Pergamon Press.
- [23] *Cornwall J.M., Tiktopoulos G.*, in preparation.
- [24] *Richardson J.L., Blankenbecler R.*// Phys. Rev. 1979. V. D19. P. 245.
- [25] *Banks T., Farrar G., Dine M., Karabali D., Sakita B.*// Nucl. Phys. 1990. V. B347. P. 581.
- [26] *Esaki E.*// IEEE J.Quantum Electron. 1986. V. 22. P. 1611; *Capasso F., Mohammed K., Cho A.Y.* // ibid 1986. V. 22. P. 1853; *Livescu G. et al*// Phys. Rev. Lett. 1989. V. 63. P. 438.
- [27] *Zakharov V.*// Nucl. Phys. 1991. V. B353. P. 683; *Veneziano G.*// unpublished note (1990) and Mod. Phys. Lett. 1992. V. A7. P. 1667; *Maggiore M., Shifman M.*// Nucl. Phys. V. B371. 1992. P. 177.
- [28] *Khlebnikov S.Yu., Rubakov V.A., Tinyakov P.G.*// Mod. Phys. Lett. 1990. V. A5. P. 1983; Nucl. Phys. 1991. V. B350. P. 441; *Yaffe L.* //Baryon Number Violation at the SSC? Proc. of the Santa Fe Workshop / M. Mattis, E. Mottola eds. World Scientific, 1990; *Arnold P., Mattis M.*// Phys. Rev. 1990. V. D42. P. 1738.
- [29] *Rubakov V.A., Tinyakov P.G.*// Phys. Lett. 1992. V. 279. P. 165; *Tinyakov P.G.*// Phys. Lett. 1992. V. 284. P. 410.
- [30] *Mueller A.* CU-TP-572 preprint (August 1992).