$P_{MB}(v_{P_{Ar}}) = 1.414\,P_{MB}(v_{P_{Ne}})$

$v_{P_{Ar}} = \sqrt{\frac{2k_BT}{m_{Ar}}} = 0.707\,v_{P_{Ne}}$

#πόσα σωματίδια

$dp = p_{MB}(v)dv$ → πιθ. v έως v+dv

$\int_0^\infty p_{MB}(v)dv = 1$

$dN = N p_{MB}(v)dv$

$p_{MB}(v) = \frac{dN}{N dv}$

$\boxed{p_{MB}(v) = \frac{1}{N}\frac{dN}{dv}}$

$\langle v\rangle = \int_0^\infty v\,p_{MB}(v)dv$

$\langle v^2\rangle = \int_0^\infty v^2 p_{MB}(v)dv$

$\langle 1\rangle = \int_0^\infty p_{MB}(v)dv = 1$

$\langle m\rangle = \int_0^\infty m\,p_{MB}(v)dv = m\int_0^\infty p_{MB}(v)dv = m\cdot 1 = m$

Ne, MB≃20, $v_p = \sqrt{\frac{2k_BT}{m}} \Rightarrow \frac{m}{2k_BT} = \frac{1}{v_p^2} = \lambda$

$\boxed{P_{MB}(v) = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 \exp\left(-\frac{v^2}{v_p^2}\right)}$

$v_{p_{Ne}} = \sqrt{\frac{2k_BT}{m_{Ne}}}$

$P_{MB}(v_p) = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v_p^2 \exp(-1) \Rightarrow$

$P_{MB}(v_p) = \frac{4\pi}{\pi^{3/2}}\left(\frac{m}{2k_BT}\right)^{1/2} e^{-1}$

$v_{p_{Ar}} = \sqrt{\frac{2k_BT}{m_{Ar}}}$

$m_{Ne} = \frac{MB_{Ne}}{N_A}$, $N_A \simeq 6.022\times10^{23}$

$m_{Ar} = \frac{MB_{Ar}}{N_A}$, $MB_{Ar} \simeq 40$

④ $\pm 1\%\, v_p$ $\Pi = \int_{0.99v_p}^{1.01v_p} P_{MB}(v)dv \simeq 0.02v_p \cdot P_{MB}(v_p) = 0.02\,v_p \cdot 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v_p^2 e^{-\frac{mv_p^2}{2k_BT}}$, $v_p = \sqrt{\frac{2k_BT}{m}}$

$\pm 0.01v_p$

$\Pi_{\pm 0.01v_p} = 0.02\frac{4}{\sqrt{\pi}}\cdot e^{-1} = 1.66\% = 0.0166 \sim \frac{1}{60}$

⑤ $\text{Prob}(v > v_{rms}) = \int_{v_{rms}}^\infty 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}dv$, $y = \left(\frac{m}{2k_BT}\right)^{1/2} v \Rightarrow$

$dy = dv\left(\frac{m}{2k_BT}\right)^{1/2}$, $y_{rms} = \left(\frac{m}{2k_BT}\right)^{1/2} v_{rms} = \sqrt{\frac{3}{2}}$

$\text{Prob}(v > v_{rms}) = \int_{y_{rms}}^\infty \frac{4}{\sqrt{\pi}} y^2 e^{-y^2}dy$, $t = y^2$ → $dt = 2y\,dy \Rightarrow$ $dy = \frac{dt}{2y}$,

$t_{rms} = y_{rms}^2 = 3/2$

$\text{Prob}(v > v_{rms}) = \frac{2}{\sqrt{\pi}}\int_{t_{rms}}^\infty \sqrt{t}\,e^{-t}dt = \frac{2}{\sqrt{\pi}}\int_{3/2}^\infty \sqrt{t}\,e^{-t}dt = 0.392$ ή 39.2%

$\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt = \int_0^\infty t^{a-1}e^{-t}dt - \int_x^\infty t^{a-1}e^{-t}dt \Rightarrow \int_x^\infty t^{a-1}\exp(-t)dt = \Gamma(a) - \gamma(a,x)$

$\Gamma(p) = \int_0^\infty x^{p-1}e^{-x}dx = (p-1)!$ $\quad \boxed{\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}}$ Gamma function

10. $dp = F(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z$

A) $F(v_x, v_y, v_z) = F\left(\sqrt{v_x^2+v_y^2+v_z^2}\right) = F(v)$

B) $F(v_x, v_y, v_z) = f(v_x) f(v_y) f(v_z)$

$f(x) = A e^{-bx^2}$, $x \to v_x, v_y, v_z$

$$\frac{\partial F}{\partial v_x} \overset{(A)}{=} \frac{\partial}{\partial v_x} F\left(\sqrt{v_x^2+v_y^2+v_z^2}\right) = \frac{dF}{dV}\frac{\partial V}{\partial v_x}$$

$V = \sqrt{v_x^2+v_y^2+v_z^2}$, $F(V) = F(v_x, v_y, v_z)$

$$\frac{\partial F}{\partial v_x} = \frac{dF}{dV} \frac{1}{\cancel{2}} \frac{\cancel{2} v_x}{\sqrt{v_x^2+v_y^2+v_z^2}} = \frac{v_x}{V}\frac{dF}{dV}$$

$$\frac{\partial F}{\partial v_x} \overset{(B)}{=} \frac{\partial}{\partial v_x}\left(f(v_x) f(v_y) f(v_z)\right) = f(v_y) f(v_z) \frac{\partial f(v_x)}{\partial v_x}$$

$\frac{df}{dv_x}$ $f'(v_x)$ $\|$

$$\frac{\partial F}{\partial v_x} = f(v_y) f(v_z) \frac{df}{dv_x} = \frac{v_x}{v}\frac{dF}{dV} \Rightarrow \frac{1}{F}\, \frac{f(v_y) f(v_z)}{F} f'(v_x) = \frac{v_x\, dF}{F\, dv}$$

$$\frac{1}{f(v_x)}\frac{df}{dv_x} = \frac{v_x}{vF}\frac{dF}{dV} \Rightarrow \frac{1}{vF}\frac{dF}{dv} = \boxed{\frac{1}{v_x f(v_x)}\frac{df}{dv_x} = -2b}$$

$c_g + \sum_{n=1}^{\infty} a_n v^n = G(v)$ $\qquad$ $c_H + \sum_{n=1}^{\infty} \beta_n v_x^n = H(v_x)$

$$\frac{1}{v_x f(v_x)}\frac{df}{dv_x} = -2b \Rightarrow \frac{df}{f} = -2b v_x dv_x \Rightarrow \ln f = -bv^2 + c'$$

$$\Rightarrow f(v_x) = e^{c'} e^{-bv^2 f}$$

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$$dp = F(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z \quad A)\ F(v_x, v_y, v_z) = F(v)$$

$$V = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\vec{V}^{\,2}}, \quad B)\ F(v_x, v_y, v_z) = f(v_x) f(v_y) f(v_z)$$

(10) $\rightarrow f(v_i) = A\, e^{-b v_i^2},\ i = x, y, z$

(7.) $b = \dfrac{m}{2k_B T} \Rightarrow (1)\ f(v_i) = A e^{-\frac{m v_i^2}{2k_B T}}, \quad \displaystyle\int_{-\infty}^{+\infty} f(v_i)\, dv_i = 1\ (2)$ $\quad v_i$ συνιστώσα του $\vec{v}$

$(1) \to (2) \Rightarrow A \displaystyle\int_{-\infty}^{+\infty} e^{-\frac{m v_i^2}{2k_B T}} dv_i = 1, \quad (3) \int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}, \quad x = \sqrt{\lambda}\, y \Rightarrow$

$(3) \displaystyle\int_{-\infty}^{+\infty} e^{-\lambda y^2} \sqrt{\lambda}\, dy = \sqrt{\pi} \Rightarrow \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} e^{-\lambda y^2} dy = 1\ (4) \quad A = \sqrt{\frac{\lambda}{\pi}} \quad \lambda = b = \frac{m}{2k_B T}$

$$\boxed{f(v_i) = \left(\frac{m}{2k_B T \pi}\right)^{1/2} \exp\left(-\frac{m v_i^2}{2k_B T}\right)} \qquad \langle K \rangle = \tfrac{3}{2} k_B T$$

$$dp = \left(\frac{m}{2k_B T \pi}\right)^{3/2} \exp\left(-\frac{m v_x^2}{2k_B T} - \frac{m v_y^2}{2k_B T} - \frac{m v_z^2}{2k_B T}\right) dv_x\, dv_y\, dv_z$$

$v^2 = v_x^2 + v_y^2 + v_z^2, \quad dp = \left(\dfrac{m}{2k_B T \pi}\right)^{3/2} e^{-\frac{m v^2}{2k_B T}}\, dv_x\, dv_y\, dv_z, \quad v \to v + dv$

$dP(v) = P_{MB}(v)\, dv \quad dP = \left(\dfrac{m}{2k_B T \pi}\right)^{3/2} e^{-\frac{m v^2}{2k_B T}}\, 4\pi v^2 dv$ $\quad$ Θεωρ

$dv_x\, dv_y\, dv_z = v^2\, d(\cos\theta)\, d\varphi\, dv \qquad \displaystyle\int_{\theta,\varphi}$

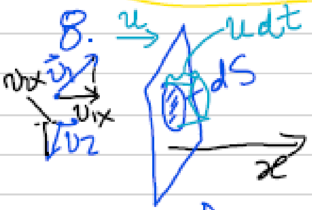
$$P_{MB}(v) = 4\pi \left(\frac{m}{2k_B T \pi}\right)^{3/2} v^2 e^{-\frac{m v^2}{2k_B T}}$$

$pV = 2K/3$

$pV = N k_B T$

Αερ

8.



$\dfrac{dN}{dt} = \dfrac{n\, dS\, u\, dt}{dt} \Rightarrow \dfrac{dN}{dt} = n u\, dS \Rightarrow$

$n = \dfrac{\#\text{μορίων}}{V}$

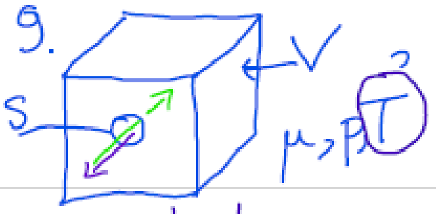
$I = \dfrac{dN}{dS\, dt} = n u \quad u = \displaystyle\int_0^{\infty} v_x f(v_x)\, dv_x = \langle v_x^+ \rangle$

$I = n \displaystyle\int_0^{\infty} v_x \left(\frac{m}{2k_B T \pi}\right)^{1/2} e^{-\frac{m v_x^2}{2k_B T}} dv_x \Rightarrow y = \left(\frac{m}{2kT}\right)^{1/2} v_x \quad \langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}}$

$I = \dfrac{n}{\sqrt{\pi}} \displaystyle\int_0^{\infty} \left(\frac{m v_x^2}{2k_B T}\right)^{1/2} e^{-\frac{m v_x^2}{2k_B T}}\, d\left[\left(\frac{m}{2kT}\right)^{1/2} v_x\right] \cdot \sqrt{\frac{2k_B T}{m}} = n \sqrt{\frac{2k_B T}{m\pi}} \int_0^{\infty} y e^{-y^2} dy \Rightarrow$

$I = \sqrt{\dfrac{2k_B T}{m\pi}}\, n \left[-\dfrac{e^{-y^2}}{2}\right]_0^{\infty} = n \sqrt{\dfrac{k_B T}{2\pi m}}, \quad \boxed{I = n\sqrt{\dfrac{k_B T}{2\pi m}}} = n \langle v_x^+ \rangle = \dfrac{n \langle v \rangle}{4}$

9.



$$pV_{εξω} = \nu RT \Rightarrow pV_{εξω} = N k_B T \Rightarrow p = \frac{N}{V_{εξω}} k_B T \Rightarrow p = n k_B T,\quad n = \frac{N}{V_{εξω}}$$

$$\boxed{n = \frac{p}{k_B T} = n_0} \qquad \boxed{\frac{dN}{dt} = I \cdot S = S\,\frac{n_0\langle v\rangle}{4}}$$

$$\frac{dN'}{dt} = I'S = S\,\frac{n(t)\langle v\rangle}{4}$$

$$\frac{d}{dt}(N-N') = \frac{n_0\langle v\rangle S}{4} - \frac{n\langle v\rangle S}{4} = (n_0-n)\,\frac{S\langle v\rangle}{4} \quad (1)\ /V$$

$$(2)\quad \frac{d}{dt}\frac{(N-N')}{V} = (n_0-n)\left[\frac{S\langle v\rangle}{4V}\right] \qquad \boxed{\lambda = \frac{S\langle v\rangle}{4V}}$$

$$d(N-N') = dN'' = d(n\cdot V) = V\,dn$$

$$\frac{dn}{dt} = (n_0-n)\lambda \Rightarrow \frac{dn}{n_0-n} = \lambda dt \Rightarrow \int_0^{n(t)} \frac{dn}{n_0-n} = \int_0^t \lambda dt \Rightarrow$$

$$\Big[-\ln(n_0-n)\Big]_0^{n(t)} = \lambda t \Rightarrow \ln\left(\frac{n_0}{n_0-n(t)}\right) = \lambda t \Rightarrow$$

$$e^{\lambda t} = \frac{n_0}{n_0-n(t)} \Rightarrow n_0 - n(t) = n_0 e^{-\lambda t} \Rightarrow n(t) = n_0(1-e^{-\lambda t})$$

$\times k_B T$

$$P(t) = P_0(1-e^{-\lambda t})$$

p ίσως εκεί.