31/3/2021 Bulk modulus $\delta p \rightarrow \frac{\delta V}{V}$ $\quad -V\frac{\delta p}{\delta V}\Big|_T = B_T$

12. $B_T = -V\left(\frac{\partial P}{\partial V}\right)_T$, $PV = nRT$, $n = \#mol$ που έχουμε από το ιδ. αέριο $\quad n = \frac{M_{\alpha\epsilon\rho}}{MB}$

$P = \frac{nRT}{V}$ $\quad \frac{dP}{dV} = \frac{d}{dV}\left(\frac{nRT}{V}\right) = nRT\frac{d}{dV}\left(\frac{1}{V}\right) = \frac{nRT}{-V^2} \Rightarrow$

$B_T = \cancel{+}\cancel{V}\frac{nRT}{\cancel{+V^2}} = \frac{nRT}{V} = P$ $\quad \boxed{B_T = P}$ $\quad \boxed{B_S = -V\frac{\delta p}{\delta V}\Big|_S}$

$P_{\alpha\tau\mu} = 1atm = 10^5 Pa$, $Pa = Nt/m^2$

$P_{\alpha\tau\mu} = 100kPa = 0.1MPa$, $B_T = P_{\alpha\tau\mu} \sim 100kPa$ ~ $0.1MPa$

→ συνάρτηση πυκνότητας πιθανότητας σππ (probability density function) pdf

dp Πιθανότητα ένα μόριο να έχει ενέργεια από E έως E+dE, $\boxed{dp = p(E)dE}$

$p(E) = \frac{e^{-E/k_BT}}{k_BT} \Rightarrow \int_0^\infty p(E)dE = 1 \longrightarrow 100\%$ ← κανονικοποίηση

$y = E/k_BT \Rightarrow dE = k_BT\,dy$

$\langle E\rangle = \int_0^\infty E\,p(E)\,dE = \int_0^\infty \frac{E}{k_BT}e^{-E/k_BT}dE = k_BT\int_0^\infty y e^{-y}dy = k_BT\,\Gamma(2) = k_BT\,1!$

$\boxed{\Gamma(p) = \int_0^\infty y^{p-1}e^{-y}dy = (p-1)!}$ $\quad 0! = 1$, $1! = 1$, $n! = 1\cdot 2\cdot 3\cdots n$ $\quad \langle E\rangle = k_BT$

$\langle E\rangle = \frac{E_1+E_2}{2}$, $\langle E\rangle = \frac{\sum_{i=1}^{N} E_i}{N}$, μέση τιμή της ενέργειας

$\langle E\rangle = k_BT$

αθροιστική συνάρτηση κατανομής (ασκ) $P(x \le x_0)$
cumulative distribution function

$P(E_0) = P(E \le E_0) \equiv Prob(0 \le E \le E_0) = \int_0^{E_0} p(E)\,dE \Rightarrow$

$P(E_0) = \int_0^{E_0} e^{-E/k_BT}\frac{dE}{k_BT} \overset{y=\frac{E}{k_BT}}{=} \int_0^{E_0/k_BT} e^{-y}dy = \left[-e^{-y}\right]_0^{E_0/k_BT} \Rightarrow$

$P(E_0) = 1 - e^{-\frac{E_0}{k_BT}} = 1 - \exp\left(-\frac{E_0}{k_BT}\right)$, $\boxed{\lim_{E_0\to\infty} P(E_0) = 1}$

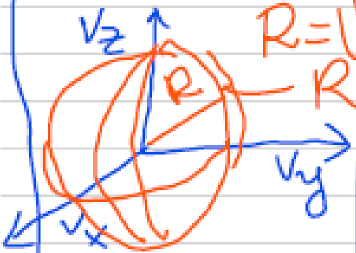
$k_B = R/N_A$

Κατανομή MB → σππ για το μέτρο της ταχύτητας ενός μορίου $dp = P_{MB}(v)\,dv \leftarrow \underline{v, v+dv}$

$$v = |\vec{v}| = \sqrt{v_x^2+v_y^2+v_z^2}$$

$$\vec{v} = (v_x, v_y, v_z), \quad P(v_x,v_y,v_z) = D_N \exp\left(-\frac{m}{2}\frac{(v_x^2+v_y^2+v_z^2)}{k_BT}\right)$$

$$\boxed{\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi}}, \quad v^2 = v_x^2+v_y^2+v_z^2$$



$$R = \sqrt{v_x^2+v_y^2+v_z^2} = v, \quad R+dR$$

$$\xrightarrow{v \to v+dv} \int dv_x dv_y dv_z = 4\pi v^2 dv$$

$$\boxed{P_{MB}(v) = K \cdot v^2 \exp\left(-\frac{m v^2}{2k_BT}\right)}$$

$$dp = P_{MB}(v)\,dv = \iint_{\text{σφαιρ. φλοιό}} p\, dv_x dv_y dv_z = \iint_{\substack{\text{σφαιρικό}\\\text{φλοιό}}} D_N e^{-\frac{mv^2}{2k_BT}} dv_x dv_y dv_z = D_N e^{-\frac{mv^2}{2k_BT}} \iint_{\substack{\text{σφαιρ.}\\\text{φλοιό}}} dv_x dv_y dv_z$$

$v \to v+dv$

$$\underbrace{dp = P_{MB}(v)dv}_{} = D_N e^{-\frac{mv^2}{2k_BT}} \iint_{\sigma\varphi} dv_x dv_y dv_z = \underbrace{D_N \exp\left(-\frac{mv^2}{2k_BT}\right) 4\pi v^2 dv}_{}, \quad \boxed{K = D_N 4\pi}$$

$$x = \sqrt{\lambda}\, y \qquad \int_{-\infty}^{+\infty} \sqrt{\lambda}\, e^{-\lambda y^2} dy = \sqrt{\pi} \Rightarrow \boxed{I(\lambda) = \int_{-\infty}^{+\infty} e^{-\lambda y^2} dy = \sqrt{\frac{\pi}{\lambda}}}$$

$$dx = \sqrt{\lambda}\, dy$$

$$\frac{dI(\lambda)}{d\lambda} = \int_{-\infty}^{+\infty} -y^2 e^{-\lambda y^2} dy = -\frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}} \Rightarrow$$

$$\int_{-\infty}^{+\infty} y^2 e^{-\lambda y^2} dy = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

$$K \int_0^{\infty} v^2 \exp\left(-\frac{mv^2}{2k_BT}\right) dv = 1 \Rightarrow \text{Prob}(0 < v < \infty)$$

$$\lambda = \frac{m}{2k_BT}$$

$$\int_{-\infty}^{+\infty} y^2 e^{-\lambda y^2} dy = 2\int_0^{\infty} y^2 \exp(-\lambda y^2) dy = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

$$4\sqrt{\frac{\lambda^3}{\pi}} \int_0^{\infty} y^2 e^{-\lambda y^2} dy = 1. \quad K = 4\sqrt{\frac{\lambda^3}{\pi}} = 4\pi \left(\frac{m}{2\pi k_BT}\right)^{3/2}$$

2/4/2021

2/4/2021 $\quad dp = P_{MB}(v)\,dv \Rightarrow dN = N P_{MB}(v)\,dv \Rightarrow$

$$\boxed{\frac{dN}{N} = P_{MB}(v)\,dv} \quad \langle v \rangle = \sum_{i=1}^{N} \frac{v_i}{N} = \frac{1}{N}\sum_{i=1} v_i = \frac{1}{N}\sum_{i=1} |\vec{v}_i|$$

$$\langle v^2 \rangle = \sum_{i=1}^{N} \frac{v_i^2}{N} = \frac{1}{N}\sum_{i=1}^{N} \vec{v}_i^{\,2} \qquad v_i^2 = \vec{v}_i^{\,2}$$

$$\langle v^2 \rangle = \int_0^\infty v^2 P_{MB}(v)\,dv, \quad P_{MB}(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_BT}}$$

$$I_1(\lambda) = \int_0^\infty x^2 e^{-\lambda x^2}\,dx = \frac{1}{4}\sqrt{\pi}\,\lambda^{-3/2} \qquad \lambda = \frac{m}{2k_BT}$$

$$-\frac{dI_1(\lambda)}{d\lambda} = \int_0^\infty x^4 e^{-\lambda x^2}\,dx = \frac{3}{8}\sqrt{\pi}\,\lambda^{-5/2} \Longrightarrow$$

$$4\sqrt{\frac{\lambda^3}{\pi}}\int_0^\infty x^4 e^{-\lambda x^2}\,dx = \frac{3}{2\lambda} \Longleftarrow$$

$$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_0^\infty v^2 \cdot v^2 e^{-\frac{mv^2}{2k_BT}}\,dv = \frac{3\,\cancel{2}k_BT}{\cancel{2}m}$$

$$\boxed{\langle v^2 \rangle = \int_0^\infty v^2\,dp = \int_0^\infty v^2 P_{MB}(v)\,dv = \frac{3k_BT}{m}}$$

$$-\frac{d}{d\lambda}I_1(\lambda) = -\int_0^\infty \frac{\partial}{\partial\lambda} x^2 e^{-\lambda x^2}\,dx = -\int_0^\infty \frac{\partial}{\partial\lambda}\left(x^2 e^{-\lambda x^2}\right)dx =$$

$$= -\int_0^\infty x^2 \frac{\partial}{\partial\lambda} e^{-\lambda x^2}\,dx = -\int_0^\infty x^2(-x^2)e^{-\lambda x^2}\,dx = \int_0^\infty x^4 e^{-\lambda x^2}\,dx$$

$$\langle v^2 \rangle = \frac{3k_BT}{m} \Longrightarrow v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_BT}{m}}$$

$$\boxed{\langle K \rangle = \left\langle \frac{mv^2}{2} \right\rangle = \frac{m}{2}\langle v^2 \rangle = \frac{3}{2}k_BT}$$

$$P_{MB}(v) = 4\pi\underbrace{\left(\frac{m}{2\pi k_B T}\right)^{3/2}}_{D} v^2 e^{-\frac{mv^2}{2k_BT}} , \quad \left.\frac{dP_{MB}}{dv}\right|_{v=v_p}=0$$

$$\underline{\lambda = \frac{m}{2k_BT}}$$

$$P_{MB}(v) = D v^2 e^{-\lambda v^2}$$

$$\frac{d}{dv}P_{MB}(v) = \frac{d}{dv}\left(Dv^2e^{-\lambda v^2}\right) = D\frac{d}{dv}\left(v^2e^{-\lambda v^2}\right) = D\left(2ve^{-\lambda v^2} + v^2\lambda(-2v)e^{-\lambda v^2}\right)$$

$$\frac{d}{dv}P_{MB}(v) = D2v(1-\lambda v^2)e^{-\lambda v^2}\Big|_{v=v_p} = 0 \to v_p=0, \quad \to 1-\lambda v_p^2 = 0 \Rightarrow v_p^2 = \frac{1}{\lambda}$$

$$\frac{d}{dv}e^{-\lambda v^2} = e^{-\lambda v^2}(-\lambda v^2)' = (-2\lambda v)\exp(-\lambda v^2) \qquad \frac{d}{dx}f(g(x)) = \frac{\partial f}{\partial g}\frac{dg}{dx}, \quad e^x, \quad g(x)=-\lambda x^2$$

$$\to \boxed{v_p = \sqrt{1/\lambda} = \sqrt{\frac{2k_BT}{m}}}$$

$$\langle v\rangle = \int_0^\infty v P_{MB}(v)\,dv = 4\sqrt{\frac{\lambda^3}{\pi}}\int_0^\infty v^3 e^{-\lambda v^2}dv \Rightarrow$$

$$y=\lambda v^2 \Rightarrow dy = 2\lambda v\,dv, \quad v^2 = y/\lambda \Rightarrow$$

$$\langle v\rangle = \frac{2}{\sqrt{\lambda\pi}}\int_0^\infty y e^{-y}dy, \qquad \Gamma(p) = \int_0^\infty x^{p-1}e^{-x}dx = (p-1)!$$

$$\to \Gamma(2) = 1! = 1$$

$$\langle v\rangle = \frac{2}{\sqrt{\lambda\pi}} = \sqrt{\frac{8k_BT}{\pi m}}$$

Ασκ. 1. Σετ2, $p(E) = Ae^{-E/kT}$, $E>0$, Να εξασφαλίσουμε $\int_0^\infty p(E)dE=1$

$$\int_0^\infty Ae^{-E/kT}dE = 1 \Rightarrow kTA\int_0^\infty e^{-E/kT}\frac{dE}{kT} = 1 \Rightarrow kTA\int_0^\infty e^{-x}dx = 1 \Rightarrow kTA=1$$

$$x = E/kT$$

$$\int_0^\infty e^{-x}dx = \left[-e^{-x}\right]_0^\infty = 0-(-1) = 1 \qquad \to A = \frac{1}{kT} \qquad \int_0^\infty dp = 1$$

$$dp = P(E)\,dE$$

$$\langle E\rangle = \int_0^\infty E\underbrace{Ae^{-E/kT}dE}_{dp} = kT\int_0^\infty \frac{E}{kT}e^{-E/kT}\frac{dE}{kT} \overset{x=E/kT}{=} kT\underbrace{\int_0^\infty xe^{-x}dx}_{1} = kT$$

$$\langle E\rangle = \int_0^\infty E\,p(E)\,dE$$

$$\langle E\rangle = \sum_i E_i\,p(E_i)$$

$$\langle E\rangle = \frac{E_1+E_2+E_3}{3}, \qquad E_1=E_3=E_0$$

$$\langle E\rangle = \frac{E_0 2 + E_2}{3}$$

$$\Delta E^2 = \langle E^2\rangle - \langle E\rangle^2 = \langle (E-\langle E\rangle)^2\rangle$$

$$\langle E\rangle = \mu, \quad \Delta E^2 = \langle (E-\mu)^2\rangle = \langle E^2 - 2\mu E + \mu^2\rangle = \langle E^2\rangle - 2\mu\langle E\rangle + \mu^2\langle 1\rangle$$

$$\Delta E^2 = \langle E^2\rangle - \mu^2, \quad \langle E^2\rangle = \int_0^\infty \frac{E^2}{(kT)^2}e^{-E/kT}\frac{dE}{kT}(kT)^3 = (kT)^2\int_0^\infty x^2e^{-x}dx = (kT)^2\,\Gamma(3) = (kT)^2 2! = 2(kT)^2$$

$$\Delta E^2 = (kT)^2$$