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ασκήσεις ΘΕΡΜΟΔΥΝΑΜΙΚΗ Σειρά 1η

2. $a = \frac{1}{\ell}\frac{d\ell}{dT}\Big|_P$, ℓ_0, T_0 , $\ell(T), T>T_0$, $(\ln(\ell))' = \frac{1}{\ell}$

$a\,dT = \frac{d\ell}{\ell} = d(\ln(\ell))$, $a = const.$ $\Rightarrow$

$\int_{T_0}^{T} a\,dT = \int_{\ell(T_0)}^{\ell(T)} d[\ln(\ell)] \rightarrow a(T-T_0) = \ln\left(\frac{\ell(T)}{\ell(T_0)}\right) \Rightarrow$

$\boxed{\ell(T) = \ell(T_0)e^{a(T-T_0)}}$

Όταν $a(T-T_0) \ll 1$ $x = (0.05)$, $e^x \approx \sum_{n=0} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, $e^{a(T-T_0)} = 1 + a(T-T_0)$

$\boxed{\ell(T) = \ell(T_0)\,(1 + a(T-T_0))}$, $\boxed{T - T_0 = \Theta - \Theta_0}$

3. (3.1) $\ell_a(\Theta) = \ell_0(1 + a_a(\Theta - \Theta_0)) = \ell_0(1 + a_a\Delta\Theta)$, $\Delta\Theta = 100K$ $\ell_0 = \ell_a(0°C) = \ell_\gamma(0°C)$

(3.2) $\ell_\gamma(\Theta) = \ell_0(1 + a_\gamma\Delta\Theta)$, $a_a = 12\times10^{-6}/K$, $a_\gamma = 8\times10^{-6}/K$, $3m$

$\Delta\ell = \ell_a(\Theta) - \ell_\gamma(\Theta) = 1.2mm$, $(3.1)-(3.2) \Rightarrow \Delta\ell = \ell_0(a_a - a_\gamma)\Delta\Theta \Rightarrow \ell_0 = \frac{\Delta\ell}{(a_a - a_\gamma)\Delta\Theta}$

$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$, $|x| < R$, $f(x) = \exp(x)$

4. 0°C έχει βαθμονομηθεί (από ατσάλι) $20cm$

$\Delta\Theta = 30K$

a $\ell_a(0) = 20cm$, $\Theta_0 = 0°C$

a $\ell_a(\Theta)$, $20cm$, $\Theta_1 = 30°C$

$\ell_a(30°C) = \ell_a(0°C)\cdot(1 + a_a\Delta\Theta)$

$\ell_{cu}(30°C) = \ell_{cu}(0°C)\cdot(1 + a_{cu}\Delta\Theta)$ $\rightarrow$ $\boxed{\ell_{cu}(0°C) = \frac{\ell_a(0°C)(1 + a_a\Delta\Theta)}{(1 + a_{cu}\Delta\Theta)} = 19.972\ cm}$

5. $|x| \ll 1$, $\sin x \simeq x$, $\cos x \simeq 1$

$T\cos\theta = W$, $F = T\sin\theta$, $m\frac{d^2x}{dt^2} = -F = -T\sin\theta$

$\theta \ll 1$, $\theta \lesssim 0.05\,rad$

$T = W$, $F = T\theta \Rightarrow m\frac{d^2x}{dt^2} = -W\theta = -mg\theta$

$W = mg$, $x \simeq \ell\theta$

$m\ell\frac{d^2\theta}{dt^2} = -mg\theta \Rightarrow \boxed{\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0}$, $\Omega^2 = \frac{g}{\ell}$, $\Omega = \frac{2\pi}{T_{περ}}$, $T_{περ} = 2\pi\sqrt{\frac{\ell(T)}{g}}$

$\Delta T = T' - T$

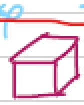
$T'_{περ} = 2\pi\sqrt{\frac{\ell(T')}{g}}$, $\frac{T'_{περ} - T_{περ}}{T_{περ}} = \frac{\sqrt{\ell(T')} - \sqrt{\ell(T)}}{\sqrt{\ell(T)}} = \sqrt{\frac{\ell(T')}{\ell(T)}} - 1$

$\ell(T') = \ell(T)(1 + a\Delta T)$

$T'_{περ} = 2\pi\sqrt{\frac{\ell(T)(1 + a\Delta T)}{g}} = 2\pi\sqrt{\frac{\ell(T)}{g}}\sqrt{1 + a\Delta T} = T_{περ}\sqrt{1 + a\Delta T}$

$|x| \ll 1$, $\sqrt{1+x} = (1+x)^{1/2} \approx 1 + \frac{x}{2}$. $T'_{περ} = T_{περ}\left(1 + \frac{a\Delta T}{2}\right)$, $\boxed{\frac{T'_{περ} - T_{περ}}{T_{περ}} = \frac{a\Delta T}{2}}$

6. $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\frac{dV}{dT}\Big|_P$, $a = \frac{1}{\ell}\frac{d\ell}{dT}\Big|_P$, $V = \ell_x\ell_y\ell_z$



$dV = V(T+dT) - V(T) = \ell_x(T+dT)\,\ell_y(T+dT)\,\ell_z(T+dT) - \ell_x\ell_y\ell_z$, $d\ell = \ell_x(T+dT) - \ell_x(T) = a\ell_x dT$

$\Rightarrow \ell_x(T+dT) = \ell_x(T) + a\ell_x dT$

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$\ell_x(T+dT)=\ell_x+a\ell_x\,dT$, $\ell_y(T+dT)=\ell_y+a\ell_y\,dT$, $\ell_x\equiv\ell_x(T)$

$\ell_z(T+dT)=\ell_z+a\ell_z\,dT$.

$dV=\ell_x(T+dT)\ell_y(T+dT)\ell_z(T+dT)-\ell_x\ell_y\ell_z$

$dV=(\ell_x+a\ell_x dT)(\ell_y+a\ell_y dT)(\ell_z+a\ell_z dT)-\ell_x\ell_y\ell_z \Rightarrow$

$dV=\cancel{\ell_x\ell_y\ell_z}+a\ell_x dT\ell_y\ell_z+\ell_x a\ell_y dT\ell_z+\ell_x\ell_y a\ell_z dT+$

$+\cancel{a\ell_x dT a\ell_y dT\ell_z}\ldots+a\ell_x dT\,a\ell_y dT\,a\ell_z dT-\cancel{\ell_x\ell_y\ell_z}$

$a\sim10^{-5}/K$

$a^2\sim10^{-10}/K^2$

$a^3\sim10^{-15}/K^3$

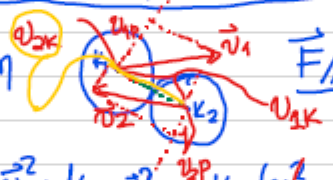
$dV=a\ell_x\ell_y\ell_z dT+a\ell_x\ell_y\ell_z dT+a\ell_x\ell_y\ell_z dT=3a\ell_x\ell_y\ell_z dT$

$V=\ell_x\ell_y\ell_z \Rightarrow dV=3aVdT\to\boxed{\beta=\frac{1}{V}\frac{dV}{dT}=3a}$

$a=\frac{1}{\ell_x}\frac{d\ell_x}{dT}\Rightarrow d\ell_x=a\ell_x dT$ $\quad d\ell_x=\ell_x(T+dT)-\ell_x=a\ell_x dT$

$V=\ell_x\ell_y\ell_z \quad \frac{1}{V}\frac{dV}{dT}=\frac{d\ell_x}{dT}\cdot\frac{\ell_y\ell_z}{\ell_x\ell_y\ell_z}+\ell_x\frac{d\ell_y}{dT}\frac{\ell_z}{\ell_x\ell_y\ell_z}+\ell_x\ell_y\frac{d\ell_z}{dT}/\ell_x\ell_y\ell_z$

8.



$\vec F // K_1K_2$, $\boxed{d\vec p=\vec F dt \quad d\vec p//\vec F \quad v'_{1p}=v_{1p},\ v'_{2p}=v_{2p}}$

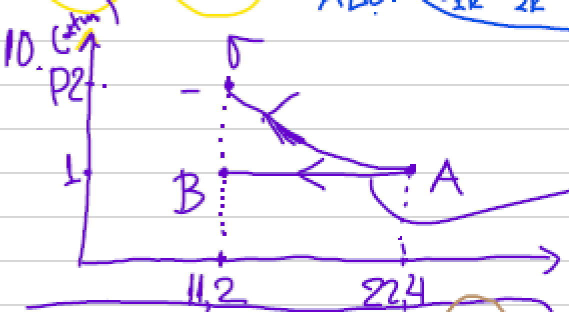
$\frac12 m\vec v_1^2+\frac12 m\vec v_2^2=\frac12 m(\cancel{v_{1p}^2}+v_{1K}^2+\cancel{v_{2p}^2}+v_{2K}^2)=\frac12 m(\vec v_1'^2+\vec v_2'^2)=\frac{m}{2}(\cancel{v_{1p}'^2}+v_{1K}'^2+\cancel{v_{2p}'^2}+v_{2K}'^2)$

$\vec P_1+\vec P_2=\vec P_1'+\vec P_2'$, $\vec P_i=m\vec v_i$, $i=1,2$

ΑΔΟ. $v_{1K}+v_{2K}=v'_{1K}+v'_{2K}$ $\quad v_{1K}^2+v_{2K}^2=v_{1K}'^2+v_{2K}'^2$ ΑΔΕ

$\Rightarrow v'_{1K}=v_{2K}$, $v'_{2K}=v_{1K}$

10.



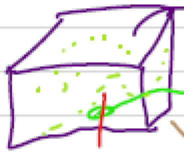
$T=273.15K$, $P=1atm$, $V_{mol}=22.4lt$

ισοβαρής $P=const.$ $A\to B$

$pV=const$ ισόθερμη, $A\to\Gamma$

$P_AV_A=pV\Rightarrow\boxed{P(V)=\frac{(P_AV_A)}{V}}$

11.



P_a, T, V

υγρό μάζας m, $\mu=MB$

$\to T'$

$P(T')=P_a(T')+P_v(T')$ $\quad \nu=n=\frac{m}{\mu}$

$\frac{V P_a(T')}{T'}=\frac{P_aV}{T}\Rightarrow P_a(T')=P_a\frac{T'}{T}$ $\quad P_v(T')\cdot V=nRT'$ $\quad P_v(T')=\frac{nRT'}{V}$

$+$

$P(T')''$

12.